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On the dynamics of the three dimensional planetary systems

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Abstract: Over the last decades, there has been a tremendous increase in research on extrasolar planets. Many exosolar systems, which consist of a Star and two inclined Planets, seem to be locked in 4/3, 3/2, 2/1, 5/2, 3/1 and 4/1 mean motion resonance (MMR). We herewith present the model used to simulate three dimensional planetary systems and provide planar families of periodic orbits (PO), which belong to all possible configurations that each MMR has, along with their linear horizontal and vertical stability. We focus on depicting stable spatial families (most of them up to mutual inclination of 60°) generated by PO of planar circular families, because the trapping in MMR could be a consequence of planetary migration process. We attempt to connect the linear stability of PO with long-term stability of a planetary system close to them. This can stimulate the search of real planetary systems in the vicinity of stable spatial PO-counterbalanced by the planets' orbital elements, masses and MMR; all of which could constitute a suitable environment convenient to host them.

1 Model

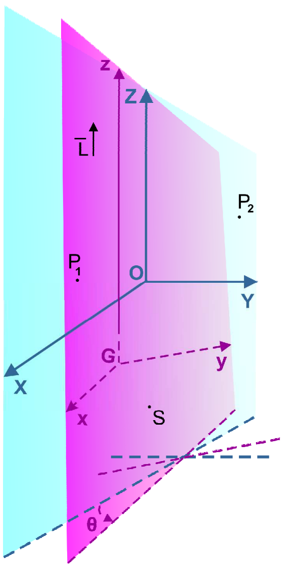

Figure 1: Inertial and rotating frame of reference.

We introduce a three dimensional system that consists of a Star, S and two inclined planets, P_1 and P_2, of masses m_0, m_1 and m_2, respectively, which are considered as point masses. The three bodies move in space $OXYZ$ (inertial frame) under their mutual gravitational attraction, where the origin O is their fixed center of mass and its Z-axis is parallel to the constant angular momentum vector, $\mathbf{L}$, of the system. The system is described by six degrees of freedom, which can be reduced to four by introducing a suitable rotating frame of reference, $Gxyz$, (Fig. 1) [1, 2], such that:

1. Its origin coincides with the center of mass G of the bodies S and P_1.
2. Its z-axis is always parallel to the Z-axis.
3. S and P_1 move always on xz-plane.

The Lagrangian of the system in the rotating frame of reference is:

$$\mathfrak{L} = \frac{1}{2}\mu[a(\dot{x}_1^2 + \dot{z}_1^2 + x_1^2\dot{\theta}^2) + b[(\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2) + \dot{\theta}^2(x_2^2 + y_2^2) + 2\dot{\theta}(x_2\dot{y}_2 - \dot{x}_2 y_2)]] - V,$$

where

$$V = -\frac{m_0 m_1}{r_{01}} - \frac{m_0 m_2}{r_{02}} - \frac{m_1 m_2}{r_{12}},\ a = m_1/m_0,\ b = m_2/m,\ \mu = m_0 + m_1,$$

and

$$r_{01}^2 = (1+a)^2(x_1^2 + z_1^2),\ r_{02}^2 = (a x_1 + x_2)^2 + y_2^2 + (a z_1 + z_2)^2,$$
$$r_{12}^2 = (x_1 - x_2)^2 + y_2^2 + (z_1 - z_2)^2.$$

2 Continuation of periodic orbits

Having defined the Poincaré section plane $\Pi = \{y_2 = 0, \dot{y}_2 > 0\}$ in $Gxyz$, the periodic orbits are the fixed or periodic points of this plane and they satisfy the conditions $\mathbf{q}(T) = \mathbf{q}_0$, where T is the period, $\mathbf{q} = \{x_1, x_2, z_2, \dot{x}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2\}$ and $\mathbf{q}_0 = \mathbf{q}(0)$. We consider the symmetries with respect to the xz-plane and the x-axis [4] and as a result the initial conditions of a xz-symmetric periodic orbit are

$$\{x_{10}, x_{20}, z_{20}, \dot{y}_{20}\} \text{ and } \dot{x}_{10} = \dot{x}_{20} = \dot{z}_{20} = 0.$$

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If $\Delta(T) = \{\xi_{ij}\}$, $i, j = 1, 2$, is the monodromy matrix of

$$\dot{\zeta}_1 = \zeta_2, \quad \dot{\zeta}_2 = A\zeta_1 + B\zeta_2$$

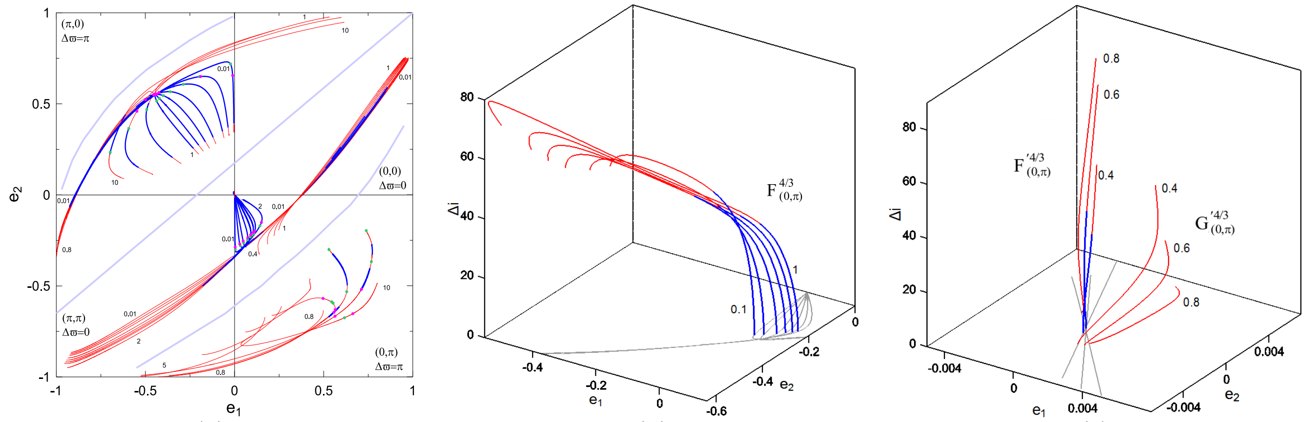
where $A = -\frac{m m_0}{\mu}[(1-\gamma)d_{02}^{-3} + (a+\gamma)d_{12}^{-3}]$, $B = -\frac{m_0 m_2 y_2}{\mu x_1 \dot{\theta}}(d_{02}^{-3} - d_{12}^{-3})$, $d_{12}^2 = (x_1 - x_2)^2 + y_2^2$, $d_{02}^2 = (a x_1 + x_2)^2 + y_2^2$ and $\gamma = b\frac{x_2\dot{\theta} + \dot{y}_2}{x_1\dot{\theta}}$, we can define the *vertical stability index* [3] of a planar periodic orbit of period T, as

$$a_v = \frac{1}{2}(\xi_{11} + \xi_{22})$$

If $|a_v| < 1$ or $|a_v| > 1$ the orbit is vertical stable or unstable, respectively.

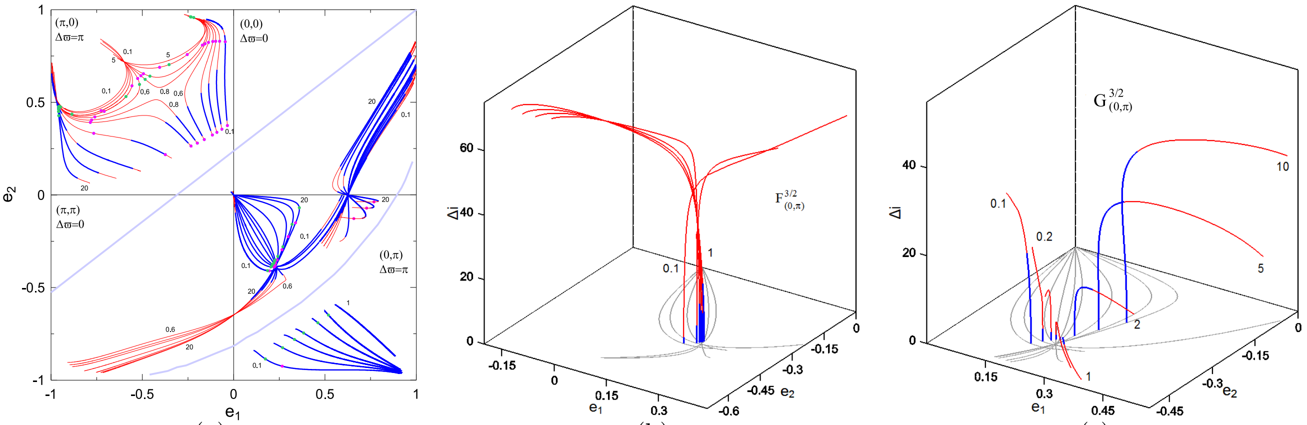
3 Results

Following the literature [5], we project the families of planar periodic orbits in the eccentricity plane (e_1, e_2). In order to distinguish the families of different configurations in the projection plane we use both positive and negative values for the eccentricities. The positive values of e_i correspond to $\theta_i = 0$ and the negative ones to $\theta_i = \pi$. We present the spatial families in the $3D$ projection space $(e_1, e_2, \Delta i)$, where Δi is the mutual inclination of the planets given by the cosine rule $\cos \Delta i = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_1 - \Omega_2)$. We chose some examples of 4/3 and 3/2 MMRs (Figs. 2,3).



(a) (b) (c)

Figure 2: Planar and spatial families of symmetric periodic orbits in 4/3 resonance.



(a) (b) (c)

Figure 3: Planar and spatial families of symmetric periodic orbits in 3/2 resonance.

4 Conclusions

We have performed an extensive study of 4/3, 3/2, 5/2, 3/1 and 4/1 MMRs in the planar and spatial case of the GTBP, in an attempt to connect the dynamics of three dimensional periodic orbits with the evolution of multiplanetary exosystems found nowadays. The complete results of our study are given in [2]. Particularly, we provided the planar families of symmetric periodic orbits, which belong to all possible configurations that each MMR has, along with their linear horizontal and vertical stability. We computed the v.c.o. of these families and then, focused on continuing to space mainly the stable ones. We observe that both xz- and x-symmetric periodic orbits were found to be stable up to values of mutual inclination 50°-60°, which could stimulate research of real systems whose planets are inclined. Also, we provided clues that could relate the required long-term stability of exoplanetary systems with three dimensional stable periodic orbits. In the neighbourhood of a stable periodic orbit, real planetary systems can be hosted and their long-term stability can be guaranteed. In contrary, if a planetary system is positioned in the vicinity of an unstable periodic orbit, due to the existence of chaotic regions around it, it will eventually destabilize.

If families of periodic orbits can constitute paths that can drive the migration process of planets and finally, lock them in MMRs, this work can help determine and understand the reasons why, the exoplanets are discovered possessing certain attributes.

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