Accuracy of the IWM–CFC approximation in differentially rotating relativistic stars

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Outline

- Motivation and background
- Why study differentially rotating relativistic stars?
- Overview of Conformal Flatness Condition (CFC)
- Construction of diagnostics
- Results and summary

Why are differentially rotating relativistic stars interesting?

- Possible outcome of binary NS merger is a long–lived compact remnant ($\tau > 10ms$)
- The remnant is a hypermassive neutron star (HMNS) exhibiting differential rotation
- Binary NS stages: inspiral, merger, post–merger ringdown
- Oscillations of the HMNS in the post–merger phase can be used to constrain the EoS

How can we constrain the EoS?

- GW asteroseismology (Gaertig, Kokkotas, 2011)
- Identification of two potentially observable frequencies of the HMNS (Stergioulas, Bauswein, Zagkouris, Janka, 2011)

currently in progress:

- Construct equilibrium models of HMNS
- Obtain oscillation modes through eigenvalue approach
- Find empirical formulas for GW frequencies

CFC and $3+1$ formalism

- line element in $3+1$ formalism: $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$
- α: lapse function, β^α: shift vector
- 3–dimensional metric tensor: $\gamma_{ab} = \psi^4 n_{ab}$

In spherical coordinates:

$$\gamma_{ab} = \psi^4 n_{ab} = \begin{pmatrix} \psi^4 & 0 & 0 \\ 0 & \psi^4 r^2 & 0 \\ 0 & 0 & \psi^4 r^2 \sin^2\theta \end{pmatrix}$$

Stationary, axisymmetric, equilibrium models

- line element in CFC

$$ds^2 = -\alpha^2 dt^2 + \psi^4(dr^2 + r^2 d\theta^2) + \psi^4 r^2 \sin^2\theta(d\phi + \beta^\phi dt)^2$$

- line element in full General Relativity

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{\gamma-\rho}r^2\sin^2\theta(d\phi - \omega dt)^2 + e^{2\mu}(dr^2 + r^2 d\theta^2)$$

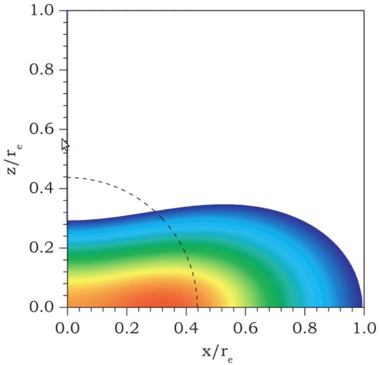
Inspection of above gives:

$$\alpha = e^{(\gamma+\rho)/2} \quad , \quad \psi = e^{\mu/2} = e^{(\gamma-\rho)/4} \quad , \quad \beta^\phi = -\omega$$

$$\mu = \frac{\gamma - \rho}{2}$$

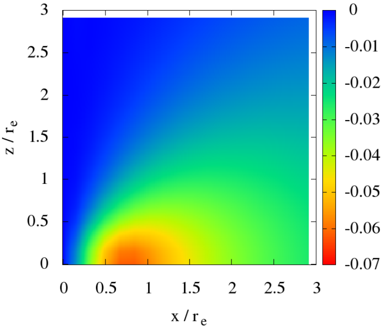
Specifics of models

- Sequence B (Stergioulas, Apostolatos, Font, 2004)
- constant central energy density $\epsilon_c = 1.444 \times 10^{-3}$
- polytropic EoS: $N = 1$, $K = 100$
- computations made with `rns` code (original version by Stergioulas, 1995)



A simple error indicator

- $\Delta c = \frac{\mu_{GR} - \mu_{CFC}}{\mu_{GR}}$
- maximum deviation at $\frac{2}{3} r_e$
- Good correlation with maximum errors in physical quantities
- fastest rotating model B13: $r_p / r_e = 0.34$



A more sophisticated diagnostic

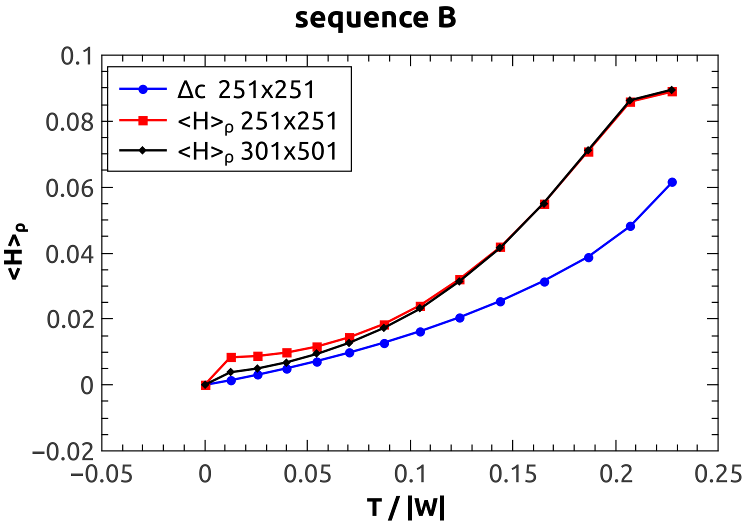
Miller, Gressman, Suen (2004):

- Cotton–York tensor $C^{ij} = \epsilon^{ik\ell} D_k \left(R^j{}_\ell - \frac{1}{4} R \delta^j{}_\ell \right)$
- calculate Cotton–York's matrix norm normalized by the covariant derivative of the 3–Ricci tensor

$$H = \frac{|H_{ij}|}{\sqrt{D_i R_{jk} D^i R^{jk}}}$$

- construct the rest mass density weighted integral quantity

$$\langle H \rangle_\rho = \frac{\int d^3x H \sqrt{\gamma} \rho W}{\int d^3x \sqrt{\gamma} \rho W}$$


sequence B
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.05
0
0.05
0.1
0.15
0.2
0.25
<H>ρ
T / |W|
Δc 251x251
<H>ρ 251x251
<H>ρ 301x501

Summary

- Δc at $2/3$ of r_e provides an easy way to evaluate CFC
- MGS diagnostic advantage: vanishes in CFC
- MGS diagnostic disadvantage: involves 3rd derivatives of metric tensor, slightly increases computational cost
- Application of both for the constructed HMNS to monitor validity of CFC

Thank you!