Extracting science from surveys of our Galaxy

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Outline

- Surveys challenge galactic dynamics
- Why a model should yield a pdf, not a discrete realisation
- Why steady-state models are fundamental and Jeans theorem is invaluable
- Why observational errors require models to be fitted in space of observables
- Introduction to torus modelling
- Some surprisingly useful worked examples
- Binney 2010, Binney & McMillan 2011, Schoenrich B & Dehnen 2010, Burnett 2010

Surveys

- Near-IR point-source catalogues
  - 2MASS, DENIS, UKIDS, VHS, ….
- Spectroscopic surveys
  - SDSS, RAVE, SEGUE, HERMES, APOGEE, VLT, WHT, …
- Astrometry
  - Hipparcos, UCAC-3, Pan-Starrs, Gaia, Jasmine, …
- Already have photometry of $\sim 10^8$ star, proper motions of $\sim 10^7$ stars, spectra of $\sim 10^6$ stars, trig parallaxes of $\sim 10^5$ stars
- By end of decade will have trig parallaxes for $\sim 10^9$ stars and spectra of 10^8 stars
- We are already data-rich & model-poor

Need for models

- Our position in the disc makes models essential for interpretation of data
  - models provide the means to compensate for strong selection effects in survey data
- Models facilitate compensation for large observational errors
- The complexity of the MW calls for a hierarchy of models of increasing sophistication
  - The bar
  - Spiral structure
  - Halo substructure

On pdfs & realisations

- ¤CDM model gives rise to increasingly detailed models of the LG
- Key physics is “sub-grid” so fudged
- Models are discrete realisations of some underlying probability density function (pdf) – we don’t expect to find a star exactly where the model has one
- The Galaxy is another discrete realisation
- How to ask if 2 realisations are consistent with same (unknown) pdf?
- Much better to formulate the model as a pdf – then can ask if Galaxy is consistent with this pdf – or in what respects the Galaxy materially differs from it – by calculating likelihoods
- Hence reject N-body & similar models

Strategy

- The galaxy is not in perfect equilibrium
- But we must start from equilibrium models:
  - First target is ©(x), which will be an important ingredient of our final model
  - Without the assumption of equilibrium, any distribution of stars in (x,v) is consistent with *any* ©(x)
  - From ©(x) we can infer ½$_{DM}$(x)
  - Can only infer ½$_{DM}$(x) to the extent that the Galaxy is in dynamical equilibrium
- Non-equilibrium structure (spiral arms, tidal streams,..) will show up as differences between best equilibrium model and the Galaxy
- The Galaxy is not axisymmetric, but it is sensible to start with axisymmetric models for related reasons

Jeans theorem

- Jeans (1915) pointed out that the distribution function (DF) of a steady-state Galaxy must be a function of integrals of motion $f(I_1,I_2,..)$
- Jeans theorem simplifies our problem: 6d ! 3d
- Already in 1915 observations implied that f must depend on I_3 in addition to E, L_z
- The Galaxy's ©(x) will not admit an analytic form of $I_3(x,v)$ – must use numerical approximations
- Unfortunately, we need a large set of DFs: one for each physically distinguisable type of star:
  - f(E,L_z,I_3,m,¿,Fe/H,®/Fe,..)
  - High-resolution spectroscopy further enlarges the space inhabited by Galaxy models

Observational error

- The quantities of interest, E, L_z,… depend in complex ways on observables that may have large observational errors
- Observational errors ! correlated errors in E, L_z,..
- For example
  - error in distance s ! errors in v_t = s¹ and thus errors in E, L,…
- Measured value of parallax ϖ can be negative – even a negative ϖ conveys information although 1/ϖ ≠ s!
- Conclude: must match model to data in space of observables
  - u = (®,±,ϖ,¹ ®,¹ ±,v_{los},V,V-I,log g,Fe/H,®/Fe,..)
  - recognising that log g, Fe/H, .. not raw observables
- Calculate the likelihood of the data given a model by calculating for each star the integral
  - P_*(u) = s dm d¿ dZ d^3x d^3v f(x,v) $\prod_i$G(u_i-u_i(m,¿,Z,x,v),$¾_i$)
  - where G(u,¾) is the normal distribution
  - in practice the integral can be greatly simplified
- Finally the model is adjusted to maximise $\sum_*$ ln(P_*)

Torus modelling

- Schwarzschild modelling is the standard technique for fitting dynamical models to external galaxies (Gebhardt + 03, Krajnovic + 05)
- For assumed ©(x), obtain a “library” of representative orbits & calculate the contribution of each orbit to each observable
- Then seek weights of orbits such that weighted sum of contributions is consistent with measured values of each observable
- Torus modelling is based on this idea but replaces time series x(t), v(t) of orbit integrated from specified initial conditions by an orbital torus T

Orbital tori

- T is the image under a canonical transformation of the 3-d surface in 6-d phase space to which an analytic orbit is confined
- Position within T is specified by three angle variables μ_i, which increase linearly in time: $\mu_i(t)=\mu_i(0)+\Omega_i t$
- T is labelled by its action integrals J_i = (2¼)$^{-1}$s∘$_i$ v.dx, which are specified up front
- In an axisymmetric ©, L_z is one of the actions

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- Instead of integrating the orbit's equations of motion, the computer determines the canonical transformation that maps an analytic torus with the specified J_i into the torus T on which the actual Hamiltonian is ~constant
- The bottom line is that for any J_i we get analytic expressions for $x(\mu_1,\mu_2,\mu_3)$ and $v(\mu_1,\mu_2,\mu_3)$

Advantages of tori

- Systematic exploration of phase space is easy
- Action integrals:
  - Are essentially unique
  - Are Adiabatic invariants
  - Have clear physical interpretation
  - Make integral (action) space a true representation of phase space: $d^3x\, d^3v = (2½)^3 d^3J$
  - Make choice of analytic DF easy
- Given T, for any x we can easily find the μ_i at which the star reaches x and determine the corresponding velocities v
- Knowledge of the μ_i of stars key to unravelling mergers (McMillan & Binney 2008)
- Angle-action variables (μ,J) are the key to Hamiltonian perturbation theory
- Kaasalainen (1995) showed that perturbation theory works wonderfully well when the integrable H is provided by tori

Example: thin/thick interface

- Local stellar population can be broken down into
  - A “thick disc” of >10 Gyr old stars with high ®/Fe and mostly low Fe/H
  - A “thin disc” with low ®/Fe and mostly quite high Fe/H in which SFR has continued for ~ 10 Gyr at a slowly declining rate
- Thick-disc stars have quite large random velocities
- The random velocities of thin-disc stars increase steadily with age

Disc DFs
(Binney 2010)

- Build full DF from "quasi-isothermal" DFs

$$f_{\sigma_z}(J_z) \equiv \frac{e^{-\Omega_z J_z/\sigma_z^2}}{2\pi \int_0^\infty dJ_z \, e^{-\Omega_z J_z/\sigma_z^2}}$$

$$f_{\sigma_r}(J_r, L_z) \equiv \left.\frac{\Omega \Sigma}{\pi \sigma_r^2 \kappa}\right|_{R_c} [1 + \tanh(L_z/L_0)] e^{-\kappa J_r/\sigma_r^2}$$

  – The disc is "hotter" at small radii:

$$\sigma_r(L_z) = \sigma_{r0}\, e^{q(R_0 - R_c)/R_d}$$
$$\sigma_z(L_z) = \sigma_{z0}\, e^{q(R_0 - R_c)/R_d}$$

  – Only two significant parameters ¾$_{r0}$, ¾$_{z0}$

- The full thick-d DF is then

$$f_{\text{thk}}(J_r, J_z, L_z) = f_{\sigma_r}(J_r, L_z) f_{\sigma_z}(J_z)$$

Thin-disc DF

- Assume that all stars of a given age ¿ are described by a “quasi isothermal” DF
- Assuming an exponentially declining SFR and ¾ » ¿¯, the thin-d DF is

$$f_{\rm thn}(J_r, J_z, L_z) = \frac{\int_0^{\tau_{\rm m}} {\rm d}\tau\, {\rm e}^{\tau/t_0} f_{\sigma_r}(J_r, L_z) f_{\sigma_z}(J_z)}{t_0({\rm e}^{\tau_{\rm m}/t_0} - 1)}$$

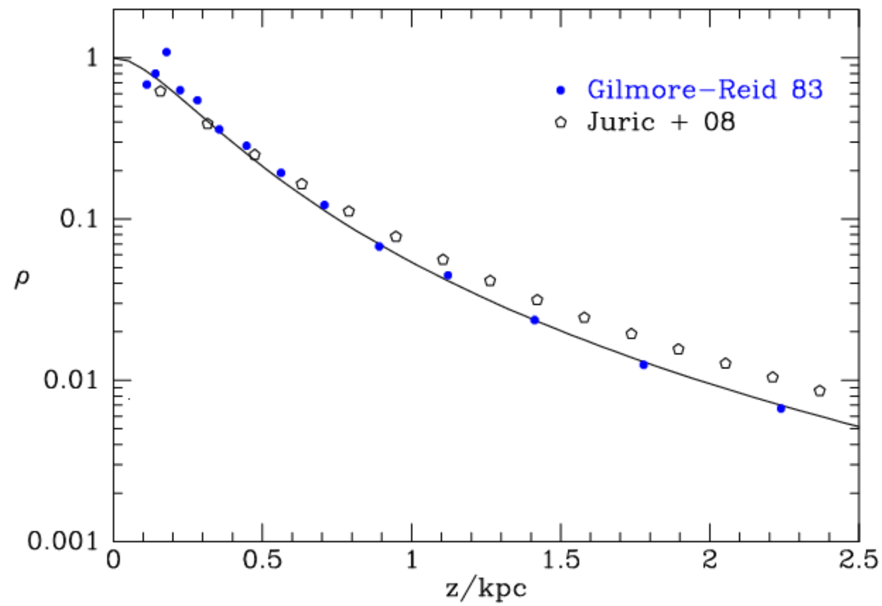
$$\sigma_r(L_z, \tau) = \sigma_{r0}\left(\frac{\tau + \tau_1}{\tau_{\rm m} + \tau_1}\right)^{\beta} {\rm e}^{q(R_0 - R_{\rm c})/R_{\rm d}}$$

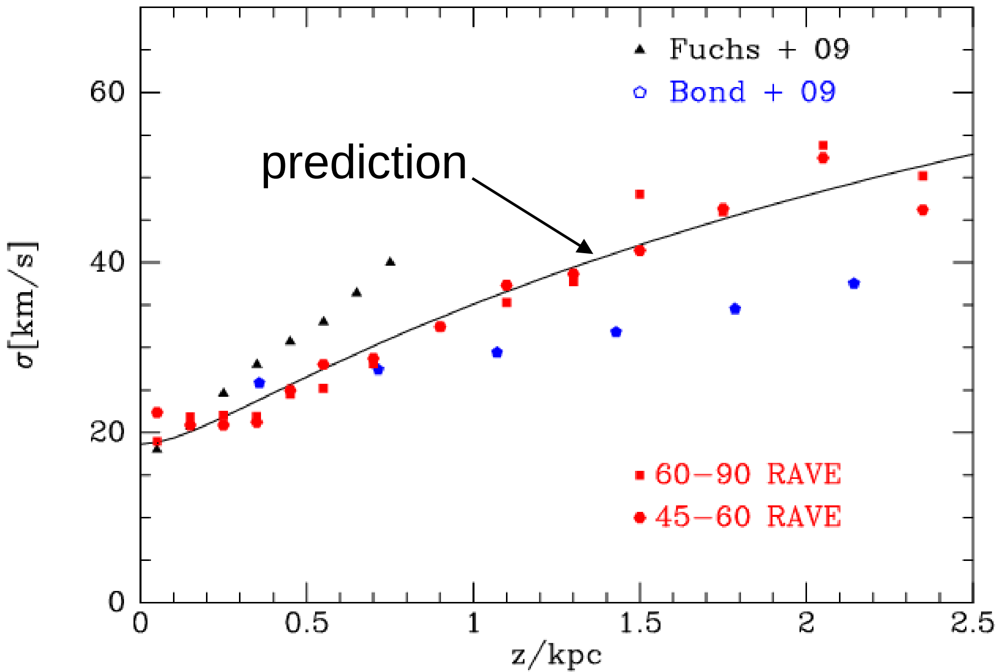
$$\sigma_z(L_z, \tau) = \sigma_{z0}\left(\frac{\tau + \tau_1}{\tau_{\rm m} + \tau_1}\right)^{\beta} {\rm e}^{q(R_0 - R_{\rm c})/R_{\rm d}}$$

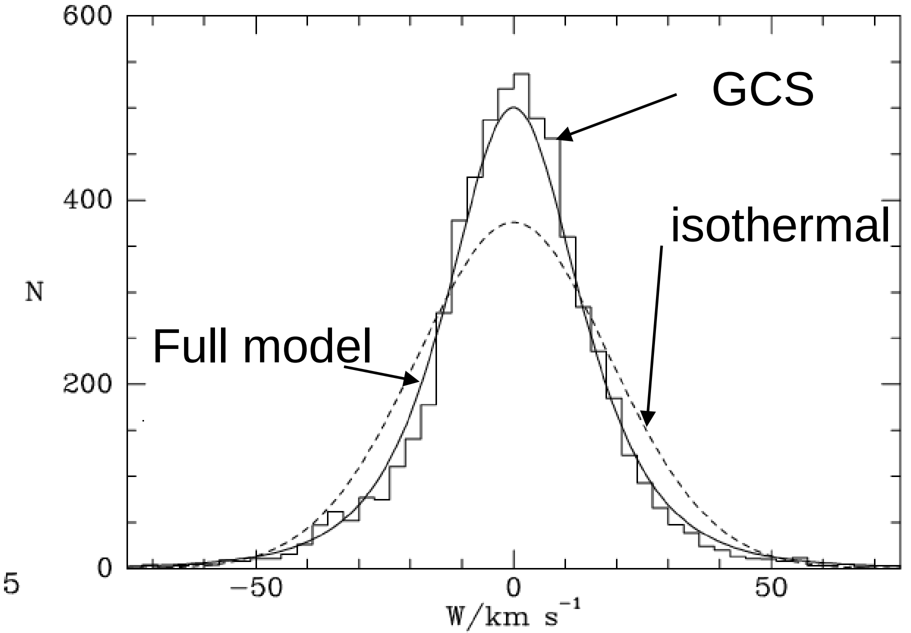
- Adjusting the parameters we fit the data

Comparison with Geneva-Copenhagen Survey (GCS)

- Choose parameters to fit U & V distributions of stars



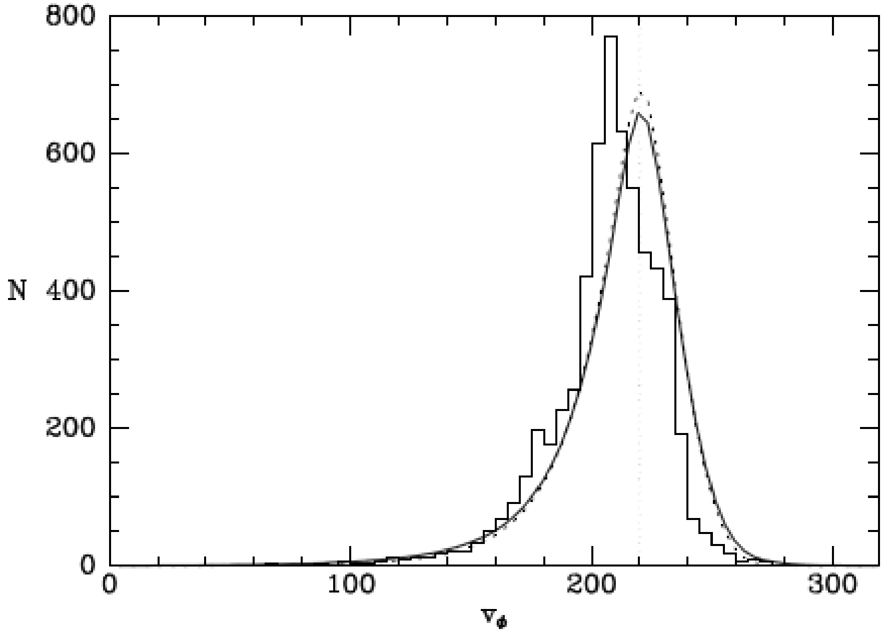


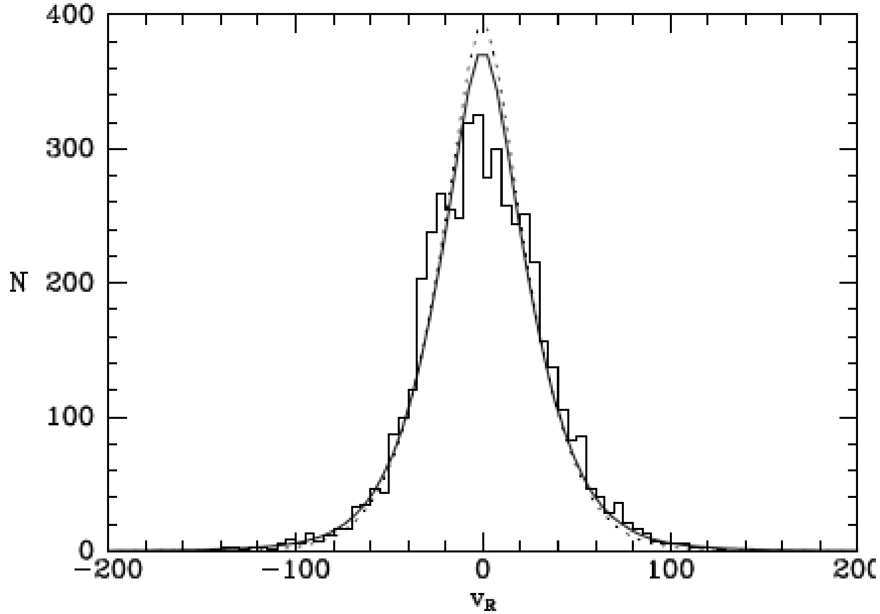


U & V distributions

(Binney 2010; Schoenrich + 2010)

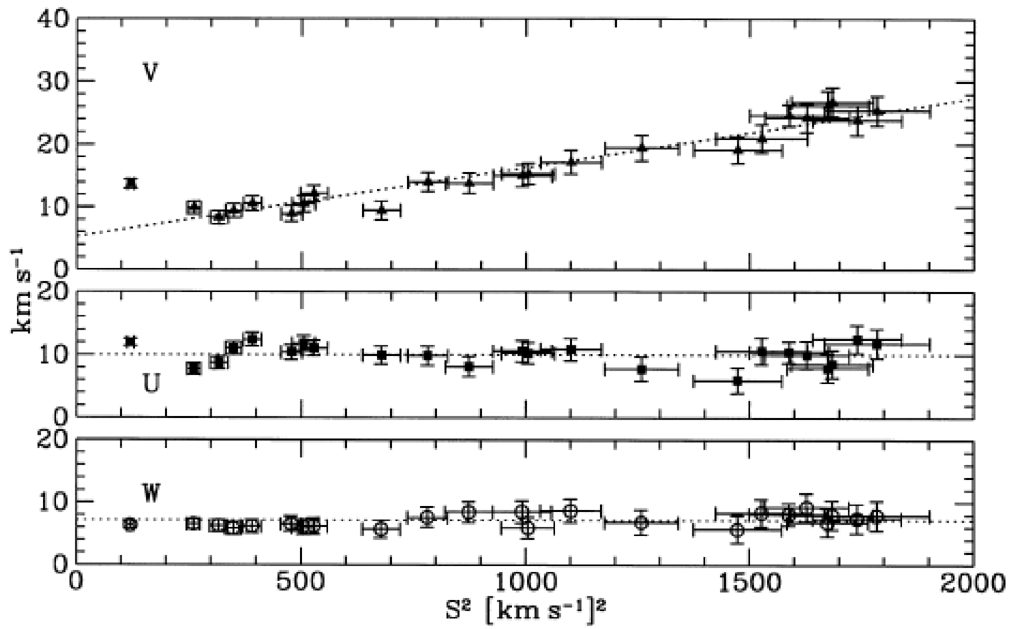
- Shapes of U and V distributions related by dynamics
- If U right, persistent need to shift observed V distribution to right by ~6 km/s
- Problem would be resolved by increasing V–





Revising V–

- Standard value by extrapolating to ¾=0 correlation ¾(B-V) for MS Hipparcos stars (Dehnen & B 1998)
- Stars selected by colour
- Stars made blue either by youth or metal-poverty
- So groups with lower ¾ also have lower [Fe/H]
- [Fe/H] decreases outwards, so groups with lower ¾ have larger L_z, so larger $v_Á$

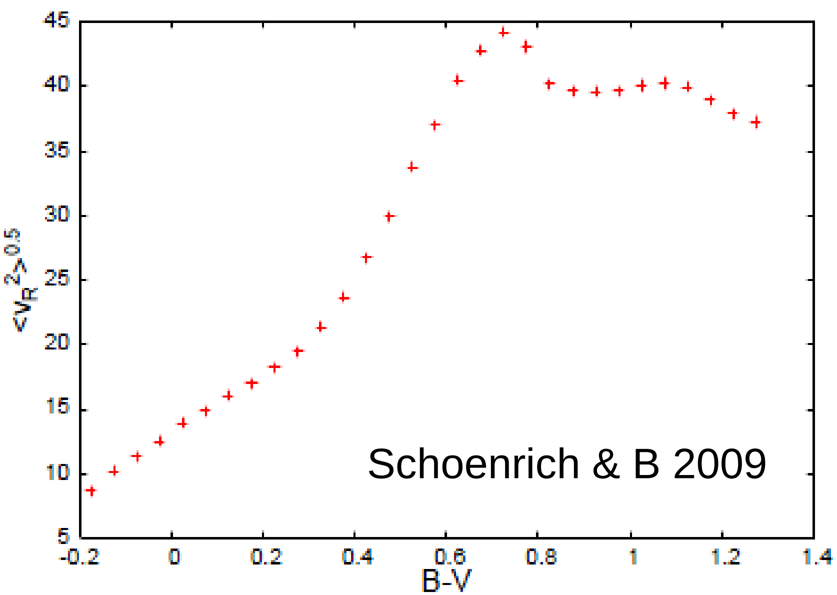


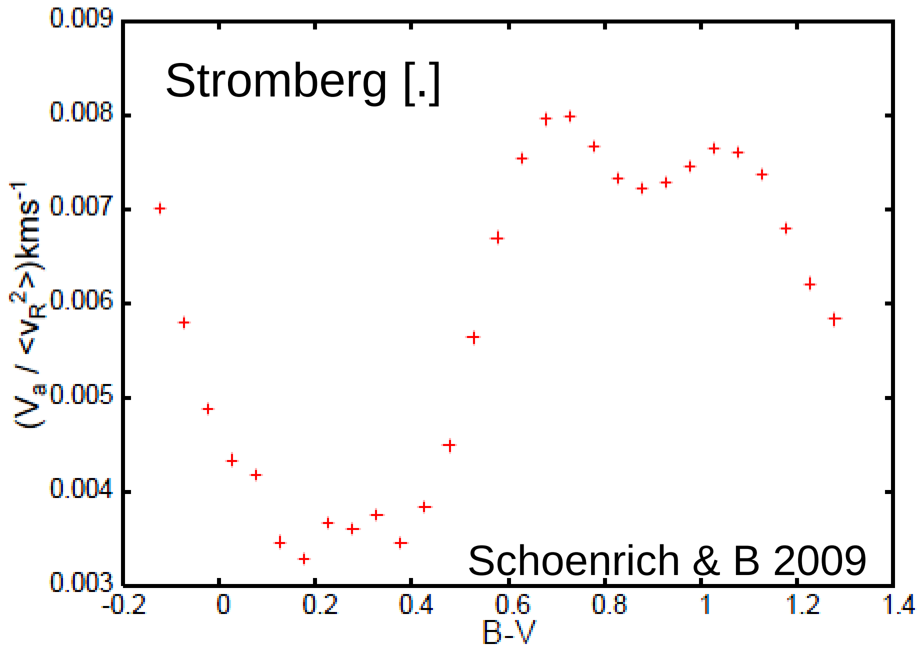
Revising V–
(Schoenrich + 2010)

- Underpinned by Stromberg's eqn

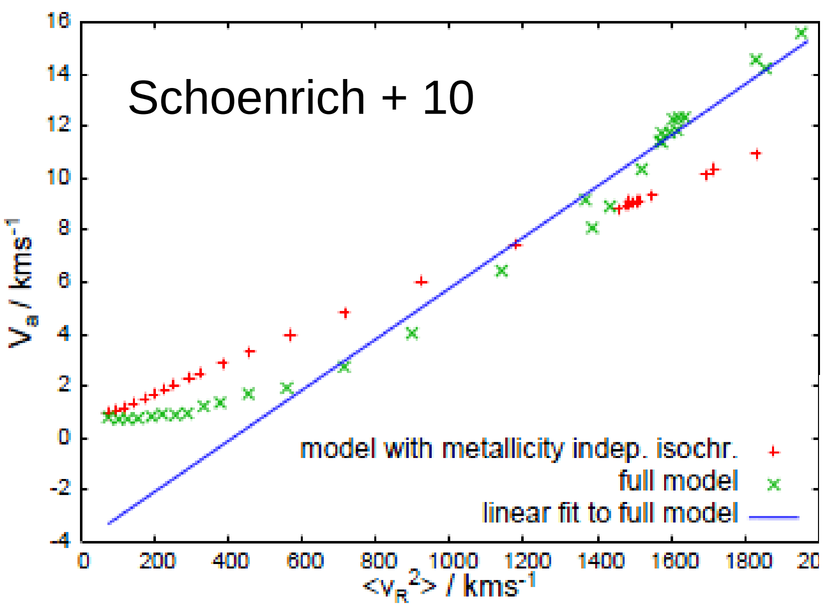
$$\overline{v_s} = V_\odot + \frac{\overline{v_R^2}}{2v_c}\left[\frac{\sigma_\phi^2}{\overline{v_R^2}} - 1 - \frac{\partial \ln(\nu \overline{v_R^2})}{\partial \ln R} - \frac{R}{\overline{v_R^2}}\frac{\partial(\overline{v_R v_z})}{\partial z}\right]$$

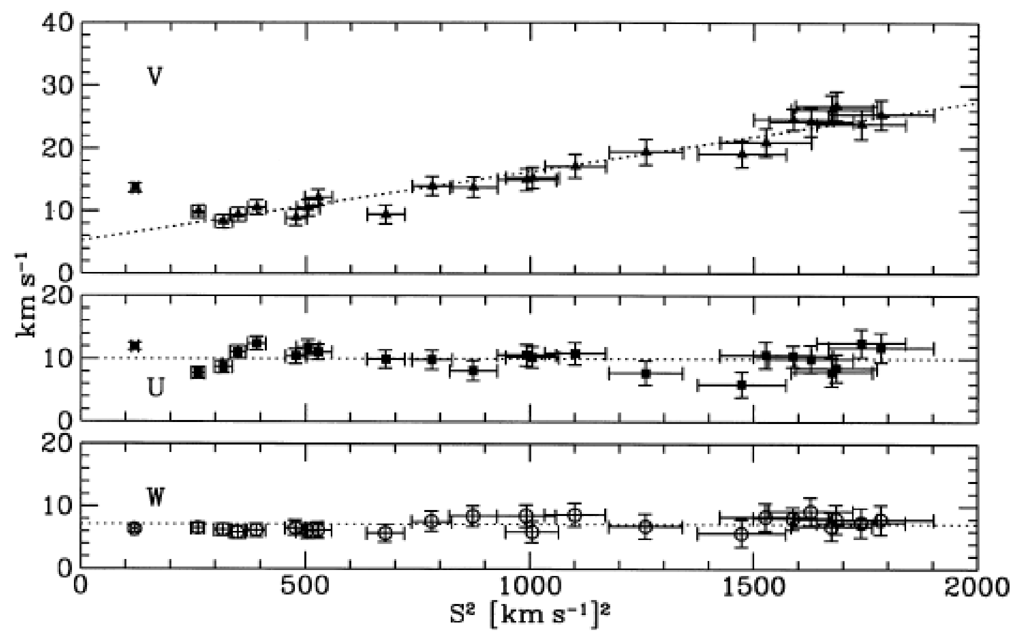
- But in model of Schoenrich & Binney (2009) [..] is not a constant!





- Conclude V– = 12§ 2 km/s not 5.2§ 0.6 km/s

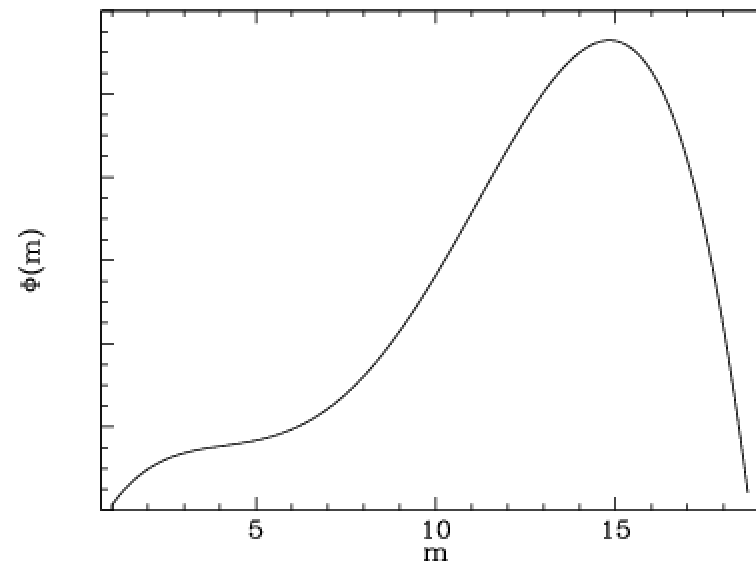




Working in data space

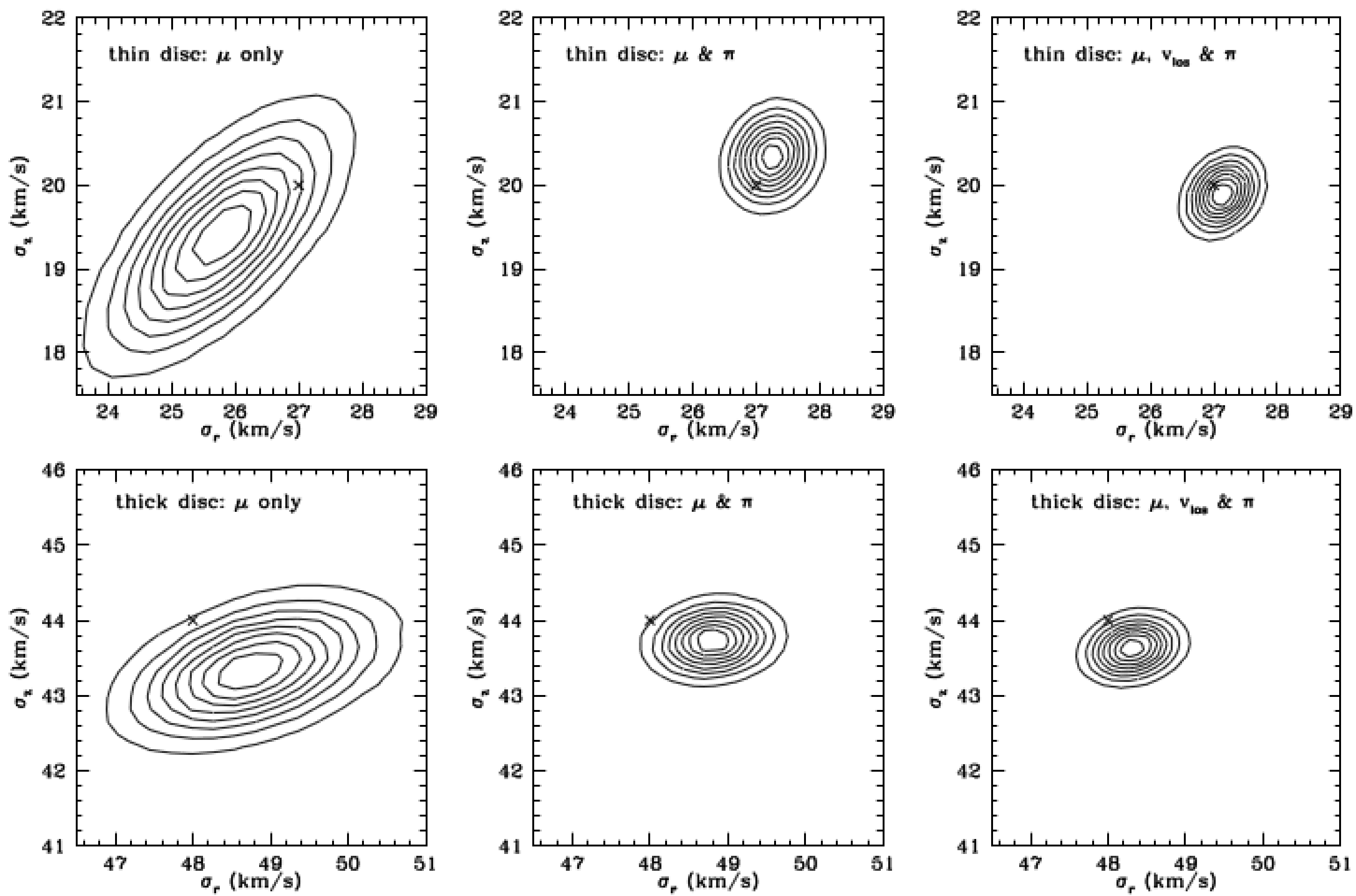
McMillan & Binney (2011)

- Use disc model to create pseudo-catalogue (®,±,ϖ,v_{los},¹ ®,¹ ±,V) for 10,000 stars
- Include Gaussian errors ±ϖ=0.2 mas, ±¹ =0.2 mas/yr, ±v_{los}=5 km/s
- Assume RAVE-like sky area
- Assume general ©(M) and RAVE-like apparent magnitude limits
- For similar models calculate probability distributions of stars in data
- Hence determine likelihood of data given model and thus determine probability distribution of model parameters



pdf of the model parameters

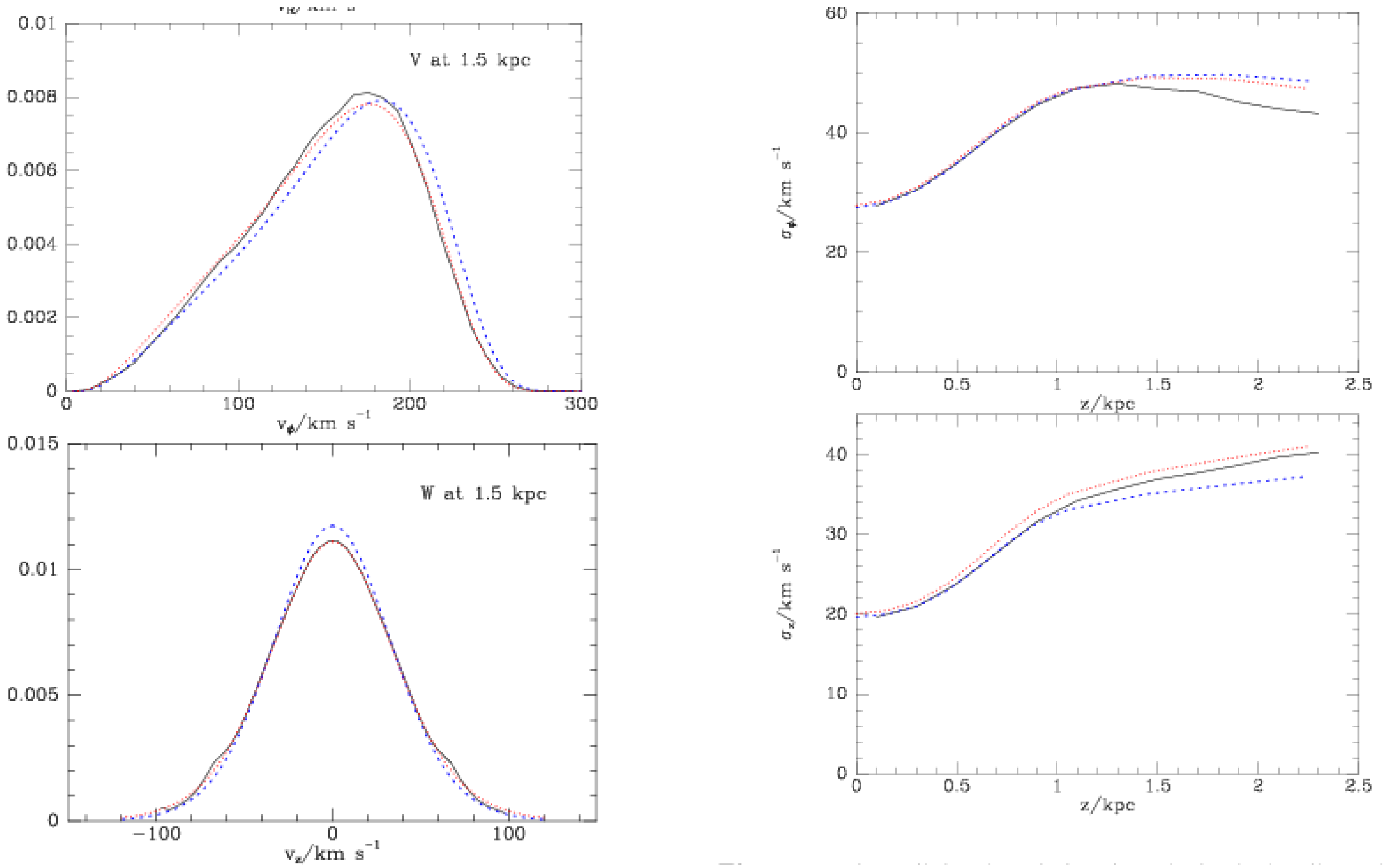
- Recovered from 10,000 stars
- Errors are remarkably small even with only [1]



Thin disc

Thick disc

Further comparisons

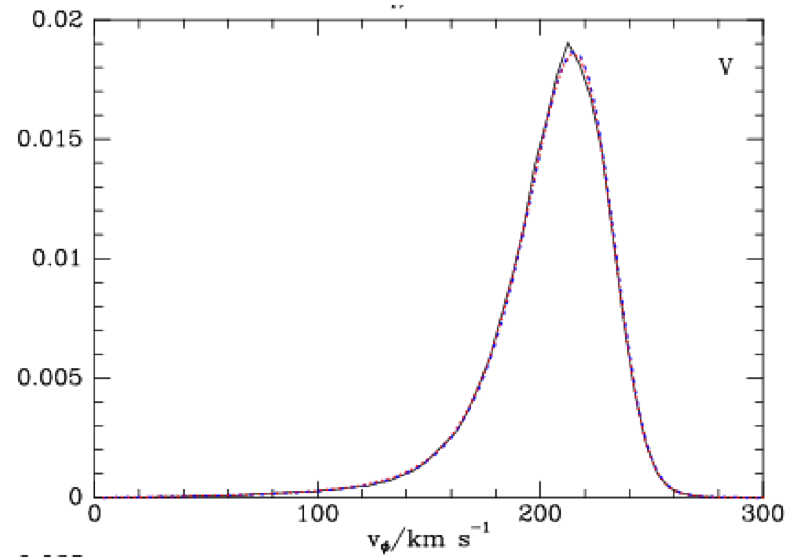


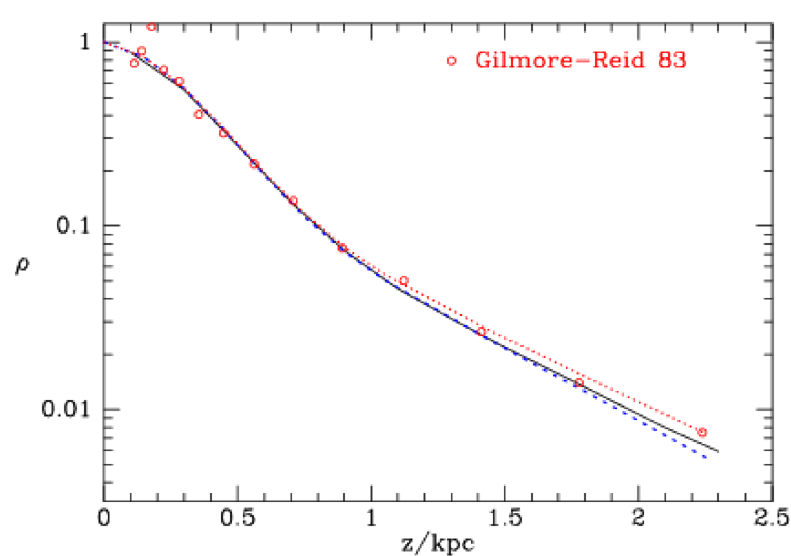
- Black: full torus model
- Red: adiabatic approx
- Conclude: adiabatic approx perfect in plane and very good for |z| < 2 kpc

Adiabatic approximation

(Binney & McMillan 2011)

- To evaluate observables such as ½(R,z) or $¾_z$(R,z), we have to integrate over velocities
- This is most easily done if we can quickly evaluate J_i(x,v)
- Torus machine yields x(J,µ) & v(J,µ)
- The integrals can be evaluated using tori, but the "adiabatic approximation" greatly speeds evaluation
- Given (x,v) we define E_z= ½ v_z^2+©(R,z) and estimate J_z = (2/¼)$s_0^{z_{max}}$ dz [2(E_z-©(R,z))]$^{1/2}$
- Then we set L = |L_z| + J_z and estimate J_r = (1/¼)$s_{r_p}^{r_a}$dR [2(E-L^2/2r^2-Phi(R,0))]$^{1/2}$

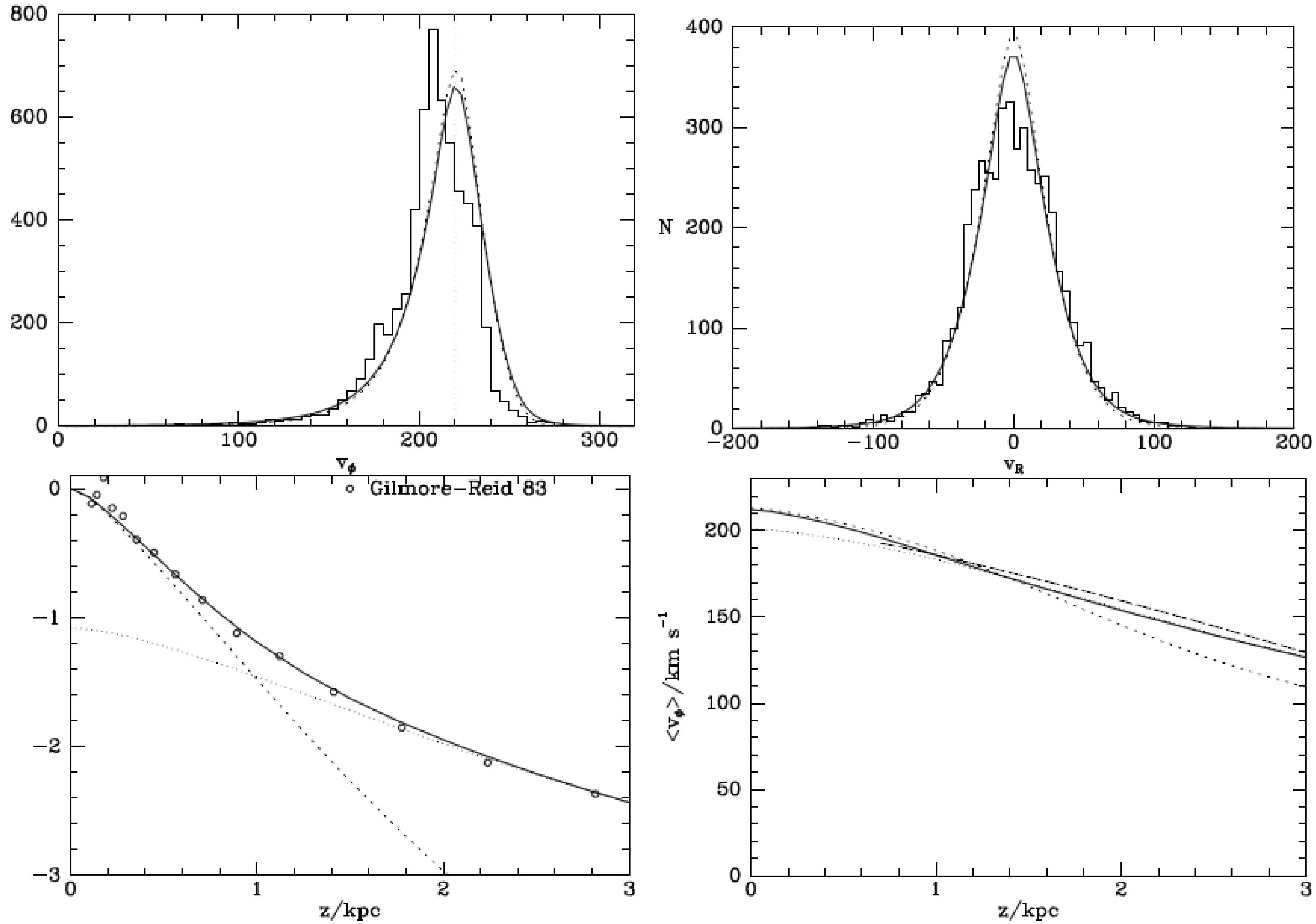




Conclusions

- There's a huge and rapidly growing volume of survey data for MW
- Key observables (ϖ,^{1}) are far removed from quantities of physical interest
- Errors in s corrupt estimates of all physical quantities
- Inversion of data to physical model ill-advised ($\varpi < 0$)
- Should fit model to data in (>=10d) space of observables
- To do this the model should deliver a pdf
- Torus modelling can be considered a variant of Schwarzschild modelling in which time series x(t) v(t) replaced by analytic 3d tori in 6d phase space
- Advantageous to weight orbits by parameterised analytic DF rather than varying weights of orbits independently
- Early models have already
  - revealed a subtle error in standard value of solar motion
  - correctly predicted ¾$_z$(z)
- We are now doing fits in space of observables – errors on model parameters remarkably small
- Adiabatic approximation yields very simple & useful expressions for J(x,v) that are remarkably accurate for thin & thick discs

Comparison with Geneva-Copenhagen Survey (GCS)



Prediction of full 2010 model

Preliminary RAVE data (Burnett 2010)