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Magnetic helicity and free energy in solar active regions

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Abstract: We study the evolution of the non-potential free magnetic energy and relative magnetic helicity budgets in solar active regions (ARs). For this we use time-series of three-dimensional, synthetic ARs produced by magnetohydrodynamical (MHD) simulations. As a first step, we calculate the potential magnetic field that has the same normal components with the MHD field along all boundaries of the AR, by solving Laplace's equation in the modelled volume. The free magnetic energy of the AR is then easily derived. From the two fields, MHD and the electric-current-free (i.e. potential) one, we calculate the generating vector potentials with a recently proposed integration method. The knowledge of both fields and their respective vector potentials throughout the AR allows us to estimate the relative magnetic helicity budget of the AR. Following this procedure for each snapshot of the AR, we reconstruct the evolution of free energy and helicity in the AR. Our method reproduces, for the synthetic AR, the energy/helicity relations known to hold in real active regions.

1 Methodology

We are interested in two important parameters for solar flare prediction, namely the free magnetic energy and the relative magnetic helicity [1, 2]. Free energy is simply the excess energy of a field $\mathbf{B}$ (occupying a volume $\mathcal{V}$) relative to that of the minimum-energy potential field $\mathbf{B}_\mathrm{p}$

$$\mathcal{E}_\mathrm{f} = \mathcal{E} - \mathcal{E}_\mathrm{p} = \int_\mathcal{V} dV \, \frac{\mathbf{B}^2 - \mathbf{B}_\mathrm{p}^2}{8\pi}. \tag{1}$$

The relative (to the reference field $\mathbf{B}_\mathrm{p}$) magnetic helicity [3] of the field $\mathbf{B}$ is given by

$$H = \int_\mathcal{V} dV \, (\mathbf{A} + \mathbf{A}_\mathrm{p}) \cdot (\mathbf{B} - \mathbf{B}_\mathrm{p}) \tag{2}$$

with $\mathbf{A}$, $\mathbf{A}_\mathrm{p}$ the corresponding vector potentials of the two fields. This quantity is gauge-independent as long as $\mathbf{B}_\mathrm{p}$ has the same normal components with $\mathbf{B}$ along the boundaries of the volume, $\vartheta\mathcal{V}$. Helicity can be split into two parts. Self helicity, owing to the twist and writhe of individual flux-tubes, and mutual helicity, representing the interaction of pairs of flux-tubes [4], given by

$$H_\mathrm{self} = \int_\mathcal{V} dV \, (\mathbf{A} - \mathbf{A}_\mathrm{p}) \cdot (\mathbf{B} - \mathbf{B}_\mathrm{p}) \text{ and } H_\mathrm{mut} = 2 \int_\mathcal{V} dV \, \mathbf{A}_\mathrm{p} \cdot (\mathbf{B} - \mathbf{B}_\mathrm{p}). \tag{3}$$

In order to evaluate the energies and helicities we follow two steps:
Step 1 Calculation of the potential field: The current-free field results from a scalar potential φ satisfying Laplace's equation $\nabla^2\varphi = 0$ in $\mathcal{V} = (x_1, x_2) \times (y_1, y_2) \times (z_1, z_2)$. We solve numerically this equation under Neumann boundary conditions $\partial\varphi/\partial\hat{n}|_{\vartheta\mathcal{V}} = -\, \hat{n} \cdot \mathbf{B}|_{\vartheta\mathcal{V}}$, so that the calculated potential field $\mathbf{B}_\mathrm{p} = -\nabla\varphi$ has the same normal components with $\mathbf{B}$ along the boundaries of the volume.
Step 2 Calculation of vector potentials: We follow the method proposed by [5] and select the gauge $\hat{z} \cdot \mathbf{A} = 0$, so that $\mathbf{B} = \nabla \times \mathbf{A}$ can be integrated to

$$\mathbf{A} = \mathbf{A}_0 - \hat{z} \times \int_{z_1}^{z} dz' \, \mathbf{B}(x, y, z') \tag{4}$$

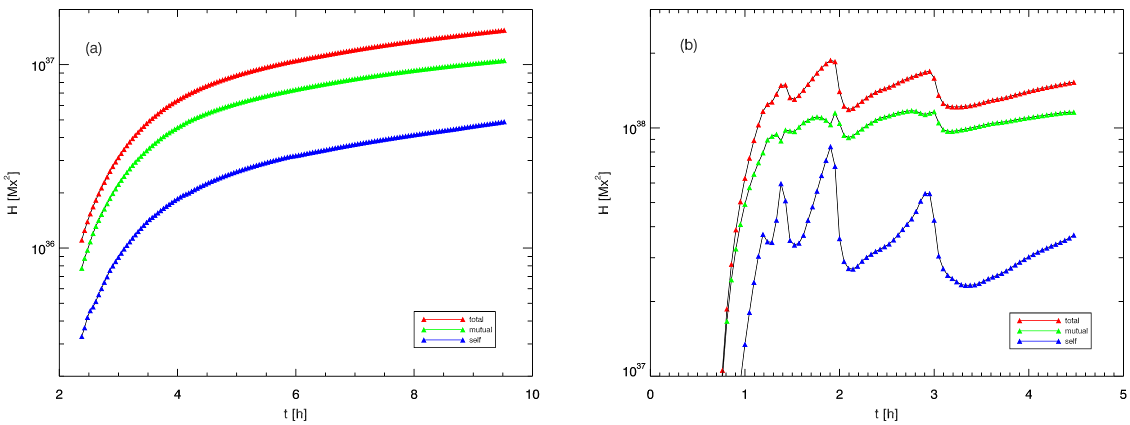

Figure 1: Evolution of relative, self and mutual helicities for a non-eruptive (a) and an eruptive active region (b). Free energy follows similar evolution patterns in both cases.

where $A_{0x} = -\frac{1}{2}\int_{y_1}^{y} dy'\, B_z(x, y', z = z_1)$ and $A_{0y} = \frac{1}{2}\int_{x_1}^{x} dx'\, B_z(x', y, z = z_1)$. A modified Simpson's rule is used in evaluating all the integrals and the same procedure is followed for the calculation of $\mathbf{A}_{\mathrm{p}}$.

2 Results

We apply this methodology to two different synthetic ARs produced in MHD simulations of the photospheric emergence of a twisted flux-tube [6]. The first simulated AR is non-eruptive, has dimensions $50 \times 50 \times 45$ Mm and is modelled for ~ 9.5 h of real solar time. The second simulation is for an eruptive AR of size $65 \times 65 \times 55$ Mm and covers its evolution up to ~ 4.5 h of real time.

As an example, we plot in Fig. 1 the evolution of relative, self and mutual helicities for the two cases. In the non-eruptive case, helicity budgets build up gradually and smoothly. In the eruptive case on the other hand, three flaring events can be identified where helicity is ejected outside the AR. At the same time, the free magnetic energy of the AR is decreased, as is expected during a flare. Additionally, the relation of the total helicity with free energy in this case agrees quite well with the one found by [2]. We also see indications for hysteresis between the peaks of self and mutual helicity terms, as suggested by [7].

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