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The network of the families of symmetric periodic orbits of the photogravitational restricted four-body problem

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Abstract: We consider three bodies of masses m1, m2 and m3 correspondingly, called primaries, which move in circular orbits around their center of mass under their mutual Newtonian gravitational attraction, keeping an equilateral triangle configuration, i.e. lie always at the apices of an equilateral triangle. A fourth massless particle moving in the same plane is acted upon the Newtonian gravitational attraction of the primary bodies. We study numerically the families of the simple symmetric periodic orbits of the problem when the first primary body, which is the bigger (Sun) than the other two, is a source of radiation. The network of these families, for various values of the radiation pressure of the Sun, is illustrated. We found that the radiation pressure has a strong effect on the network of the families. Sample of simple symmetric periodic solutions of the families are presented.

1 Introduction

In the present paper we consider the restricted photogravitational four-body problem where the motion of the system is referred to axes rotating with uniform angular velocity. The three bodies move in the same plane and their mutual distances remain unchanged with respect to time. The motion of the primaries consists of circular orbits around their center of gravity. The equations of motion of the problem, in the usual dimensionless rectangular rotating coordinate system are written as [1], [2],

$$\ddot{x} - 2\dot{y} = x - \sum_{i=1}^{3} \frac{q_i m_i (x - x_i)}{r_i^3}, \ddot{y} + 2\dot{x} = y - \sum_{i=1}^{3} \frac{q_i m_i (y - y_i)}{r_i^3}, \ddot{z} = - \sum_{i=1}^{3} \frac{q_i m_i (z - z_i)}{r_i^3}$$

when q_i are the radiation pressure parameters, the distance of the fourth particle from each of the three primaries is $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$, $i = 1, 2, 3$ where x_i, y_i and $z_1 = z_2 = 0$ (plane case), are the coordinates of the primaries

$$x_1 = -\frac{|K|\sqrt{m_2^2 + m_2 m_3 + m_3^2}}{K}, \qquad y_1 = 0, \qquad x_2 = \frac{|K|[(m_2 - m_3)m_3 + m_1(2m_2 + m_3)]}{2K\sqrt{m_2^2 + m_2 m_3 + m_3^2}},$$

$$y_2 = -\frac{m_3}{m_2^{3/2}} M, \qquad x_3 = \frac{|K|}{2\sqrt{m_2^2 + m_2 m_3 + m_3^2}}, \qquad y_3 = \frac{1}{m_2^{1/2}} M$$

where we have abbreviated $K = m_2(m_3 - m_2) + m_1(m_2 + 2m_3)$ and $M = \frac{\sqrt{3}}{2}\left(\frac{m_2^3}{m_2^2 + m_2 m_3 + m_3^2}\right)^{1/2}$. The equations of motion admit a Jacobian type of integral $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - C$ where $\Omega = \frac{1}{2}(x^2 + y^2) + \frac{q_1 m_1}{r_1} + \frac{q_2 m_2}{r_2} + \frac{q_3 m_3}{r_3}$ and C is the Jacobian constant.

2 Results

We consider that the first primary body m_1 is dominant (Sun) with mass $m_1 = 0.99$ while the other two primaries are equal with masses $m_2 = m_3 = 0.005$. This mass distribution provides a stable

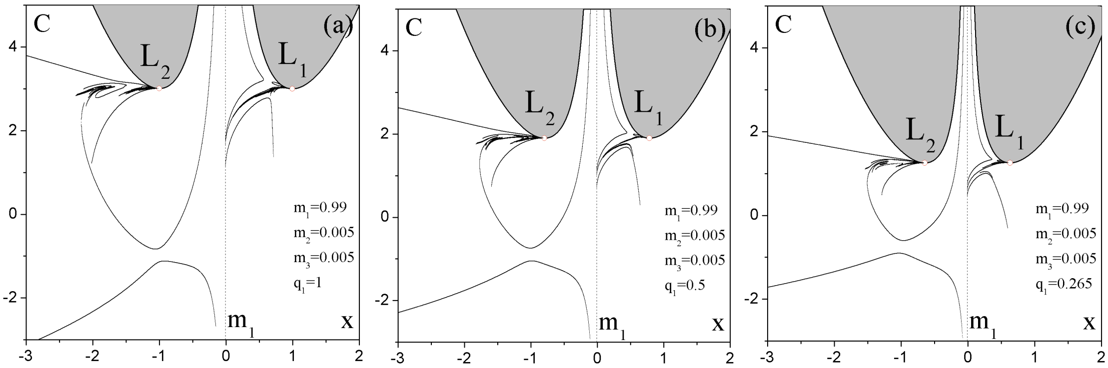

Figure 1: The network of the families of the simple symmetric periodic orbits when the dominant primary body m_1 is a source of radiation.

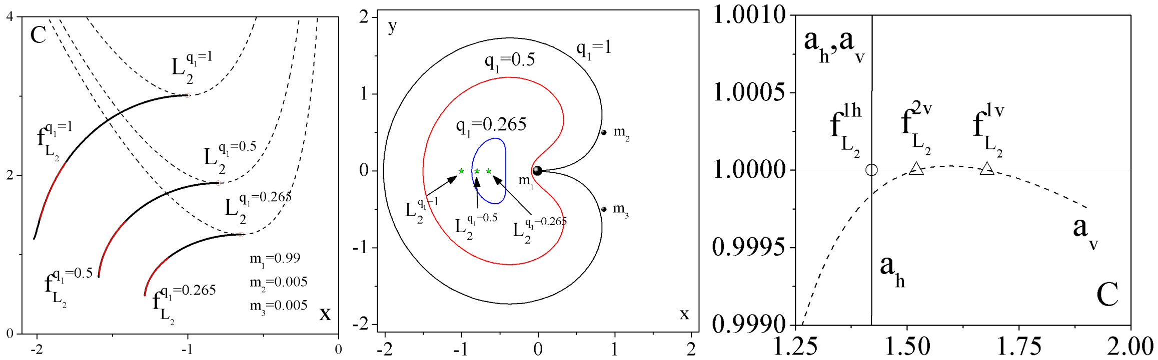

Figure 2: Left: Characteristic curves of families f_{L_2} of simple symmetric periodic orbits around the equilibrium point L_2 for $q_1 = 1$, $q_1 = 0.5$ and $q_1 = 0.265$. Middle: Sample of periodic orbits of f_{L_2} for the same $q_1 = 1, 0.5, 0.265$ and for fixed value of the Jacobian constant $C = 1.2$. Right: The stability diagram of family f_{L_2} for $q_1 = 0.5$.

Lagrangian triangle configuration of the problem (see for details [2]). In the photogravitational case, and for the previous set of masses, we have found in paper [2] that only if $q_1 \in [1, 0.254827)$, then the Lagrangian triangle configuration is linearly stable. In Fig. 1 we present the network of the families of the simple symmetric periodic orbits when the Sun is a source of radiation. In Fig. 1a we have no radiation i.e. we illustrate the network of the families of the gravitational restricted four-body problem. In Figs. 1b and 1c the dominant primary body radiates and the radiation parameter is $q_1 = 0.5$ and $q_1 = 0.265$ correspondingly. The shaded areas are non-accessible to motion due to the Jacobi integral. We calculated, using a standard corrector-predictor procedure, the families emanate from the Lagrangian collinear equilibrium point L_2 for $q_1 = 1$, $q_1 = 0.5$ and $q_1 = 0.265$ correspondingly. In Fig. 2 (left) we present the characteristic curves of these families. The stability of each periodic solution of the families are also studied. The stability arcs of these families are presented by red lines (Fig. 2 (left)). In Fig. 2 (middle) we illustrate three periodic orbits one of each family but all of them they have the same value of the Jacobi constant ($C = 1$) for comparison reasons. A zoomed area of the stability diagram, using the horizontal and the vertical stability parameters, of family f_{L_2} for $q_1 = 0.5$ is also shown in the last figure. It is obvious from the above results that the radiation pressure has a strong effect on the network of the families of the simple symmetric periodic solutions of the problem.

References

[1] Moulton, F. R., 1900, "On a class of particular solutions of the problem of four bodies", *Trans. of the American Math. Soc.*, **1**, pp. 17–29.

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