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Critical Rotation of General-Relativistic Polytropic Models Revisited

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Abstract: We develop a numerical method for computing critical rotation parameters of rigidly rotating polytropic models in the "post-Newtonian approximation" (PNA). By treating our models as initial value problems of ordinary differential equations in the complex plane, all syndromes known to be present in this class of initial value problems are removed. In our computations we take into account the complete solution due to the relativity parameter, this issue being a significant improvement compared to the classical PNA.

1 Introduction

A rapidly rotating neutron star in hydrostatic equilibrium is studied by considering the rotational and relativistic effects υ and σ, respectively, as decoupled perturbations acting on a nonrotating Newtonian configuration obeying the polytropic equation of state.

Numerical integrations are performed by the code `DCRKF54` [1], which is a Runge-Kutta-Fehlberg code of fourth and fifth order modified so that to integrate initial value problems established on systems of first-order ordinary differential equations of complex-valued functions in one complex variable along prescribed complex paths. It seems that integration in the complex plane can avoid the difficulties of the original PNA, since it proceeds independently of the particular perturbation approach (to be) used.

2 The Numerical Method

According to PNA, the polytropic density function can be expanded as in [2] (Eqs. (26), (35), and (62)). We proceed with a new numerical approach, assuming that the relativistic distortion is represented by its complete solution as studied in [3]. By substituting the complete solution for the relativistic effects, Θ_σ, in the place of the Lane–Emden function θ_{00} (plus the first-order relativistic correction θ_{30}), i.e. $\Theta_\sigma(\xi) = \theta_{00}(\xi) + \sigma\,\theta_{30}(\xi)$, we obtain

$$\begin{aligned}\Theta(\xi,\mu) = {} & \Theta_\sigma\, P_0(\mu) + \upsilon\,[\theta_{10}(\xi)\,P_0(\mu) + A_{12}\theta_{12}(\xi)P_2(\mu)] \\ & + \upsilon^2\,\{\theta_{20}(\xi)\,P_0(\mu) + [\theta_{22}(\xi) + A_{22}\theta_{12}(\xi)]\,P_2(\mu) + [\theta_{24}(\xi) + A_{24}\theta_{14}(\xi)]\,P_4(\mu)\}\,.\end{aligned} \tag{1}$$

The functions θ_{ij} obey Eqs. (37) and (38) of [2]. The parameters A_{ij} ([2], Eq. (59)) multiply properly the homogeneous solutions of θ_{ij} ([2], Eqs. (42), (43)), so that certain boundary conditions be satisfied. To compute the function Θ_σ, we use the Oppenheimer–Volkoff equations of hydrostatic equilibrium ([3], Eqs. (19) and (20)),

$$\frac{d\Theta_\sigma}{d\xi} = -\frac{1}{\xi^2}\left(\Upsilon_\sigma + \sigma\xi^3\Theta_\sigma^{n+1}\right)\frac{[1+(n+1)\,\sigma\,\Theta_\sigma]}{1-2\,(n+1)\,\sigma\,(\Upsilon_\sigma/\xi)}, \qquad \Upsilon'_\sigma = \xi^2\,\Theta_\sigma^n\,(1+\sigma\,n\,\Theta_\sigma)\,, \tag{2}$$

where the function Υ_σ is defined by (cf. [3], Eq. (18)).

PNA includes terms of first order in σ and, thus, the sum has the single term $\sigma\,\theta_{30}$. With infinite terms, the sum should be equal to $\Theta_\sigma - \theta_{00}$. The basis of our numerical approach consists in using the complete solution in the relativistic distortion, and perturbation terms of up to second order in υ with respect to the rotational distortion.

3 Discussion

Our numerical experiments concern certain critically rotating maximum-mass models. By applying our numerical method to such models, we have computed results, which, to the extent of the knowledge of the authors, show the smallest deviations among similar noniterative methods with respect to corresponding results of the well-known RNS package [4]. A full treatment of the numerical method described briefly here will be the subject of a subsequent investigation.

References

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