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Solving the wind equation for relativistic magnetized jets

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Abstract: We approach the problem of bulk acceleration in relativistic, cold, magnetized outflows, by solving the momentum equation along the flow, a.k.a. the wind equation, under the assumptions of steady-state and axisymmetry. The bulk Lorentz factor of the flow depends on the geometry of the field/streamlines and by extension, on the form of the "bunching function" $S = h_\phi^2 B_p/A$ where h_ϕ is the cylindrical distance, B_p the poloidal magnetic field, and A the magnetic flux function. We investigate the general characteristics of the S function and how its choice affects the terminal Lorentz factor γ_∞ and the acceleration efficiency γ_∞/μ, where μ is the total energy to mass flux ratio (which equals the maximum possible Lorentz factor of the outflow). Various fast-rise, slow-decay examples are selected for S, each one with a corresponding field/streamline geometry, with a global maximum near the fast magnetosonic critical point, as required from the regularity condition. As it is proved, proper choices of S can lead to efficiencies greater than 50%. The results of this work, depending on the choices of the flow integral μ, can be applied to relativistic GRB or AGN jets.

1 Introduction

The steady-state magnetohydrodynamic (MHD) equations for Kerr spacetime, after partial integration, reduce to two equations for the proper speed $u = \gamma V_p/c$ and the magnetic flux function A. Projecting the momentum equation along the flow, we derive the so called wind equation, while across the flow we get the transfield or Grad–Shafranov (GS) equation. To simplify our problem, we focus on the solution of the wind equation alone, which gives u as function of distance for a prescribed A function. Working with the 3+1 formalism as in [1, 2], we choose Zero Angular Momentum Observers (ZAMOs), who are at fixed distances from the horizon and are dragged by the hole's rotation, ω being their angular velocity. The wind equation for cold flows is written in dimensionless units as

$$\mu^2\left[\frac{\sigma_{\rm M}S\alpha(1-x_A^2)+u(1-\nu x_A^2)x^2}{\sigma_{\rm M}S\alpha^2-ux^2\alpha-x^2(1-\nu)^2\sigma_{\rm M}S}\right]^2-u^2-1=\mu^2\left[\frac{\alpha x x_A^2 u+x(1-\nu)\sigma_{\rm M}S(x_A^2-1)}{\sigma_{\rm M}S\alpha^2-ux^2\alpha-x^2(1-\nu)^2\sigma_{\rm M}S}\right]^2, \qquad (1)$$

where μ is the total energy to mass flux ratio, $\nu = \omega/\Omega$ with Ω the "field angular velocity", $x = h_\phi\Omega/c$ is the "cylindrical" radius expressed in light surface units, x_A is the Alfvénic lever arm, and $\sigma_{\rm M}$ is the Michel's magnetization. The lapse function α measures the gravitational redshift of the ZAMO clocks. Note that the bunching function is connected to the function A through $S = h_\phi|\nabla A|/A$.

A simplified version of the wind equation can be found as follows: Assuming that the ϕ-component of the velocity is $V_\phi \ll h_\phi\Omega$ far from the source, we may write the total energy to mass flux ratio as

$$\mu \approx \gamma + \sigma_{\rm M}S\frac{\gamma}{\sqrt{\gamma^2-1}}\,. \qquad (2)$$

Differentiating the above equation we get

$$\frac{d\gamma}{dx} = -\gamma^2\sigma_{\rm M}(\gamma^2-1)^{1/2}\frac{dS/dx}{\gamma^3-\mu}\,, \qquad (3)$$

an expression which becomes 0/0 at the fast magnetosonic critical point. As we can see from Eq. (3), for $\gamma < \mu^{1/3}$ the S function increases, while for $\gamma > \mu^{1/3}$ it decreases, for an accelerating flow. Therefore, the most realistic approach is to use a fast-rise & slow-decay function $S(x)$.

Finally we note that working with Eq. (2) in the superfast region, where $\gamma \gg 1$, we find the accelaration efficiency $\mathbf{a} = \gamma_\infty/\mu \approx (1 - S_\infty/S_{max})$.

2 Results

We used the following forms of A, all corresponding to asymptotically cylindrical jets:

Flux Function	μ	$\sigma_{\rm M}$	$\mathbf{a}\,(\%)$
$A_1 = r\left\{\left(\csc^2\theta\right)\left(\tan\frac{\theta}{2}\right)^{3\cos\theta}\right\}$	100	55.23	53.1
$A_2 = A_1$ (special relativistic case)	100	56.28	53.4
$A_3 = r\tan\theta$	100	58.81	50.1
$A_4 = r\left\{e^{3.3-3.3ln\lvert\tan(\theta/2)\rvert\tan(\theta/2)}\sin\theta\right\}$	100	49.32	66.63

The above Table also shows the values of μ, $\sigma_{\rm M}$ and efficiency $\mathbf{a}\,(\%)$ for each flux function. The corresponding bunching functions $S = h_\phi|\nabla A|/A$ are shown in the left panel of Fig. 1. In all cases except A_3 they have a fast-rise & slow-decay form, with the maximum corresponding to the approximate location of the fast magnetosonic point, where $S_{max} \approx \mu/\sigma_M$. In the case A_3 the bunching function monotonically decreases, as in [3]. In this case the fast magnetosonic point is located close to the Alfvén point and the approximations that led to Eq. (2) do not hold.

As seen both in Fig.1 and the Table, efficiencies $\gamma_\infty/\mu \approx u_\infty/\mu$ of the order of $\sim 50\%$ are possible, with the corresponding outflow to be cylindrical jets, as expected. It is obvious that the greater (/lower) S_{max} ($/S_\infty$) becomes, the greater efficiency gets. As in our previous work (see [4]), similar results are shown in solutions of the full set of MHD equations, including the transfield equation (see [5] and [6]).

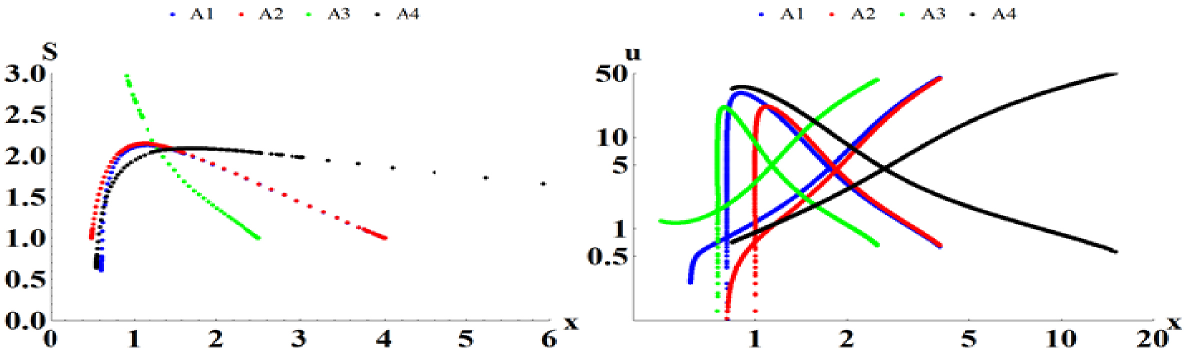

Figure 1: *Left panel*: The bunching functions for the above choices of A. *Right panel*: The corresponding solutions of the wind equation in the u–x plane. The solutions $u(x)$ of interest go from low values of u at small distances, via the fast point (an X-type critical point), to the terminal values u_∞ at x_∞.

References

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