Relativistic Magnetized Jets without Current Sheets

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Outline

- Motivation
- Mathematical Setup
- Static Solution
- Relativistic Solutions
- Simulation
- Stability
- Observational Signature

Motivation

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- We describe the structure of jets containing both plasma and magnetic field

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- We focus at the region further from the jet engine, where the structure is mainly affected by the boundaries and not very sensitive to the source (i.e. Spruit 2010)
- The external timescales are slow enough, compared to the internal ones, so that the system can reach an equilibrium state
- If the reconnection takes place fast enough we expect that any current sheets will be dissipated

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  - Endpoints of dissipative evolution
  - Simulation trial solutions - initial conditions

Mathematical formulation

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- We assume cylindrical symmetry
- Some hot plasma confines the cylinder
- What is the equilibrium solution for this structure?

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The system consists of two regions:
(I) The magnetized one, which contains magnetic field and hot gas
(II) The external one which contains only hot gas and confines the jet

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The system is everywhere in force equilibrium: the sum of the Lorentz and the pressure gradient is zero (Grad-Shranov Equation)

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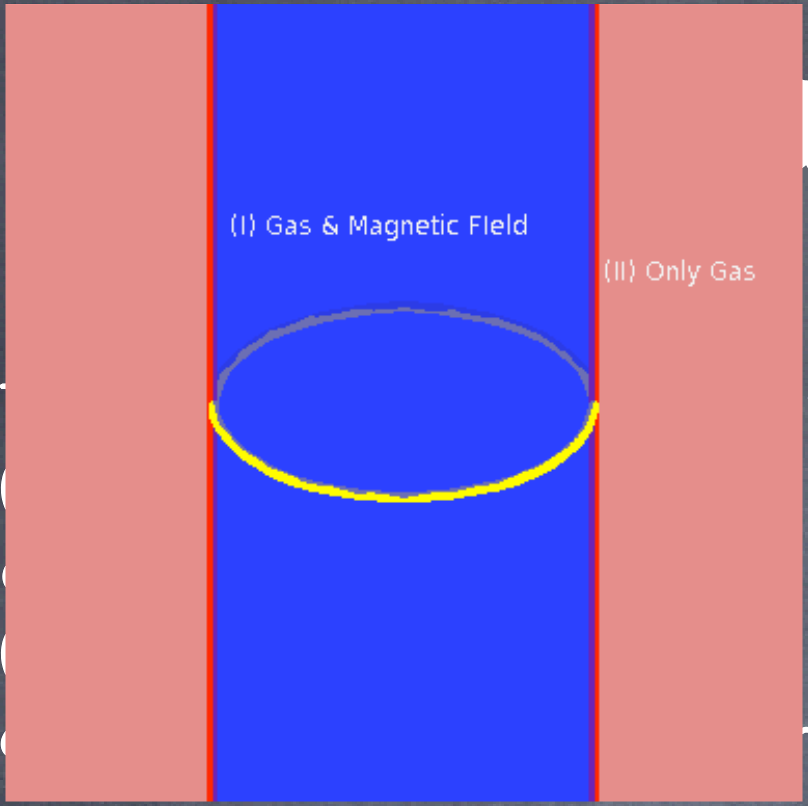

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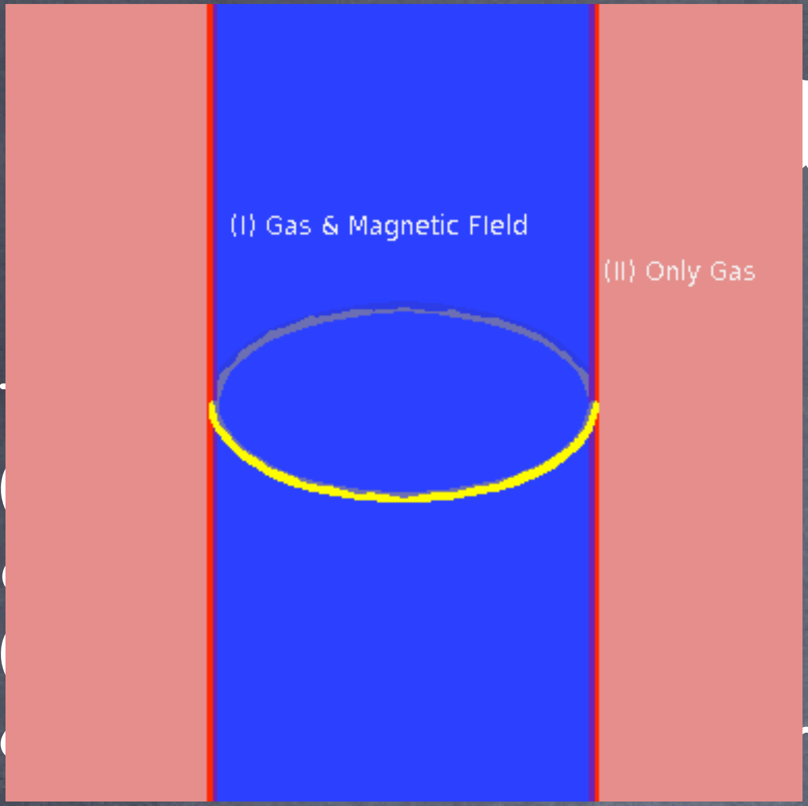

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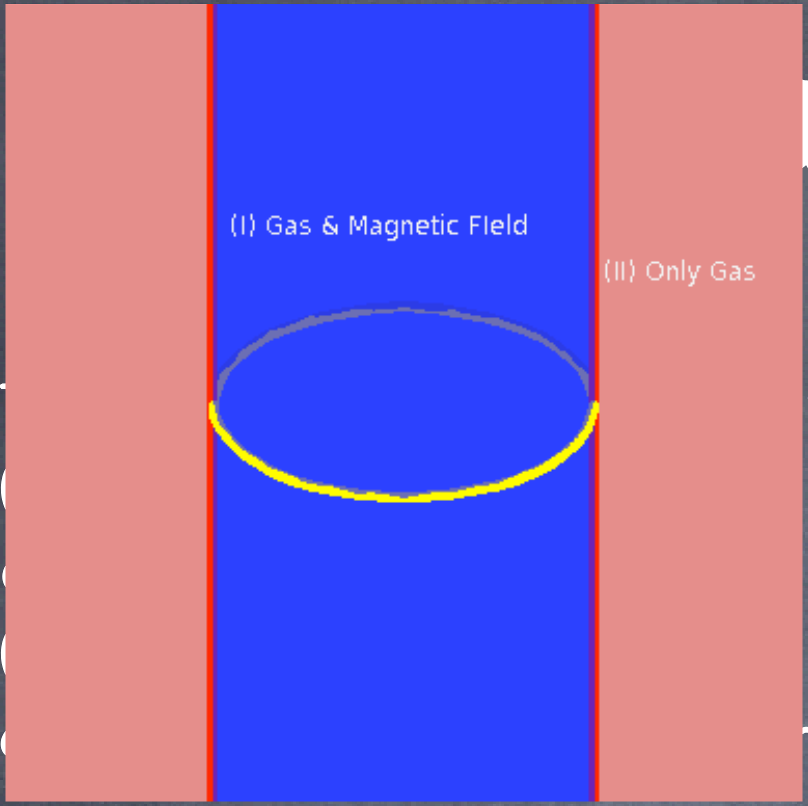

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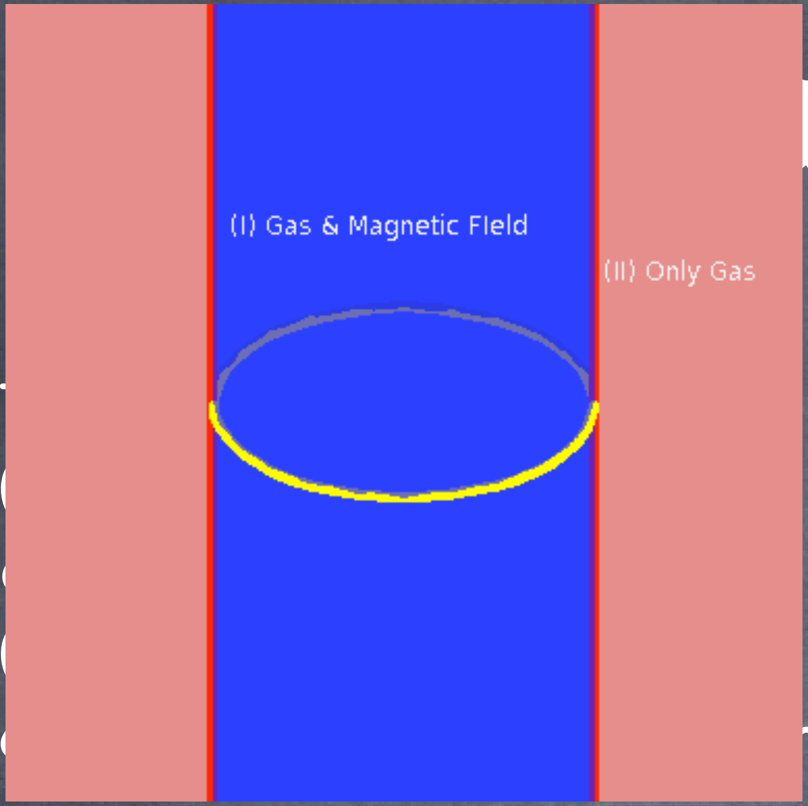

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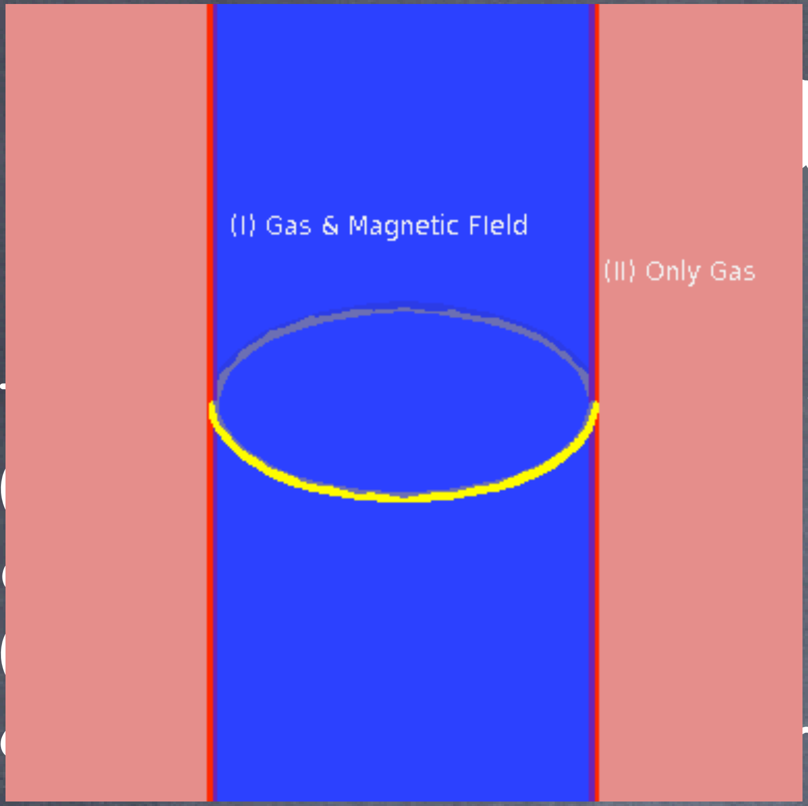

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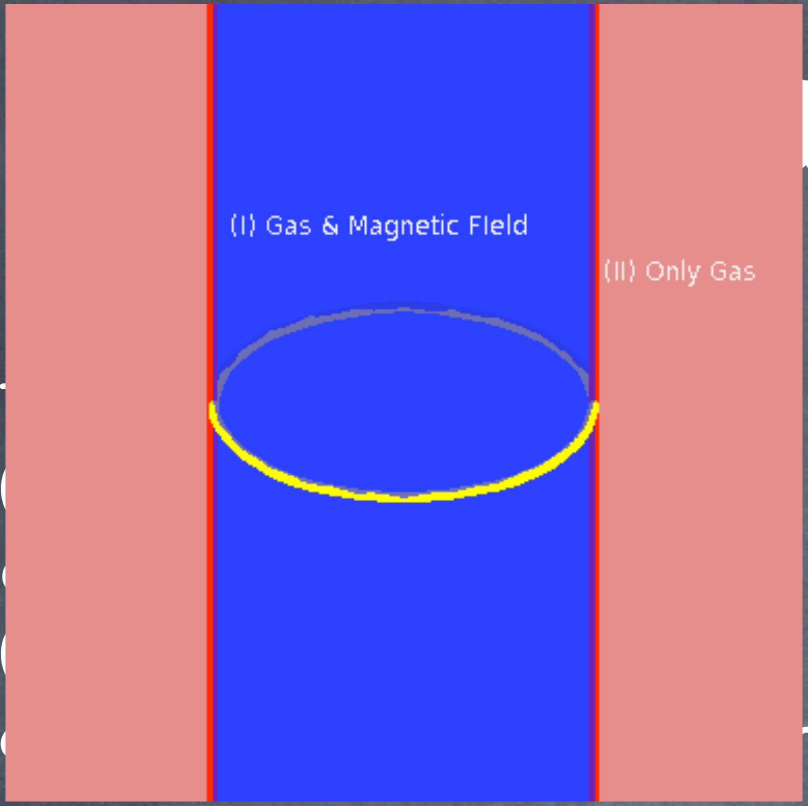

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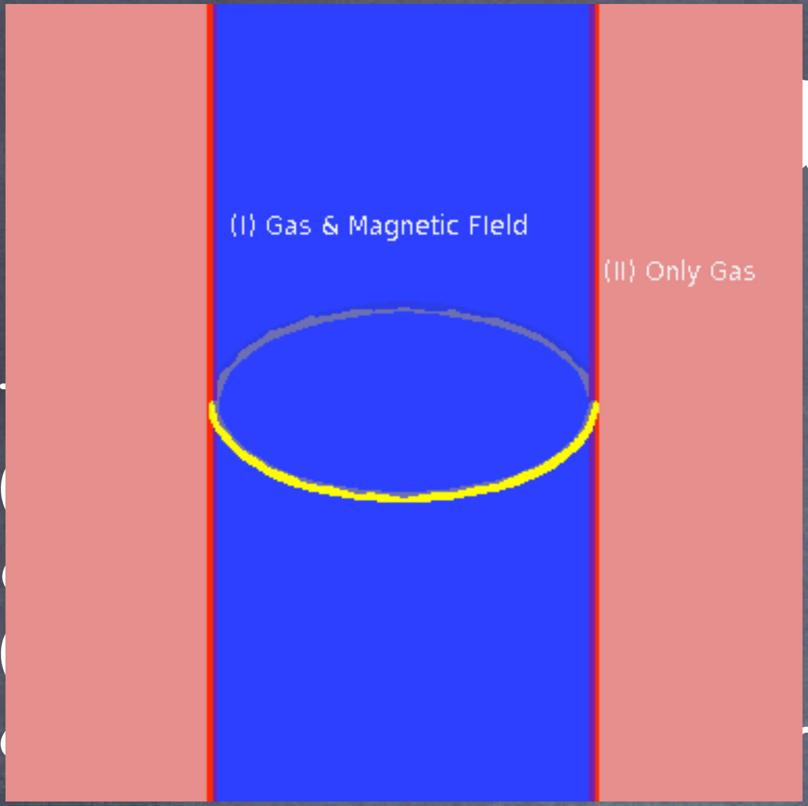

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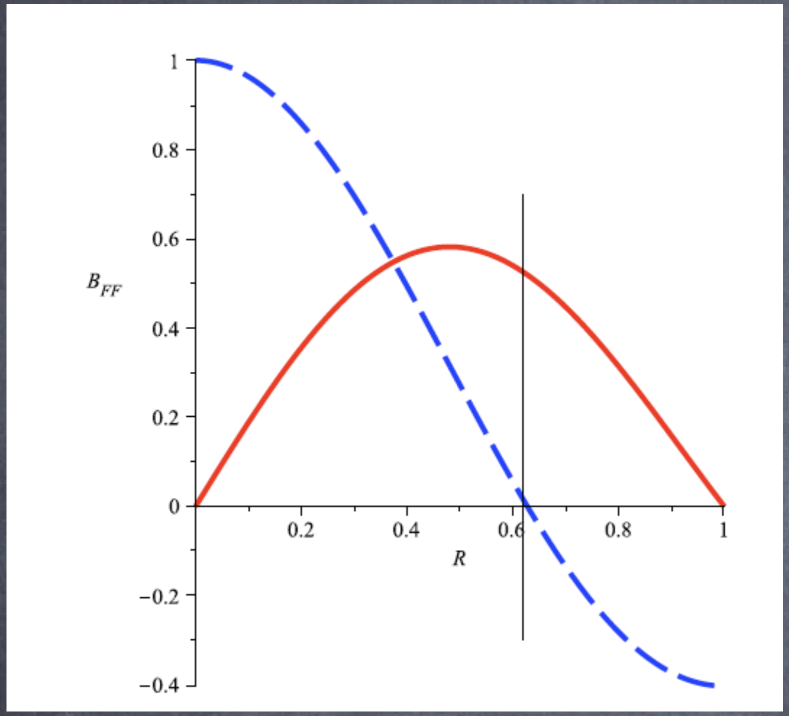

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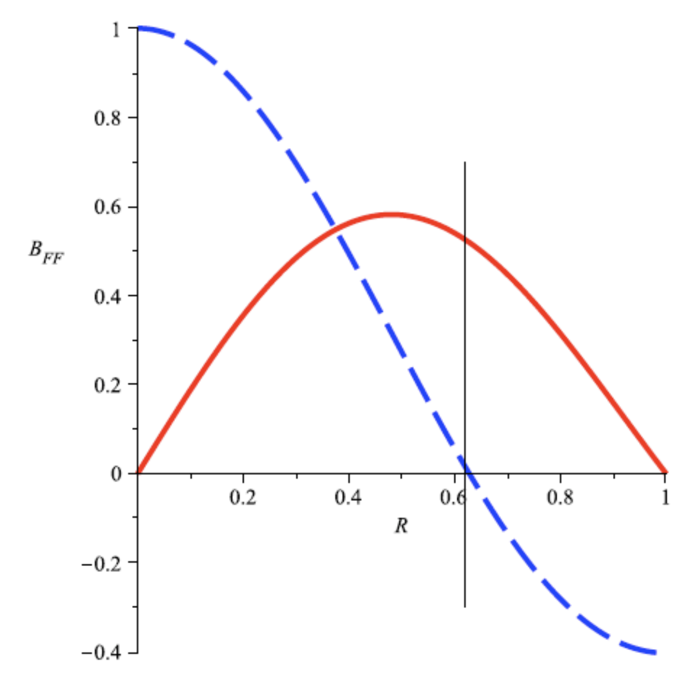

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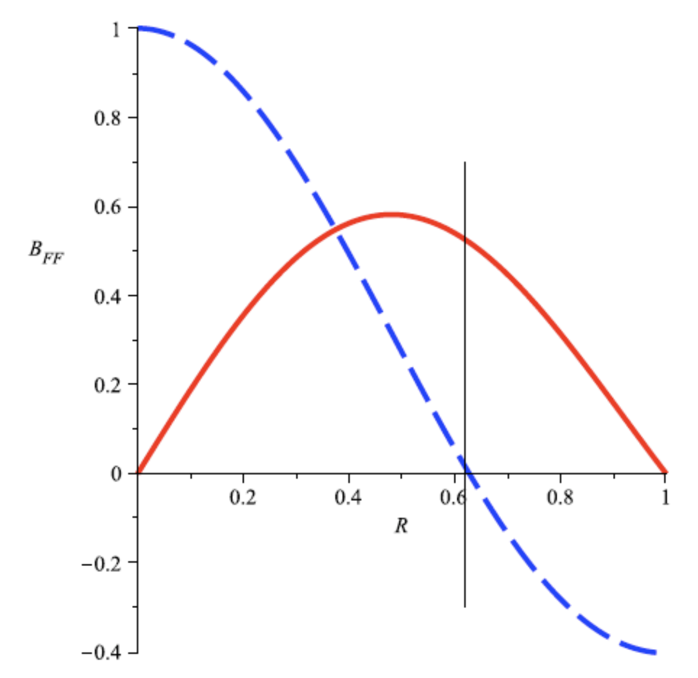

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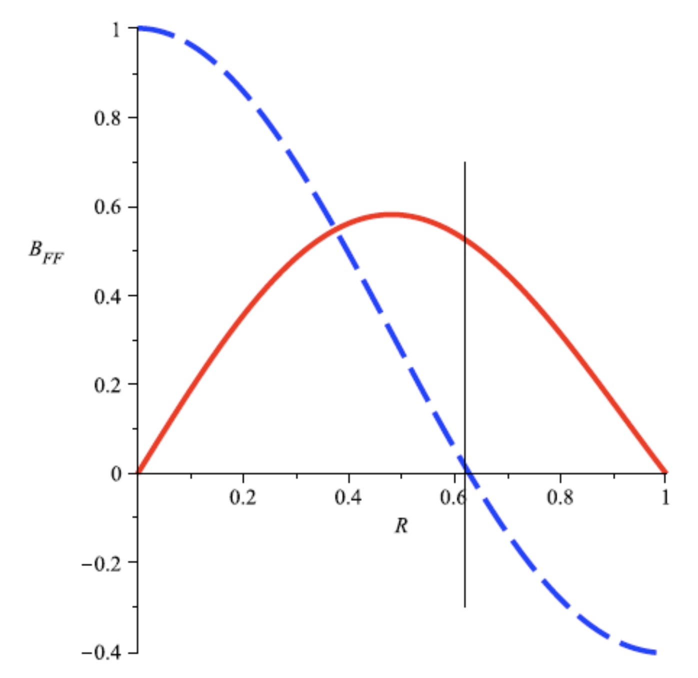

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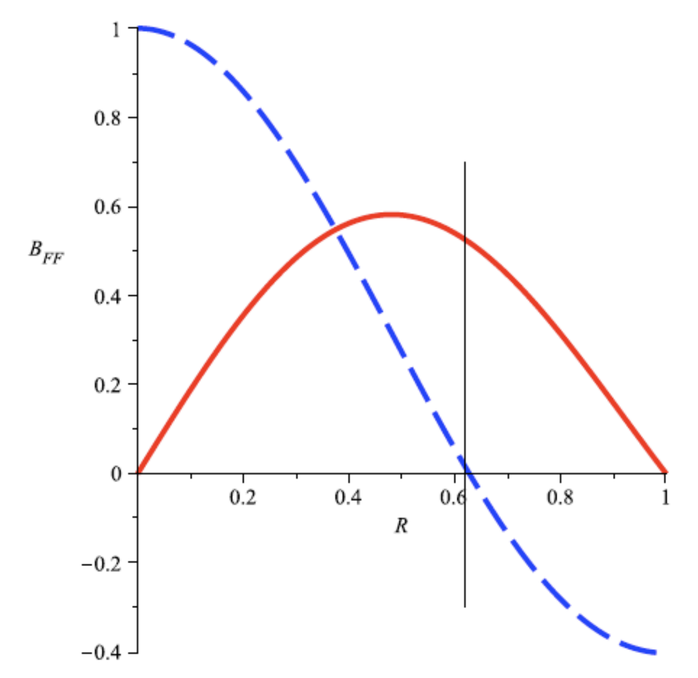

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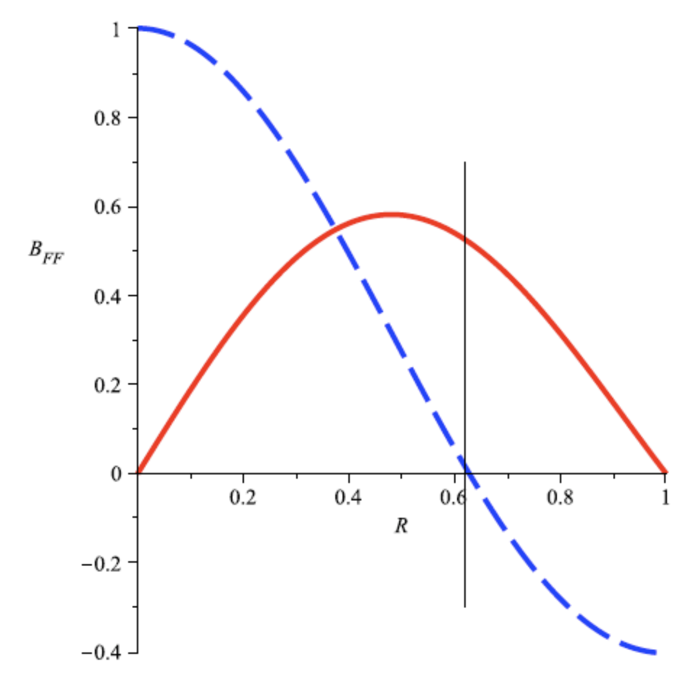

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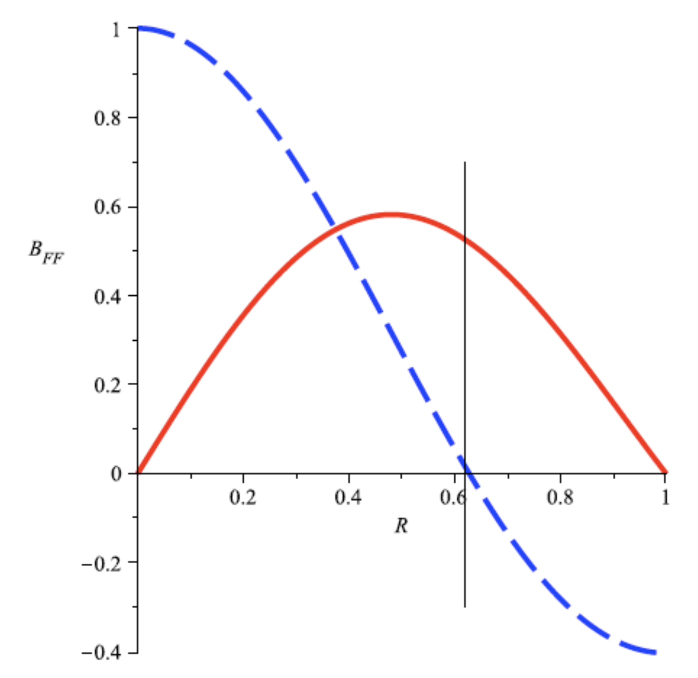

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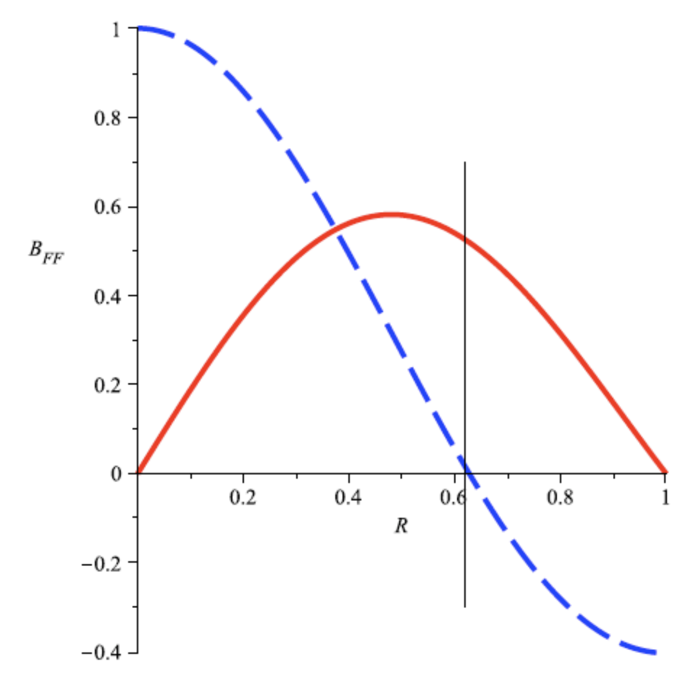

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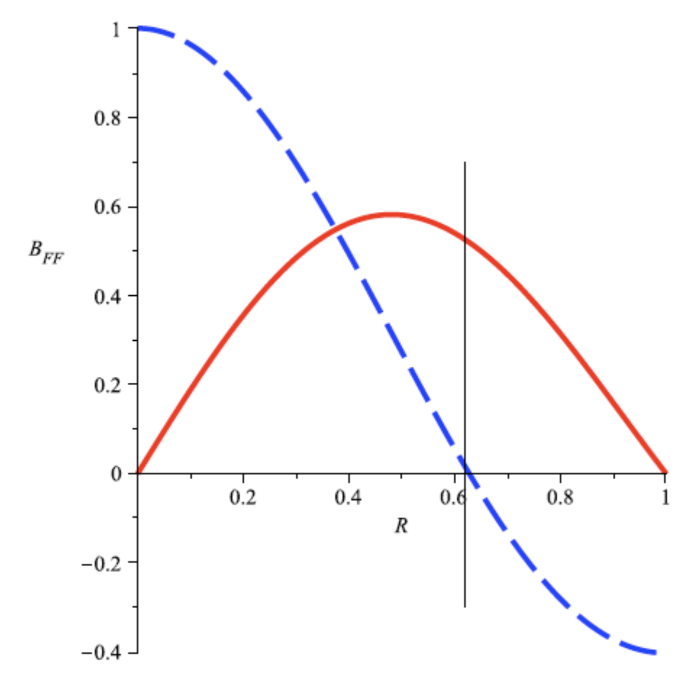

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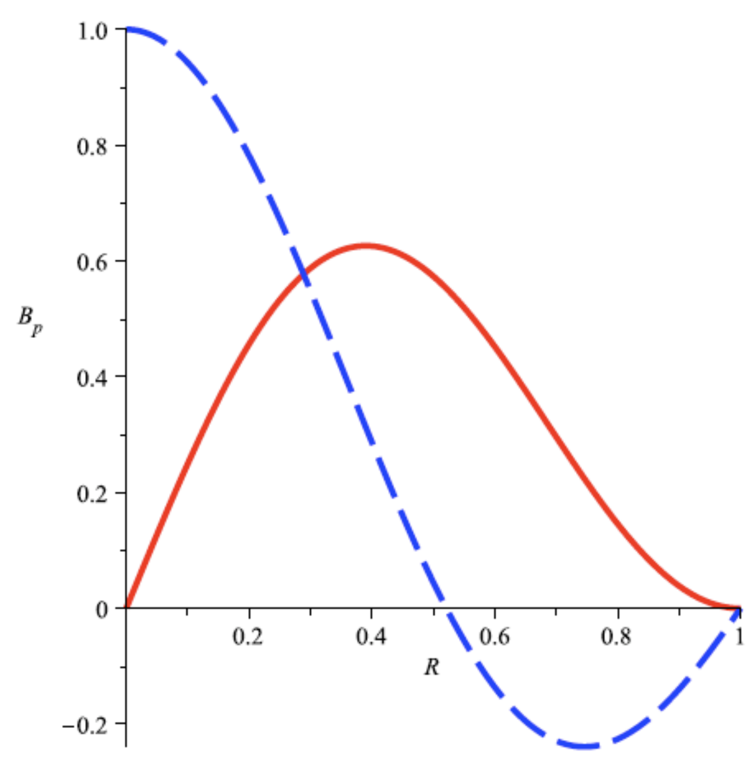

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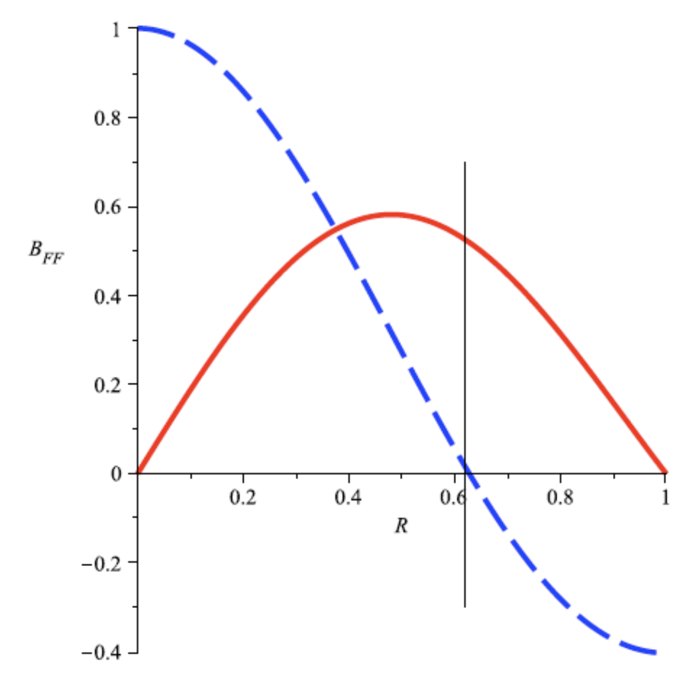

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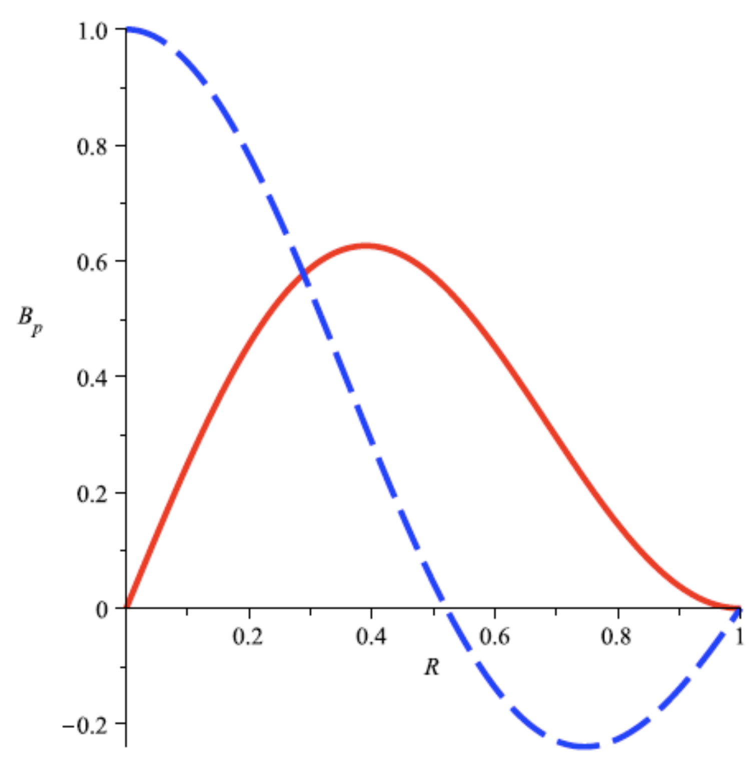

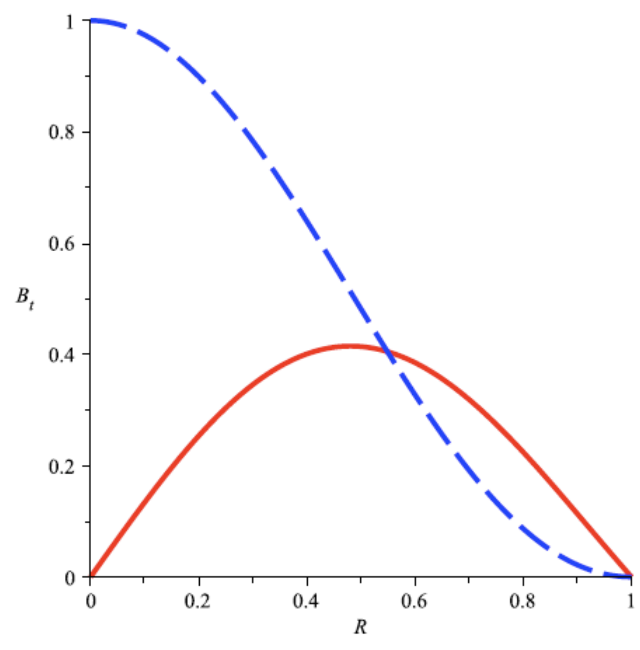

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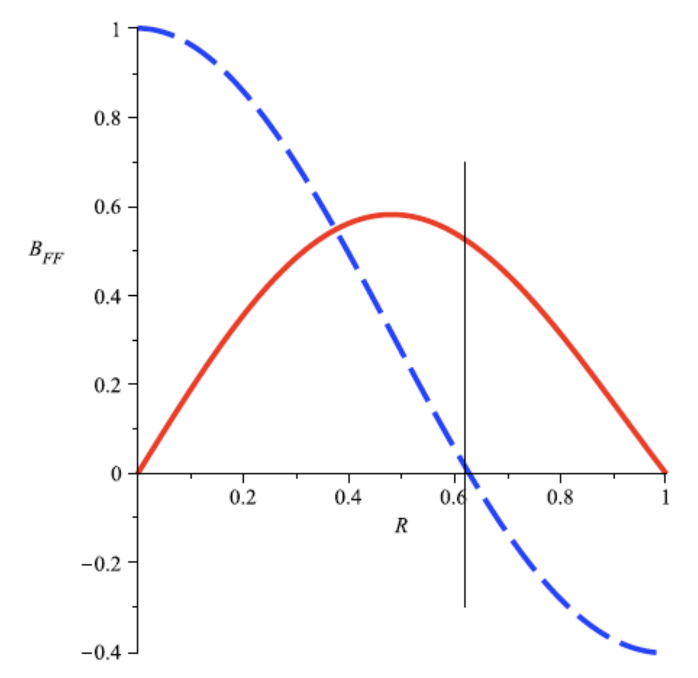

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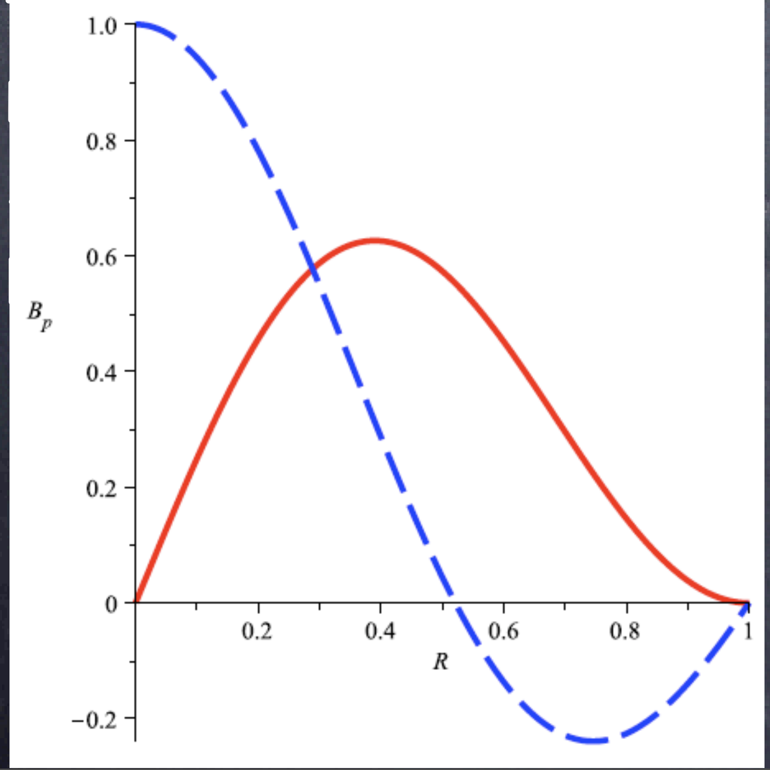

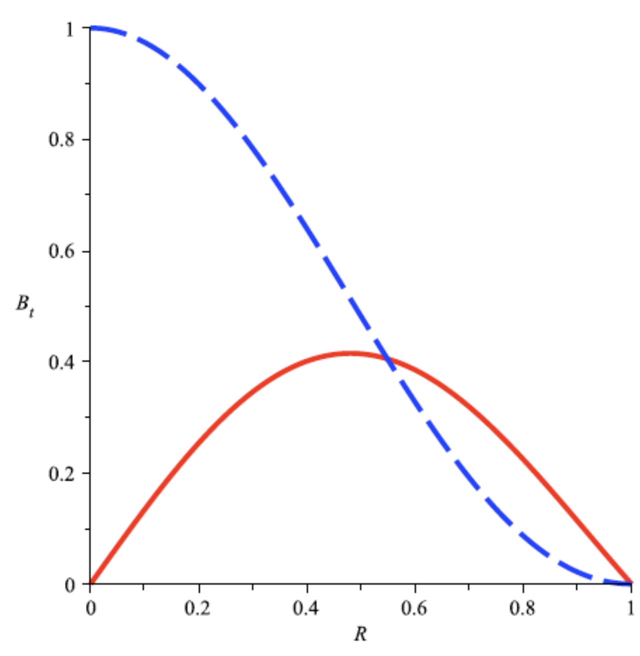

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To ensure the absence of current sheets we need a minimum pressure

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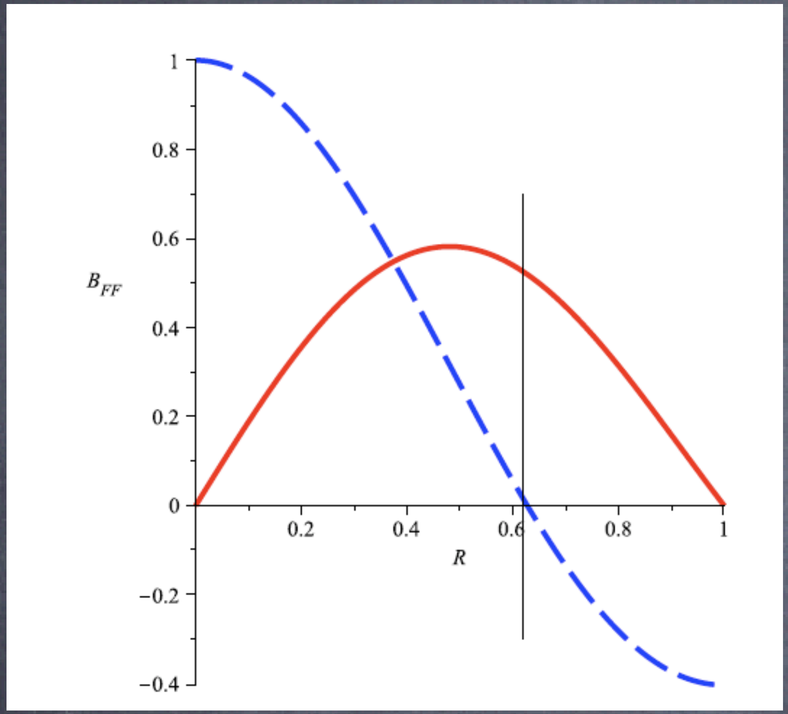

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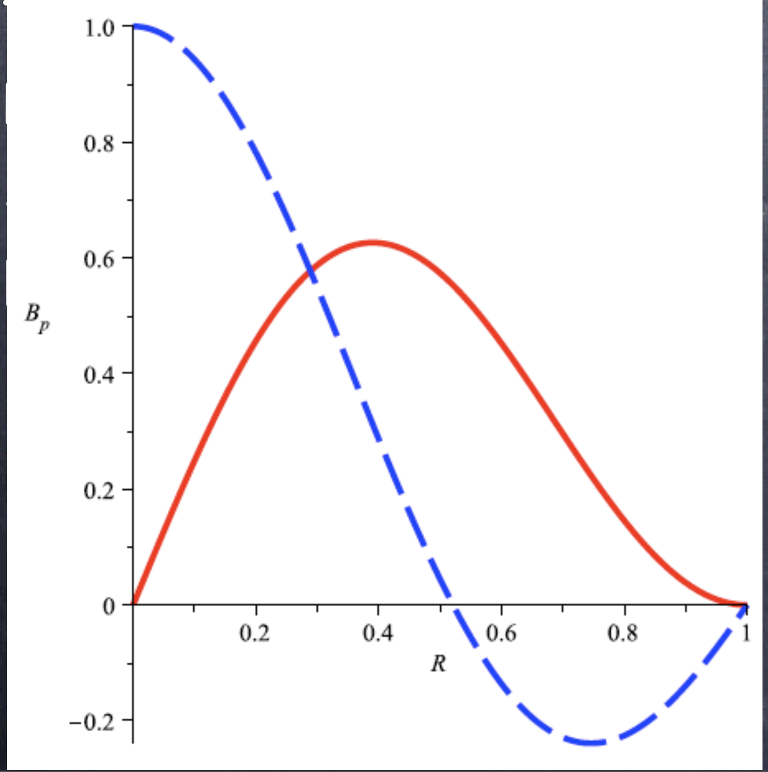
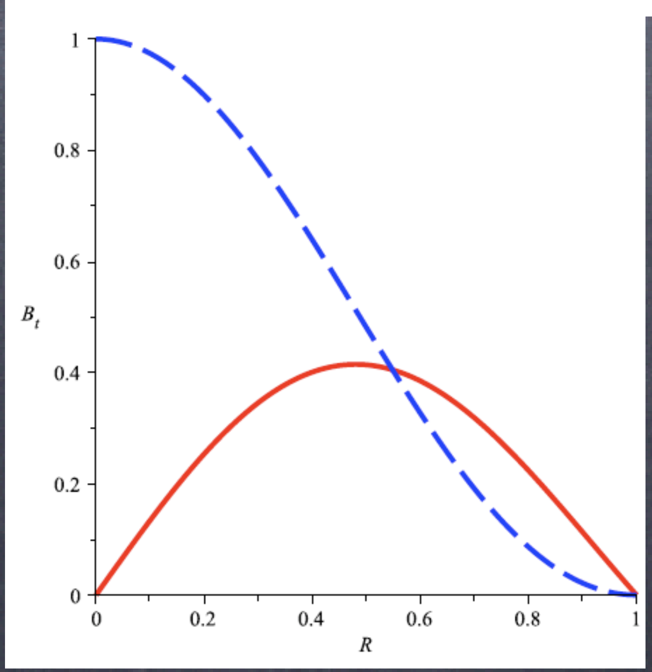
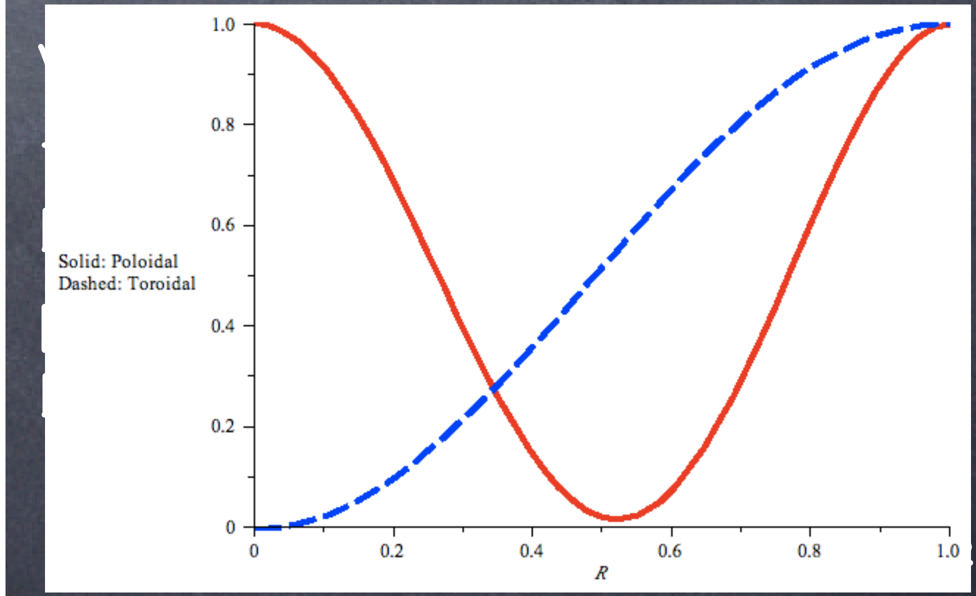






nt

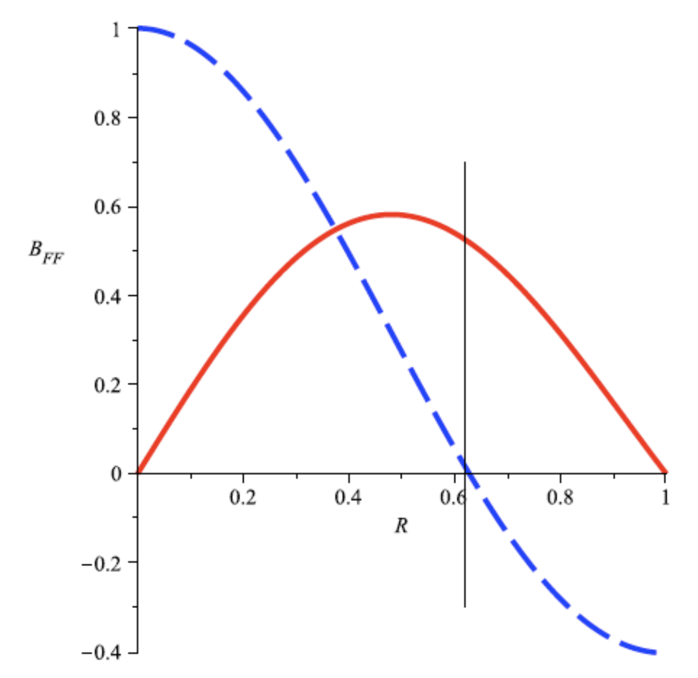
sheets we need a minimum pressure

For the Force-Free case, for a linear relation between *P* and *I*, there is the Lundquist (1951) solution

$$\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$$

$$I_{p,t} = \alpha_{p,t} P_{p,t}$$

$$\boldsymbol{B}_{FF} = c_0 \alpha (J_1(\alpha R)\hat{\boldsymbol{\phi}} + J_0(\alpha R)\hat{\boldsymbol{z}})$$



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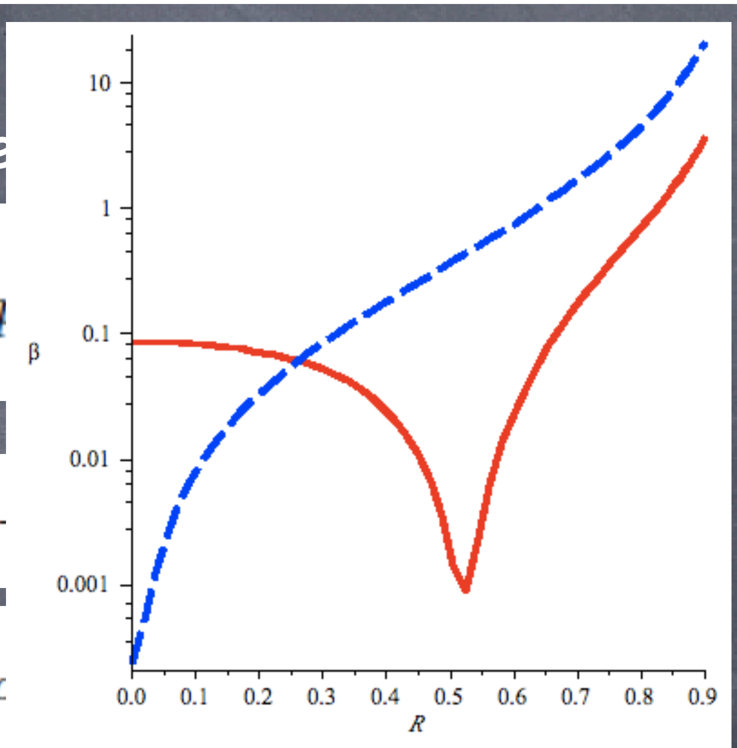

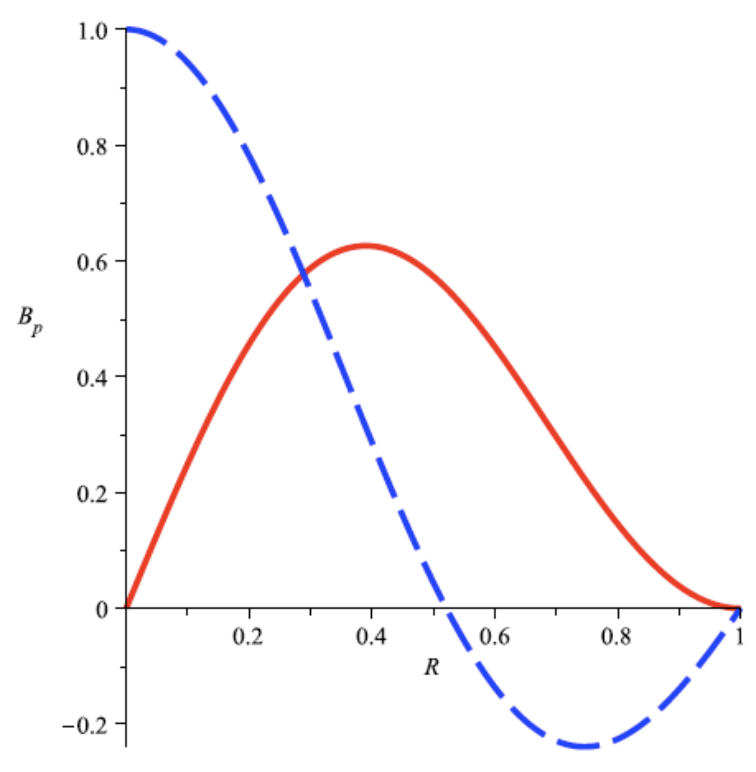

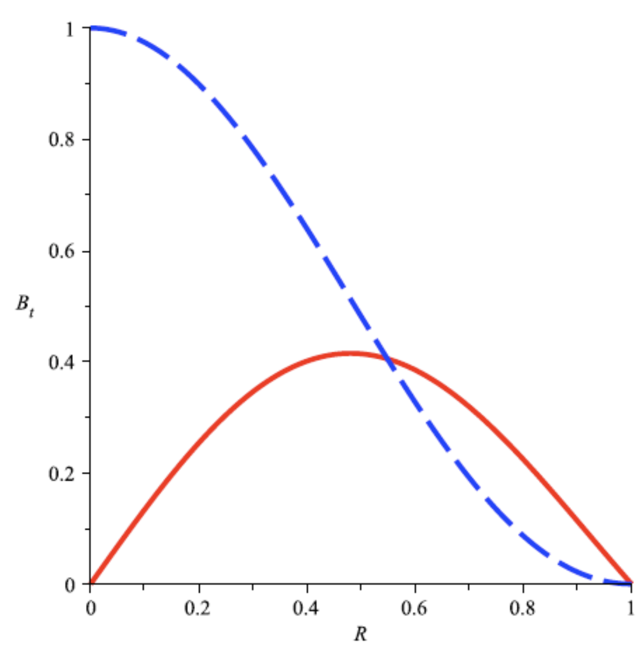

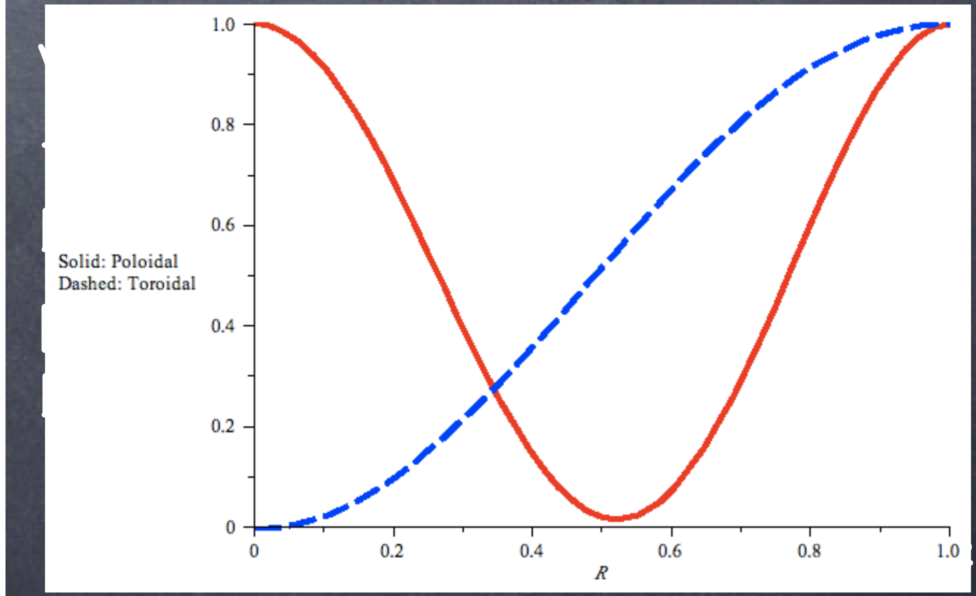

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Relativistic Case

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Motion along the z axis
induces electric field

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Using the same means as in the static case we can solve the relativistic

Solution

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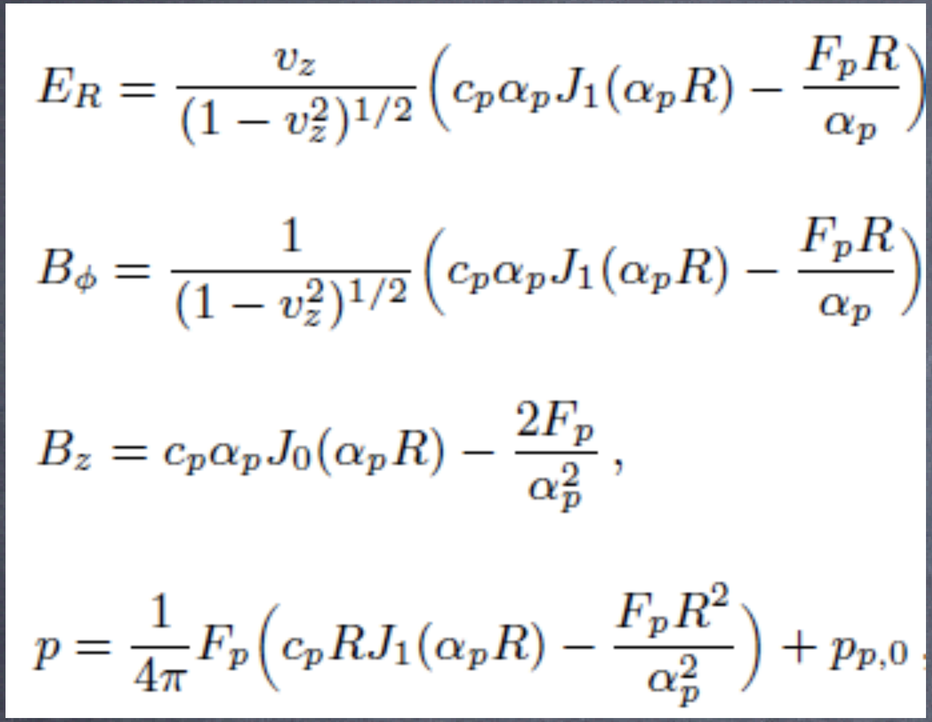

$$E_R = \frac{v_z}{(1 - v_z^2)^{1/2}} \left(c_p \alpha_p J_1(\alpha_p R) - \frac{F_p R}{\alpha_p} \right)$$

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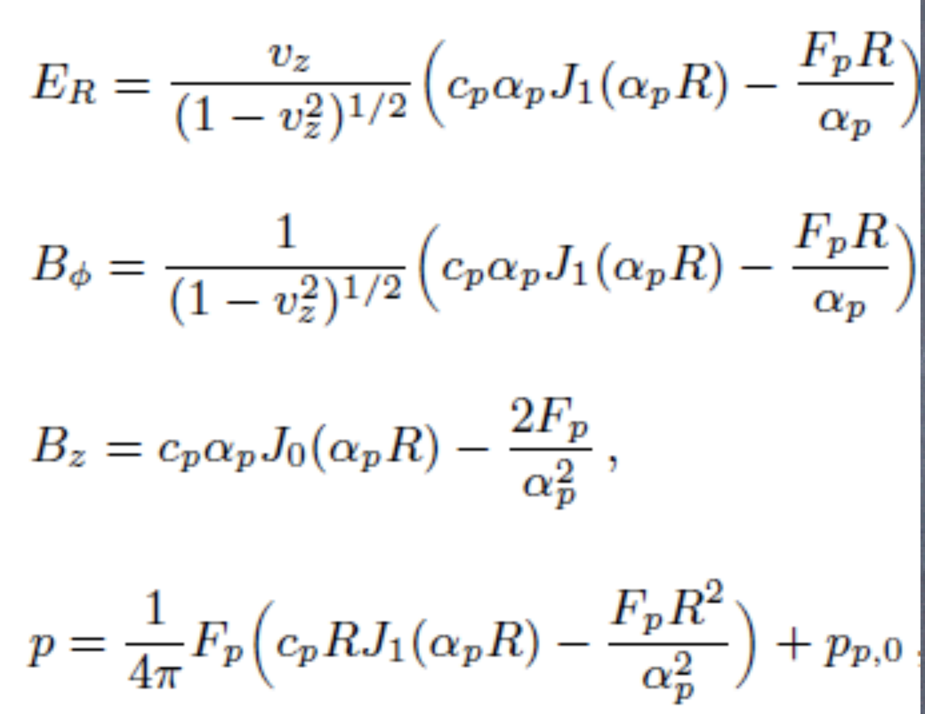

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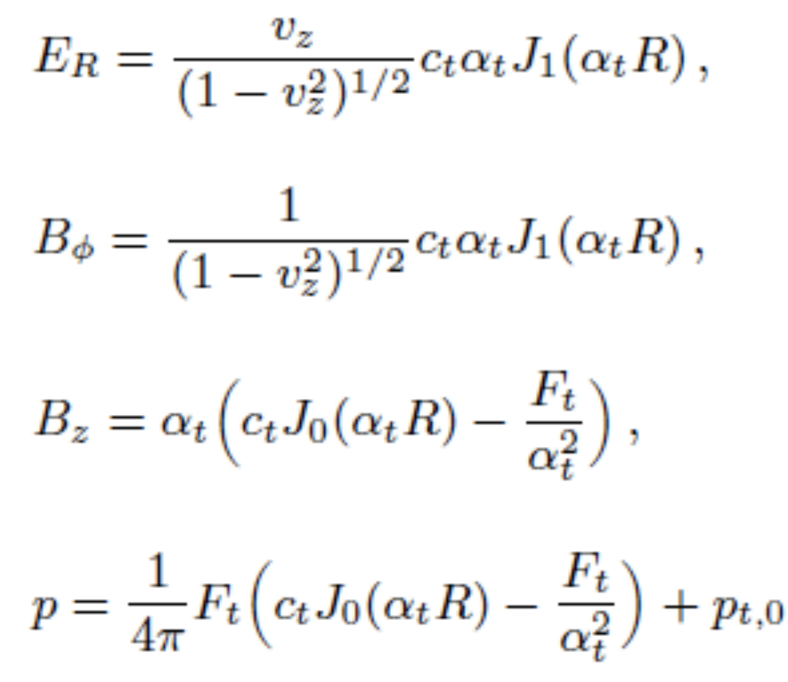

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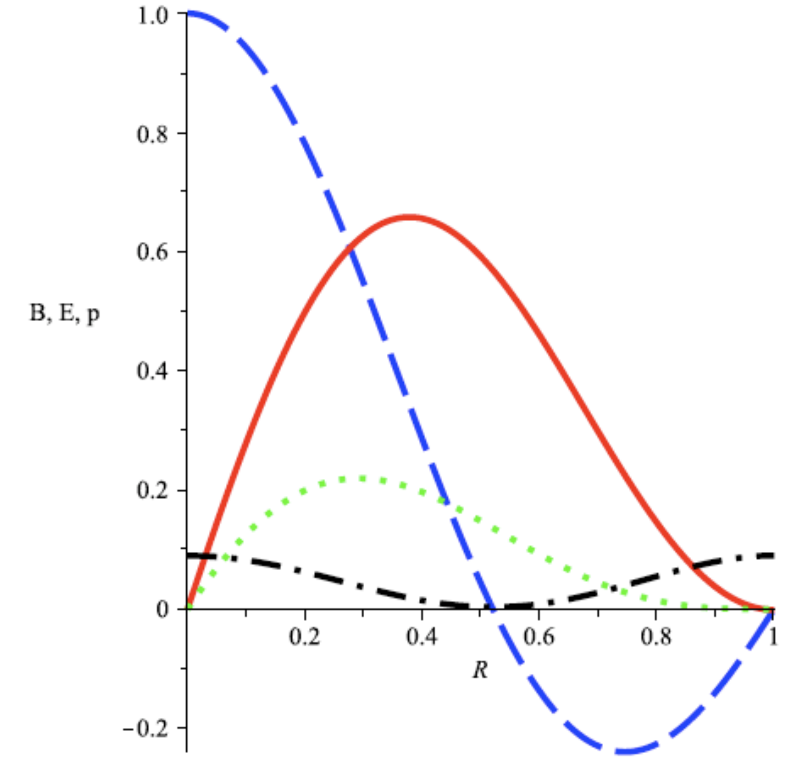

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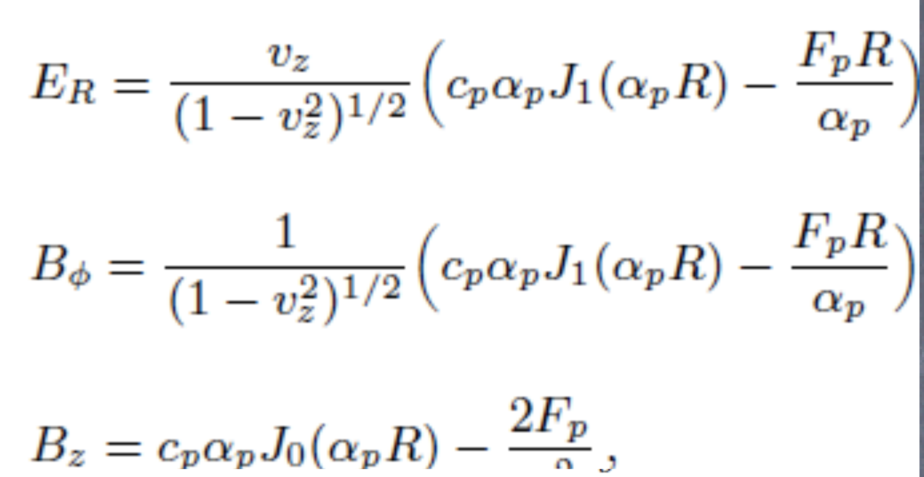

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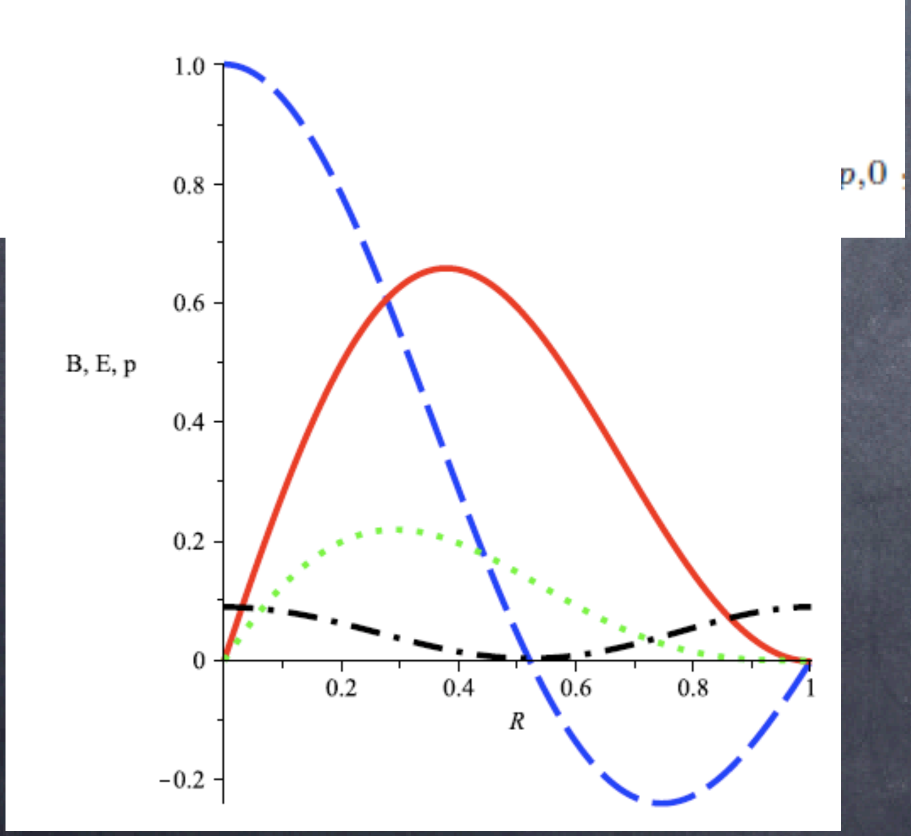

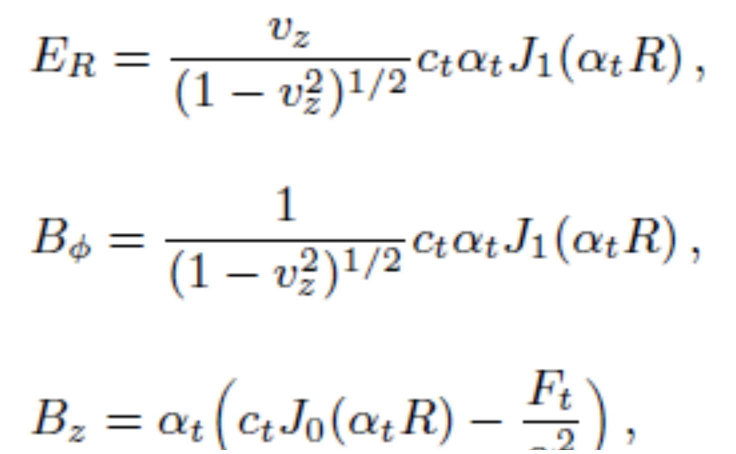

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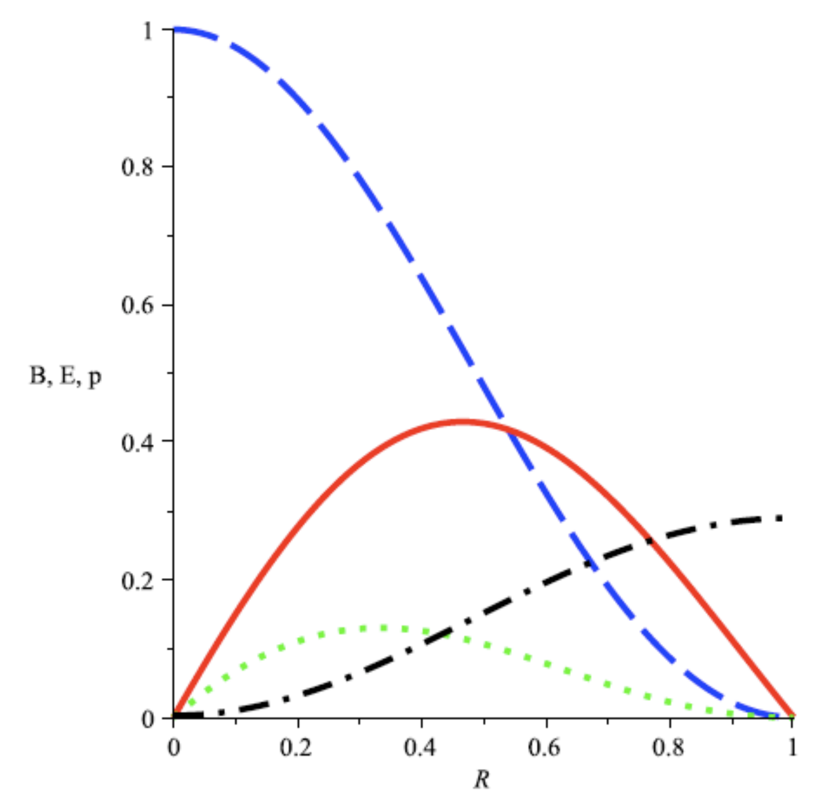

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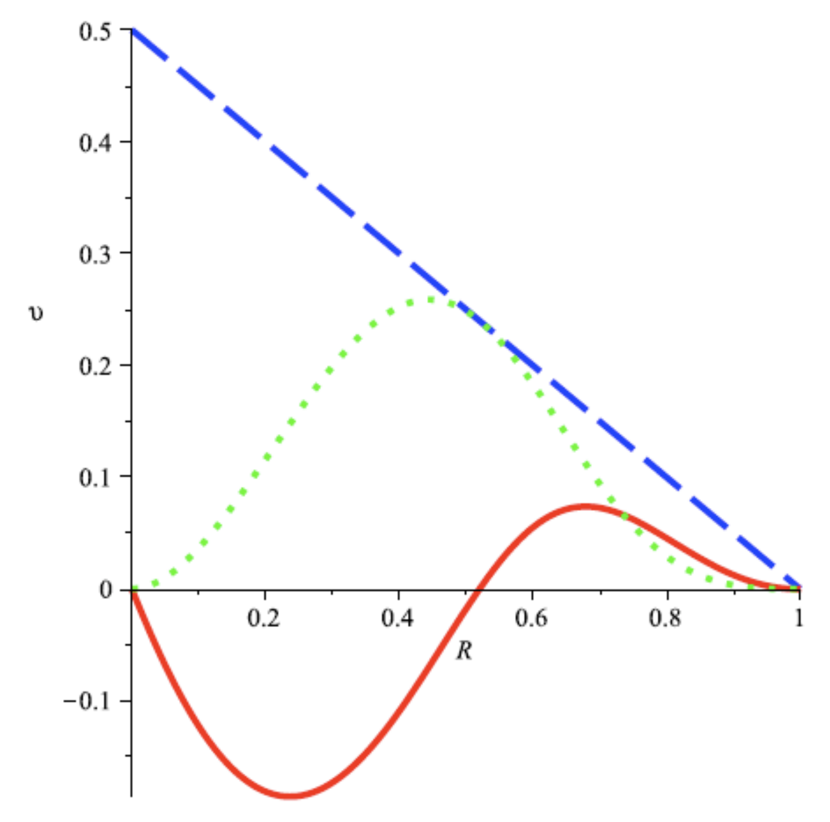

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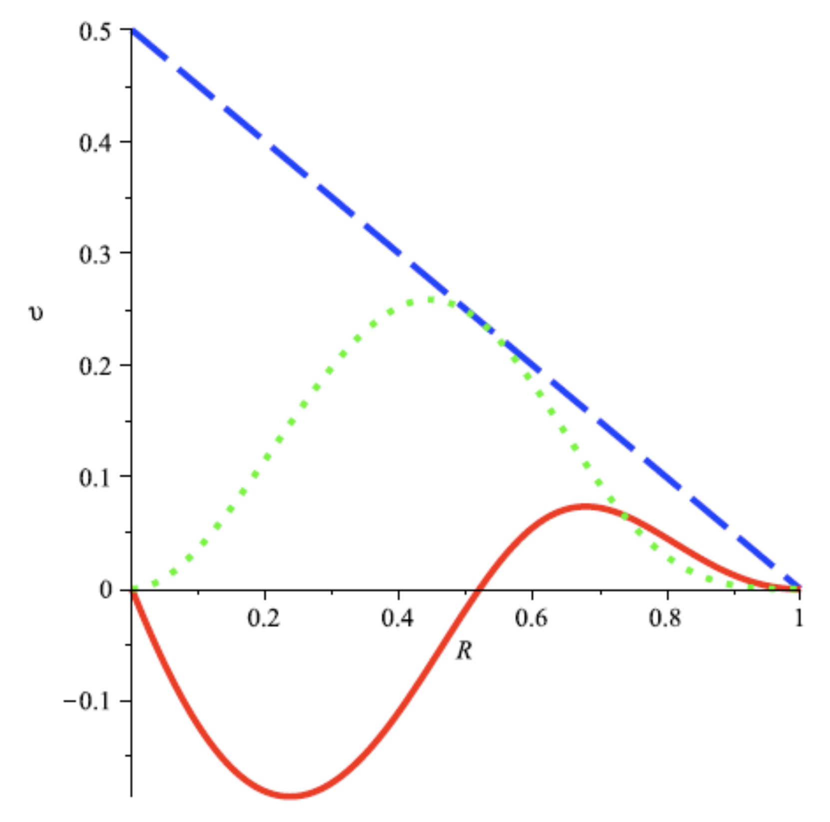

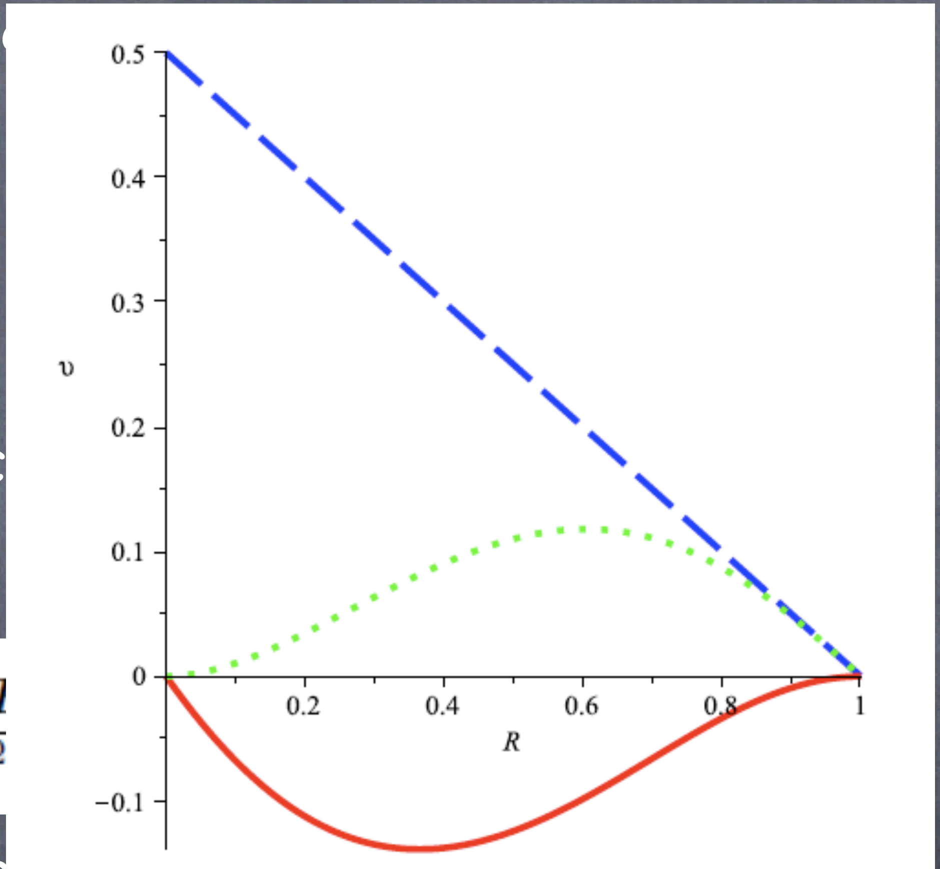

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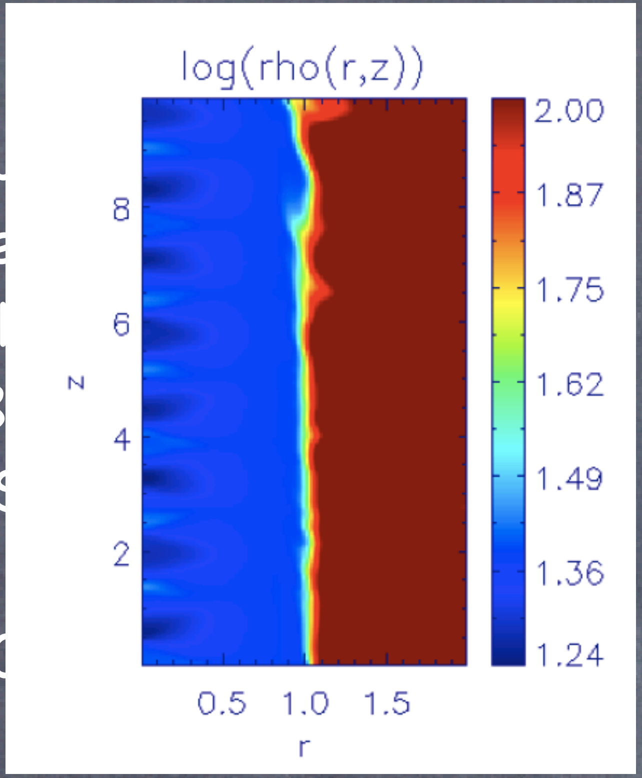

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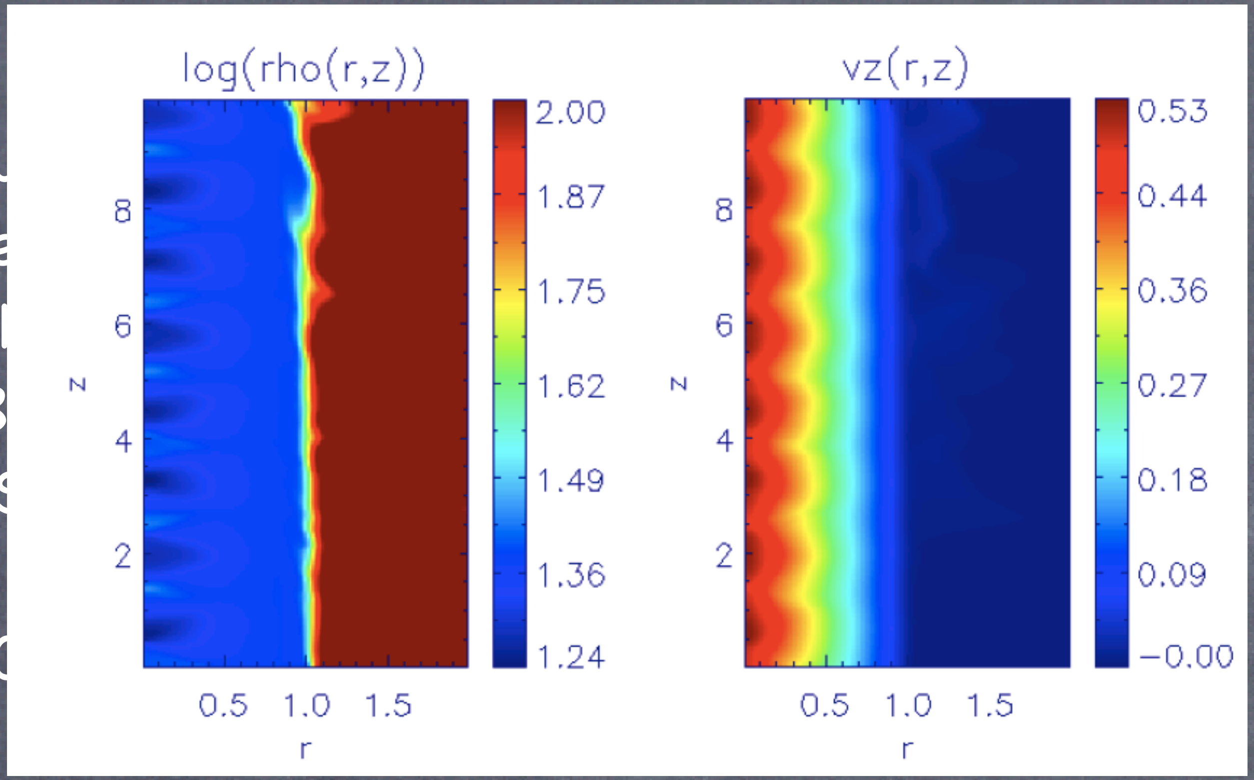

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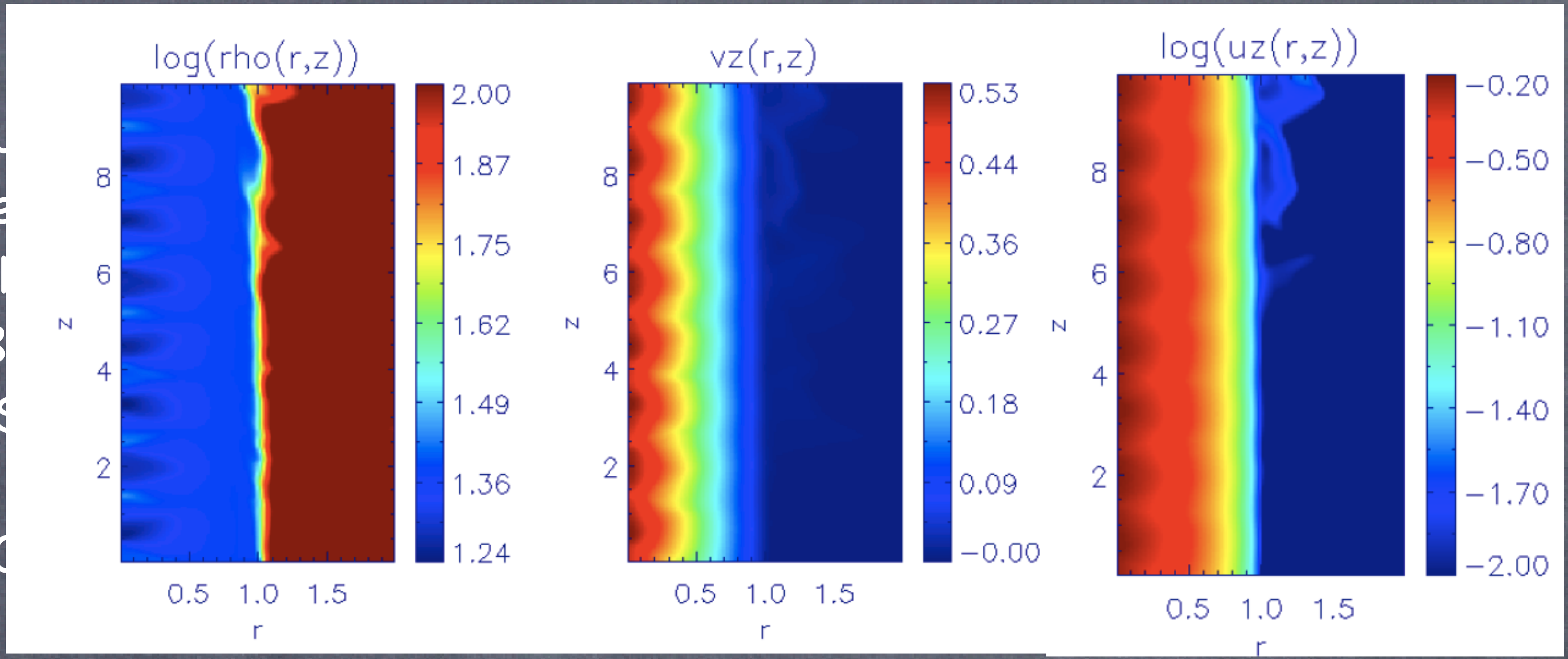

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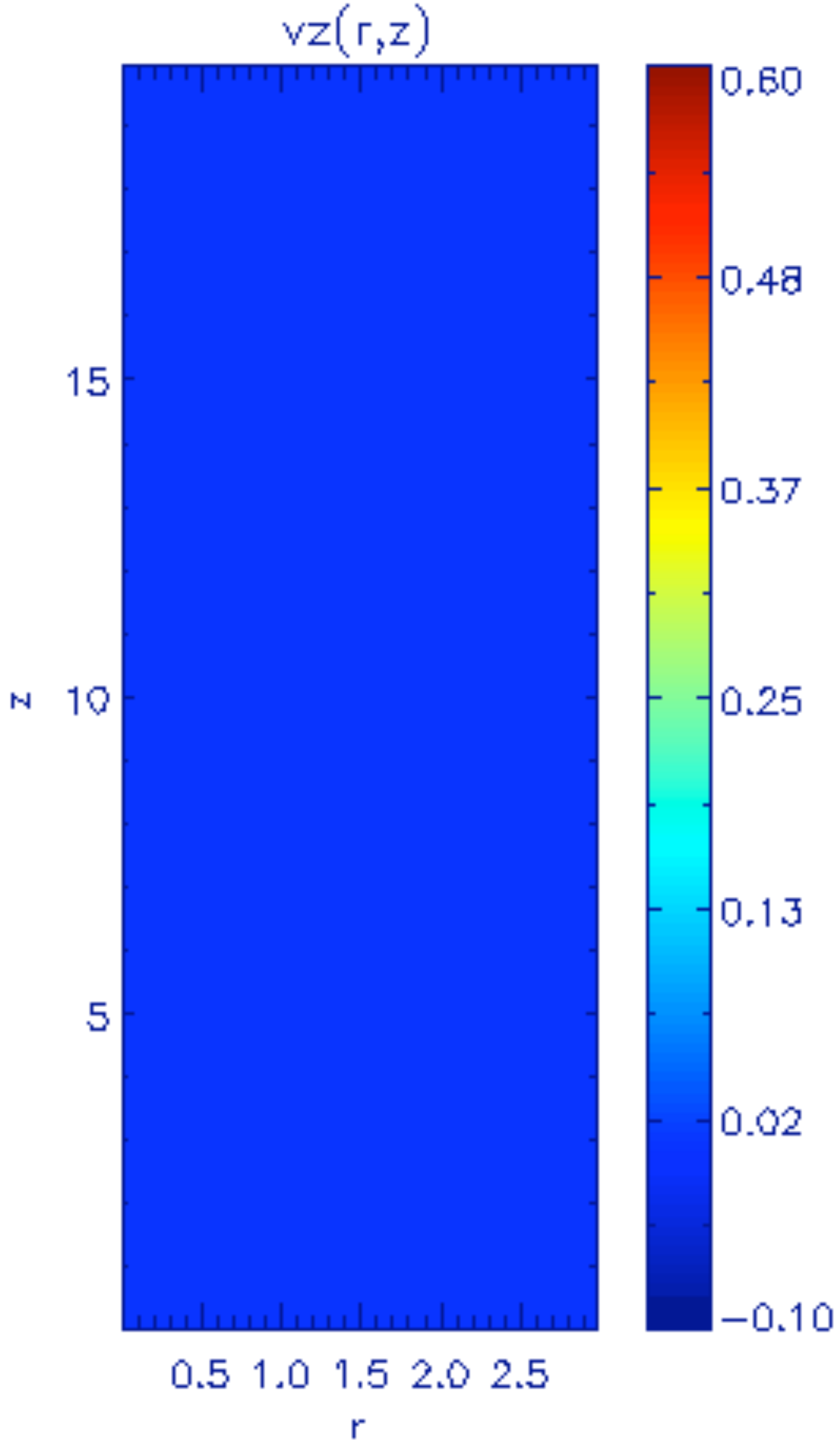
vz(r,z)
z
15
10
5
r
0.5
1.0
1.5
2.0
2.5
0.60
0.48
0.37
0.25
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0.02
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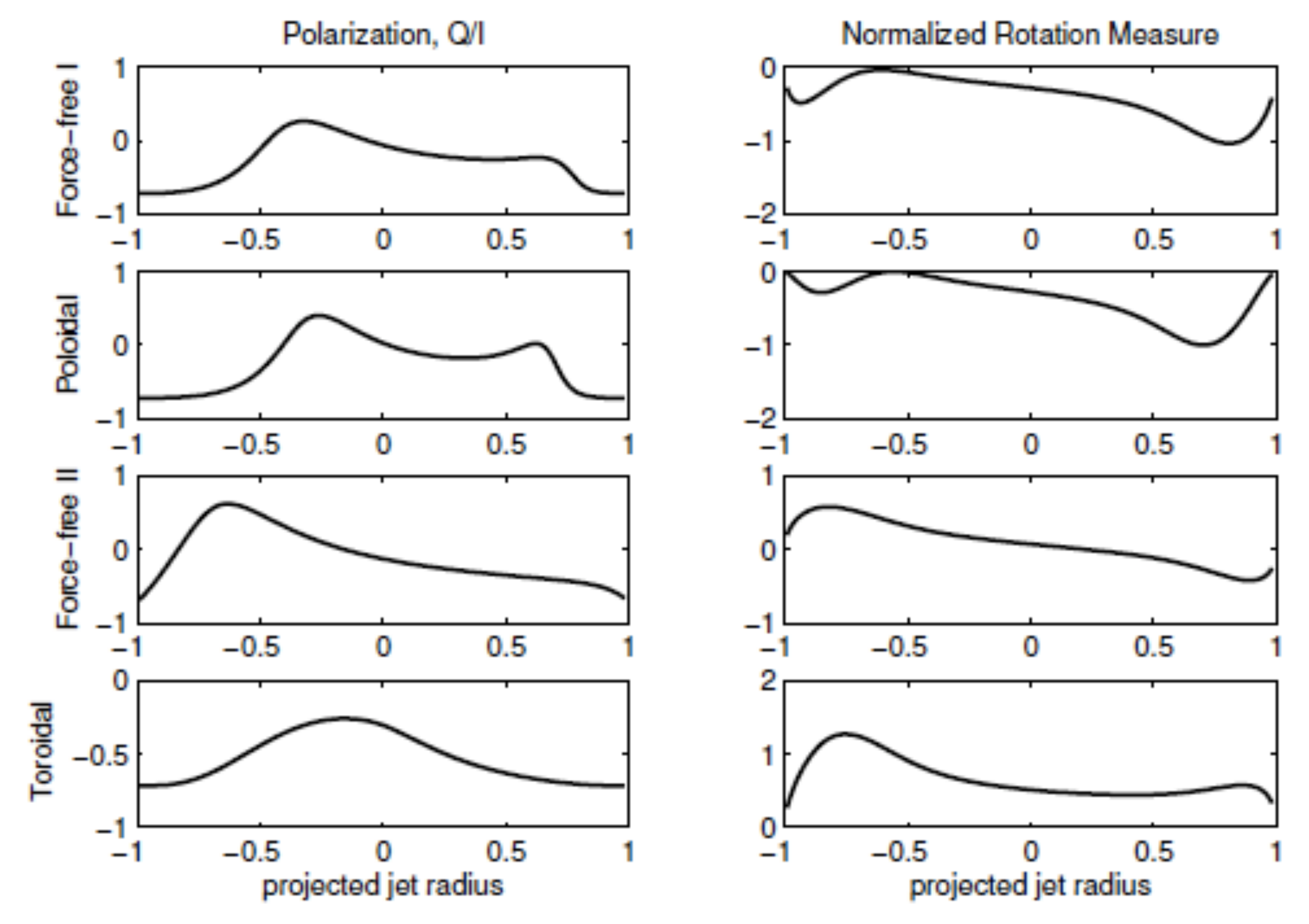

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Thanks