11th Hel.A.S Conference
Athens, 9-12 September, 2013
CONTRIBUTED POSTER

Computing polytropic models obeying specific metrics

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Abstract: We apply zatrikean pregeometry to polytropic stars, find the corresponding zatrikean Lane-Emden equations, solve them numerically and present the results on the boundary conditions and the interior structure of the zatrikean polytropes.

Summary

Applying the hydrostatic equilibrium in a spherical symmetrical body, we get the classic equations of ref. [1] §3.1. By further applying the pregeometric transformation and the mass-energy equivalence, finally leads to:

$$\frac{dP}{dr} = \frac{\rho c^2 + P}{\Psi_0 \gamma} \frac{d\Psi}{dr} \tag{1}$$

$$\frac{dm}{dr} = 4\pi r^2 (\rho + \frac{P}{c^2})(1 + \Psi - r\frac{d\Psi}{dr})(1+\Psi)^{-4} \tag{2}$$

$$\frac{d\Psi}{dr} = \Psi_0 \gamma \frac{G}{c^2 r}(\frac{dm}{dr} - \frac{m}{r} - 4\pi r^2(\rho + \frac{P}{c^2})) \tag{3}$$

The eqs. (1), (2), (3) are the equations of zatrikean hydrostatic equilibrium for a spherical symmetrical body.

We now limit our study to objects where the polytropic equations of states holds (ref. [1] §4.1) and introduce the well-known dimensionless polytropic units θ and ξ. We derive the equations:

$$\frac{d\theta}{d\xi} = \frac{1 + \sigma\theta}{\Psi_0 \gamma \sigma (n+1)} \frac{d\Psi}{d\xi} \tag{4}$$

$$\frac{d\upsilon}{d\xi} = \xi^2 \theta^n (1 - \frac{\xi}{1+\Psi} \frac{d\Psi}{d\xi})(1+\Psi)^{-3} \tag{5}$$

$$\frac{d\Psi}{d\xi} = \Psi_0 \gamma \sigma (n+1) \frac{1}{\xi}(\frac{d\upsilon}{d\xi} - \frac{\upsilon}{\xi} - \xi^2 \theta^n (1 + \sigma\theta)) \tag{6}$$

From the definition of θ and υ we have:

$$\theta(0) = 1 \tag{7}$$

$$\upsilon(0) = 0 \tag{8}$$

and we can deduce that for the centre of the polytrope:

$$\Psi(0) = \Psi_0 \gamma \sigma (n+1)(\frac{\upsilon(\xi_1)}{\xi} + \frac{\ln(1+\sigma)}{\sigma}) \tag{9}$$

The eqs. (4), (5), (6), with the initial conditions (7), (8), (9), are the zatrikean polytropic equations. Of course the system cannot be solved analytically. The numerical solution was tackled with the modification of a program originally used to calculate the boundary conditions of polytropic relativistic spheres (ref. [2]). It uses a Adams-Bashford-Moulton method with variable step and order (ref. [4])

modified to work on the complex plane. To find ξ_1 it uses the Secant method on the real part of the complex values calculated by the integration algorithm.

The mean density of zatrikean polytropic stars is:

$$\frac{\rho_0}{\bar{\rho}} = \frac{\xi_1^3}{3\upsilon(\xi_1)} \tag{10}$$

While from the speed of sound in the interior we deduce:

$$(\frac{v}{c})^2 = \frac{n+1}{n}\sigma\theta \tag{11}$$

This ratio should be less than 1. Thus, for all the solutions of the zatrikean polytropic equations, only the ones with $v/c \leq 1$ are valid. As θ is maximum at the centre of the star we get the limit:

$$\sigma \leq \frac{n}{n+1} \tag{12}$$

Contrary to the classical case, there are combinations of mass and radius for a zatrikean polytrope that are just not possible. The polytrope has the maximum ration of M/R at the maximum σ allowed for its polytropic index n. The dynamic energy of a zatrikean polytrope is:

$$-\Omega = \frac{3}{5-n}\frac{(\sigma c^2)^{(3-n)/2}(n+1)^{3/2}K^{n/2}}{(4\pi)^{1/2}G^{3/2}}\frac{\upsilon^2(\xi_1)}{\xi_1}$$

which in Mc^2 units becomes:

$$-\frac{\Omega}{Mc^2} = \frac{3}{5-n}\sigma(n+1)\frac{\upsilon(\xi_1)}{\xi_1} \tag{13}$$

Although this is valid only in the classic limit $\sigma \to 0$, it is a good first order approximation.

The interior structure of polytropes with $n < 3$ has small differences even for large variations of σ. Increasing σ increases the dilation Ψ in the interior of the polytrope, while the pressure P and the density ρ are decreasing faster from the centre of the polytrope towards its surface. However, the mass distribution does not change drastically. The variability due to n follows the same pattern. The interior structure of polytropes with $n > 3$ has large differences even for small variations of σ. Changes in mass distribution and dilation are faster for $n > 3$ than for $n < 3$, while the pressure P and the density ρ are changing slower for $n > 3$ than for $n < 3$.

References

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