Weak Gravitational Lensing as a Tool for Cosmology

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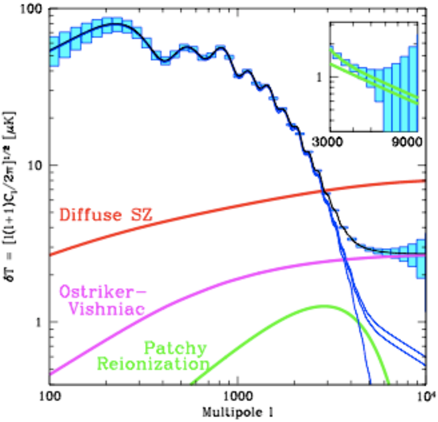
Information encoded in the CMB

At large angular scales, $\theta > 5'$ or $\ell < 2500$:

- primary anisotropies: small linear perturbations in the primordial plasma's density and velocity.

At small angular scales:

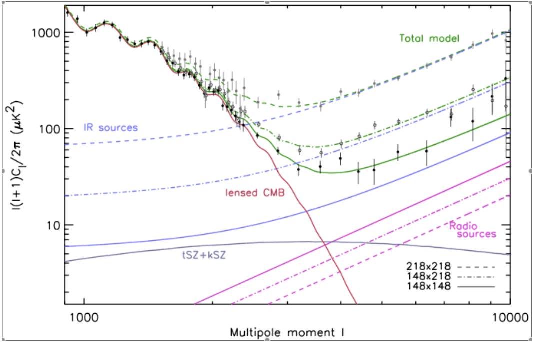
- primary anisotropies negligible due to Silk damping: sets scale for thickness of last-scattering surface
- non-linear effects from more recent epochs: secondary sources of perturbations from primordial potential–matter coupling.



[ACT research proposal]

Information measured in the CMB

[Dunkley et al., arXiv:1009.0866]



- **thermal Sunyaev-Zel'dovich**
  $\Rightarrow$ spectral distortion of photons to higher energies.
- **Ostriker-Vishniac (cosmological velocity) and kinetic Sunyaev-Zel'dovich (peculiar motion of cluster)**
  $\Rightarrow$ velocity–density coupling.
- **gravitational lensing** $\Rightarrow$ smearing of acoustic peaks
  + generation of small scale power at temperature

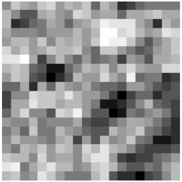
Questions call for more experiments

To answer questions such as:

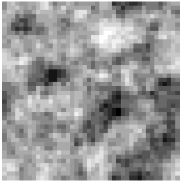
- Did cosmic structure form solely via gravitational instability?
- What is the mass of the neutrino and how does it affect the structure formation?
- How does galaxy distribution relate to the mass distribution?
- What is the nature of the dark matter and the dark energy?
- Are we missing any important ingredient from the cosmological model?

need experiments that can probe the corresponding scales.

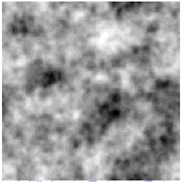
WMAP: $15'$/pixel
[http://lambda.gsfc.nasa.gov/]



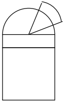
Planck space probe: $7.2'$/pixel
[http://www.rssd.esa.int/index.php?project=Planck]

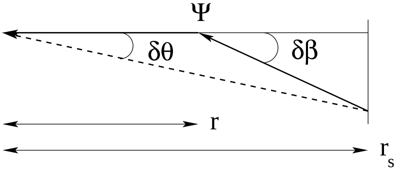


Atacama Cosmology Telescope: $1.4'$/pixel
[http://www.physics.princeton.edu/act/about.html]
South Pole Telescope: $1.0'$/pixel
[http://pole.uchicago.edu/]



The lens equation





For a line element

$$ds^2 = a^2(\eta)\left[-(1-2\Psi)d\eta^2 + (1+2\Psi)\left[dr^2 + f_k^2(r)\left(d\theta_x^2 + d\theta_y^2\right)\right]\right]$$

find the lens equation by setting

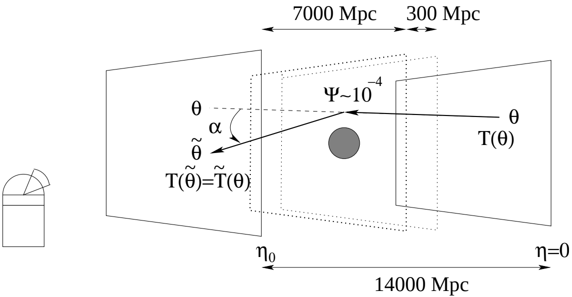
$$f_k(r_s)\,\delta\theta = f_k(r_s - r)\,\delta\beta, \text{ where } \delta\beta = -2\delta r[1/f_k(r)]\nabla_{\boldsymbol{\theta}}\Psi(r\boldsymbol{\theta}, r)$$

Adding up the deflections between observer and source

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = -2\int_0^{r_s} dr\,\frac{f_k(r_s - r)}{f_k(r_s)\,f_k(r)}\nabla_{\boldsymbol{\theta}}\Psi(r\boldsymbol{\theta}, r), \;\; f_k(r) = \begin{cases} \sin[r] : k = +1, \\ r : k = \;\;0, \\ \sinh[r] : k = -1. \end{cases}$$

[Challinor & Lewis, PhysRep 429, 1 (2006)]

Lensing of the CMB: probe of large-scale structure



The lensing potential ψ deflects the photons by $\boldsymbol{\alpha}(\boldsymbol{\theta}) = \nabla\psi$
$\Leftrightarrow$ remap of the temperature anisotropy according to

$$\tilde{T}(\boldsymbol{\theta}) = T(\boldsymbol{\theta} + \nabla\psi) = T(\boldsymbol{\theta}) + \nabla\psi \cdot \nabla T(\boldsymbol{\theta}) + O[(\nabla\psi)^2].$$

$14000/300 \sim 50$ deflections

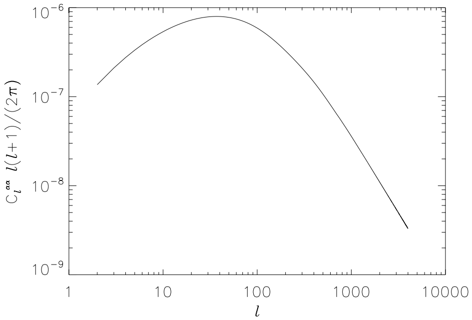
Assume random walk:
$\alpha(\theta) \simeq \sqrt{50} \times 10^{-4} \simeq 7 \times 10^{-4} = 2.4$ arcmin

Deflections are coherent over $\theta_{corr} \simeq 300/7000 \simeq 2$ deg

Power spectrum of the lensing deflection

Deflection power spectrum peaks at $\ell \sim 60 \Rightarrow \theta \sim 3$ deg

$$C_\ell^{\alpha\alpha} = [\ell(\ell+1)]^2 C_\ell^{\psi\psi}$$



$$C_\ell^{\psi\psi} \approx \frac{8\pi^2}{\ell^3} \int_0^{\eta_0} d\eta\, \eta \left[\frac{f_k(\eta_0 - \eta)}{f_k(\eta) f_k(\eta_0)} \right]^2 P_\Psi\left(\frac{\ell}{f_k(\eta)}, \eta \right)$$

and

$$P_\Psi(k,\eta) = \frac{9}{4} \Omega_m^2(\eta) H^4(\eta) \frac{P_\delta(k,\eta)}{k^4}$$

Relevance of lensing reconstruction for cosmology

$\hat{\alpha} \times \hat{\alpha}$:

- Subtract lensing B-modes [Seljak & Zaldarriaga, PRL 78, 2054 (1997)]
- Probe density power spectrum [Seljak & Zaldarriaga, PRL 82, 2636 (1999)]
- Constrain sum of neutrino masses and dark energy equation of state [de Putter et al., PRD 79, 065033 (2009); Sherwin et al., PRL 107, 021302 (2011)]

$\hat{\alpha} \times$ Galaxies:

- Estimate galaxy bias [Suginohra et al., ApJ 495, 511 (1998)]

$\hat{\alpha} \times$ Weak lensing:

- Constrain growth of structure [Bean & Tangmatitham, PRD 81 083534 (2010)]
- Constrain distance ratios and curvature [Das & Spergel, PRD 79, 043509 (2009)]

Detections of the lensing signal

- Cross-correlation of WMAP with large-scale structure data:
  $Corr = 1.15 \pm 0.34$ at 3.4σ
  [Smith et al., PRD 76, 043510 (2007)]
  $Corr = 1.06 \pm 0.42$ at 2.5σ
  [Hirata et al., PRD 78, 043520 (2008)]

- Direct detection from WMAP data:
  $A_L = \{0.96 \pm 0.60, 1.06 \pm 0.69, 0.97 \pm 0.47\}$ at 2.5σ
  [Smidt et al.,ApJL. 728, 1 (2011)]
- Direct detection from ACT data: $A_L = 1.16 \pm 0.29$ at 4σ
  [Das et al., PRL 104, 021301 (2011)]

Estimator of the lensing potential

Lensing effects are apparent in the CMB power spectrum

$$\left\langle \tilde{T}(\boldsymbol{\ell}')\ \tilde{T}(\boldsymbol{\ell}'-\boldsymbol{\ell})\right\rangle = \delta(\boldsymbol{\ell})\ C_{\ell'}^{TT} + \boldsymbol{\ell}\cdot\left[\boldsymbol{\ell}'\ C_{\ell'}^{TT} + (\boldsymbol{\ell}-\boldsymbol{\ell}')\ C_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}^{TT}\right]\psi(\boldsymbol{\ell})$$

The optimal estimator is the convolution of $\tilde{T}(\boldsymbol{\ell})$ by $Q^{\psi}(\boldsymbol{\ell},\boldsymbol{\ell}')$

$$\hat{\psi}(\boldsymbol{\ell}) = \int d^2\ell'\ \tilde{T}(\boldsymbol{\ell}')\ \tilde{T}(\boldsymbol{\ell}-\boldsymbol{\ell}')\ Q^{\psi}(\boldsymbol{\ell},\boldsymbol{\ell}')$$

where

$$Q^{\psi}(\boldsymbol{\ell},\boldsymbol{\ell}') = \mathcal{N}_{\ell}\ \frac{1}{2}\frac{\boldsymbol{\ell}\cdot\boldsymbol{\ell}' C_{\ell'} + \boldsymbol{\ell}\cdot(\boldsymbol{\ell}-\boldsymbol{\ell}')C_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}}{\tilde{C}_{\ell'}\tilde{C}_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}}$$

$$\mathcal{N}_{\ell} = \left[\int \frac{d^2\ell'}{(2\pi)^2}\ \frac{1}{2}\frac{[\boldsymbol{\ell}\cdot\boldsymbol{\ell}'\ C_{\ell'} + \boldsymbol{\ell}\cdot(\boldsymbol{\ell}-\boldsymbol{\ell}')\ C_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}]^2}{\tilde{C}_{\ell'}\ \tilde{C}_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|}}\right]^{-1} \equiv C_{\ell}^{\hat{\psi}(\ell)}.$$

Do we need a new estimator?

Optimal reconstruction in harmonic space [Hu, ApJ 557, 79 (2001); Okamoto & Hu, PRD 67, 083002 (2003)] implicitly assumes full-sky coverage without:

- galactic cuts,
- bad pixels due to excision of point sources,
- nonuniform weighting for uneven sky coverage.

We consider a slightly less optimal estimator which:

- has compact support,
- acts in real space.

[Carvalho & Moodley, PRD 81, 123010 (2010)]
[Bucher, Carvalho, Moodley & Remazzeilles, arXiv:1004.3285]

A new estimator for the lensing potential

An estimator for the convergence $\boldsymbol{\kappa}_0 = -\frac{1}{2}\nabla^2\psi$ in real space

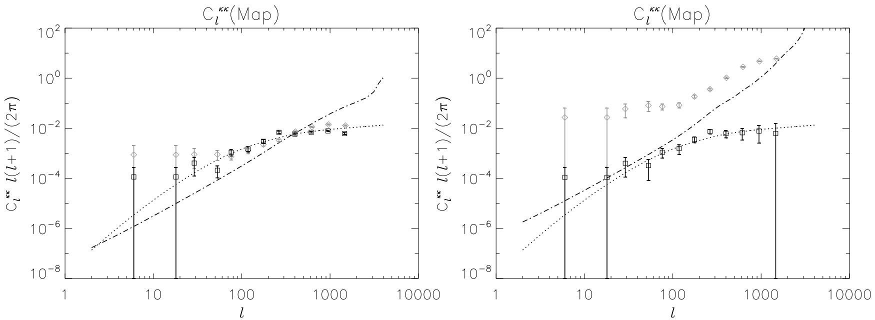
$$\begin{aligned}\hat{\kappa}_0(\boldsymbol{\theta}) &= \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2}\ \exp[i\boldsymbol{\ell}\cdot\boldsymbol{\theta}] \int d^2\boldsymbol{\ell}'\ \tilde{T}(\boldsymbol{\ell}')\ \tilde{T}(\boldsymbol{\ell}-\boldsymbol{\ell}')\ Q(\boldsymbol{\ell},\boldsymbol{\ell}') \\ &\equiv \int d^2\boldsymbol{\theta}'\ \tilde{T}(\boldsymbol{\theta}') \int d^2\boldsymbol{\theta}''\ \tilde{T}(\boldsymbol{\theta}'')\ Q(\boldsymbol{\theta},\boldsymbol{\theta}',\boldsymbol{\theta}''),\end{aligned}$$

where we define the weight function in real space by

$$\begin{aligned}Q(\boldsymbol{\theta},\boldsymbol{\theta}',\boldsymbol{\theta}'') &= \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2}\ \exp[i\boldsymbol{\ell}\cdot\boldsymbol{\theta}] \\ &\times \int \frac{d^2\boldsymbol{\ell}'}{(2\pi)^2}\ \exp[-i\boldsymbol{\ell}'\cdot\boldsymbol{\theta}']\exp[-i(\boldsymbol{\ell}-\boldsymbol{\ell}')\cdot\boldsymbol{\theta}'']\ Q(\boldsymbol{\ell},\boldsymbol{\ell}').\end{aligned}$$

and $Q(\boldsymbol{\ell},\boldsymbol{\ell}') = \ell^2 Q^{\psi}(\boldsymbol{\ell},\boldsymbol{\ell}')$.
[Carvalho & Moodley, PRD 81, 123010 (2010)]

Noiseless | Noisy



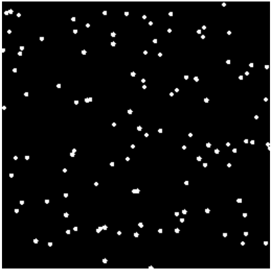
- Loss of power at small scales: in theory $\ell_{max} \sim 1/\theta_{fwhm}$ but in practice limited by $1/\theta_{kernel}$ which measures the smallest wavelengths probed by the kernel

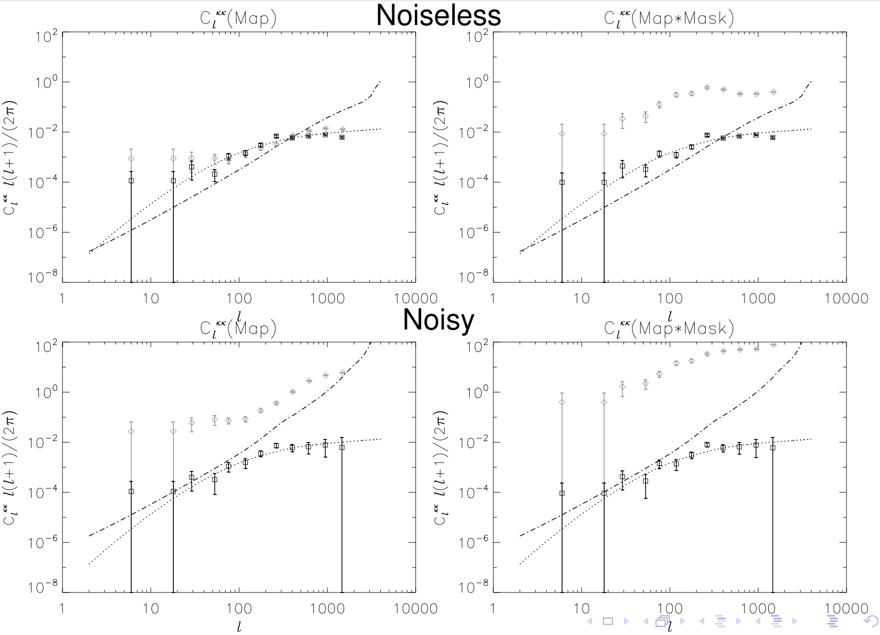
- No sensitivity to experimental noise: test with white noise only $\Rightarrow$ noise, independent in each pixel, is averaged out.

Application to maps with excisions of points

- Foreground signals can generate spurious correlations.
- Weak lensing induces non-Gaussianities in the form of mode couplings $\Rightarrow$ sensitive to foreground signals.

To which extent does masking small areas at the locations of point sources affect the weak lensing convergence extracted from CMB maps? [Carvalho & Tereno, PRD 84, 063001 (2011)]

[Lensed Map$\times$Beam+Noise] $\times$ 


Noiseless
Noisy
$C_l^{\kappa\kappa}$(Map)
$C_l^{\kappa\kappa}$(Map*Mask)
$C_l^{\kappa\kappa}\, l(l+1)/(2\pi)$
l
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
100
1000
10000

Lensing: probe of dark matter and dark energy

- Convergence of the CMB:

$$\boldsymbol{\kappa}(\hat{\boldsymbol{n}}) = \frac{3}{2}\Omega_m H_0^2 \int_0^{\eta_0} d\eta \, \frac{\eta}{a(\eta)} \, \frac{\eta_0 - \eta}{\eta_0} \, \delta(\eta\hat{\boldsymbol{n}}, \eta)$$

dependent on total mass
$\Rightarrow$ probes statistical properties of large scale structure

- Convergence of distant galaxies:

$$\boldsymbol{\kappa}(\hat{\boldsymbol{n}}) = \frac{3}{2}\Omega_m H_0^2 \int_{\eta_{gal}}^{\eta_0} d\eta \, \frac{\eta}{a(\eta)} \, g(\eta) \, \delta(\eta\hat{\boldsymbol{n}}, \eta)$$

$$g(\eta) = \int_{\eta_{gal}}^{\eta} d\eta' \, (dN/d\eta') \frac{\eta' - \eta}{\eta'}$$

dependent on (total mass $+$ source redshift distribution)
$\Rightarrow$ probes dark energy properties.

  - affects distant-redshift relation
  - suppresses time-dependent growth of structures

Lensing: probe of modifications of gravity

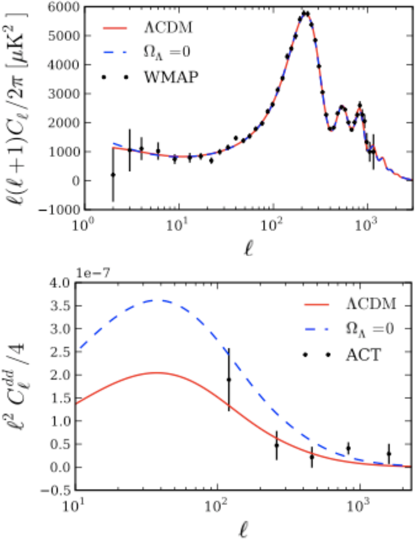
Modified gravity must converge to General Relativity:

- at high z, when expansion is not accelerated
- at small scales, where GR is well tested.

To distinguish between effects of MG and (GR+dark energy):

- impossible with background expansion history only
- need also growth history.

Weak lensing probes the growth
⇒ can be used to distinguish between models.



[Blake et al., PRL 107, 021301 (2011)]

Parametrization of modified gravity

Parametric Post Friedmann prescription: a model independent description of deviations from GR:

- gravitational split function $\phi - \psi = \zeta\phi + \boxed{\text{g/k}}\,\dot{\phi}$
- mass screening function: $G_{eff} = G\mu^{-2} = G(1 + H\boxed{\text{g/k}})$

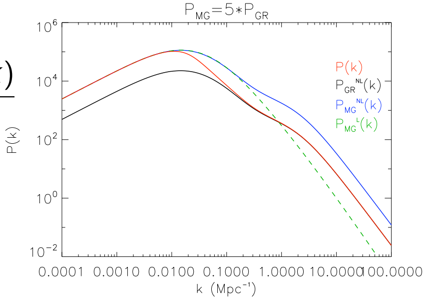
[Ferreira & Skordis, PRD 81, 1045020 (2010)]

At small scales expect deviations from the linear regime of the matter power spectrum

$$P_{HS}(k) = \frac{P^{nl}_{MG}(k) + c_{nl}\Sigma^2(k)P^{nl}_{GR}(k)}{1 + c_{nl}\Sigma^2(k)}$$

$\Sigma^2(k)$: picks up non-linear scales
c_{nl} : defines scale $P_{HS} \rightarrow P_{GR}$

[Hu & Sawicki, PRD 76, 104043 (2007)]



In progress... [Carvalho, Tereno & Basilakos]

Constrain models of modified gravity with the lensing of galaxies, the **cosmic shear**

$$\boldsymbol{\kappa}_{+} = -\frac{1}{2}(\partial_x^2 + \partial_y^2)\psi, \quad \boldsymbol{\kappa}_{\times} = -\partial_x \partial_y \psi,$$

and

- Test on COSMOS data: shear of galaxies over $z \in [0, 5]$ [Schrabback et al.,A&A 516 (2010)]
- Prepare for the space survey EUCLID: will measure the shape and spectra of galaxies over the entire extragalactic sky in the visible and NIR, out to $z = 2$ [http://www.ias.u-psud.fr/imEuclid]
- Combine with lensing of the CMB

Summary

- The CMB reaches us from the surface of last scattering
  $\Rightarrow$ contains information about the early Universe and its subsequent evolution.
- Lensing of the CMB imprints in the primordial fluctuations the cumulative deflection by the large-scale structure along the line of sight
  $\Rightarrow$ on its own or combined with other effects contributes to clearing out the picture of the Universe.
- A novel approach to extract the lensing field from CMB maps in real space appears to be a promising tool to deal with noise, bias, point source excisions.

Current studies:

- constrain modified gravity models with weak lensing, both cosmic and galactic