Order and chaos in a triaxial galaxy model with a dark halo component

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1. Summary

We study the regular or chaotic nature of orbits in a 3D potential describing a triaxial galaxy surrounded by a spherical dark halo component. Our numerical calculations show that the percentage of chaotic orbits decreases exponentially as the mass of the dark halo increases. A linear increase of the percentage of chaotic orbits was observed as the scale length of the halo component increases. In order to distinguish between regular and chaotic character of orbits we use the total angular momentum L_{tot}, as a new indicator. Comparison of this new dynamical parameter, with other, previously used, chaos indicators, shows that the L_{tot} gives very fast and reliable results in order to detect the character of orbits in galactic potentials.

2. The model

We shall study the motion in a 3D composite galactic model described by the potential

$$V_t(x,y,z) = V_g(x,y,z) + V_h(x,y,z) \quad , \qquad (1)$$

where

$$V_g(x,y,z) = \frac{\upsilon_0^2}{2} \ln\left[x^2 - \lambda x^3 + a y^2 + b z^2 + c_b^2\right] \; , \qquad (2)$$

while

$$V_h(x,y,z) = \frac{-M_h}{\left(x^2 + y^2 + z^2 + c_h^2\right)^{1/2}} \quad . \qquad (3)$$

Potential (2) describes a triaxial elliptical galaxy with a dense nucleus and a small asymmetry introduced by the term $-\lambda x^3$, $\lambda << 1$. The parameters α and b describe the geometry of the galaxy, while c_b is the scale length of the bulge. The parameter υ_0 is used for the consistency of the galactic units. To this potential we add a spherically symmetric dark halo described by the potential (3). Here M_h and c_h are the mass and the scale length of the dark halo component respectively.

3. Aim

The aim of this article is: (i) To investigate the character of orbits in the potential (1) and to determine the role played by the dark halo. (ii) To introduce, use and check a new and fast detector, the total angular momentum L_{tot}, in order to obtain a reliable criterion to distinguish between order and chaos in galactic potentials.

The Hamiltonian to the potential (1) writes

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) + V_t\left(x, y, z\right) = h_3 , \qquad (4)$$

where p_x, p_y and p_z are the momenta per unit mass conjugate to x, y and z, while h_3 is the numerical value of the Hamiltonian.

The system of galactic units

Length unit : 1 kpc

Mass unit : $2.325 \times 10^7 M_\odot$

Time unit : 0.97748×10^8 yr

Velocity unit : 10 km s^{-1}

Energy unit (per unit mass) : 100 km^2 s^{-2}

G is equal to unity

In the above units we use the values :

$\upsilon_0=15$, $c_b=2.5$, $a=1.5$, $b=1.8$ and $\lambda=0.03$,

while M_h and c_h are treated as parameters.

4. Results of the 2D system

Here we study the properties of the 2D system, which comes from the potential (1) if we set *z=0*.

The corresponding 2D Hamiltonian writes

$$H_2 = \frac{1}{2}\left(p_x^2 + p_y^2\right) + V_t\left(x, y\right) = h_2 \quad , \qquad (5)$$

where h_2 is the numerical value of the Hamiltonian. We do this, in order to use the results obtained from the 2D model for the study of the more complicated 3D model, which will be presented in the next Section.

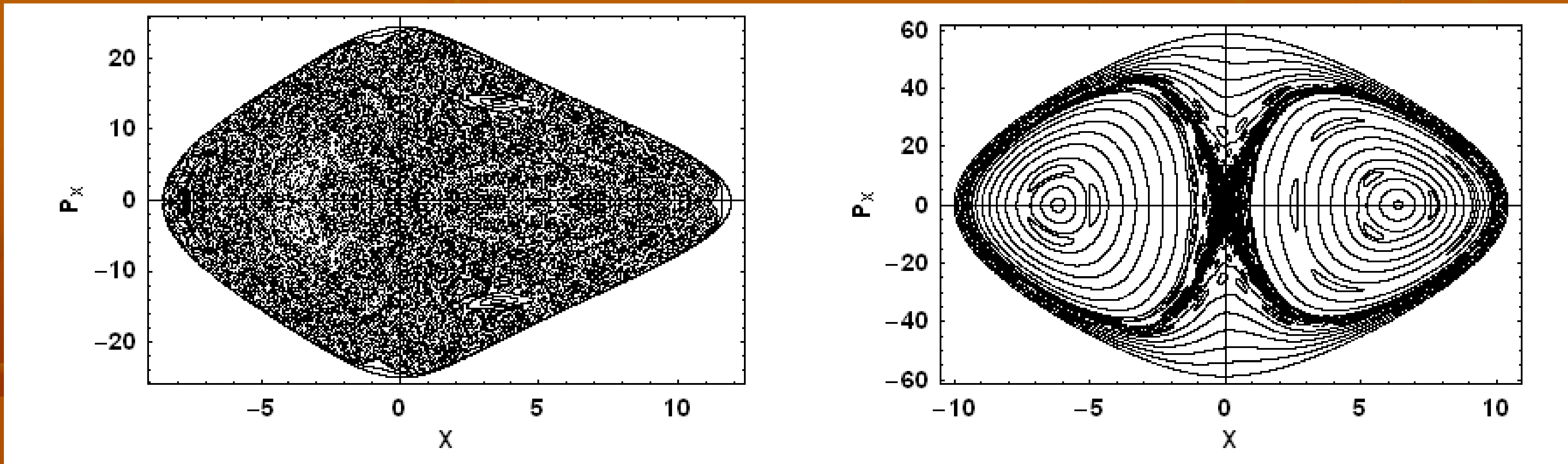


Figure 1 a-b shows the x - p_x, $y=0$, $p_y>0$ phase planes for two different values of the mass of the halo. The values of all the other parameters are: $\upsilon_0=15$, $c_b=2.5$, $\alpha=1.5$, $b=1.8$, $\lambda=0.03$ and $c_h=8$. Fig. 1a shows the phase plane, when the system has no halo component, that is when M_h=0. The value of the energy h_2 is 516. Fig. 1d shows the phase plane when $M_h=30000$ and $h_2=-1788$.

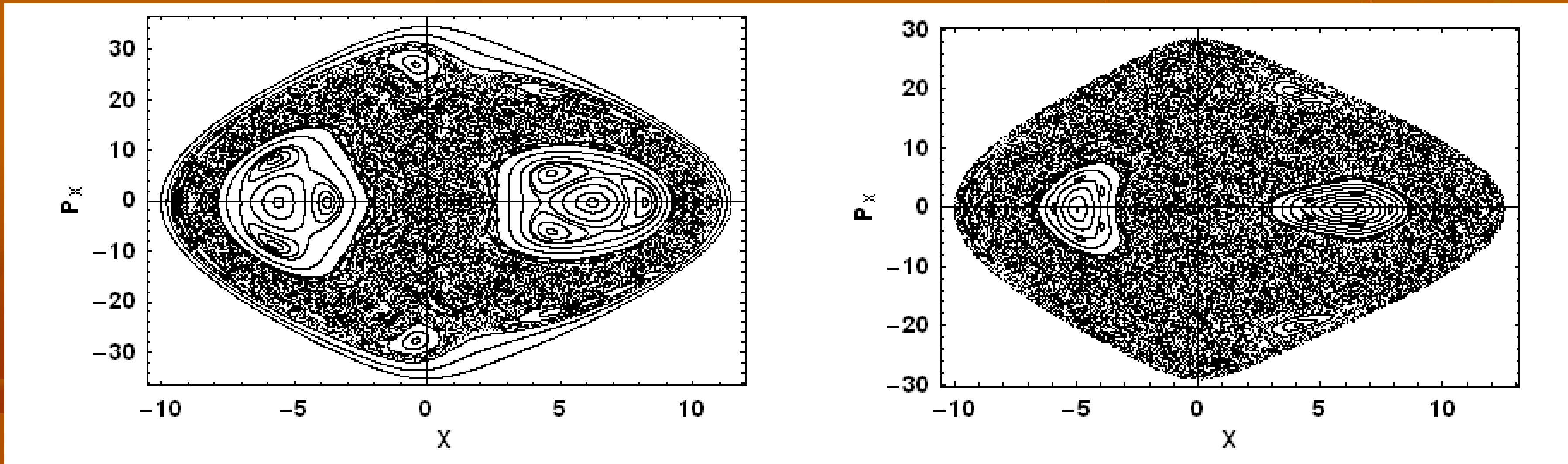


Figure 2a-b is similar to Fig. 1a-b when M_h is 10000, while c_h is treated now as a parameter. The values of all the other parameters are as in Fig. 1. In Fig. 2a we have $c_h=10.5$ and $h_2=-135$, while in Fig. 2b we have the case where $c_h=18$ and $h_2= 68$.

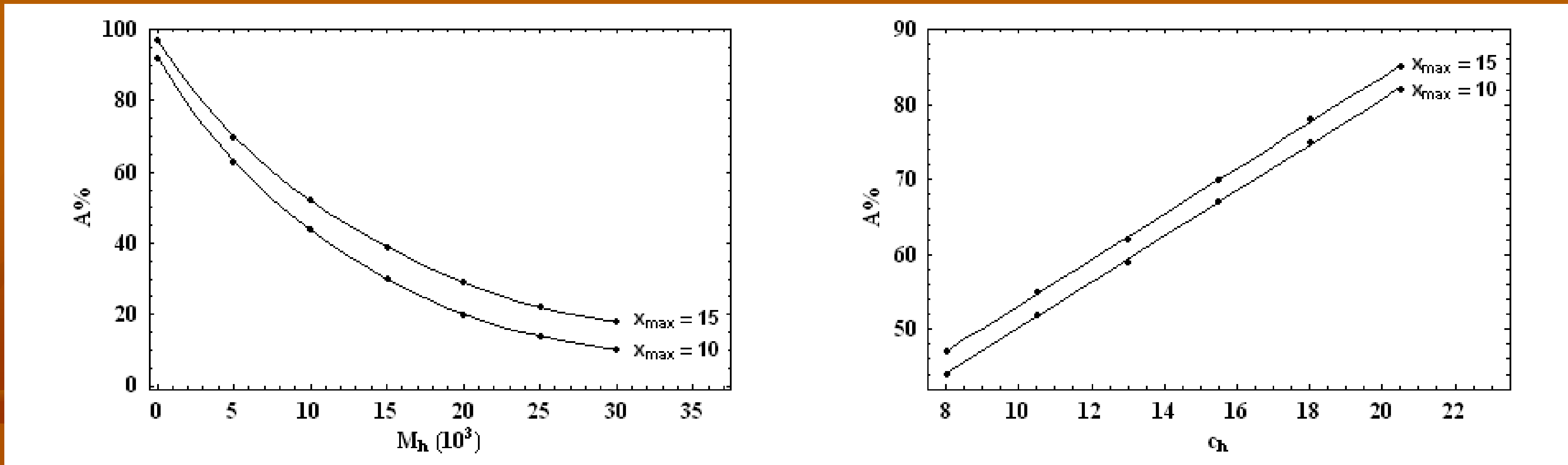
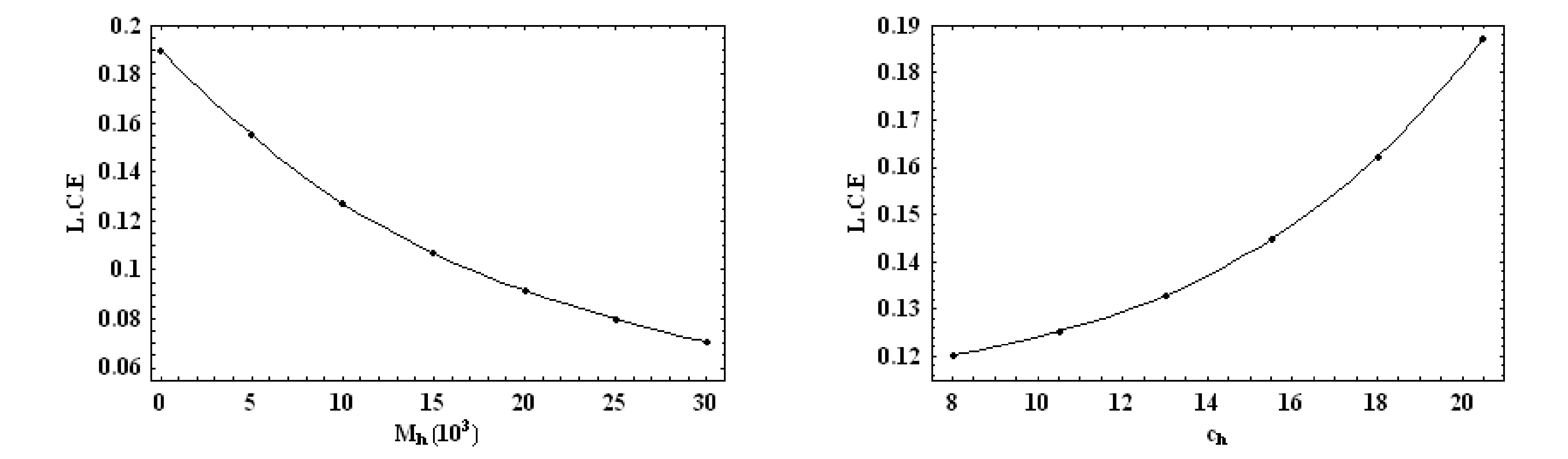


Figure 3a shows the percentage of the phase plane *A%* covered by chaotic orbits as a function of the mass of the halo. The values of all other parameters and energy are as in Fig. 1. Figure 3b shows a plot between *A%* and c_h when $M_h=10000$.



Figures 4a and 4b show a plot of the maximum Lyapunov Characteristic Exponent (L.C.E) vs M_h or vs c_h respectively. One can see, that the L.C.E decreases exponentially when M_h increases, while the L.C.E increases exponentially when c_h increases.

In what follows, we shall investigate the regular or chaotic character of orbits in the 2D Hamiltonian (5) using the new dynamical detector L_{tot}. In order to see the effectiveness of the new method, we shall compared the results with two other indicators, the classical method of the L.C.E and the method *P(f)* used by Karanis and Vozikis (2008). This spectral method, uses the Fast Fourier Transform (F.F.T) of a series of time intervals, each one representing the time that elapsed between two successive points on the Poincaré x - p_x phase plane for 2D systems, while for 3D systems they take two successive points on the plane $z=0$.

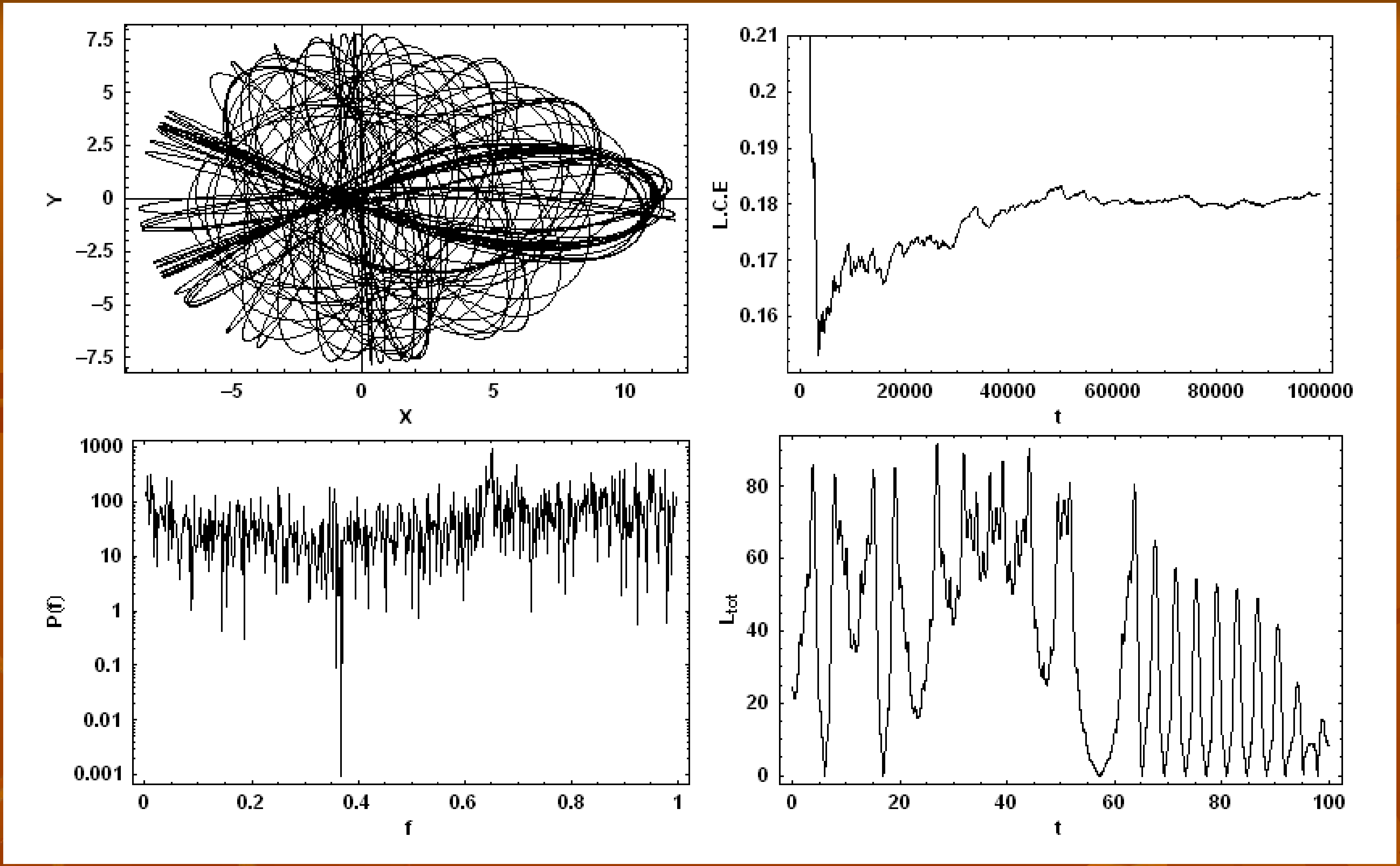


Figure 5 shows a 2D chaotic orbit with initial conditions: x_0=*-1.0*, y_0= p_{x0}=*0*, while the value of p_{y0} is found from the energy integral for all orbits. The values of all other parameters and energy are as in Fig. 1a.

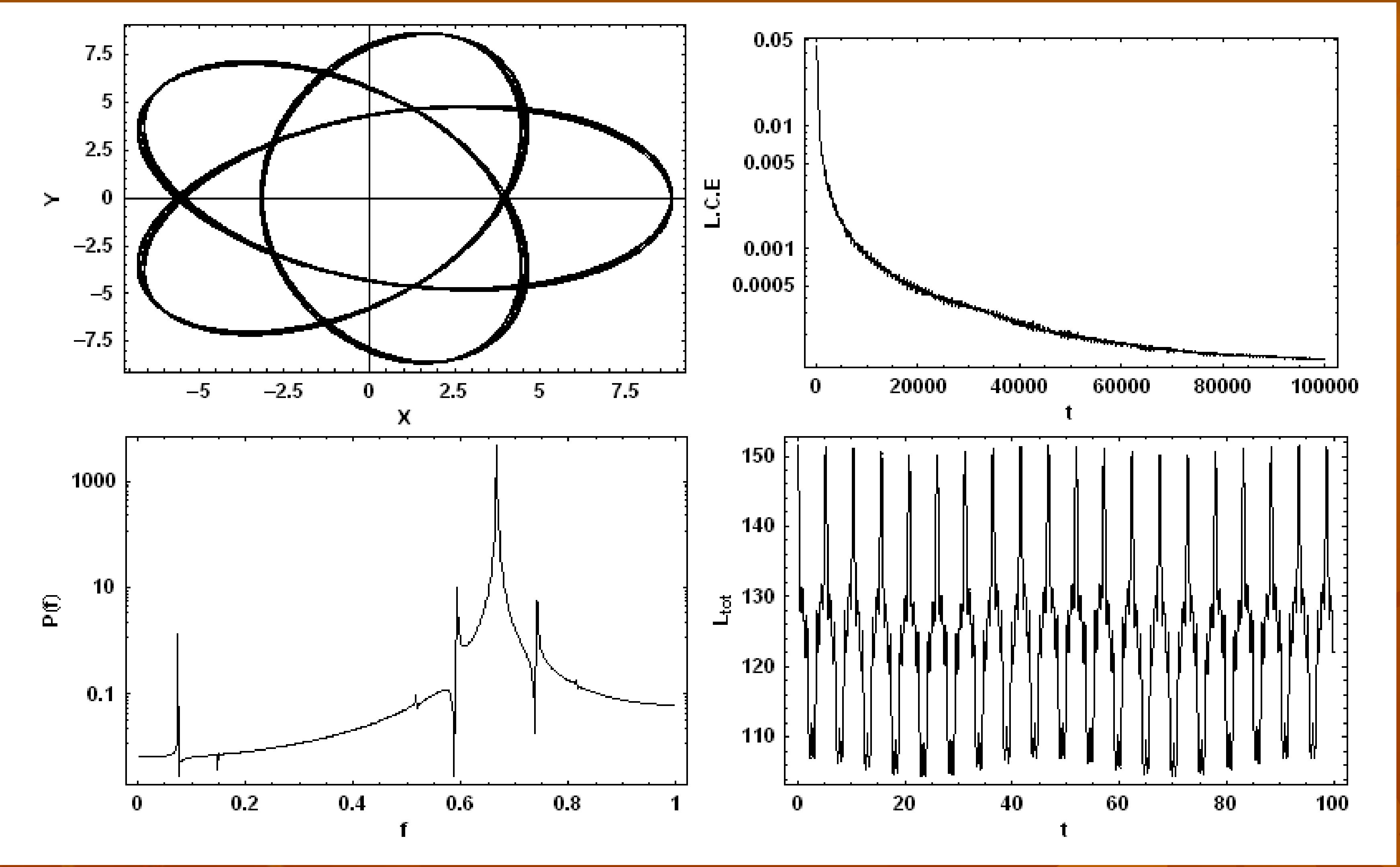


Figure 6 shows a regular 2D orbit with initial conditions: $x_0=8.8$, $y_0= p_{x0}=0$. The values of all other parameters and energy are as in Fig. 1b.

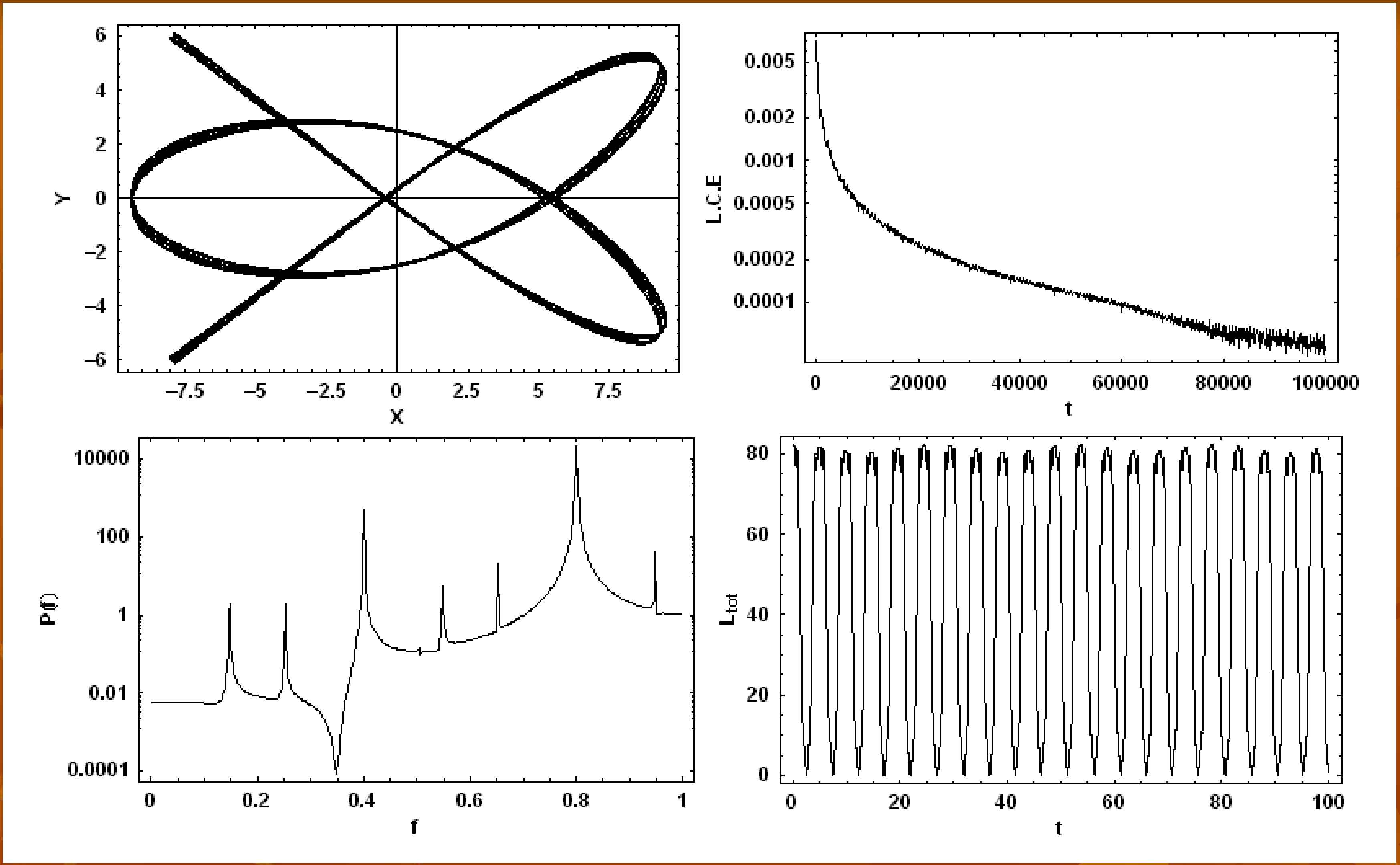


Figure 7 shows a resonant 2D orbit with initial conditions: x_0=-9.36, y_0= p_{x0}=0. The values of all other parameters and energy are as in Fig. 2a.

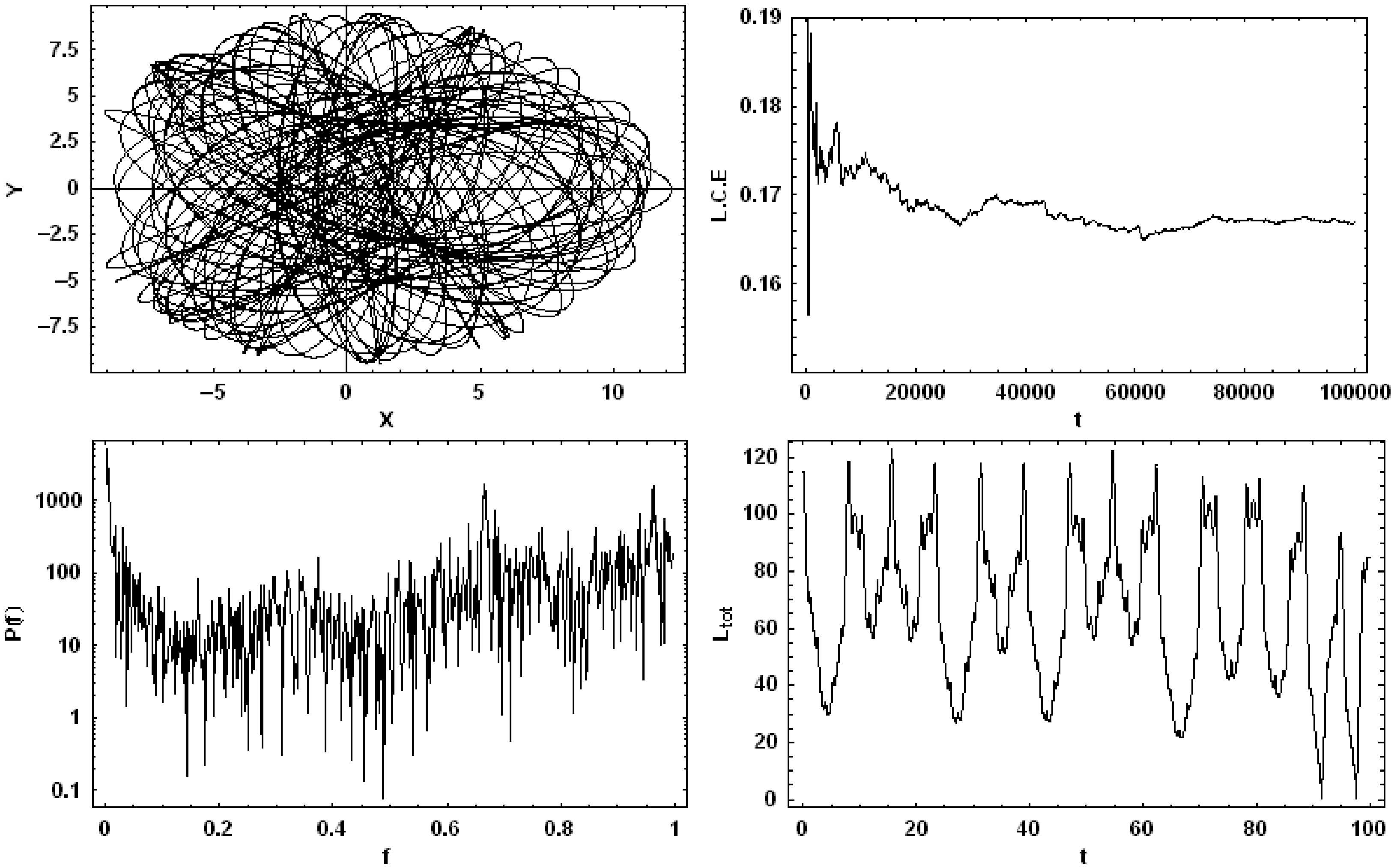


Figure 8 shows a chaotic 2D orbit with initial conditions: $x_0=10$, $y_0= p_{x0}=0$. The values of all other parameters and energy are as in Fig. 2d.

A large number of orbits in the 2D system were calculated for different values of the parameters and also for different initial conditions *(x_0, p_{x0})*. All numerically obtained results suggested, that the L_{tot} is a very fast and reliable dynamical parameter and can be safely used, in order to distinguish ordered from chaotic motion in 2D dynamical systems.

5. Results of the 3D system

The regular or chaotic nature of the 3D orbits is found as follows: we choose initial conditions (x_0, p_{x0}, z_0) such as (x_0, p_{x0}) is a point on the phase plane of the 2D system. The point (x_0, p_{x0}) lies inside the limiting curve

$$\frac{1}{2} p_x^2 + V(x) = h_2 \quad , \qquad (6)$$

which is the curve containing all the invariant curves of the 2D system. We choose $h_3 = h_2$ and the value of p_{y0}, for all orbits, is obtained from the energy integral (4). Our numerical experiments suggest, that orbits with initial conditions (x_0, p_{x0}, z_0), $y_0 = p_{z0} = 0$, such as (x_0, p_{x0}) is a point in the chaotic regions of Figs. 1a-b or 2a-b and for all permissible values of z_0, produce chaotic orbits.

Our next step is to study the character of orbits, with initial conditions (x_0, p_{x0}, z_0), $y_0=p_{z0}=0$ such as (x_0, p_{x0}) is a point in the regular regions of Figs. 1a-b or 2a-b. It was found that in all cases the regular or chaotic character of the above 3D orbits, depends strongly on the initial value of z_0. Orbits with small values of z_0 remain regular, while for large values of z_0 they change their character and become chaotic. The general conclusion which is based on the results derived from a large number of tested orbits, suggests that orbits with values of $z_0 \geq 0.75$ are chaotic, while orbits with values of $z_0 < 0.75$ are regular.

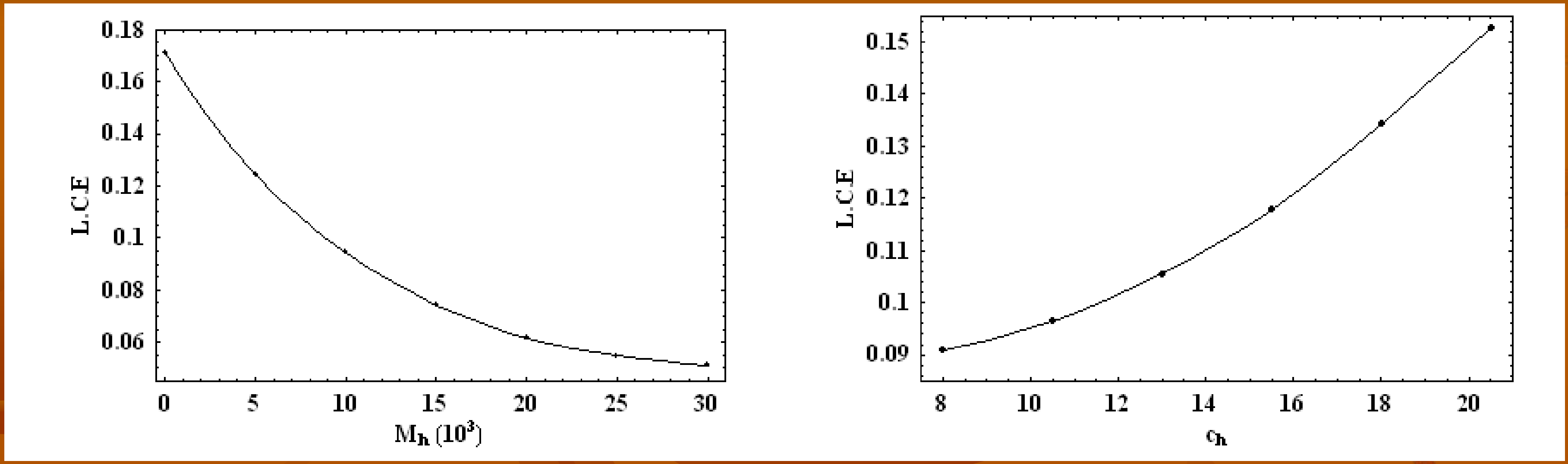
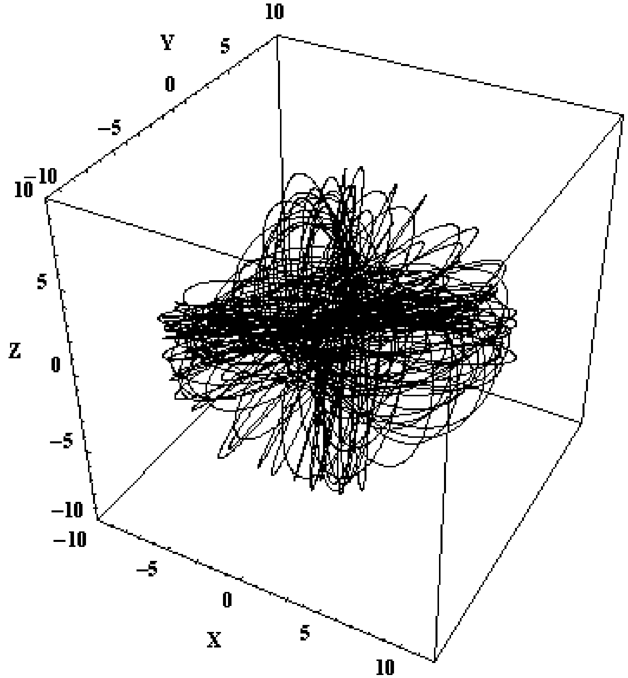
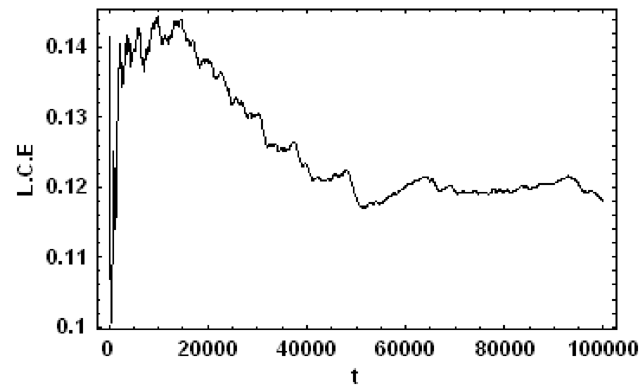
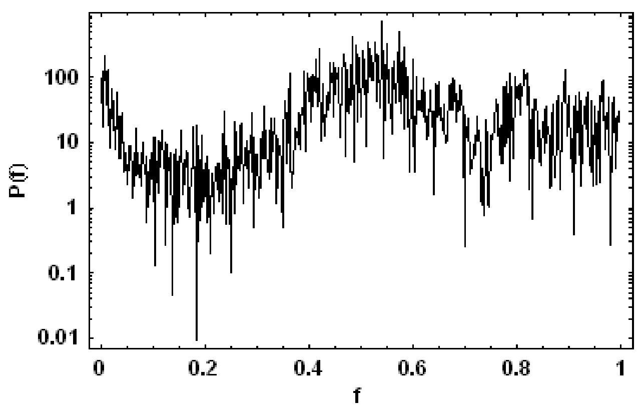


Figure 9a shows the L.C.E of the 3D system as a function of the mass of halo for a large number of chaotic orbits, when $c_h=8$. The values of all other parameters are as in Fig. 4a. Figure 9b shows he L.C.E as a function of c_h, when $M_h=10000$. The values of all the other parameters are as in Fig. 4b.


Y
X
Z
10
5
0
−5
−10


L.C.E
t
0.14
0.13
0.12
0.11
0.1
0
20000
40000
60000
80000
100000


P(f)
f
100
10
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1

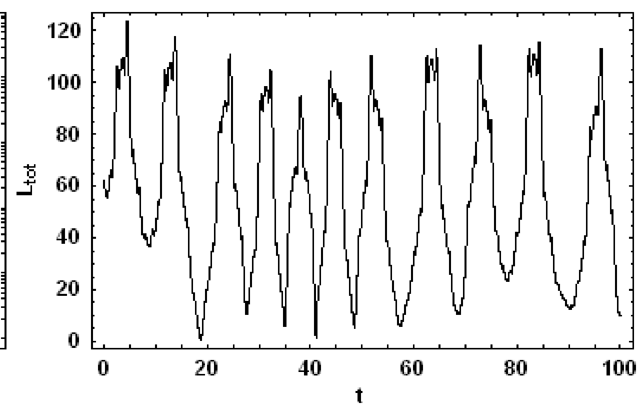
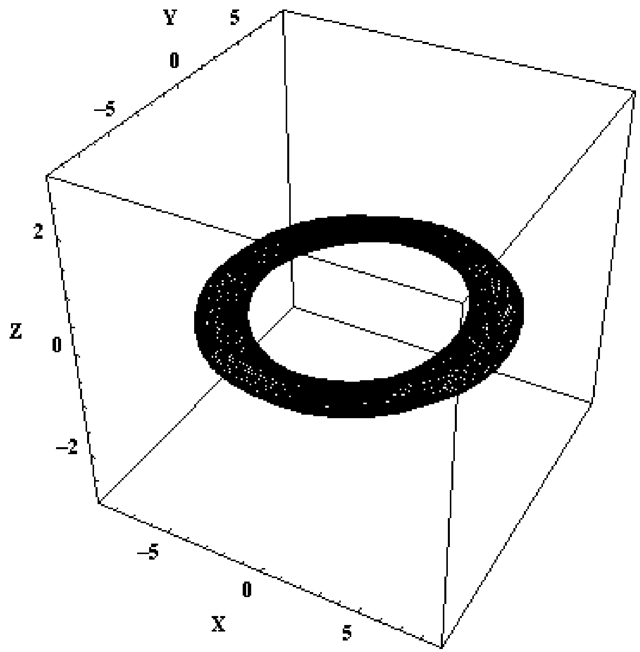
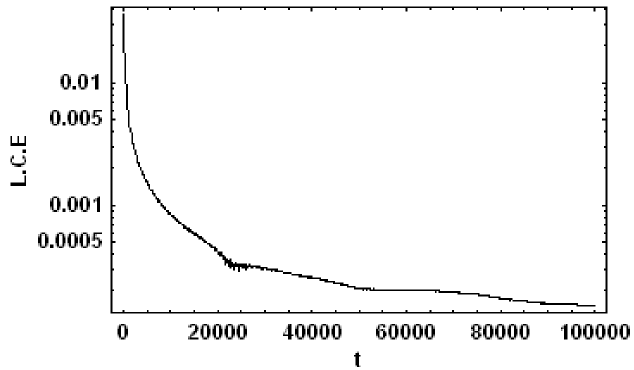

L_{tot}
t
120
100
80
60
40
20
0
0
20
40
60
80
100

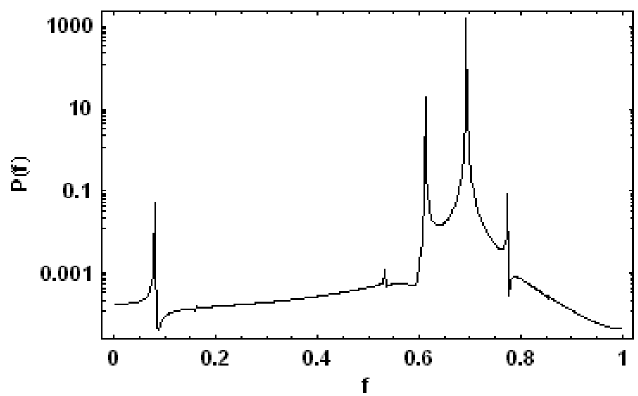
Fig. 10a-d is similar to Fig. 8a-d but for a 3D orbit. The orbit shown in Fig. 10a looks chaotic. The initial conditions are: $x_0=2.0$, $p_{x0}=0$, $z_0=0.5$. Remember that all 3D orbits have $y_0=p_{z0}=0$, while the value of p_{y0} is found from the energy integral. The values of all other parameters and energy h_3 are as in Fig. 2b. The L.C.E shown in Fig. 10b, also assures the chaotic character of the orbit. The $P(f)$ spectrum given in Fig. 10c also suggests chaotic motion. The same conclusion comes from the L_{tot} which is shown in Fig. 10d.



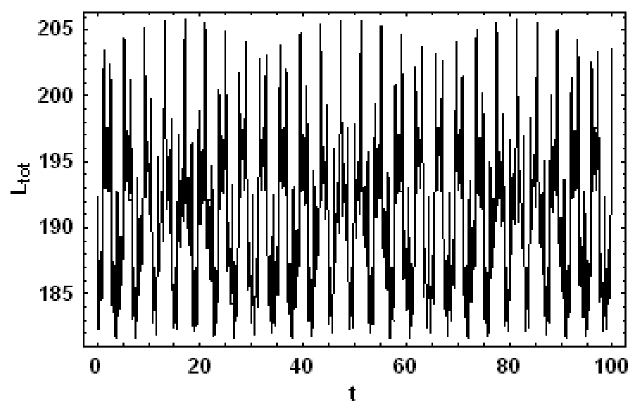
Y
X
Z



L.C.E
t



P(f)
f



L_{tot}
t

Figure 11a-d is similar to Fig. 10a-d but for a quasi periodic 3D orbit. The initial conditions are: $x_0=5.0$, $p_{x0}=0$, $z_0=0.1$. The values of all the other parameters and the energy h_3 are as in Fig. 1. Here we can observe, that all three indicators support the regular character of the 3D orbit.

The main conclusion from the study of the 3D model, is that the L_{tot} detector, can provide reliable and very fast results for the character of the orbits. There is no doubt, that the L_{tot} is much faster than the two other indicators used in this research. Therefore, we can say that this indicator, is a very useful tool for a quick inspection of the character of orbits in galactic dynamical systems.

6. Discussion and conclusions

The main conclusions of this research are the following:

1. The percentage of the chaotic orbits decreases as mass of the spherical halo increases. Therefore, the mass of the halo can be considered as an important physical quantity, acting as a controller of chaos in galaxies showing small asymmetries.

2. We expect to observe a smaller fraction of chaotic orbits in asymmetric triaxial galaxies with a dense spherical halo, while the fraction of chaotic orbits would increase in asymmetric triaxial galaxies surrounded by less dense spherical halo components.

3. It was found that the L.C.E in both the 2D and the 3D models decreases as the mass of halo increases, while the L.C.E increases as the scale length c_h of the halo decreases. This means that not only the percentage of chaotic orbits, but also the degree of chaos is affected by the mass or the scale length of the spherically symmetric halo component.

4. The L_{tot} gives fast and reliable results regarding the nature of motion, both in 2D and 3D galactic potentials. For all calculated orbits the results given by the L_{tot}, coincide with the outcomes obtained using the L.C.E or the *P(f)* spectral method. The advantage of the L_{tot}. is that it is much faster than the above two methods.

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THANK YOU !