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The global polytropic model for the solar and jovian systems revisited

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Abstract: The so-called "global polytropic model" is based on the assumption of hydrostatic equilibrium for the solar/jovian system, described by the Lane-Emden differential equation. A polytropic sphere of polytropic index n and radius R_1 represents the central component S_1 (Sun/Jupiter) of a polytropic configuration with further components the polytropic spherical shells S_2, S_3, ..., defined by the pairs of radii (R_1, R_2), (R_2, R_3), ..., respectively. $R_1, R_2, R_3, \ldots$, are the roots of the real part $\mathrm{Re}(\theta(R))$ of the complex Lane-Emden function $\theta(R)$. Each polytropic shell is assumed to be an appropriate place for a planet/satellite to be "born" and/or "live". This scenario has been studied numerically for the cases of the solar and the jovian systems. In the present paper, the Lane-Emden differential equation is solved numerically in the complex plane by using the Fortran code `DCRKF54` (modified Runge-Kutta-Fehlberg code of fourth and fifth order for solving initial value problems in the complex plane along complex paths). We include in our numerical study, some trans-Neptunian objects.

Summary

The so-called "complex-plane strategy" (CPS) proposes and applies numerical integration of "ordinary differential equations" (ODE, ODEs) in the complex plane, either along an interval $\mathbb{I}_r \subset \mathbb{R}$ when the independent variable r is real, or along a contour $\mathfrak{C} \subset \mathbb{C}$ when r is complex. Integrating in $\mathbb{C}$ is necessary when the "initial value problem" (IVP, IVPs) under consideration is defined on ODEs: (i) suffering from singularities and/or indeterminate forms in $\mathbb{R}$, and/or (ii) involving terms that become undefined in $\mathbb{R}$ when the independent variable r exceeds a particular value.

CPS extends numerical integration of the differential equations of an IVP well beyond the radius R of the nonrotating model instead of terminating integration just below R. Thus CPS knows the distortion caused by rotation over a sufficiently extended space surrounding the initially spherical configuration. So, to compute a particular rotating model, CPS does not extrapolate beyond the end of the function tables constructed by such extended numerical integrations. It is exactly the avoidance of any extrapolation which keeps the error in the computations appreciably small.

In the so-called "global polytropic model" for the solar system ([1], Sec. 1), the primary assumption is hydrostatic equilibrium. The polytropic sphere of polytropic index n and radius $\bar{\xi}_1$ is the central component or central body S_1 of a "resultant polytropic configuration" of which further components are the polytropic spherical shells defined by their respective pairs of radii. Each polytropic shell can be considered as an appropriate place for a planet/satellite to be born and live. We speak for a planet when the central body S_1 simulates the Sun [1]; in this case, the resultant polytropic configuration represents the solar system. On the other hand, we speak for a satellite when S_1 simulates a planet, say the Jupiter [2]; then the resultant polytropic configutation represents the jovian system. The most appropriate location for a planet/satellite to settle inside a polytropic shell S_j is the place $\bar{\Xi}_j$ at which $|\bar{\theta}|$ takes its maximum value inside S_j, $\max|\bar{\theta}[S_j]| = |\bar{\theta}(\bar{\Xi}_j + i\,\breve{\xi}_0)|$.

To solve the complex IVPs involved in this investigation, we use the code `DCRKF54` included in the Fortran package dcrkf54.f95 [3]. `DCRKF54` is a Runge–Kutta–Fehlberg code of fourth and fifth order modified for the purpose of solving complex IVPs, which are allowed to have high complexity in the definition of their ODEs, along contours (not necessarily simple and/or closed) prescribed as continuous chains of straight-line segments; interested readers can find full details on dcrkf54.f95 in [3].

The jovian system of satellites constitutes a short-distance integration problem; the code has been proved very accurate, since the global percentage error has been found to be $\%\mathcal{E}(\bar{\theta}) \leq 2 \times 10^{-9}$.

Treating planets of the solar system is a long-distance integration problem; numerical integration gives very accurate results, since $\%\mathcal{E}(\bar{\theta}) \leq 2 \times 10^{-9}$.

Computing quantities related to certain TNOs constitutes a very-long-distance integration problem; a global percentage error $\%\mathcal{E}(\bar{\theta}) \leq 9 \times 10^{-8}$ has been verified, which is quite satisfactory for this case.

References

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