11th Hel.A.S Conference
Athens, 8-12 September, 2013.
CONTRIBUTED POSTER

3D periodic orbits in the restricted four-body problem

A. N. Baltagiannis and K. E. Papadakis
Department of Engineering Sciences, Division of Applied Mathematics and Mechanics,
University of Patras, Patras, GR-26504 Greece

Abstract: One big body (Sun) of mass m1 and two other small bodies of masses m2 and m3 correspondingly, move in circular orbits keeping an equilateral triangle configuration, about the center of mass of the system fixed at the origin of the coordinate system. A massless particle is moving under the Newtonian gravitational attraction of the primaries and does not affect the motion of the three bodies. Using the vertical-critical orbits of planar families of symmetric periodic orbits as starting points, we determine and present in this paper, families of three-dimensional periodic solutions of the problem. Characteristic curves of the 3D-families which emanate from the plane are presented. The stability of every three-dimensional periodic orbit which numerically calculated is also studied.

1 Introduction

We consider that the dominant primary body m_1, is on the negative x-axis at the origin of time and the three point masses moving in circular periodic orbits around their center of mass. The equations of motion of the massless fourth body referred to a synodic rotating coordinate system with the same origin as the primaries are [2], [1],

$$\ddot{x} - 2\dot{y} = x - \sum_{i=1}^{3} \frac{m_i(x - x_i)}{r_i^3}, \ddot{y} + 2\dot{x} = y - \sum_{i=1}^{3} \frac{m_i(y - y_i)}{r_i^3}, \ddot{z} = -\sum_{i=1}^{3} \frac{m_i(z - z_i)}{r_i^3}$$

when $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$, $i = 1, 2, 3$ and x_i, y_i are the coordinates of the primaries

$$x_1 = -\frac{|K|\sqrt{m_2^2 + m_2 m_3 + m_3^2}}{K}, \qquad y_1 = 0, \qquad x_2 = \frac{|K|[(m_2 - m_3)m_3 + m_1(2m_2 + m_3)]}{2K\sqrt{m_2^2 + m_2 m_3 + m_3^2}},$$

$$y_2 = -\frac{m_3}{m_2^{3/2}} M, \qquad x_3 = \frac{|K|}{2\sqrt{m_2^2 + m_2 m_3 + m_3^2}}, \qquad y_3 = \frac{1}{m_2^{1/2}} M$$

where we have abbreviated $K = m_2(m_3 - m_2) + m_1(m_2 + 2m_3)$ and $M = \frac{\sqrt{3}}{2}\left(\frac{m_2^3}{m_2^2 + m_2 m_3 + m_3^2}\right)^{1/2}$. The gravitational potential in synodic coordinates is given by the equation $\Omega = \frac{1}{2}(x^2 + y^2) + \frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}$ and a Jacobian type of integral of the problem is $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - C$ where C is the Jacobian constant.

2 Results

In Fig. 1 (left) we present the network of all the families of the planar symmetric simple periodic solutions, i.e. these having two perpendicular intersections with the $x-$axis per period for $m_1 = 0.99$ and $m_2 = m_3 = 0.005$. Using a standard corrector-predictor procedure we calculated the family f_1 which consists of retrograde periodic orbits around the primary bodies. The stability of these periodic solutions are also computed and the arc of the stable periodic orbits are presented with red color (Fig. 1 (left)). We calculated the vertical-critical periodic solutions and found that family f_1 has five

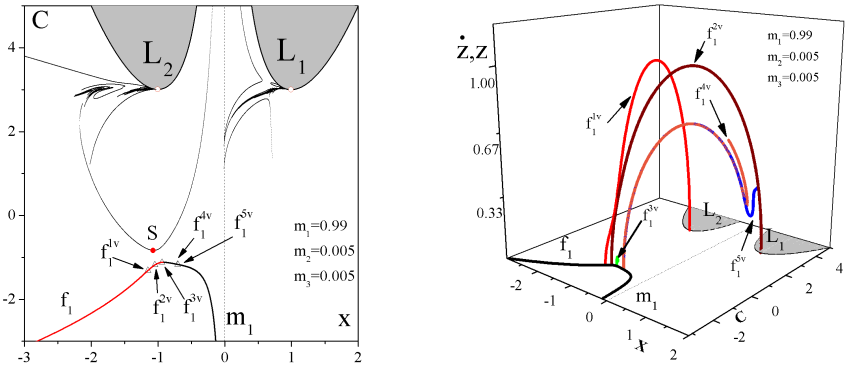

Figure 1: Left: The network of the families of the simple symmetric periodic orbits for $m_1 = 0.99$ and $m_2 = m_3 = 0.005$. The small triangles indicate the five vertical-critical periodic orbits of family f_1. Right: The five three-dimensional families which emanate from the plane vertical critical periodic orbits of family f_1.

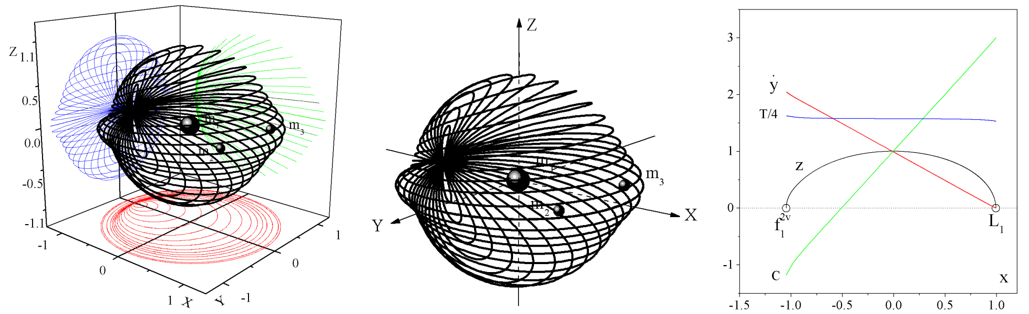

Figure 2: Left: Three-dimensional doubly-symmetric periodic orbits around the primary bodies. Middle: The same 3D-orbits without projections. Right: Characteristic curves of the 3D-family emanates from the planar vertical-critical orbit f_1^{2v}

ones (small triangle in the same figure), namely f_1^{iv}, $i = 1, \ldots, 5$. It is well known that these vertical-critical orbits are starting points for the determination of the families of three-dimensional periodic orbits. So, we calculated the five 3D families emanate from them. The first 3D family, emanates from the vertical-critical periodic orbit f_1^{1v}, has members three-dimensional periodic orbits doubly symmetric with respect to the $x-$axis and the xz plane. The second and the fifth 3D families have orbits doubly symmetric with respect to xz plane and to the $x-$axis while the third is a 3D family with periodic orbits symmetric with respect to the $x-$axis and the fourth one has three-dimensional periodic orbits symmetric with respect to xz plane. Three of the 3D families go up until the parameter z or the velocity $\dot{z}$ become maximum and then go down again to the plane, namely on the equilibrium point L_2 (family f_1^{1v}), on L_1 (family f_1^{2v}) and on point S (Fig. 1 (left)) of an other plane family (family f_1^{3v}). The other two 3D families go up and change multiplicity. In Fig. 1 (right) the three-dimensional characteristic curves of the five families which emanate from the planar family f_1 are illustrated. In Fig. 2 we plot three dimensional periodic orbits of family f_1^{1v} with (left) and without (middle) their projections. In Fig. 2 (right) the characteristic curves in the (x, C), (x, z), (x, T) and $(x, \dot{y})$ planes, are also illustrated. All the families have stable and critical 3D periodic orbits except family f_1^{4v} where is entirely unstable.

References

[1] Baltagiannis, A. N. & Papadakis, K. E., 2011, "Families of periodic orbits in the restricted four-body problem", *Astrophys Space Sci*, **336**, pp. 357–367.

[2] Moulton, F. R., 1900, "On a class of particular solutions of the problem of four bodies", *Trans. of the American Math. Soc.*, **1**, pp. 17–29.