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CONTRIBUTED POSTER

On the dynamics of a small body in a post Newtonian potential field created by a regular polygon formation of N bodies

D.Gn.Fakis[1] *and T.J. Kalvouridis*[2]
Department of Mechanics, Faculty of Applied Sciences
National Technical University of Athens
[1]fakisdim@yahoo.gr
[2]tkalvouridis@gmail.com

Abstract: We present an improved version of the $(N+1)$-body regular polygon model where the central body creates a post-Newtonian Manev-type potential of the form $1/r + B/r^2$, where B is a constant ($B = e\alpha$) and α is the side of the regular polygon (see also [1] and [2]). The problem is characterized by three parameters; (i) the number ν $(= N-1)$ of the peripheral primaries, (ii) the mass parameter $\beta = m_0/m$ and (iii) the Manev coefficient e.

1 Equations of motion - Equilibrium points and zones

The motion of the particle is described by the following dimensionless differential equations

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} = U_x, \qquad \ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y} = U_y, \qquad \ddot{z} = \frac{\partial U}{\partial z} = U_z \tag{1}$$

where

$$U(x,y,z) = \frac{1}{2}(x^2+y^2) + \frac{1}{\Delta}\left[\beta\left(\frac{1}{r_0} + \frac{e}{r_0^2}\right) + \sum_{i=1}^{\nu}\frac{1}{r_i}\right] \tag{2}$$

$$r_0 = (x^2+y^2+z^2)^{1/2}, \qquad r_i = \left((x_i-x)^2 + (y_i-y)^2 + z^2\right)^{1/2} \tag{3}$$

$$\Delta = M(\Lambda + \beta M^2 + 2\beta e M^3), \quad \Lambda = \sum_{i=2}^{\nu}\frac{\sin^2(\pi/\nu)}{\sin(i-1)(\pi/\nu)}, \quad M = 2\sin(\pi/\nu) \tag{4}$$

There is a Jacobian-type integral of motion: $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2U(x,y,z) - C$

When $e \geq 0$, all the equilibrium points lie on the xy-plane and are grouped in either five or three equilibrium zones which are symbolized with A_1, A_2, B, C_2, C_1 or A_1, C_2 and C_1 respectively. When $e < 0$ two new zones E_1 and E_2 appear in the xy-plane, as well as two symmetric points L_{-Z} and L_{+Z} on the z-axis (Fig.1). The bifurcation diagram (e, β) of Fig.2 shows the number of the existing equilibrium zones for $\nu = 7$.

2 Regions of $2D$ and $3D$ particle's motion

When $e < 0$, a "folding" of the "chimney" around the central primary, starts to create (Fig.3) and a closed area of non permitted motion in the neighborhood of P_0 is formed. This region is surrounded by a narrow annular region of permitted motion.

Bifurcations in the topology of the zero-velocity surfaces for the $3D$ motion (Fig.4) occur at values $C = C_J$, where $C_J = C_{A1}$, C_{A2}, B, C_{C2}, C_{C1}, E_1, E_2, L_{-Z}, L_{+Z}. The way that the *zvs* evolve is directly related to the sequence of the Jacobian constants of the existing equilibrium zones.

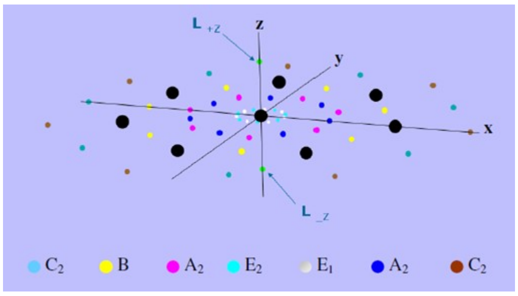

Figure 1: Equilibria for $e < 0$

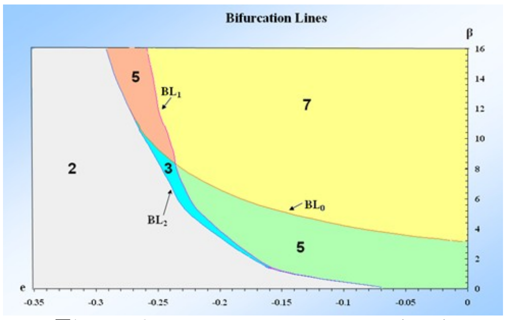

Figure 2: Bifurcation diagram (e, β)

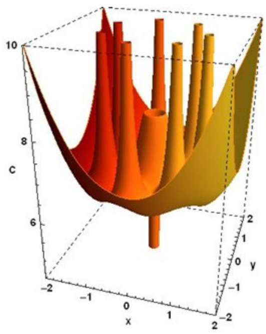

Figure 3: Zvs for $2D$ motion

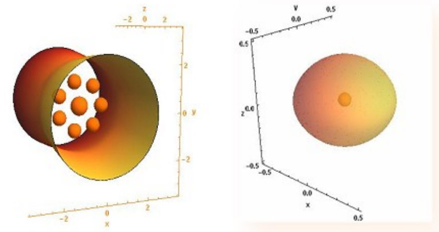

Figure 4: Regions of $3D$ motion

3 Focal points and focal curves

For any ν and for a given β, all the zero-velocity curves $C = C(x)$ drawn for $y = 0$ and for various e (> 0, < 0, or 0) pass through two focal points which are independent of the value of e (Fig.5). The same property is valid when e is kept constant ($e > 0$) and parameter β varies. However, when $e < 0$, there are four focal points (Fig.6). These properties can be extended in (x, y, C) space where all zero-velocity surfaces intersect along either one or two continuous curves.

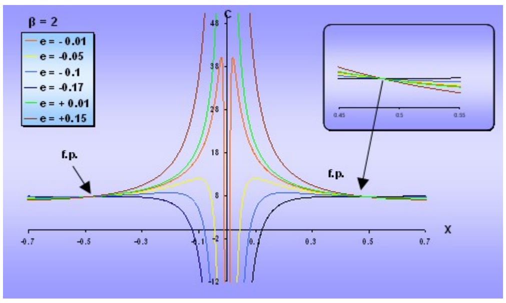

Figure 5: Focal points for $e > 0$

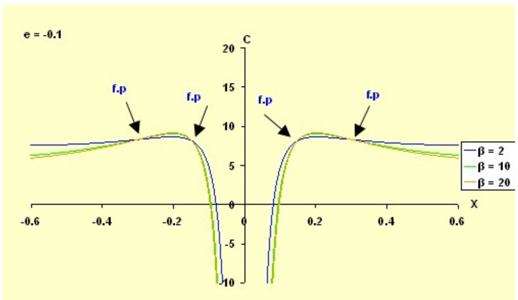

Figure 6: Focal points for $e < 0$

References

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[2] Kalvouridis, T.J., Fakis, D. 2010: *The five-body model of Ollöngren with a central Manev-type potential*. 9th HSTAM International Congress on Mechanics (Limassol-Cyprus, July 12-14, 2010).