A Data-Driven, Integrated Flare Model Based on Self-Organized Criticality

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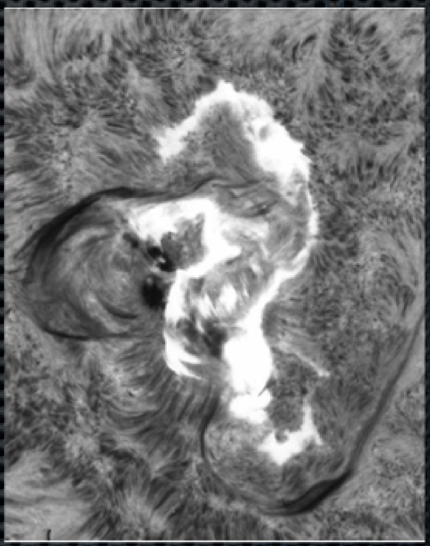
From the sandpile to the Sun…

The BTW model

Bak, Tang & Wiesenfield (1987)

The LH model

Lu, Hamilton (1991)



$\bar{h}(t) = \frac{1}{4}[h(x+1,y,t) + h(x-1,y,t) + h(x,y+1,t) + h(x,y-1,t)]$ $G(x,y,t) = h(x,y,t) - \bar{h}(t)$	**Control parameter**	$\Delta\mathbf{B}_i = \mathbf{B}_i - \frac{1}{6}\sum_{nn}\mathbf{B}_{nn}$
$h(x,y,t) \rightarrow h(x,y,t) + 1$	**Transition rules**	$\mathbf{B}_i \rightarrow \mathbf{B}_i + \delta\mathbf{B}, \; \lvert\delta\mathbf{B}\rvert \ll B_c$
$G(x,y,t) > G_c$	**Instability criterion**	$\lvert\Delta\mathbf{B}_i\rvert > B_c$
$h(x,y,t+1) = h(x,y,t) - 4$ $h(x\pm1,y,t+1) = h(x\pm1,y,t) + 1$ $h(x,y\pm1,t+1) = h(x,y\pm1,t) + 1$	**Relaxation rules**	$\mathbf{B}_i \rightarrow \mathbf{B}_i - \frac{6}{7}\Delta\mathbf{B}_i,$ $\mathbf{B}_{nn} \rightarrow \mathbf{B}_{nn} + \frac{1}{7}\Delta\mathbf{B}_i$

SOC: Concept & misconceptions

Self-Organized Criticality*: open systems self-organize in a statistical stationary state in which a phase transition* far from equilibrium *is undergone; by means of:*

- *slow-driving,*
- *critical threshold,*
- *time scale separation,*
- *metastability, and*
- *boundary dissipation*

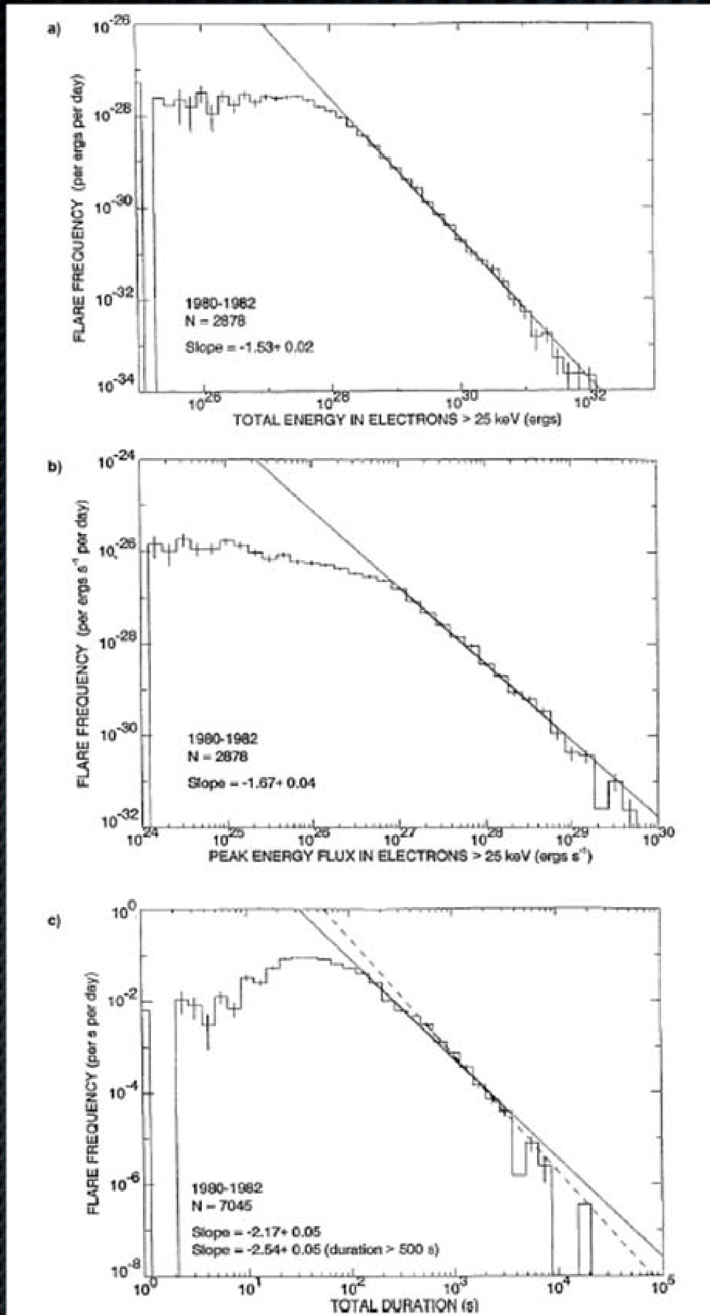
the dynamics of the system leads it to a critical state in which scale invariance is present in all the observables.

SOC Diagnostics:

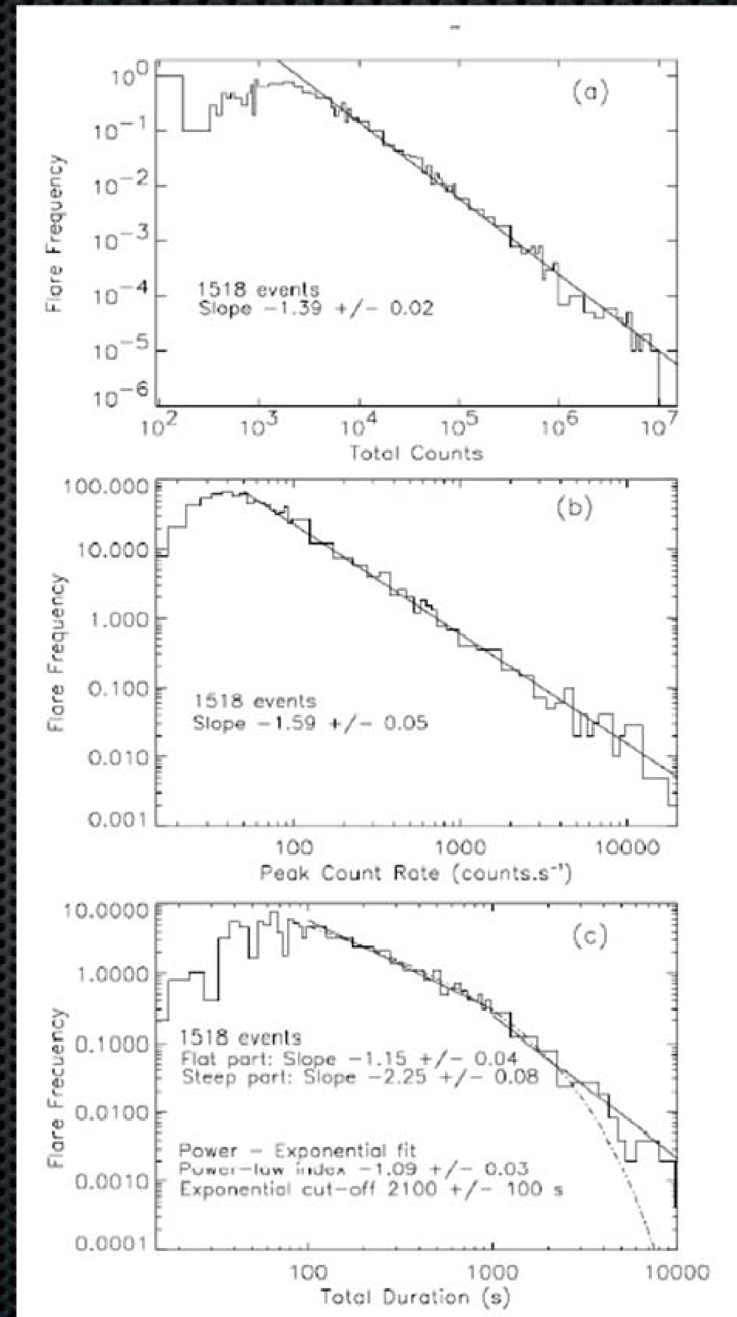
- "control parameter": stabilization of mean value just below the critical threshold
- avalanches: well defined mean number in equal time intervals
- long-term/extensive temporal/spatial two-point-correlations → scale invariance/self-similarity

long-term memory **fractality** **power-law scaling**

SOC in Solar Flares?



Crosby et al, 1993
(HXRBS/SMM)



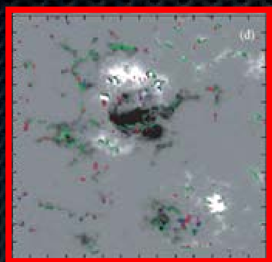
Georgoulis et al, 2001
(WATCH/GRANAT)

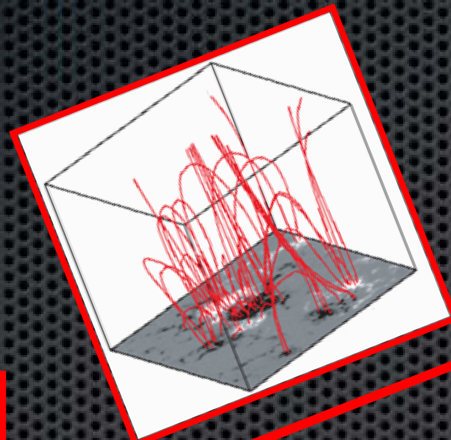
The Static IFM (S-IFM)

Dimitropoulou et al., 2011

- **Dataset:** 11 AR photospheric vector magnetograms from IVM
  - spatial resolution of 0.55 arcsec/pixel
  - 180 azimuthal ambiguity removed (Non-Potential magnetic Field Calculation)
  - rebinned into a 32x32 grid

- **Model:**

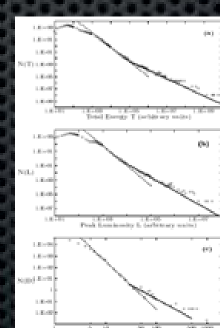




DISCO:
INSTABILITIES SCANNER
Identifies an instability when one site exceeds a critical threshold in the magnetic field Laplacian
If YES, HANDOVER to RELAX
If NO, HANDOVER to LOAD

EXTRA:
MAGNETIC FIELD EXTRAPOLATOR
Reconstructs the 3D configuration
NLFF optimization (Wiegelmann 2008):
Lorentz force + B divergence minimization
HANDOVER to DISCO

RELAX:
Magnetic Field Re-distributor (reconnection)
Redistributes the magnetic field according to predefined relaxation rules
(Lu & Hamilton 1991)
HANDOVER to DISCO



LOAD:
Driver (photospheric convection / plasma upflows)
Adds a magnetic field increment of random magnitude (though obeying to specific "physically" imposed rules) to a random site of the grid,
HANDOVER to DISCO

The physics behind the "S-IFM" (1)

1. EXTRA: a nonlinear force-free extrapolation module

$$L = \int_V w(x,y,z)[|\mathbf{B}|^{-2}|(\nabla \times \mathbf{B}) \times \mathbf{B}|^2 + |\nabla \cdot \mathbf{B}|^2]d^3x$$

- ✓ **Lorentz force minimization**
- ✓ **Magnetic field divergence minimization**

Wiegelmann, 2008

The physics behind the "S-IFM" (2)

2. DISCO: a module to identify magnetic field instabilities

- ✓ Selection of critical quantity (magnetic field stress = magnetic field Laplacian):

$$G_{av}(r) = |\mathbf{G}_{av}(\mathbf{r})|$$

where:

$$\mathbf{G}_{av}(\mathbf{r}) = \mathbf{B}(\mathbf{r}) - \frac{1}{nn}\sum_{nn}\mathbf{B}_{nn}(\mathbf{r}) \quad (1)$$

- ✓ Why this? It favors reconnection
- ✓ Is there an underlying physical meaning behind this selection?

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \rightarrow \nabla^2 \mathbf{B}(\mathbf{r}) = (\nabla^2 B_x)\hat{\mathbf{i}} + (\nabla^2 B_y)\hat{\mathbf{j}} + (\nabla^2 B_z)\hat{\mathbf{k}}$$

Induction Equation

↓ central finite difference

$$\nabla^2 \mathbf{B}(\mathbf{r}) \simeq \sum_{nn}\mathbf{B}_{nn}(\mathbf{r}) - nn\mathbf{B}(\mathbf{r})$$

↓ (1)

$$\nabla^2 \mathbf{B}(\mathbf{r}) \simeq -nn\mathbf{G}_{av}(\mathbf{r})$$

Resistive term dominates,
in the presence of local currents

- ✓ Threshold determination:

$$G_{cr} = 10G$$

The physics behind the "S-IFM" (3)

3. RELAX: a redistribution module for the magnetic energy

- ✓ **Selection of redistribution rules:**

$$B^{+}(r) \rightarrow B(r) - \frac{6}{7}G_{av}(r)$$ and $$B^{+}_{nn}(r) \rightarrow B_{nn}(r) + \frac{1}{7}G_{av}(r)$$

Lu & Hamilton 2001

- ✓ **Do these rules meet basic physical requirements, like magnetic field zero divergence?**

$$\nabla \cdot B^{+}(r) \simeq \frac{1}{7}\nabla \cdot B(r) - \frac{1}{7nn}\nabla \cdot B_{nn}(r)$$

$$\nabla \cdot B^{+}_{nn}(r) \simeq \frac{1}{7}\nabla \cdot B(r) + \frac{1}{7nn}\nabla \cdot B_{nn}(r)$$

$$\nabla \cdot B(r) \simeq \nabla \cdot B_{nn}(r) \simeq 0$$

NLFF extrapolation

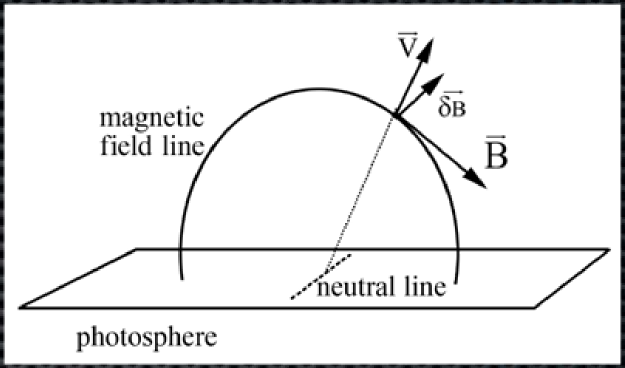
$$\nabla \cdot B^{+}(r) \simeq \nabla \cdot B^{+}_{nn}(r) \simeq 0$$

Magnetic field retains divergence freedom

The physics behind the "S-IFM" (4)

4. LOAD: the driver

✓ **Selection of driving rules:**



1.Magnetic field increment perpendicular to existing site field

$$B(r) \cdot \delta B(r) = 0$$

- localized Alfvenic waves
- convective term of induction eq:

$$\nabla \times (V \times B)$$

2.Slow driving

$$\frac{|\delta B(r)|}{|B(r)|} = \epsilon, \epsilon < 1$$

- the slower the driving, the longer the average waiting time

3. Divergent free magnetic field during the driving process

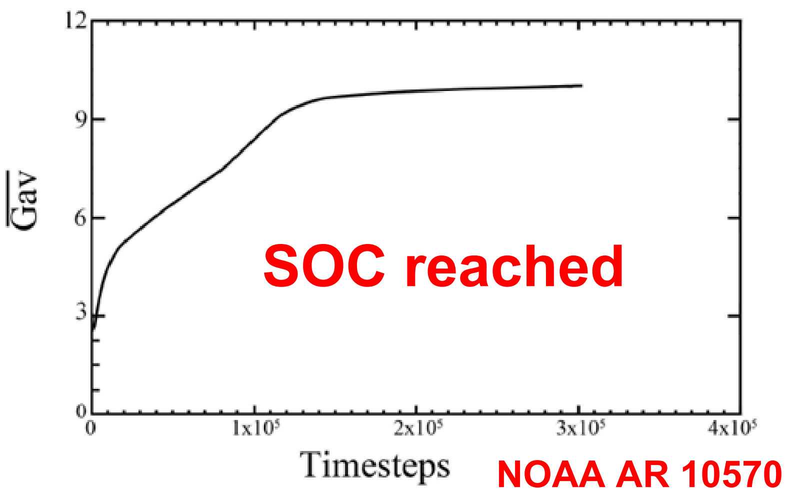
$$\nabla \cdot (B(r) + \delta B(r)) = 0$$

- only a first degree approximation → need to monitor the departure from the divergence-free condition:

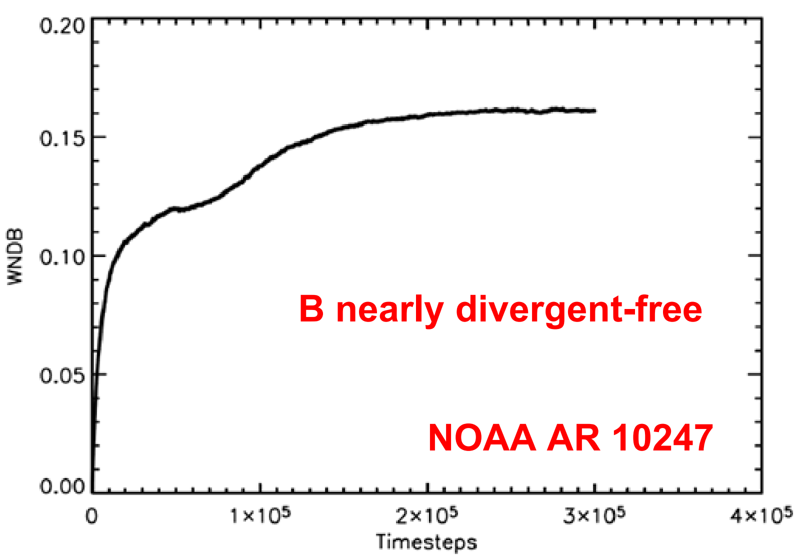
- tolerate a 20% departure

$$WNDB = \frac{|\nabla \cdot B|}{\sqrt{3}\sqrt{(\frac{\partial B_x}{\partial x})^2+(\frac{\partial B_y}{\partial y})^2+(\frac{\partial B_z}{\partial z})^2}}$$

“S-IFM” results (1)



FLARE DURATION (model)						
	Double Power Law fit				Power Law with Exponential Rollover	
	Flat PL		Steep PL			
AR	PL Index	*Probability*	PL Index	*Probability*	PL Index	*Probability*
9415	...	...	...	...	−1.42 ± 0.18	0.95
9635	−2.29 ± 0.19	0.96	−5.28 ± 0.42	0.95	...	...
9661	...	...	...	...	−0.26 ± 0.05	0.94
9684	...	...	...	...	−0.91 ± 0.09	0.95
9845	...	...	...	...	−1.12 ± 0.06	0.95
10050	−1.80 ± 0.18	0.98	−4.03 ± 0.29	0.95	...	...
10247	...	...	...	...	−1.27 ± 0.07	0.95
10306	...	...	...	...	−1.27 ± 0.09	0.94
10323	...	...	...	...	−0.98 ± 0.05	0.95
10488	−1.60 ± 0.16	0.94	−3.64 ± 0.19	0.94	...	...
10570	...	...	...	...	−0.83 ± 0.07	0.95
MEAN	−1.90		−4.32		−1.01	
σ^2	0.35		0.86		0.36	

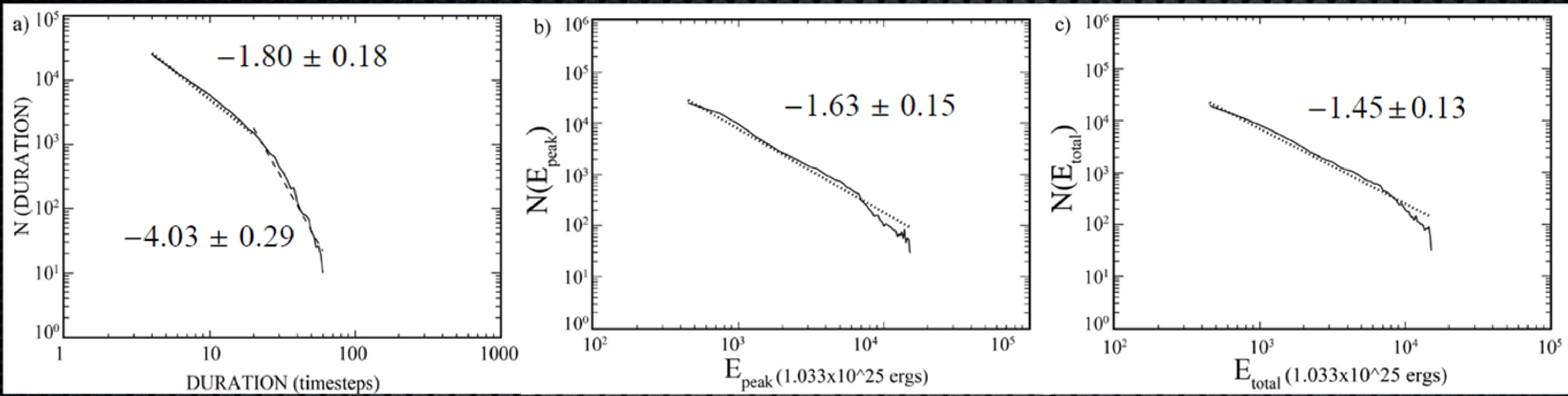


FLARE TOTAL ENERGY (model)		
Single Power Law fit		
AR	PL Index	*Probability*
9415	−1.50 ± 0.13	0.95
9635	−2.22 ± 0.19	0.98
9661	−1.27 ± 0.05	0.99
9684	−1.43 ± 0.07	0.99
9845	−1.69 ± 0.17	0.95
10050	−1.45 ± 0.13	0.95
10247	−1.89 ± 0.17	0.98
10306	−1.23 ± 0.08	0.99
10323	−1.45 ± 0.16	0.98
10488	−1.54 ± 0.13	0.95
10570	−1.45 ± 0.08	0.99
MEAN	-1.56	
σ^2	0.28	

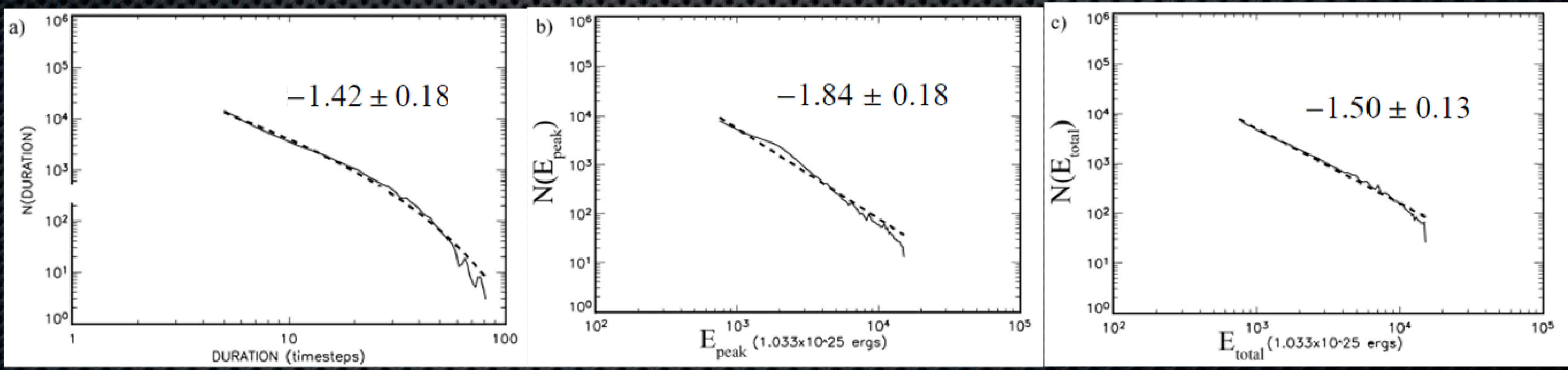
FLARE PEAK ENERGY (model)		
Single Power Law fit		
AR	PL Index	*Probability*
9415	−1.84 ± 0.18	0.95
9635	−2.62 ± 0.17	0.97
9661	−1.42 ± 0.15	0.98
9684	−1.70 ± 0.17	0.97
9845	−1.85 ± 0.12	0.95
10050	−1.63 ± 0.15	0.95
10247	−2.15 ± 0.12	0.98
10306	−1.61 ± 0.16	0.97
10323	−1.72 ± 0.17	0.97
10488	−1.59 ± 0.14	0.95
10570	−1.63 ± 0.15	0.98
MEAN	−1.80	
σ^2	0.33	

"S-IFM" results (3)

NOAA AR 10050

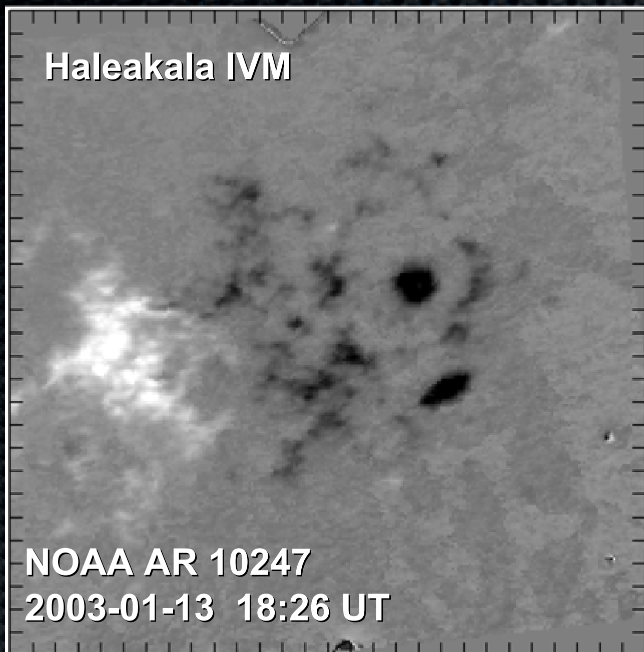


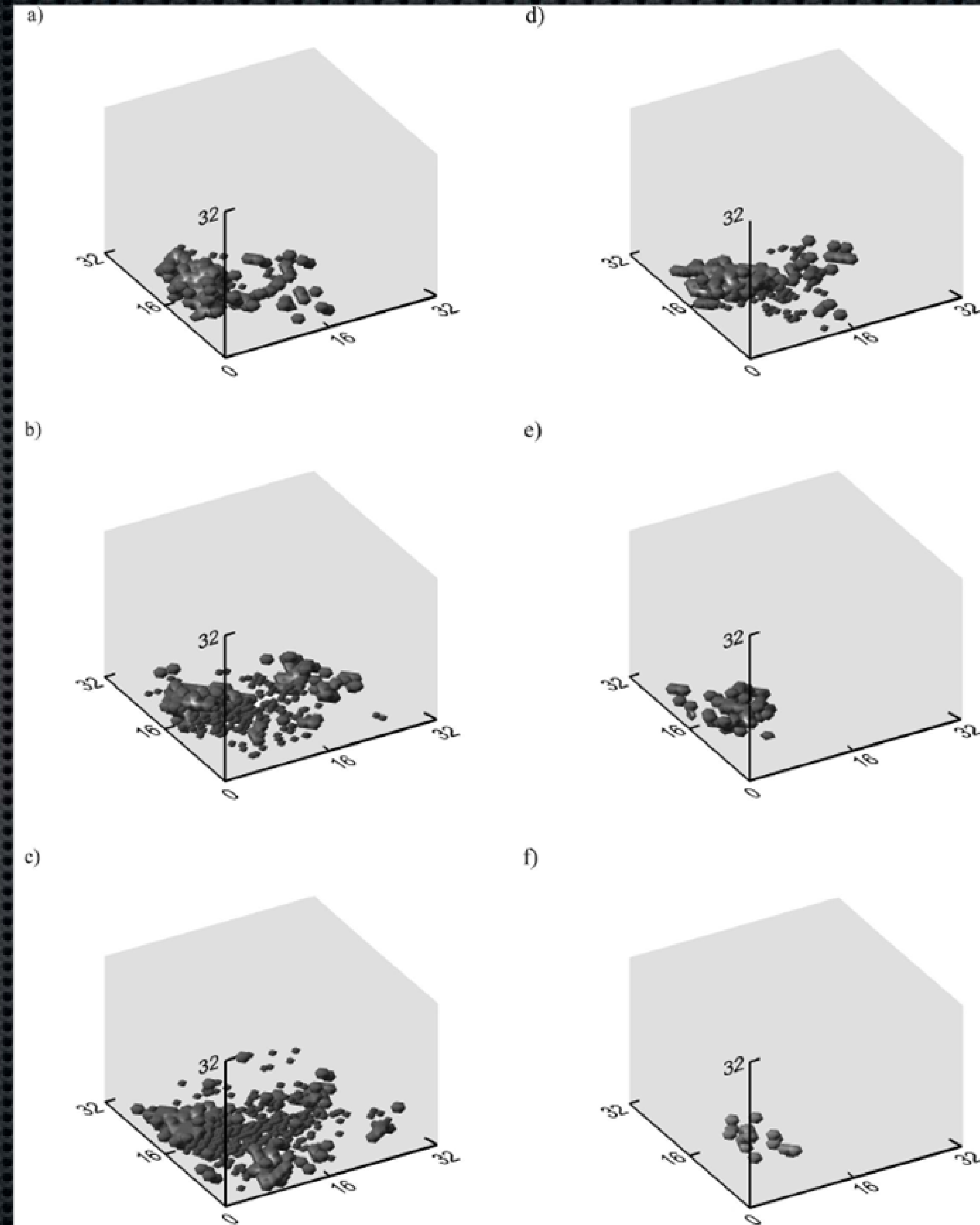
NOAA AR 9415



“S-IFM” results (4)

NOAA AR 10247





Static! (arbitrary time units → model timesteps for all time scales)

The Dynamic IFM (D-IFM)

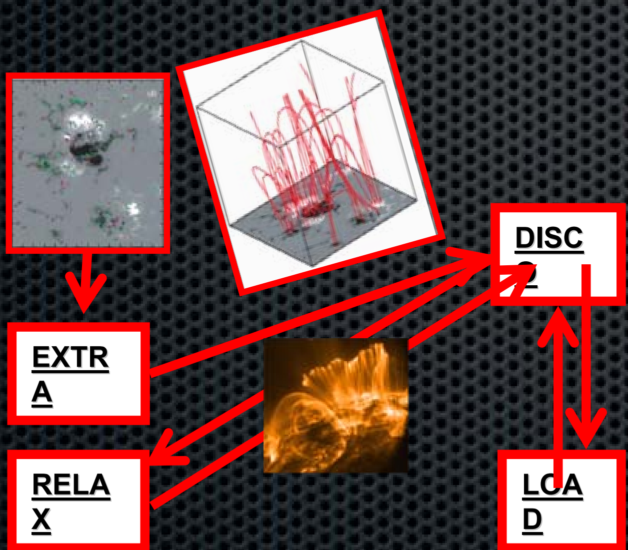
- **Dataset:** 7 subsequent photospheric vector magnetograms from IVM **(NOAA AR 8210)**
  - spatial resolution of 0.55 arcsec/pixel
  - 180 azimuthal ambiguity removed (Non-Potential magnetic Field Calculation)
  - rebinned into a 32x32 grid

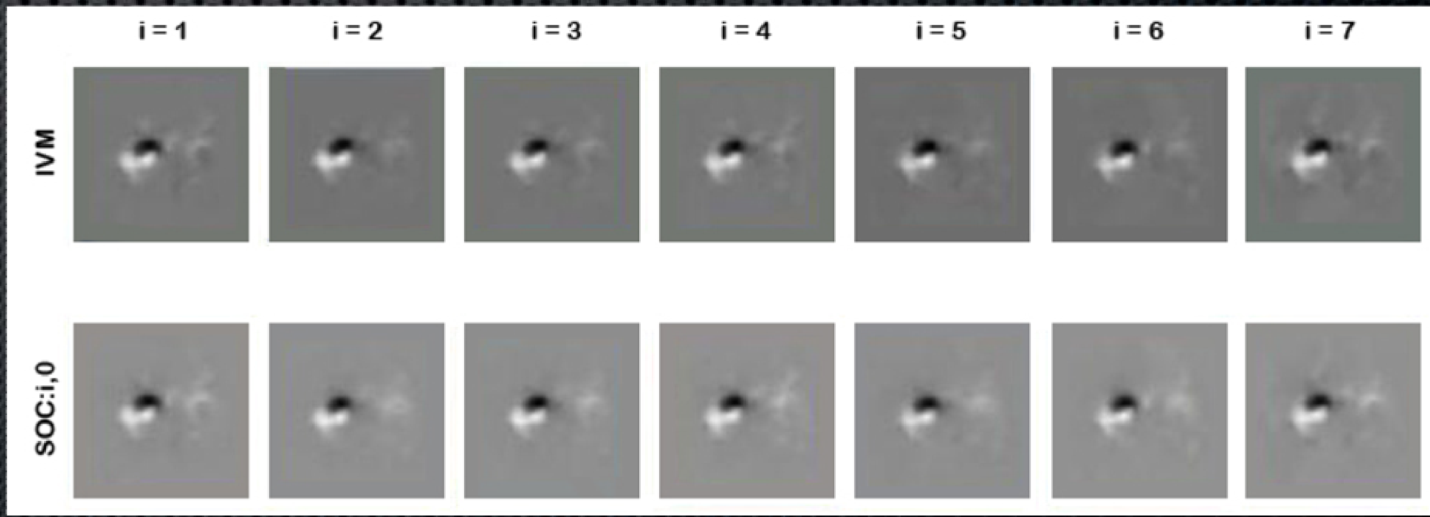
Dimitropoulou et al., 2013

Magnetogram no (i)	UT Time	D-IFM Time t (sec)
1	18 : 58	0
2	19 : 43	2700
3	20 : 14	4560
4	21 : 20	8520
5	22 : 08	11400
6	28 : 38	13200
7	23 : 16	15480

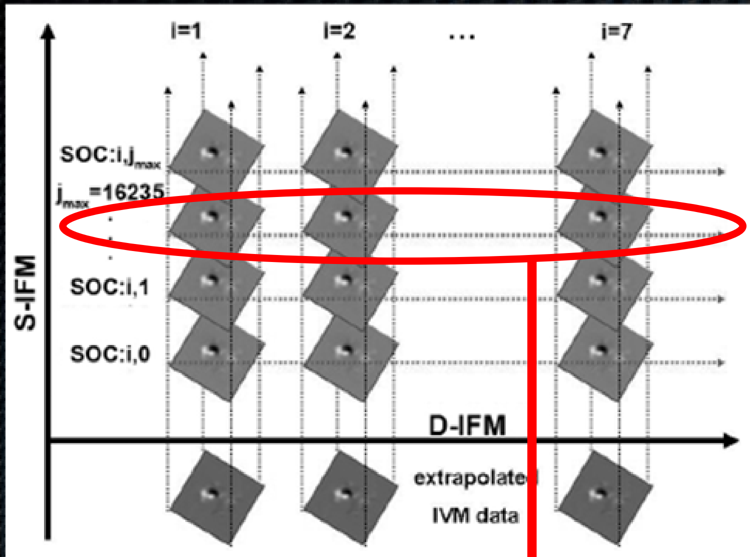
Index	Flare Start Time (UT)	Flare Peak Time (UT)	Flare End Time (UT)	Flare Class
1	20 : 08	20 : 30	20 : 35	$C2.8$
2	21 : 40	21 : 51	21 : 59	$C2.6$
3	22 : 36	22 : 54	23 : 08	$M1.2$

- **Getting going:**





The Dynamic IFM (D-IFM)



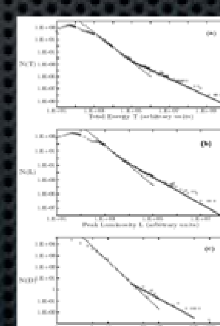
Dimitropoulou et al., 2013

50 additional S-IFM avalanches

DISCO:
INSTABILITIES SCANNER
Identifies an instability when one site exceeds a critical threshold in the magnetic field Laplacian
If YES, HANDOVER to RELAX
If NO, HANDOVER to INTER

INTER:
Driver (photospheric convection / plasma upflows)
Adds magnetic increments in multiple sites following a spline interpolation from one magnetic snapshot to the next
HANDOVER to DISCO

RELAX:
Magnetic Field Re-distributor (reconnection)
Redistributes the magnetic field according to predefined relaxation rules
(Lu & Hamilton 1991)
HANDOVER to DISCO



16236- 7- member SOC groups
i = 0,1,2,…,6
j = 0, 1,2,…16235
90971 avalanches

The physics behind the "D-IFM"

1. INTER: a magnetic field interpolator acting as driver

- ✓ Cubic spline interpolation for all transitions SOC:i,j → SOC:i+1,j of the same sequence j
- ✓ Interval τ between 2 interpolation steps:
  - ✓ 32x32x32 grid dimensions
  - ✓ IVM spatial resolution 0.55 arcsec/pixel
  - ✓ pixel size = 8.8 arcsec
  - ✓ grid site linear dimension = 6.4Mm
  - ✓ Alfven speed (1st approximation, coronal height) = 1000km/s
  - ✓ τ = 6.4 sec
- ✓ Multisite driving
- ✓ No avalanche overlapping
- ✓ MHD timestamps (t) on the avalanche onset
- ✓ Instant relaxation (MHD time t stops)

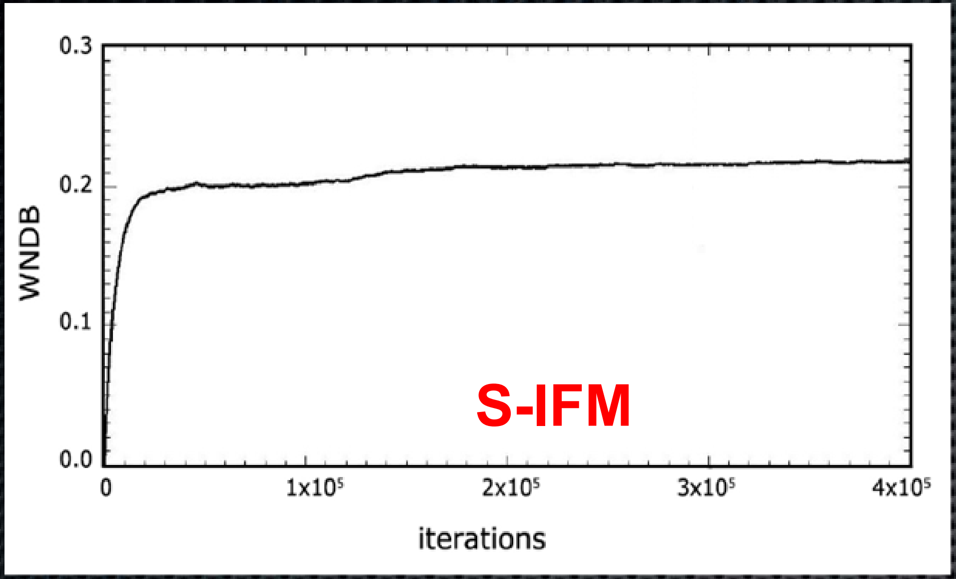
k	SOC:i,j → SOC:i+1,j	$t_{(SOC:i+1,j)} - t_{(SOC:i,j)}$ (sec)	s_k
1	1 → 2	2700	421
2	2 → 3	1860	290
3	3 → 4	3960	618
4	4 → 5	2880	450
5	5 → 6	1800	281
6	6 → 7	2280	356

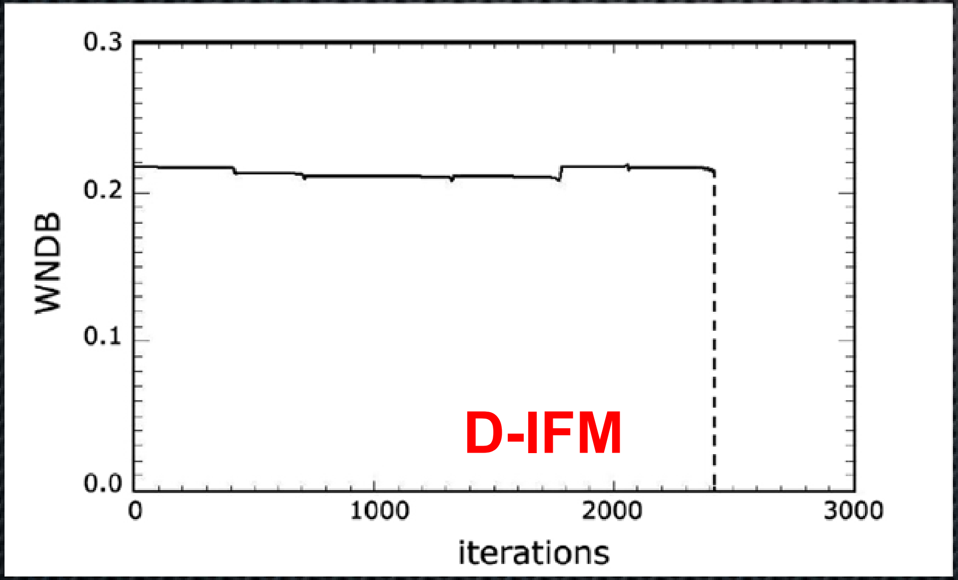
- ✓ Monitoring of the magnetic field divergence

$$WNDB = \frac{|\nabla \cdot B|}{\sqrt{3}\sqrt{(\frac{\partial B_x}{\partial x})^2+(\frac{\partial B_y}{\partial y})^2+(\frac{\partial B_z}{\partial z})^2}}$$

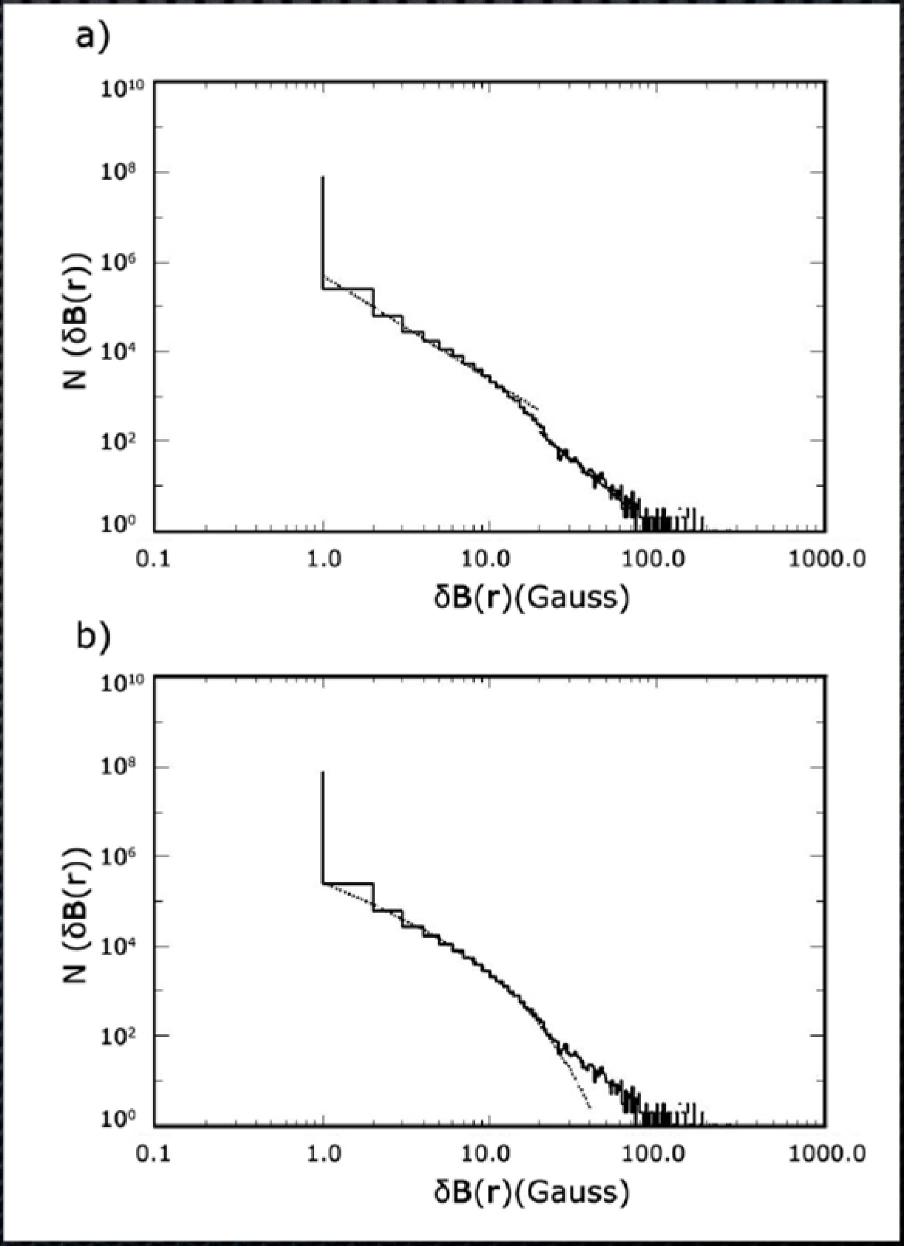
"D-IFM" results (1)

✓ Is B retained nearly divergent-free?

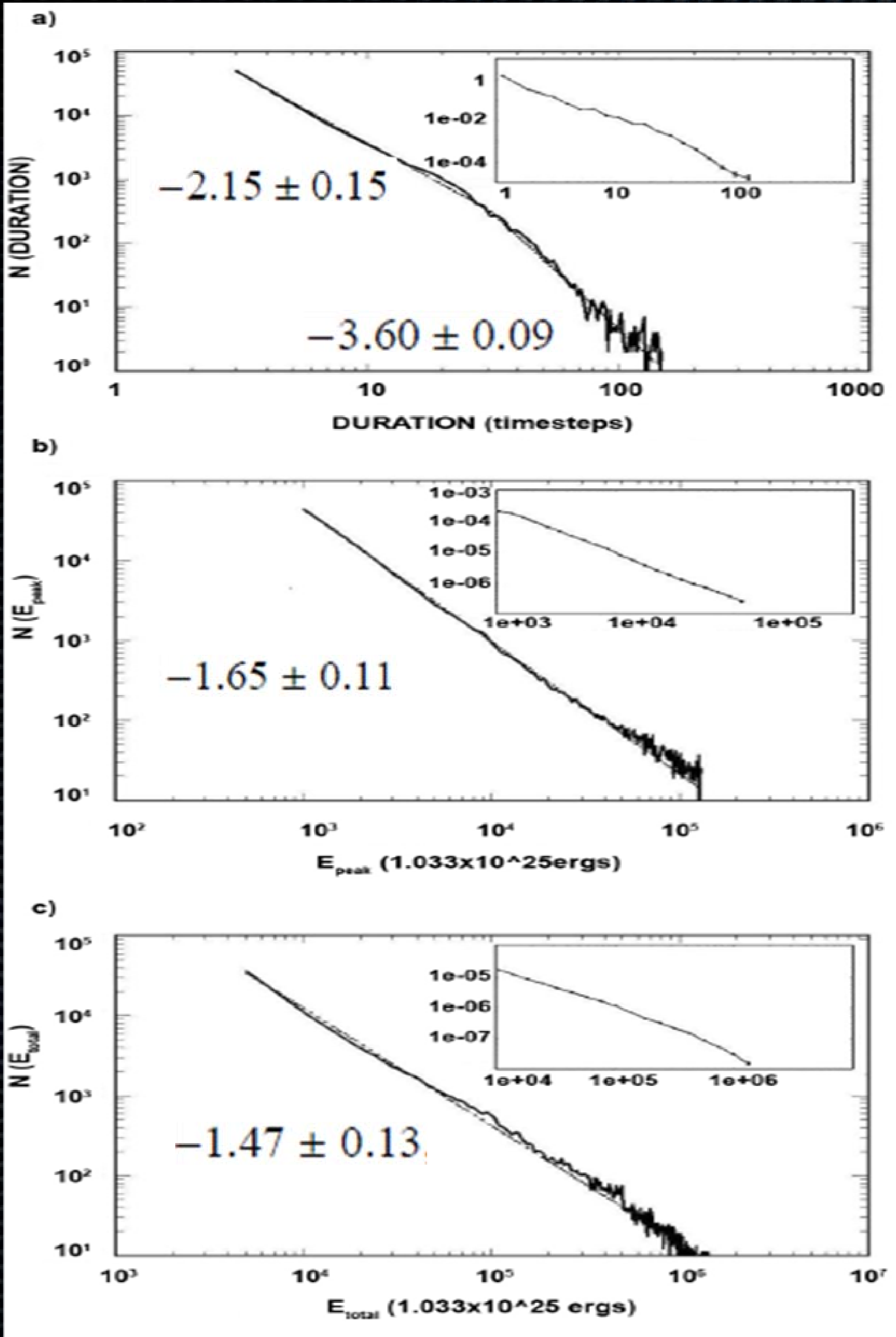


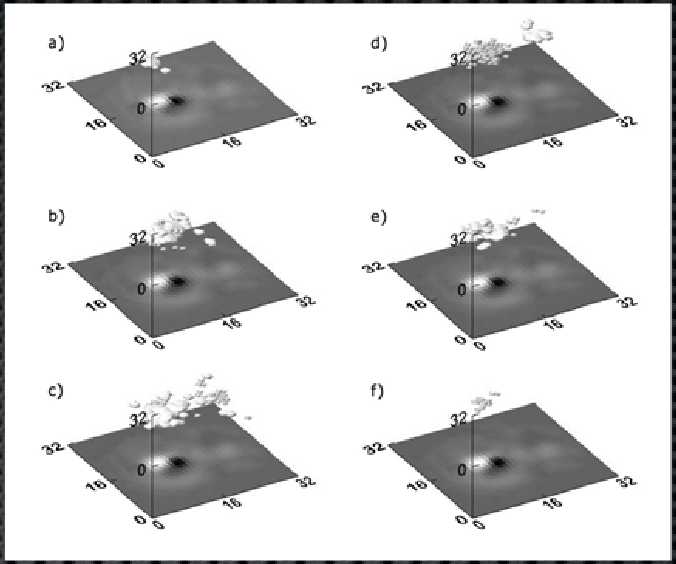


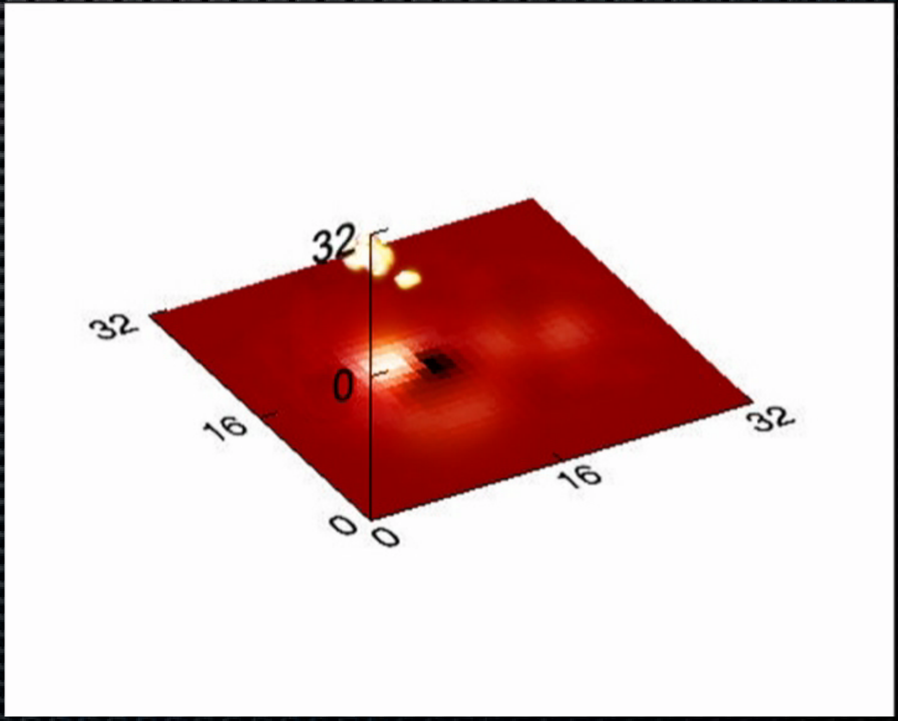
✓ How does the driver behave?



“D-IFM” results (2)







Conclusions

Why IFM?

✓ Data-driven
✓ Physical units for energies (peak and total)
✓ Physical units for the MHD time-scale (D-IFM)
✓Attempts to simulate physical processes:
    ✓ Alfvenic waves / plasma upflows
    ✓ Diffusion
    ✓ Data-based driving mechanism (D-IFM)
✓ Attempts to fulfill principal physical requirements (zero B divergence)

SOC CA models complement the MHD approach, by reproducing the global statistics of the physical processes at play