11th Hellenic Astronomical Conference
RCAAM Academy of Athens, 8-12 September 2013

Dynamical formation of resonant planetary systems with high mutual inclination

Kleomenis Tsiganis
Section of Astrophysics, Astronomy and Mechanics,
Aristotle University of Thessaloniki, GR.

Collaborators:
K. Antoniadou and **G. Voyatzis**

Dedicated to the memory of J.D. Hadjidemetriou

European Union
European Social Fund
OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS, CULTURE & SPORTS
MANAGING AUTHORITY
Co-financed by Greece and the European Union
NSRF 2007-2013
programme for development
EUROPEAN SOCIAL FUND
AUTH
Research Committee

Overview - Motivation

- There are ~240 planetary systems with 2 or more planets (~730 planets)

- Some systems have planets *in resonance* (mainly 2/1, also 3/1)

- Lee & Thommes (2009) → 2/1 inclination resonance
- Libert & Tsiganis (2009): capture and inclination excitation possible in the 2/1, 3/1, 4/1 and 5/1 resonances

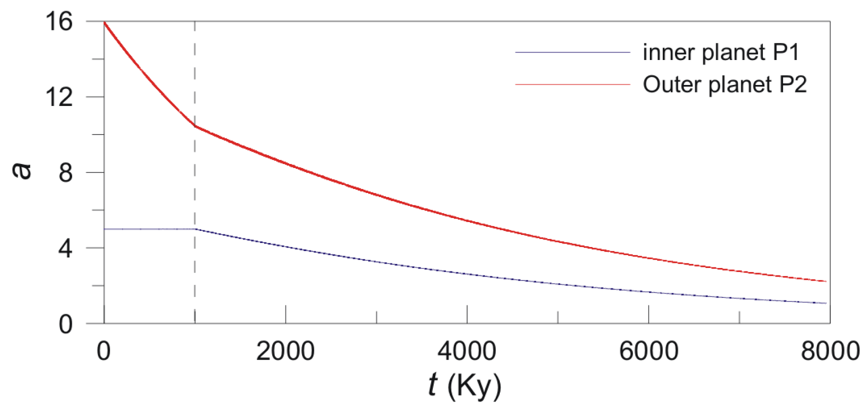
Q: do such '3-D' (stable) systems exist ?

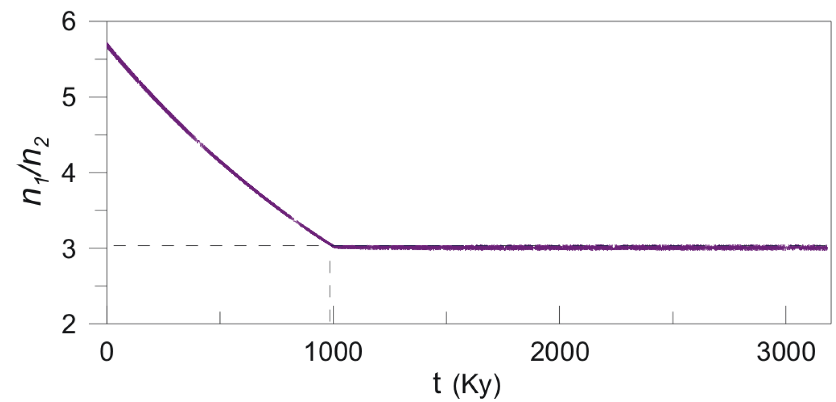
- Also important to understand the distribution of non-resonant systems (resonance → excitation → pl-pl scattering → “new” system forms)

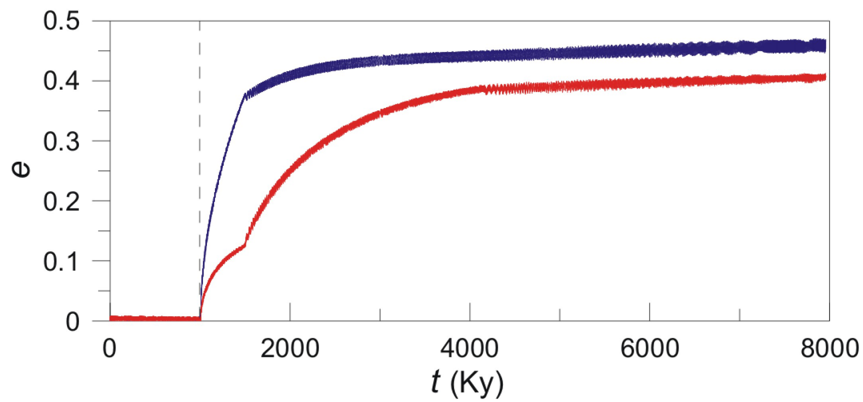
Q: could we 'recognize' such systems observationally (i.e. constraints) ?

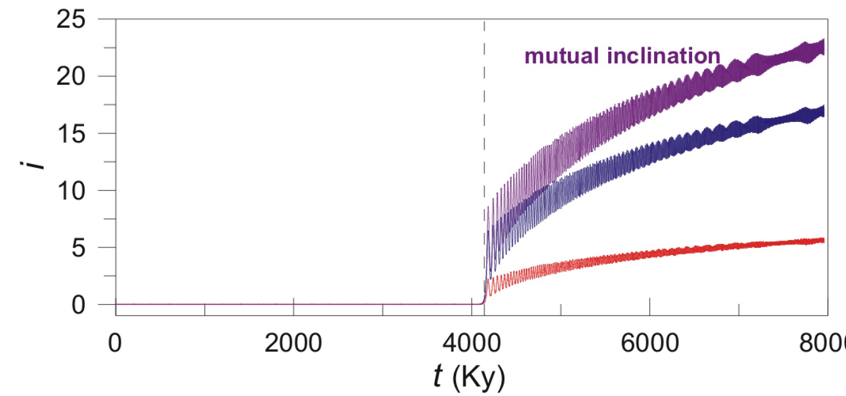
Planet migration and Resonant capture

Planets migrating at different rates → mean motion ratio changes → if converging, resonance trapping can occur → planets then migrate as a pair, while keeping the MMR n_1/n_2 ~fixed.









Adiabatic resonance response → eccentricity growth (damping?)

Goal: understand what conditions need to be met for *i* to grow

Capture and evolution in resonance: 2-D case

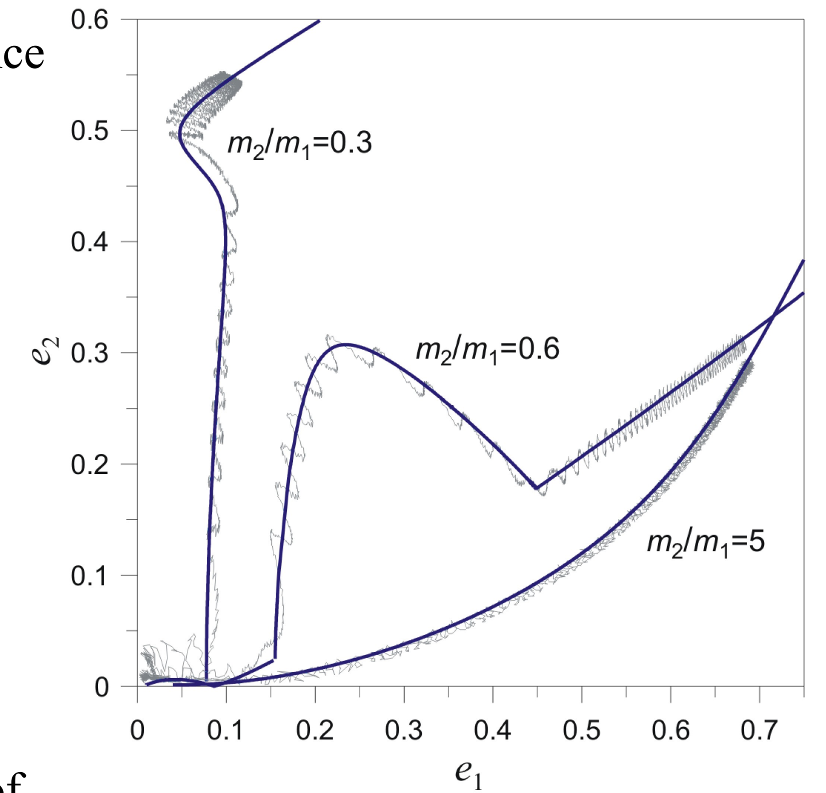
Lee & Peale (2002): GJ876 planets in 2/1 resonance

Migration rate

$$\frac{da_i/dt}{a_i} = -\nu_i,$$

Eccentricity damping

$$\frac{de_i/dt}{e_i} = -K\nu_i$$



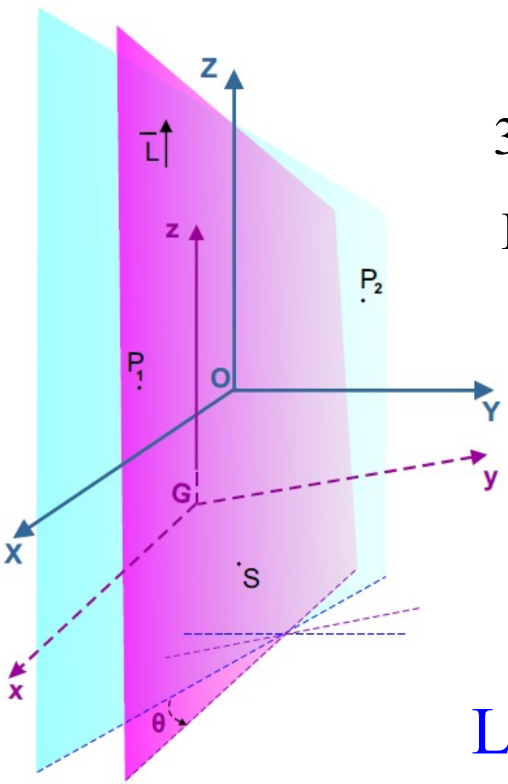
Ferraz-Mello, Beauge and Michtchenko (2003, 2006) → migration forces the resonant system to follow a specific path in phase space, along stationary resonant solutions

Hadjidemetriou & Voyatzis (2010): these paths correspond to families of stable periodic orbits of the 3BP (easily computed in a rotating frame)

→ *For given mass ratio and MMR we can predict the evolution path by computing the families of POs (and their stability character)*

***Q:** are there critical solutions that force the system to go out of plane?*

The general spatial TBP in the rotating frame



(Hadjidemetriou 1975, Ichtiaroglou et al 1978)

3-body problem: Star (**P_0**) , **P_1** (inner planet), **P_2** (outer planet)

Inertial frame **O***XYZ* (OZ//**L**) $\longrightarrow$ rotating frame **G***xyz*

G is the center of mass of **P_1** and **P_0**.
Axis **G***z* is parallel to **O***Z*
Plane **G***xz* contains always planet **P_1** and **P_0**

$$\mathbf{P}_1 = (x_1, 0, z_1), \quad \mathbf{P}_2 = (x_2, y_2, z_2), \quad \mathbf{P}_0 = (-ax_1, 0, -az_1) \qquad (a = m_1/m_0)$$

Lagrangian $L = L(x_1, z_1, x_2, y_2, z_2, \dot{x}_1, \dot{z}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2, \dot{\theta})$

Jacobi Integral $\quad E = T + V$

Angular momentum $\quad \mathbf{L} = (0, 0, L_Z) = \text{const.}$

$$z_1, \dot{z}_1, \dot{\theta} = f(q_i, L_z)$$

$$\boxed{(\mathbf{4\ d.o.f}) \quad \mathbf{q} = \{x_1, x_2, y_2, z_2, \dot{x}_1, \dot{x}_2, \dot{y}_2, \dot{z}_2\}, \quad \mathbf{q} = \mathbf{q}(t), \quad \mathbf{q}_0 = \mathbf{q}(0)}$$

Vertical stability of POs

(Henon, 1973; Michalodimitrakis, 1979)

- Take a 2-D periodic solution

$$\bar{q}(t) = (x_1, x_2, y_2, 0, \dot{x}_1, \dot{x}_2, \dot{y}_2, 0)$$

- Add a small vertical displacement $(|\zeta_{20}| \ll 1, |\dot{\zeta}_{20}| \ll 1)$

$$q(t) = (x_1, x_2, y_2, \zeta_2, \dot{x}_1, \dot{x}_2, \dot{y}_2, \dot{\zeta}_2)$$

See Poster **S1-4** by Antoniadou

- Linearize the vertical component

$$\ddot{z}_2 = A(\mathbf{q})z_2 + B(\mathbf{q})\dot{z}_2 \quad \to \quad \ddot{\zeta}_2 = A(\bar{\mathbf{q}})\zeta_2 + B(\bar{\mathbf{q}})\dot{\zeta}_2 \quad \to \quad \ddot{\zeta}_2 = \bar{A}(t)\zeta_2 + \bar{B}(t)\dot{\zeta}_2$$

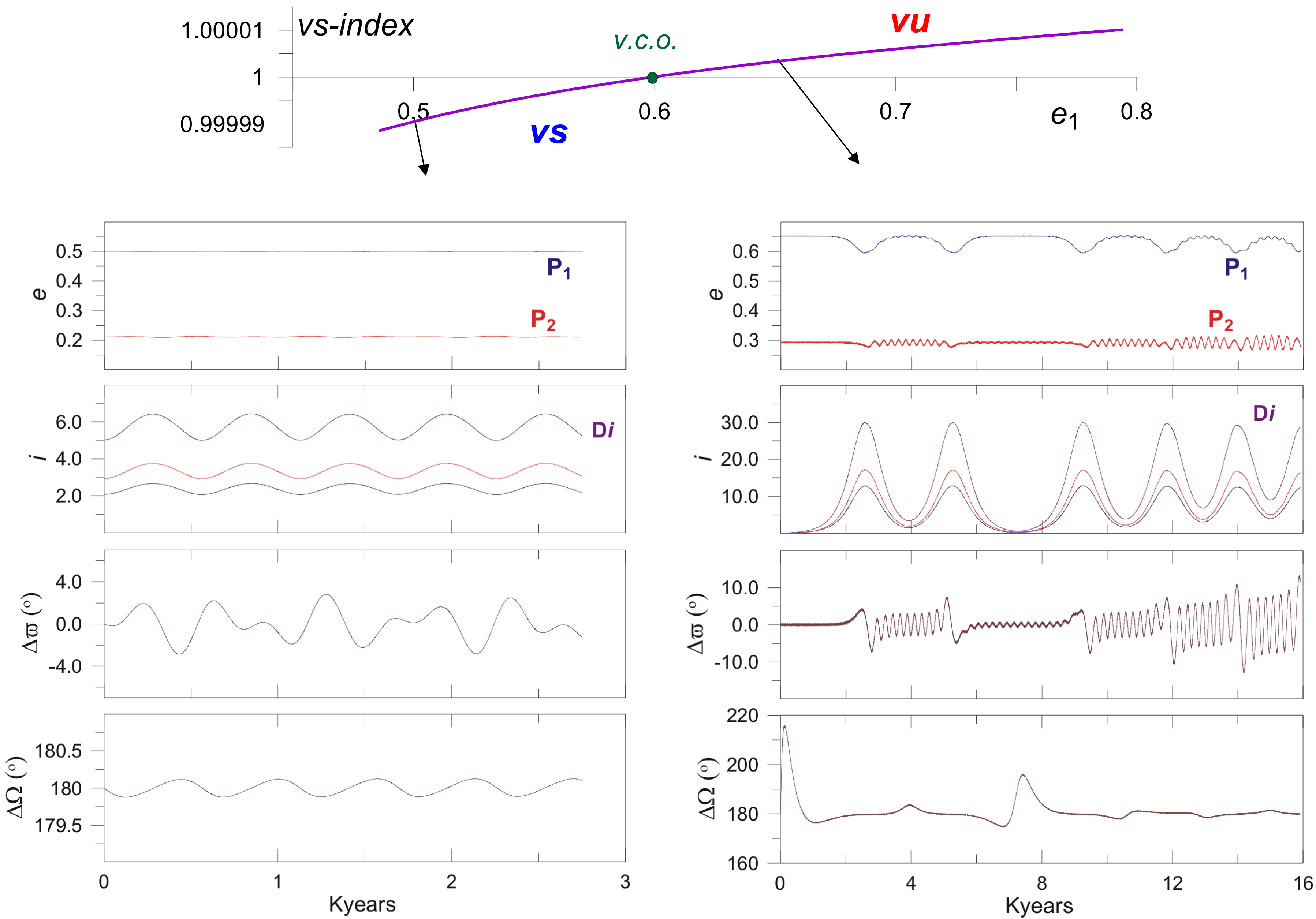
- and plug in the 2-D solution $\boldsymbol{Z} = \Delta(T)\boldsymbol{Z}_0$

$$(\det \boldsymbol{\Delta}(T) = 1) \quad \to \quad \lambda^2 - (a+d)\lambda + 1 = 0, \quad \boxed{k_v = \frac{1}{2}(a+d)}$$

Vertical stability index

- $|k_v|<1$: bounded solutions – vertical stability
- $|k_v|>1$: unbounded solutions – vertical instability
- $|k_v|=1$: vertical critical orbits (v.c.o.)

Vertical stability and orbit evolution



Numerical experiments

- To 1st order, the effects of Type II migration can be approximated by adding a Stokes-like force to the equations of motion of the 3BP:

$$\boldsymbol{F}_d = -C\left(\boldsymbol{v}_p - a\,\boldsymbol{v}_c\right)$$

this gives (Beauge&Ferraz-Mello, 1993; Beauge et al 2006)

Migration rate

$$\frac{da/dt}{a} = -\nu \,,$$

Eccentricity damping

$$\frac{de/dt}{e} = -K\nu$$

$$\nu = 2C(1-a)$$

$$K = \frac{a}{2(1-a)}$$

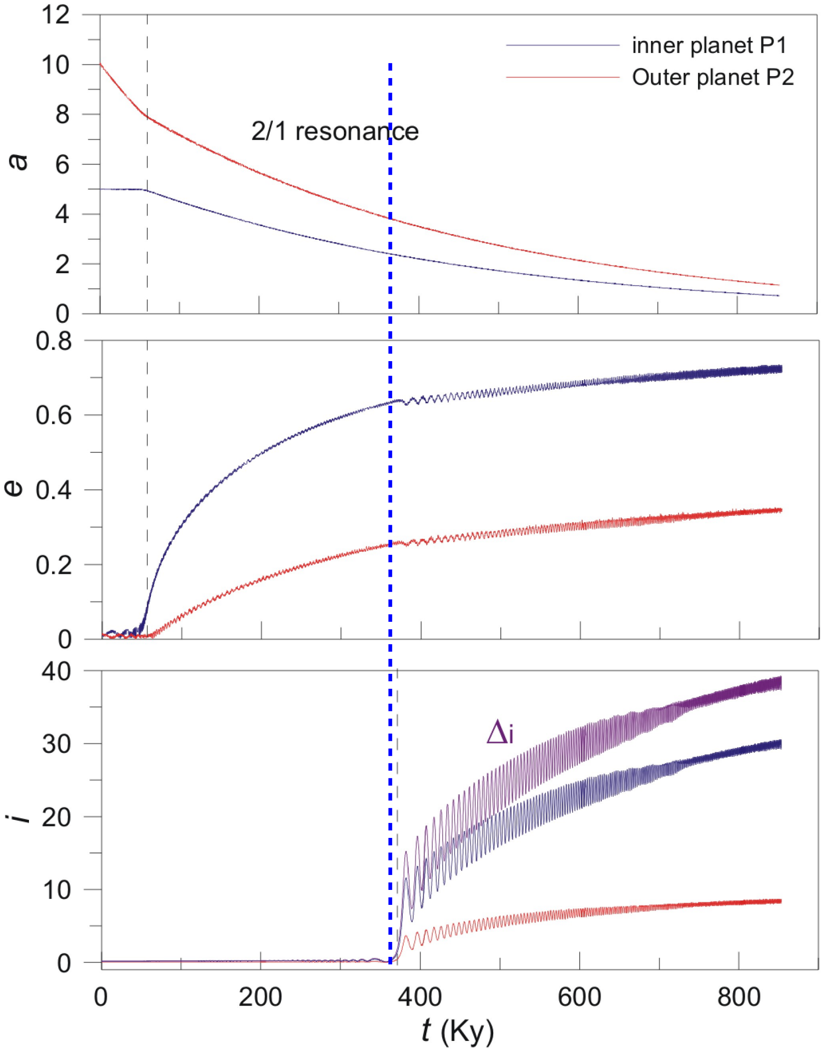
- Model : TBP + $\mathbf{F}_d$
- Starting conditions : $e_i \approx 0$, $i_i \approx 0$
- Parameters: $10^{-7} \le \nu \le 10^{-4}$ (y^{-1}), $0.5 \le K \le 100$

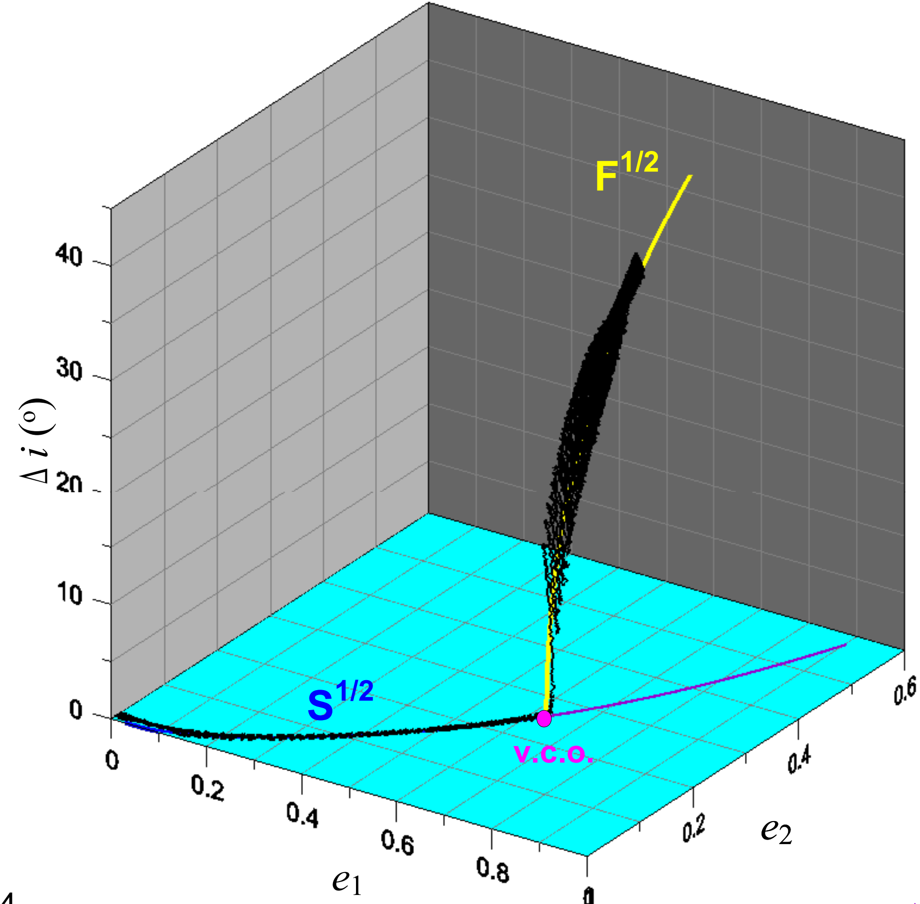
An example

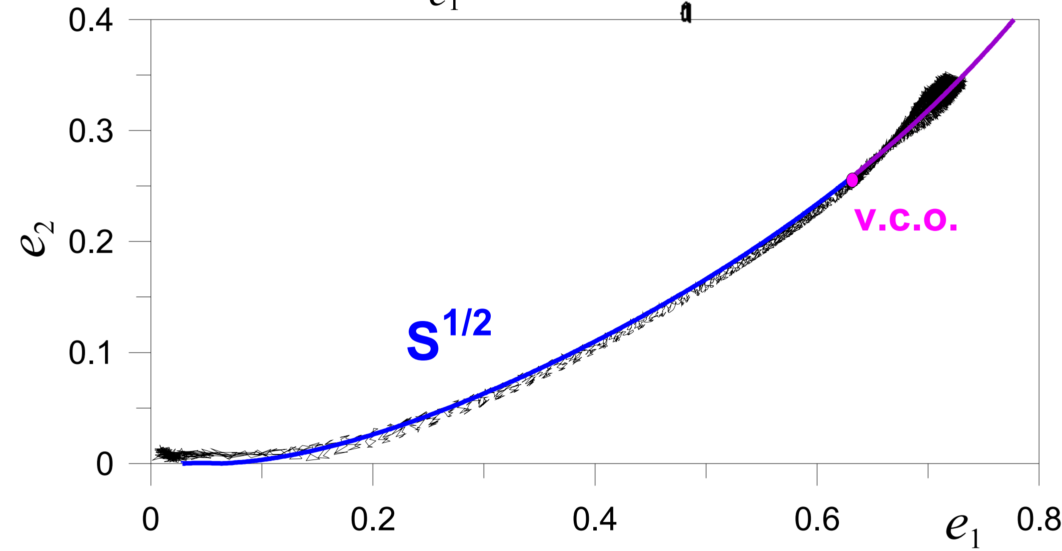
m_1=1M$_J$, m_2=2M$_J$

a_{10}=5AU a_{20}=10AU, e_1=e_2=0.01,

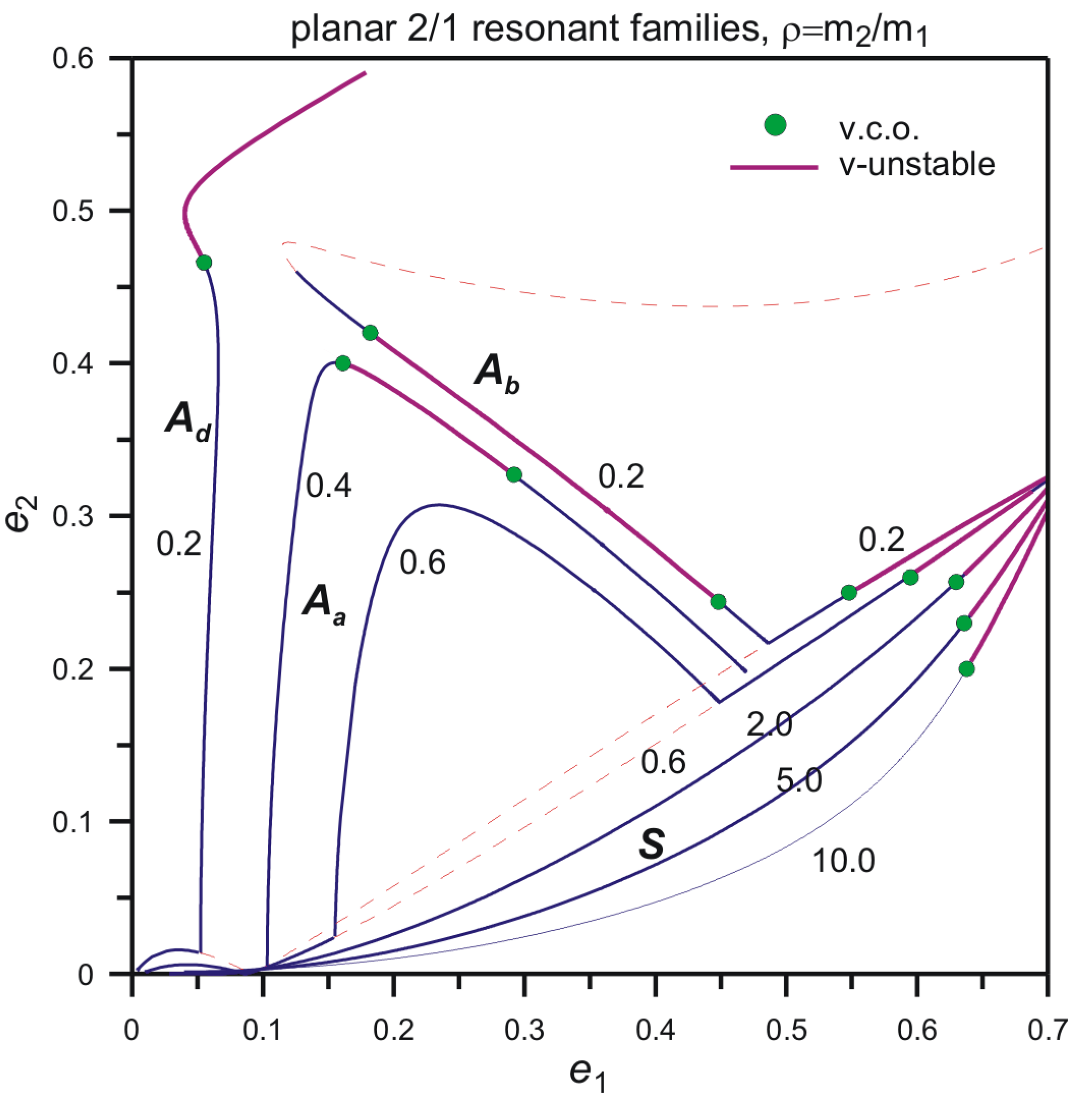
i_{i0}= i_{20}=0.1°, ν=8 10^{-6} y^{-1}, K=1







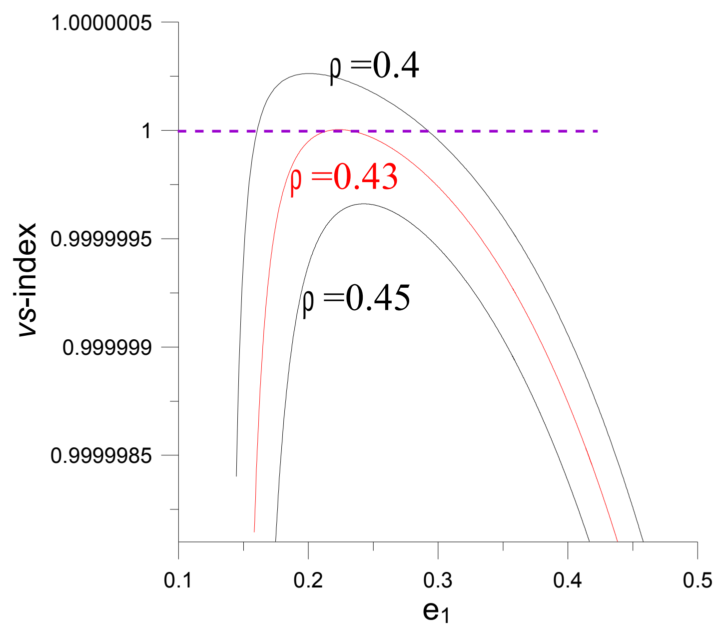
2/1 resonance : vertical critical points



Antoniadou & Voyatzis, CMDA 115, 2013 - Symmetric 3D configurations

$\rho \approx 0.43$

Bifurcation of vco in family A_a

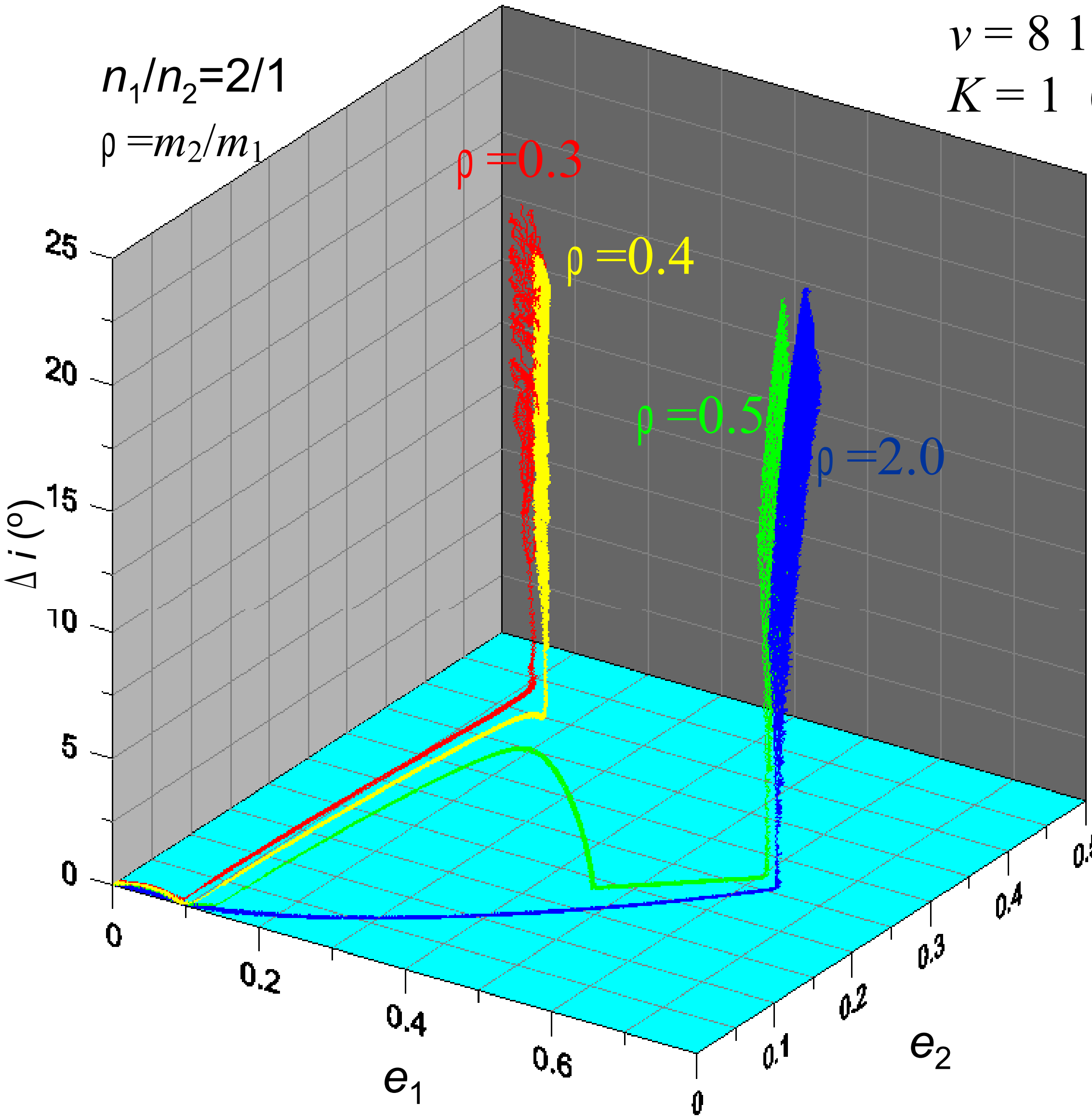


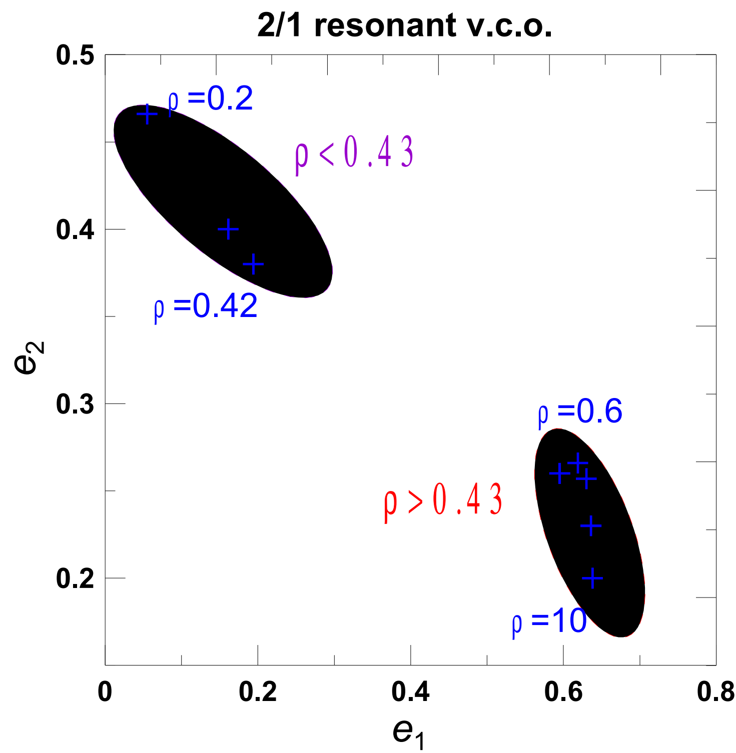
2/1 resonance : different mass ratios

Migration parameters

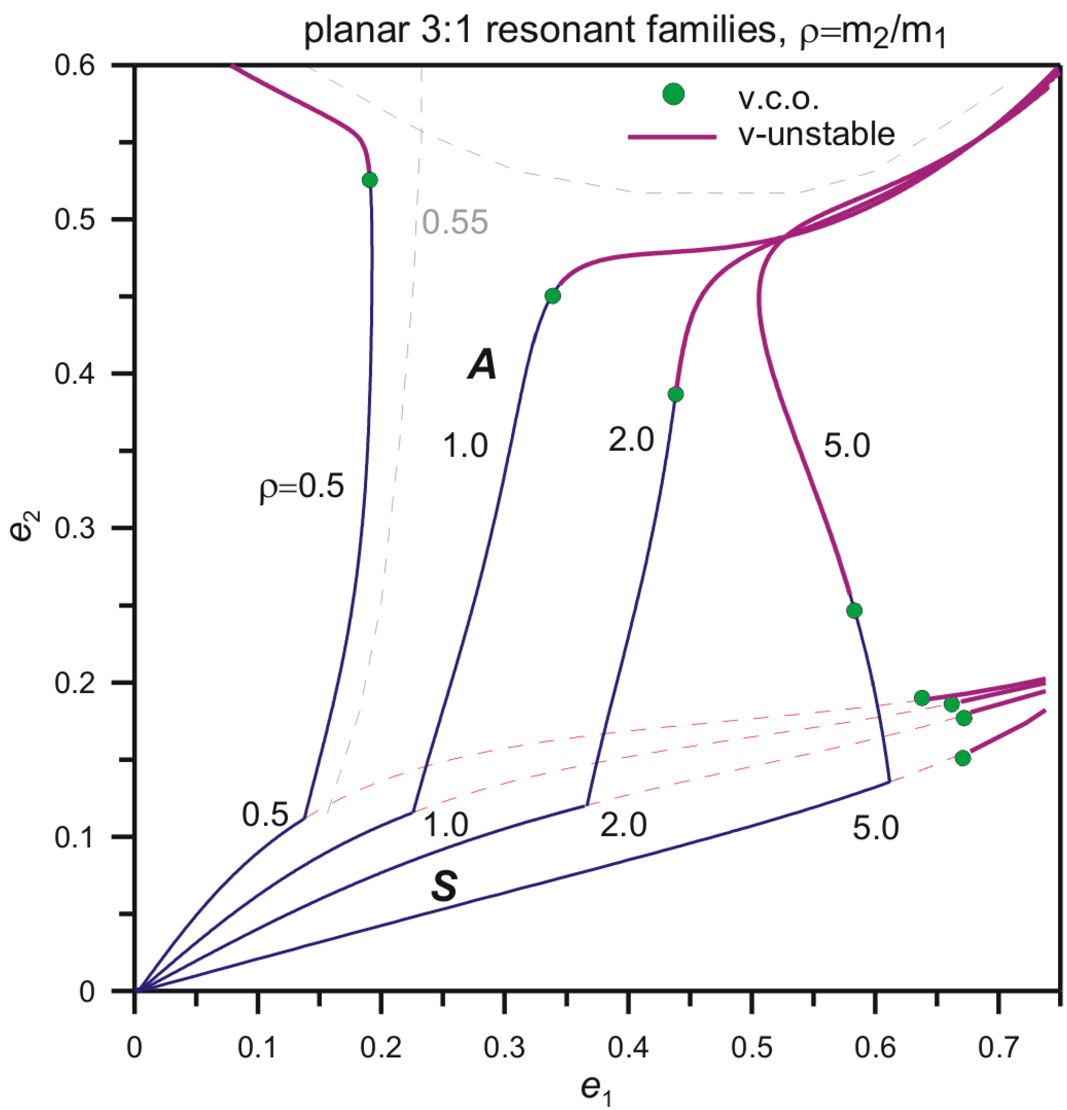
$\nu = 8\ 10^{-7}\ y^{-1}$

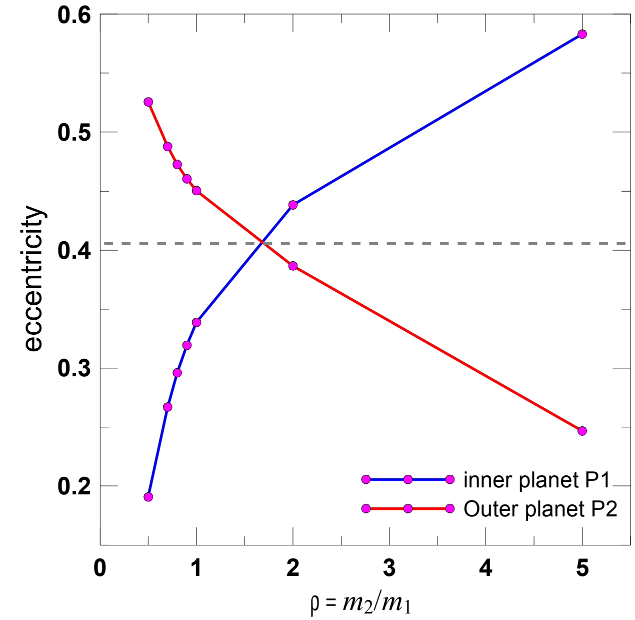
$K = 1$ (K=0.5)

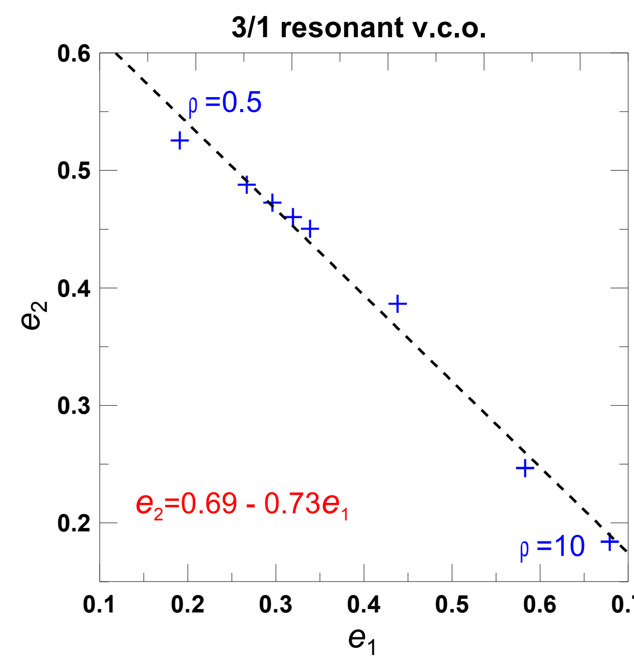




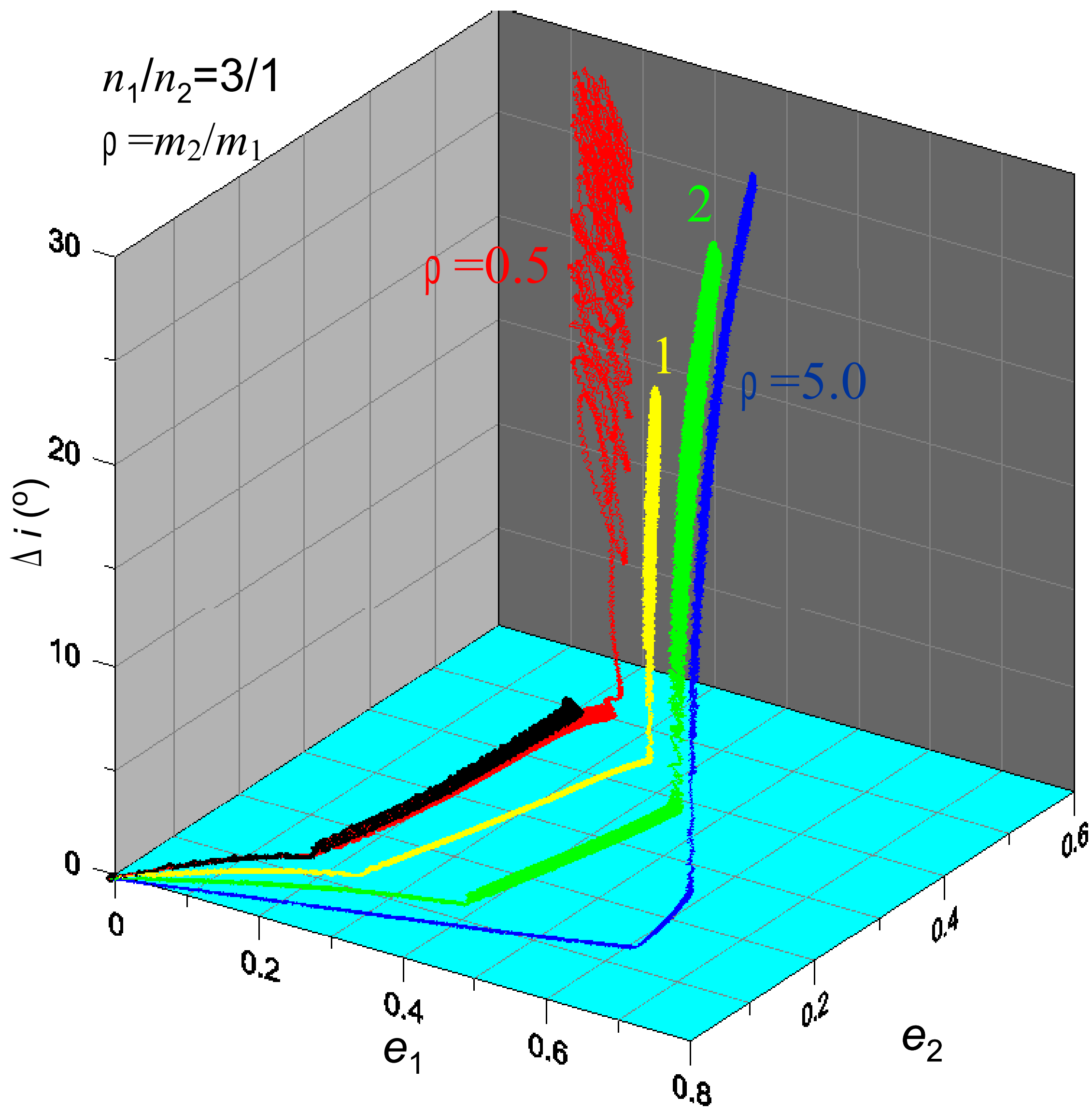
3/1 resonance : vertical critical points







3/1 resonance : different mass ratios



Migration parameters

ν = 8 10^{-7} y^{-1}

K = 1 (K=0.5)

Conclusions

- *VCOs* determine the position of *inclination resonance* in the eccentricity plane
- After crossing a VCO the system follows a path that is determined by the bifurcating *3D family of periodic orbits*

⇨ *evolution is 'predictable' (save for K ...)*

- For 2/1 resonance captures, the inclination resonance occurs in *two different eccentricity domains* (critical mass ratio $\rho=0.43$)
- For 3/1 resonance captures V.C.Os fall *on a line* in e1-e2

⇨ mass ratio, resonance ration and eccentricities tell us
if the system has reached the critical solution (none so far...)

⇨ a minimum effective *K* (damping) can be deduced for systems
that have not passed through a VCO