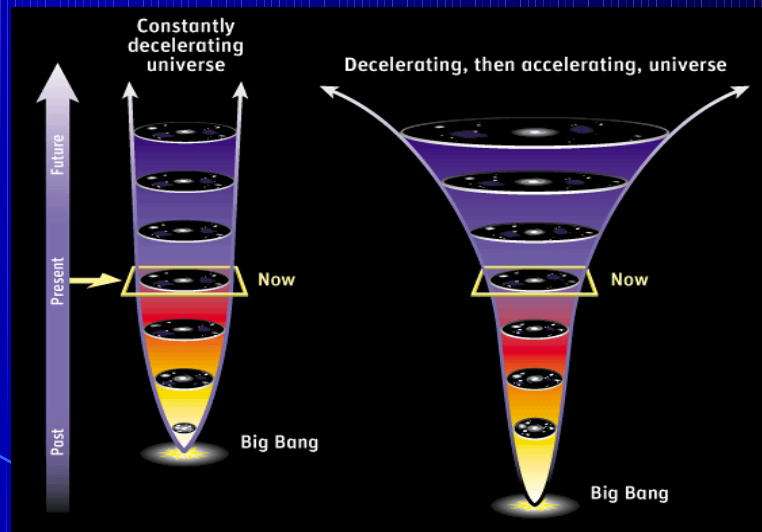


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## Testing Einstein's Gravity at Cosmological Scales

*Spyros Basilakos*

*Ioannina 5-8/09/2011*

## Collaborators

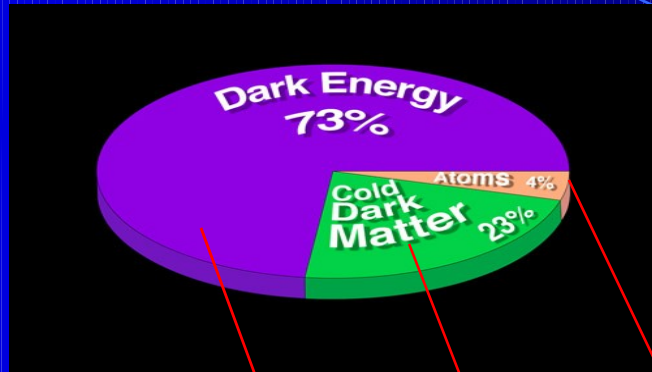
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**A part of this work can be found in Basilakos, Plionis & Pouri  
Phys. Rev. D., 83, 123525 (2011)**

## THE OUTLINE OF THE PRESENT TALK

- Current status in Cosmology
- Introduction
- Observational evidence of the cosmic acceleration
- The basic Cosmological equations
- Dark energy or modified gravity?
- Dark energy as a modification of gravity
- Testing Einstein's gravity theory
- Conclusions

# THE CURRENT COSMOLOGICAL STATUS



We live in a very exciting period for the advancement of our knowledge for the Cosmos

$$\Omega_{\text{DE}} + \Omega_{\text{DM}} + \Omega_{\text{BAR}} = 1$$

- Current observational data strongly support a flat and accelerating Universe with  $H_0 \sim 71 \text{ Km/sec/Mpc}$  and  $T_0 \sim 14 \text{ Gyr}$ .
- The mystery of dark energy poses a challenge of such magnitude that, as stated by the Dark Energy Task Force (DETF –advising DOE, NASA and NSF), *“Nothing short of a revolution in our understanding of fundamental physics will be required to achieve a full understanding of the cosmic acceleration”* (Albrecht et al. 2006).

# COSMOLOGICAL OBSERVATIONS & SPACETIME

Hubble diagram (SNIa)  $\rightarrow \Omega_m + \Omega_\Lambda$

Temperature fluctuations of the CMB  $\rightarrow$  spatial geometry  $\Omega_k$

Large-Scale Structure  $\rightarrow \Omega_m$  (independent from dark energy)

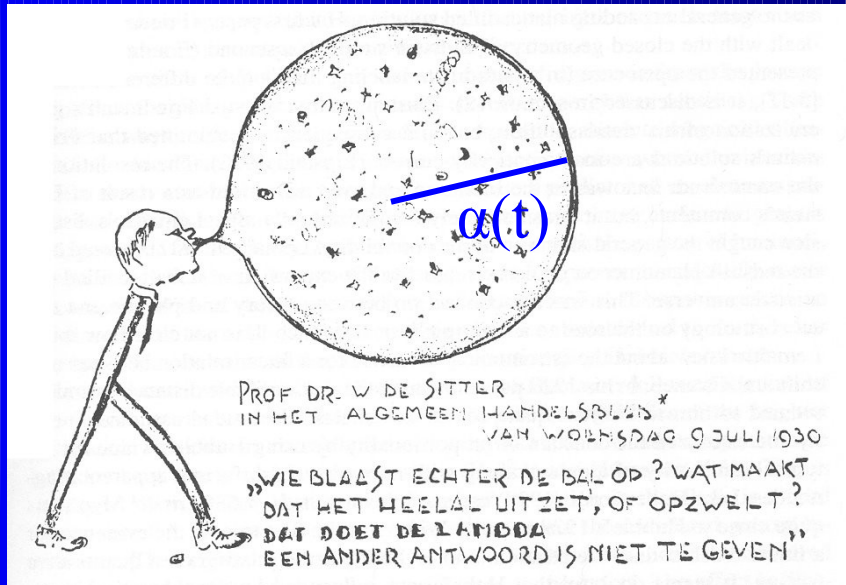
Extragalactic sources (galaxies, AGNs, GRBs, LBGs etc) at large redshifts  $\rightarrow \Omega_m + \Omega_\Lambda$

$$ds^2 = -c^2 dt^2 + \alpha^2(t) \left[ \frac{dr^2}{(1 - kr^2/R_0^2)} + r^2 d\Omega_{S^2} \right]$$

We use a spatially flat ( $k=0$ ) the FRW metric.



# THE BASIC COSMOLOGICAL EQUATION



From the energy conservation:

$$E = T + U = \frac{\dot{\alpha}^2}{2} - \frac{GM}{\alpha}$$

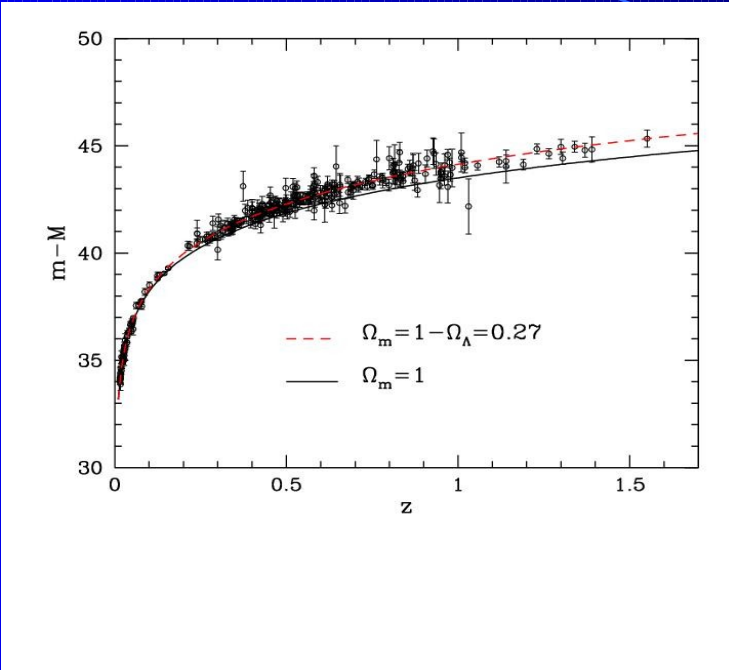
$$M = \frac{4\pi\rho}{3} \alpha^3$$

$$H^2 = \frac{\dot{\alpha}^2}{\alpha^2} = \frac{8\pi G}{3} \rho + \frac{k}{\alpha^2}$$

where  $k=2E=\text{const.}$

**The First Friedmann Equation = The first integral of motion in  
Newtonian Physics**

# Evidence of cosmic acceleration



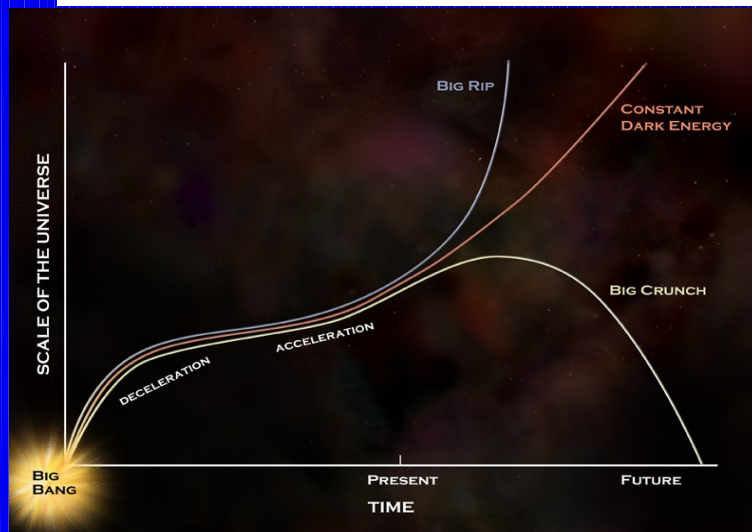
Assuming a matter dominated Universe observationally:

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 > \frac{8\pi G}{3} \rho_m$$

or

Change Gravity. GR is not valid at Cosmological scales. Modify the "law of gravity"

Change the cosmic fluid. We add a "dark energy"

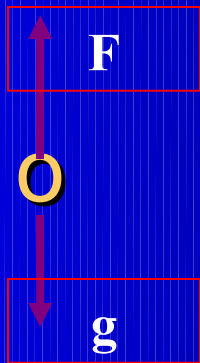


$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_{eff}}{3} \rho_m$$

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_m + \rho_Q)$$

# Gravity versus “Anti-gravity”

F is a repulsive force



At the epoch of galaxy formation F is negligible with respect to gravity,  $F \ll g$

At small scales (galaxies) F is weak with respect to gravity,  $F \sim g/100000$  ....

...but at cosmological scales F becomes strong. Indeed prior to the present time  $F \sim 5g$  and it yields the cosmic acceleration



As a general result: We call "*quintessence*" models those dark energy models that adhere to GR. These models contain an exotic new cosmic fluid (field).

Alternatively, we can modify GR by changing the *Einstein's field equations*. Note that up to galactic scales the modified gravity must be close to GR. But at Cosmological scales we have  $g_{MG} < g_{GR}$  which implies that the "cosmic acceleration" is due to the weak gravity nature ("geometrical" dark energy).

Both scenarios provide the same Hubble expansion  **$H=H(\alpha)$** .

Can we decide between the two theories? Using observational data which are based on the distance modulus (SNIa, BAOs, etc) the answer is NO!!

# Tests of Gravity

by R. Caldwell 2005

**Locally Einstein's General Relativity is the standard model of gravitation**

$$\eta = 2 \frac{a_1 - a_2}{a_1 + a_2} < 4 \times 10^{-13}$$

local acceleration of bodies of different composition  
Eot-Wash: Baessler et al, PRL 83 (1999) 3585

$$\dot{G}/G = (4 \pm 9) \times 10^{-13} / \text{yr}$$

Lunar laser ranging  
Williams et al, PRL 93 (2004) 261101

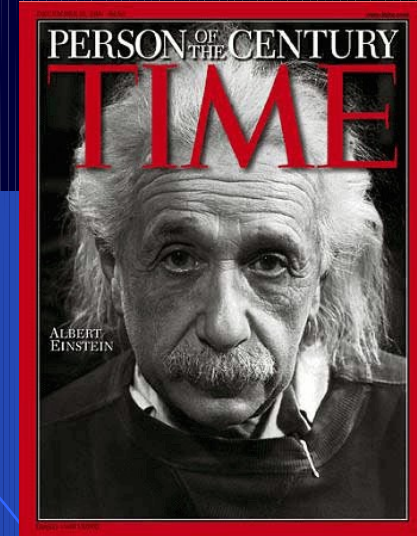
$$\frac{M_G}{M_I} |_{\text{earth}} - \frac{M_G}{M_I} |_{\text{moon}} = (-1 \pm 1.4) \times 10^{-13}$$

Nordtvedt effect: observations of the acceleration of bodies with different gravitational binding energies tests the Strong Equivalence Principle

Mass definitions:

$$\vec{F} = M_I \vec{a}$$

$$\Phi = -GM_G/r$$



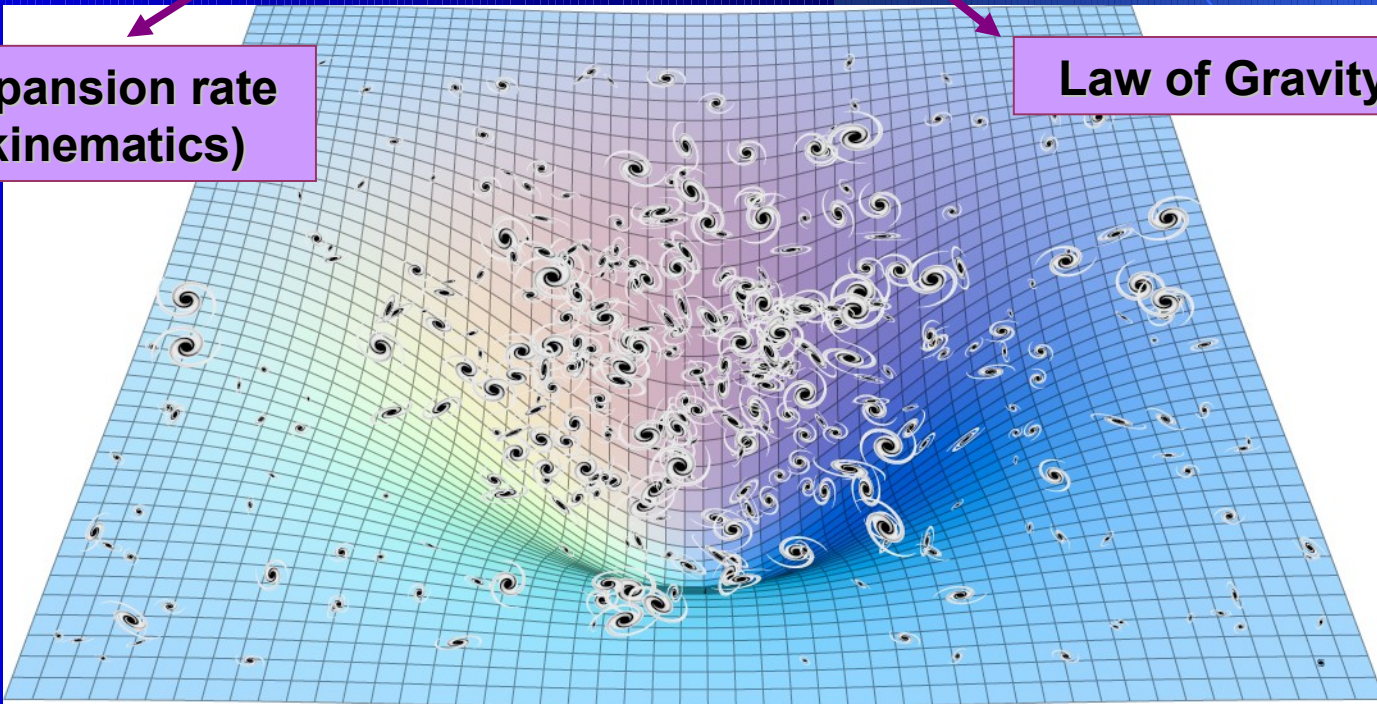
# Using Cosmology to test Gravity

The dark energy component slows the growth of inhomogeneities in the total matter: dark matter and baryons. Using linear perturbation theory (mass conservation, Euler equation, Poisson equation and Friedman equations  $\delta_m \ll 1$ ) we get:

$$\ddot{\delta}_m + 2H(t)\dot{\delta}_m = 4\pi G_{\text{eff}}\rho_m\delta_m$$

**Expansion rate  
(kinematics)**

**Law of Gravity**





**An approximate solution of the differential equation that governs the evolution of matter fluctuations is:**

$$\delta_m \propto D(a) = \exp \left[ - \int_a^1 \Omega_m^\gamma(a) d \ln a \right]$$

$$a(z) = \frac{1}{1+z}$$

**Where  $\gamma$  is the so called gravity index originally introduced by Peebles (1993)**

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{E^2(a)}$$

$$E(a) = H(a) / H_0$$

From theoretical viewpoint, it has been found that for those DE models that adhere to GR,  $\gamma=0.55$  (Linder 2004; Nesseris & Perivolaropoulos 2008).

If the derived value of  $\gamma$  shows scale or time dependence or it is inconsistent with 0.55 then this will be a hint that the nature of DE reflects in the physics of gravity.

## The role of bias

How the luminous matter trace the underlying matter distribution?

It has been found via a correlation function analysis that the mass-tracer fluctuation field is proportional to that of matter (Kaiser 19984; Bardeen et al. 2000)

$$\delta_{tr} = b\delta_m$$

If  $b > 1$  then the extragalactic tracers (galaxies, QSOs, clusters) are more clustered with respect to underlying matter

If  $b < 1$  then the underlying matter is more clustered with respect to the luminous matter fluctuation field

If  $b = 1$  then the luminous matter traces the underlying matter fluctuation field (this is the case for optical galaxies; Lahav et al. 2003; Marinnoni et al. 2005)



# Using Cosmology to test Gravity

Using the fact that the mass tracers and the underlying matter share the same gravity field and considering that the mass tracer population is conserved with time ie. The effects of hydrodynamics do not significantly alter the population mean we find :

$$\ddot{\delta}_{tr} + 2H(t)\dot{\delta}_{tr} = 4\pi G_{\text{eff}} \rho_m \delta_m$$

$$\ddot{\delta}_m + 2H(t)\dot{\delta}_m = 4\pi G_{\text{eff}} \rho_m \delta_m$$

$$\delta_{tr} = b\delta_m$$

$$\ddot{b} + 2[f(t) + H(t)]\dot{b} + 4\pi G_{\text{eff}} \rho_m \delta_m b = 4\pi G_{\text{eff}} \rho_m \delta_m$$

**This is the growth rate of clustering**

$$f(a) = \frac{d \ln D}{d \ln a} = \Omega_m^\gamma(a)$$

## A general solution of which is ...

$$b(z) = 1 + \frac{b_0 - 1}{D(z)} + C_2 \frac{J(z)}{D(z)}$$

$$J(z) = \int_0^z \frac{(1+x) dx}{E(x)}$$

The bias evolution is a function of A) the expansion rate of the Universe [ $E(z) = H(z)/H_0$ ] and B) the matter fluctuations (and thus of  $\gamma$ ).

This implies that the combination of the current bias model with data could provide an efficient way to discriminate between geometrical DE models and DE models that adhere to GR.

In other words we propose to use the above (theoretical) bias evolution and bias data (already available in the literature) to test GR.

**Basilakos, Plionis & Pouri, Phys. Rev. D. 83, 123525, 2011**

## Simulations and bias data

In this study we assume that the extragalactic tracers (galaxies, QSOs, etc) are hosted by a dark matter halo of a given mass. Thus the constants of integration  $b_0, C_2$  should be functions of mass of dark matter halos. Comparing our theoretical bias evolution model with that predicted by N-body simulations we find:

$$b_0(M) = 0.71 \left[ \left( \frac{M}{M_c} \right)^{-0.04} + \left( \frac{M}{M_c} \right)^{0.5} \right]$$
$$C_2(M) = 0.59 \left( \frac{M}{10^{13} h^{-1} M_{sol}} \right)^{0.28}$$

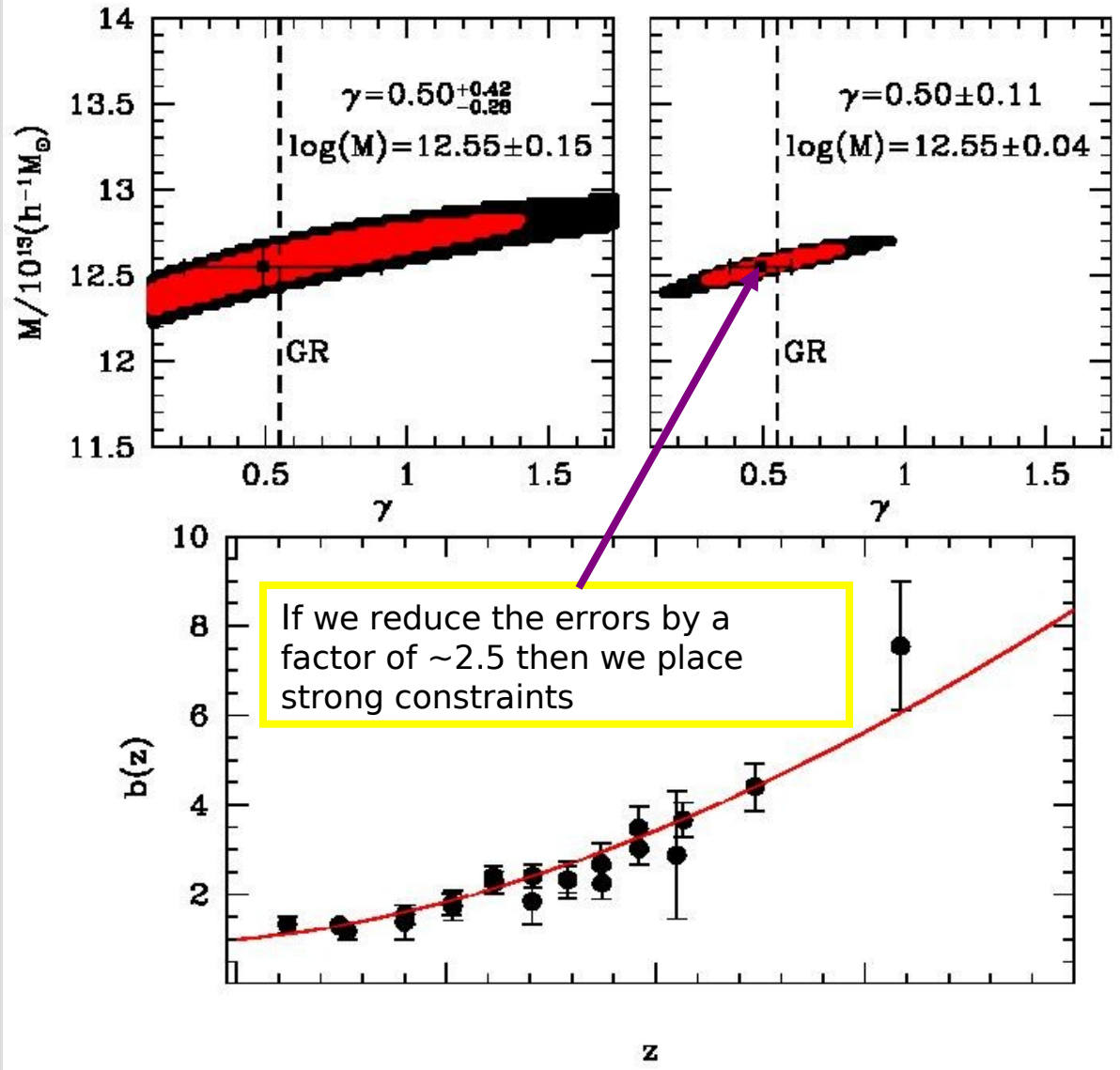
$$M_c = 4.95 \times 10^{13} h^{-1} M_{sol}$$

Finally we also use literature bias data for the case of optical QSOs (Croom et al. 2005; Ross et al. 2009; Shen et al. 2009).

# Using Cosmology to test Gravity

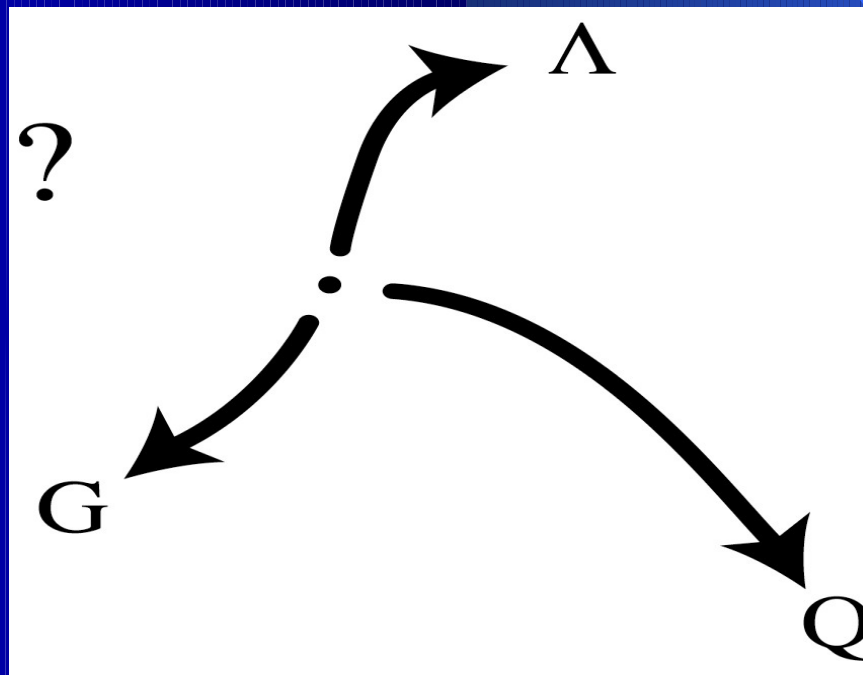
We impose  $\Omega_m = 0.27$

Performing a likelihood analysis using current bias data, we find feeble indications that GR is valid at cosmological scales.



# Conclusions

- In this work we provide a general bias evolution model, based on linear perturbation theory, which is valid for all possible non-interacting DE models, including those of modified gravity
- Comparing the current bias model with available bias data we find weak indications that GR is the correct theory at cosmological scales.
- But still we need better data in order to place strong constraints on the gravity index.





## THE BASIC COSMOLOGICAL EQUATIONS PART-II

From the 1<sup>st</sup> law of thermodynamics using adiabatic processes  
(the Universe is perfectly homogeneous):

$$dE + PdV = dQ \Rightarrow c^2 dM + PdV = 0 \quad M = \frac{4\pi\rho}{3} \alpha^3$$
$$c^2 \frac{d}{dt}(\rho \alpha^3) + P \frac{d}{dt}(\alpha^3) = 0$$

Therefore we get the continuity equation:

$$\frac{d\rho}{dt} + 3H \left( \rho + \frac{P}{c^2} \right) = 0 \quad H(t) = \frac{\dot{\alpha}(t)}{\alpha(t)}$$

where P is the total  
pressure, also from now  
on we use  $c=1$

Differentiating the 1<sup>st</sup> Friedmann  
equation and using the continuity  
equation we get:

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3} (\rho + 3P)$$

**The second Friedmann  
Equation is practically the  
Newton's second law**