10th Hel.A.S Conference
Ioannina, 5-8 September, 2011.
CONTRIBUTED POSTER

Families of Periodic Orbits in the Sun - Jupiter - Trojan Asteroid System

A. N. Baltagiannis and K. E. Papadakis
Department of Engineering Sciences, Division of Applied Mathematics and Mechanics,
University of Patras, Patras, GR-26504 Greece

Abstract: We study, numerically, families of simple non-symmetric periodic orbits of the restricted four-body problem where the three primary bodies lie at the apices of an equilateral triangle while a fourth massless body is moving under the Newtonian gravitational attraction of the primaries. More precisely, the primary bodies Sun, Jupiter and a Trojan Asteroid are set in the stable Lagrangian equilateral triangle configuration and as a massless fourth body we consider a spacecraft. The problem admits eight non-collinear equilibrium points. Four of them are close to Asteroid, two are stable and two are unstable. The network of the families of the simple periodic solutions using their characteristic curves in the (x, C) plane is presented. The linear stability of each periodic solution is also studied.

1 Introduction

The planar four-body problem describes the motion of an infinitesimal particle attracted by the gravitational field of three primary bodies. We consider that the three primaries m_1, m_2 and m_3 always lie at the vertices of an equilateral triangle and one of them, say m_1, is on the negative x-axis at the origin of time. The motion of the system is referred to axes rotating with uniform angular velocity. The three bodies move in the same plane and their mutual distances remain unchanged with respect to time. The motion of the primaries consists of circular orbits around their center of gravity. The equations of motion of the problem, in the usual dimensionless rectangular rotating coordinate system are written as [3], [1], [2],

$$\ddot{x} - 2\dot{y} = x - \sum_{i=1}^{3} \frac{m_i(x - x_i)}{r_i^3}, \ddot{y} + 2\dot{x} = y - \sum_{i=1}^{3} \frac{m_i(y - y_i)}{r_i^3}, \ddot{z} = -\sum_{i=1}^{3} \frac{m_i(z - z_i)}{r_i^3}$$

when the distance of the fourth particle from each of the three primaries is $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$, $i = 1, 2, 3$ where x_i, y_i and $z_1 = z_2 = 0$ (plane case), are the coordinates of the primaries

$$x_1 = -\frac{|K|\sqrt{m_2^2 + m_2 m_3 + m_3^2}}{K}, \qquad y_1 = 0, \qquad x_2 = \frac{|K|[(m_2 - m_3)m_3 + m_1(2m_2 + m_3)]}{2K\sqrt{m_2^2 + m_2 m_3 + m_3^2}},$$

$$y_2 = -\frac{m_3}{m_2^{3/2}} M, \qquad x_3 = \frac{|K|}{2\sqrt{m_2^2 + m_2 m_3 + m_3^2}}, \qquad y_3 = \frac{1}{m_2^{1/2}} M$$

where we have abbreviated $K = m_2(m_3 - m_2) + m_1(m_2 + 2m_3)$ and $M = \frac{\sqrt{3}}{2}\left(\frac{m_2^3}{m_2^2 + m_2 m_3 + m_3^2}\right)^{1/2}$. The equations of motion admit a Jacobian type of integral $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - C$ where $\Omega = \frac{1}{2}(x^2 + y^2) + \frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}$ and C is the Jacobian constant.

2 Results

In Fig. 1 (left) we present the eight non-collinear equilibrium points considering as $m_{Asteroid}$ the mass of an actual asteroid of the Trojan group, 624 Hektor, a main one of the group, as well as the network

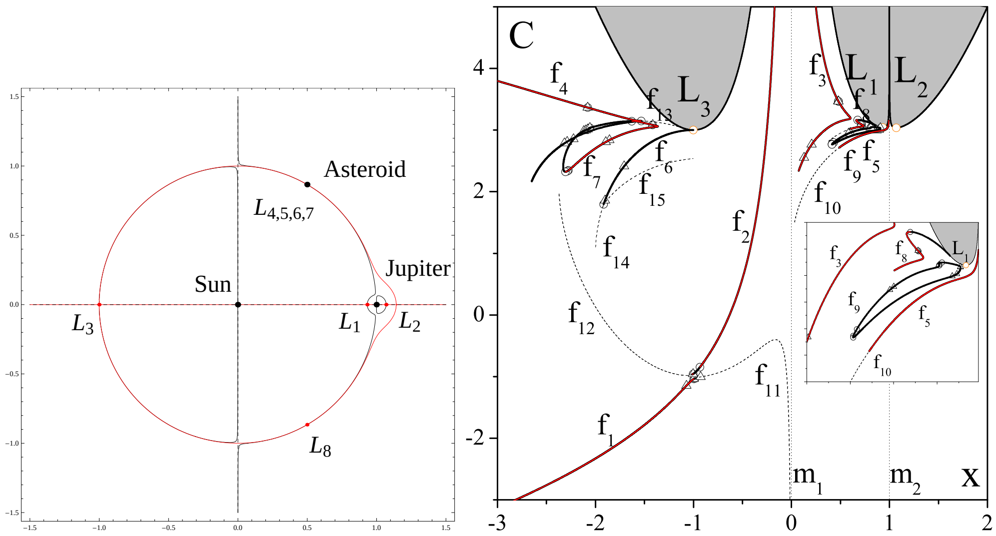

Figure 1: Left: The eight non-collinear equilibrium points in the Sun - Jupiter - Trojan Asteroid system. Three of them are linearly stable ($L_{6,7,8}$) and five are unstable. Right: The network of the families of the simple non-symmetric periodic orbits for $m_{Sun} = 0.999046$, $m_{Jupiter} = 0.000953$ and $m_{Asteroid} = 0.7032 * 10^{-11}$. The small circles and triangles indicate the horizontal and the vertical critical periodic orbits of these families correspondingly

of the families of the simple non-symmetric periodic orbits of the problem (right frame). The stability arcs of these families are presented by red lines. Because of the small value of the mass of the third primary, all calculations reported in this paper were performed using the Adams integration method and settings the allowable energy variation $\Delta C = |C_{start} - C_{end}| < 10^{-20}$ and $|x_0 - x_T| < 10^{-18}$ (initial and final conditions at $t = 0$ and $t = T$). The main majority of the periodic orbits of the families are retrograde orbits since only the families f_3, f_8 and f_9 consist of periodic solutions with direct orbits. The families f_1, f_4, f_7 and f_{13} have non-symmetric periodic orbits around the three primaries, while the families f_{10}, f_{11} and f_{12} consist of periodic orbits around the two primaries Jupiter-Asteroid, Sun-Jupiter and Sun-Asteroid correspondingly. The families f_2, f_3, f_8 and f_9 consist of orbits around Sun, while only one family, namely f_5, has orbits around Jupiter. The families f_6 and f_{14} have orbits around the equilibrium point L_3 and the family f_{15} consists of orbits around the Asteroid. All the families have stable periodic orbits except the families f_6 and f_{13} where their members are unstable. The families f_5, f_{14} and f_{15} are the only families where consist entirely of stable periodic orbits. One family, namely f_5, has as members stable (horizontally and vertically) retrograde non-symmetric periodic orbits around Jupiter and an other family, namely f_{15}, consists of stable (horizontally and vertically) retrograde non-symmetric periodic orbits around the Trojan Asteroid with obvious practical interest.

Acknowledgements: During this work the first Author was in receipt of a "K. Karatheodory" research grant.

References

[1] Baltagiannis, A. N. & Papadakis, K. E., 2011, "Equilibrium points and their stability in the restricted four-body problem", *Inter. Jour. of Bifurc. and Chaos*, **21**, pp. 2179–2193.

[2] Baltagiannis, A. N. & Papadakis, K. E., 2011, "Families of periodic orbits in the restricted four-body problem", *Astrophys Space Sci*, DOI No: 10.1007/s10509-011-0778-7.

[3] Moulton, F. R., 1900, "On a class of particular solutions of the problem of four bodies", *Trans. of the American Math. Soc.*, **1**, pp. 17–29.