Searching for chaos around black hole candidates

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Modern Physics Letters A

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TESTING THE KERR BLACK HOLE HYPOTHESIS

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It is thought that the final product of the gravitational collapse is a Kerr black hole and astronomers have discovered several good astrophysical candidates. While there is some indirect evidence suggesting that the latter have an event horizon, and therefore that they are black holes, a proof that the space-time around these objects is described by the Kerr geometry is still lacking. Recently, there has been an increasing interest in the possibility of testing the Kerr black hole hypothesis with present and future experiments. In this paper, I briefly review the state of the art of the field, focussing on some recent results and work in progress.

A black hole spacetime

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A spinning black hole (BH) of mass “m” and spin “S” is described by the Kerr spacetime when α=S/m<1.

The line element in the Boyer-Lindquist coordinates:

$$
\begin{aligned}
\mathrm{d}s^2 \;=\; & -\left[1-\frac{2mr}{r^2+a^2\cos^2\theta}\right]\mathrm{d}t^2-\frac{4mra\sin^2\theta}{r^2+a^2\cos^2\theta}\,\mathrm{d}t\,\mathrm{d}\phi \\
& +\left[\frac{r^2+a^2\cos^2\theta}{r^2-2mr+a^2}\right]\mathrm{d}r^2+(r^2+a^2\cos^2\theta)\,\mathrm{d}\theta^2 \\
& +\left[r^2+a^2+\frac{2mra^2\sin^2\theta}{r^2+a^2\cos^2\theta}\right]\sin^2\theta\,\mathrm{d}\phi^2 .
\end{aligned}
$$

Black holes have no hair theorem

Cosmic censorship hypothesis

Astrophysical Setup

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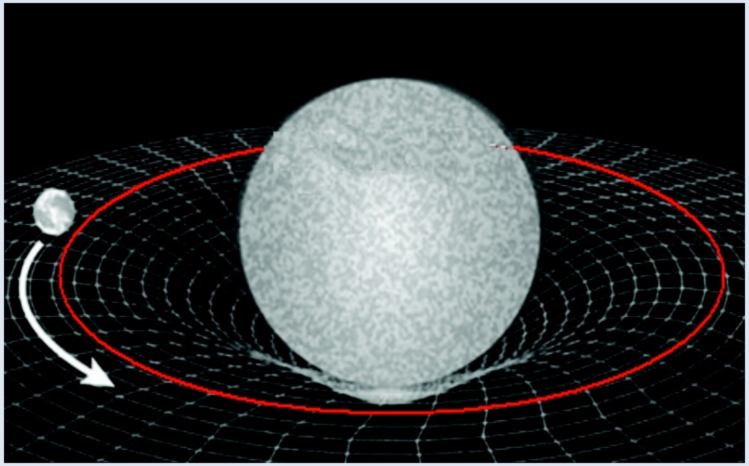
The centres of the galaxies are occupied by SuperMassive Black Holes canditates $M=10^5$-10^9 M_{Sun} .

An Extreme Mass Ratio Inspiral is the inspiral of a stellar compact object into a SMBH, and this inspiral follows approximately (adiabatically) the geodesic motion.

Laser Interferometer Space Antenna-like gravitational wave detectors designed to detect signals from EMRI.



Babak , Gair , Petiteau & Sesana, CQG (2010)
Amaro-Seoane et al. ArXiv: 1202.0839

Non-integrability approach

Kerr is integrable:

4-momentum conservation → $p^{\mu} p_{\mu} = -\mu^2$
stationary → E
axisymmetric → L_z
second order Killing tensor field → Q (Carter, PR, 1968)

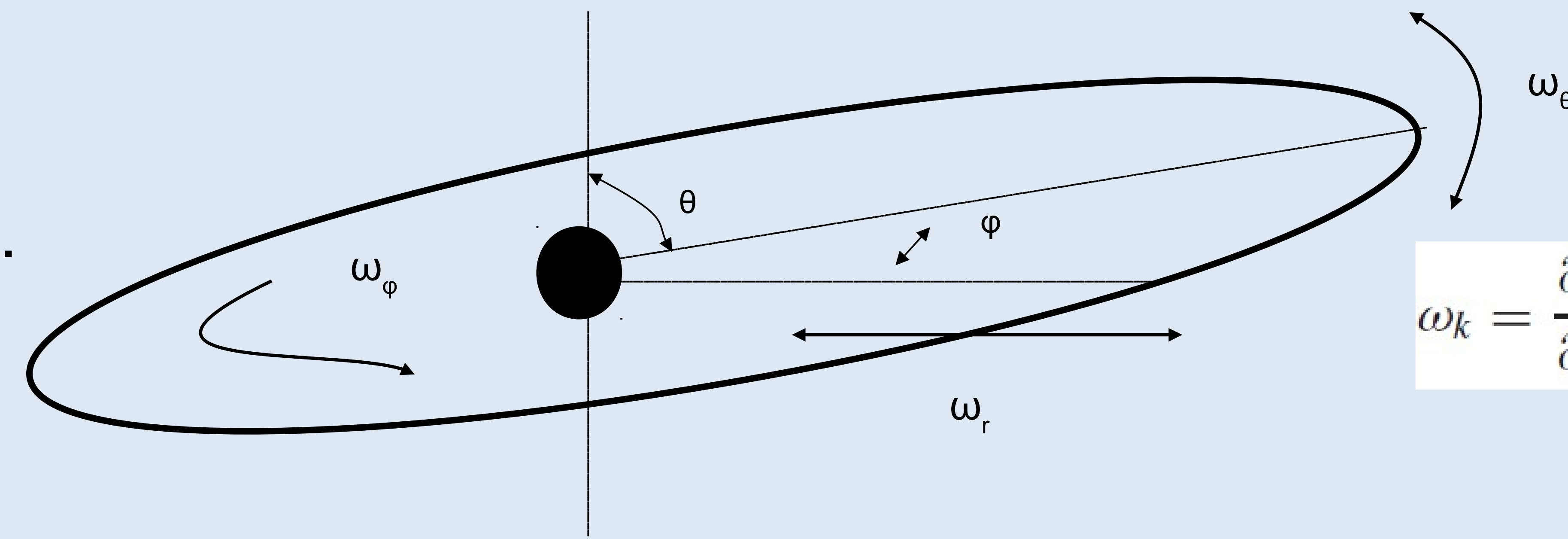
Non-integrability → non-Kerr

Characteristic frequencies

In the generic case of stationary axisymmetric spacetime background:

$$E = -g_{tt}\dot{t} - g_{t\phi}\dot{\phi}, \qquad L_z = g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi}$$

If we have an extra integral of motion, then by expressing the Hamiltonian into action-angle variables $H^{(aa)}$:



$$\omega_k = \frac{\partial H^{(aa)}}{\partial J_k}$$

Kerr in celestial mechanics

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The Kerr metric has the "extra" integral of motion, the Carter constant Q. Carter, PR (1968)

The Hamiltonian has not been yet expressed in action-angle variables, the characteristic frequencies
ω_r, ω_θ, ω_φ were found "analytically" as functions of:
the black hole spin a and mass M,
the constants of motion (E, L_z, Q) and
the geometrical characteristics of the orbit
(turning points of r and θ). Schmidt, CQG (2002)

Shaken, not stirred

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Consider a weak perturbation of a Kerr background that “breaks” the Carter constant, but leaves the background stationary and axisymmetric, e.g. a compact object more prolate or oblate than Kerr black hole.

$$H_{New}=H_{Kerr}+\varepsilon\, H_{Perturbation} \qquad (\varepsilon<<)$$

By the two remaining integrals of motion E, L_z we can reduce the number of degrees of freedom from 4 to 2.

We restrict the study of the dynamics on the meridian plane.

Metric from the Manko-Novikov family (CQG, 1992)

Weyl-Papapetrou metric element in prolate spheroidal coordinates:

Gair, Li Mandel, PRD (2008)

$$ds^2 = g_{tt}\, dt^2 + g_{xx}\, dx^2 + g_{yy}\, dy^2 + g_{\phi\phi}\, d\phi^2 + g_{t\phi}\, dt\, d\phi$$

$$g_{tt} = -f\,,$$

$$g_{xx} = \frac{k^2 e^{2\gamma}(x^2-y^2)}{f(x^2-1)}\,,$$

$$g_{yy} = \frac{k^2 e^{2\gamma}(x^2-y^2)}{f(1-y^2)}\,,$$

$$g_{\phi\phi} = \left(\frac{k^2(x^2-1)(1-y^2)}{f} - f\omega^2\right)\,,$$

$$g_{\phi t} = 2\,\omega\, f$$

$$f = e^{2\psi}\frac{A}{B},$$

$$\omega = 2ke^{-2\psi}\frac{C}{A} - 4k\frac{\alpha}{1-\alpha^2},$$

$$e^{2\gamma} = e^{2\gamma'}\frac{A}{(x^2-1)(1-\alpha^2)^2},$$

$$\alpha = \frac{-M+\sqrt{M^2-(S/M)^2}}{(S/M)}, \qquad k = M\frac{1-\alpha^2}{1+\alpha^2},$$

$$\beta = q\left(\frac{1+\alpha^2}{1-\alpha^2}\right)^3.$$

$$A = (x^2-1)(1+a\,b)^2 - (1-y^2)(b-a)^2,$$

$$B = [x+1+(x-1)a\,b]^2 + [(1+y)a+(1-y)b]^2,$$

$$C = (x^2-1)(1+a\,b)[b-a-y(a+b)] + (1-y^2)(b-a)[1+a\,b+x(1-a\,b)],$$

$$\psi = \beta\frac{P_2}{R^3},$$

$$\gamma' = \ln\sqrt{\frac{x^2-1}{x^2-y^2}} + \frac{3\beta^2}{2R^6}(P_3^2-P_2^2) + \beta\left(\sum_{\ell=0}^{2}\frac{x-y+(-1)^{2-\ell}(x+y)}{R^{\ell+1}}P_\ell - 2\right)$$

$$a = -\alpha\exp\left[-2\beta\left(-1+\sum_{\ell=0}^{2}\frac{(x-y)P_\ell}{R^{\ell+1}}\right)\right],$$

$$b = \alpha\exp\left[2\beta\left(1+\sum_{\ell=0}^{2}\frac{(-1)^{3-\ell}(x+y)P_\ell}{R^{\ell+1}}\right)\right],$$

$$R = \sqrt{x^2+y^2-1},$$

$$P_\ell = P_\ell\left(\frac{x\,y}{R}\right)\,,\quad P_\ell(w) = \frac{1}{2^\ell \ell!}\left(\frac{d}{dw}\right)^\ell (w^2-1)^\ell$$

For q>0 the compact object is more oblate than Kerr and for q<0 more prolate.

For q=0 we get the Kerr metric.

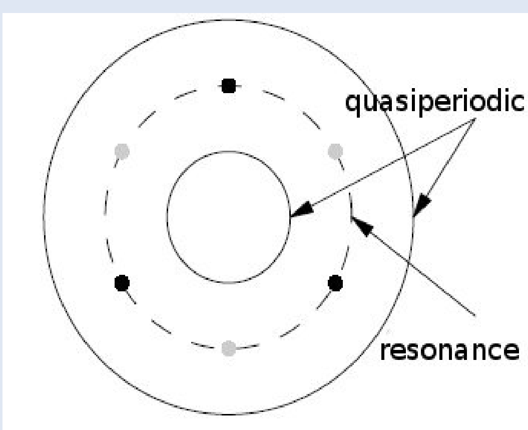
Two theorems to rule them all

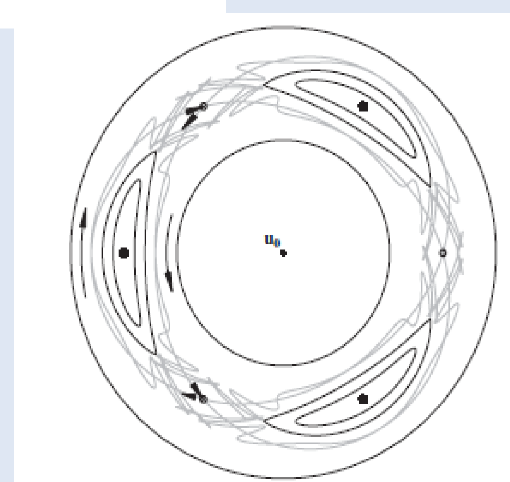
If the perturbation is weak enough, then most of the non-resonant tori survive (KAM theorem) and the resonant tori transform into the Birkhoff chain (Poincaré-Birkhoff theorem).

Resonance condition:

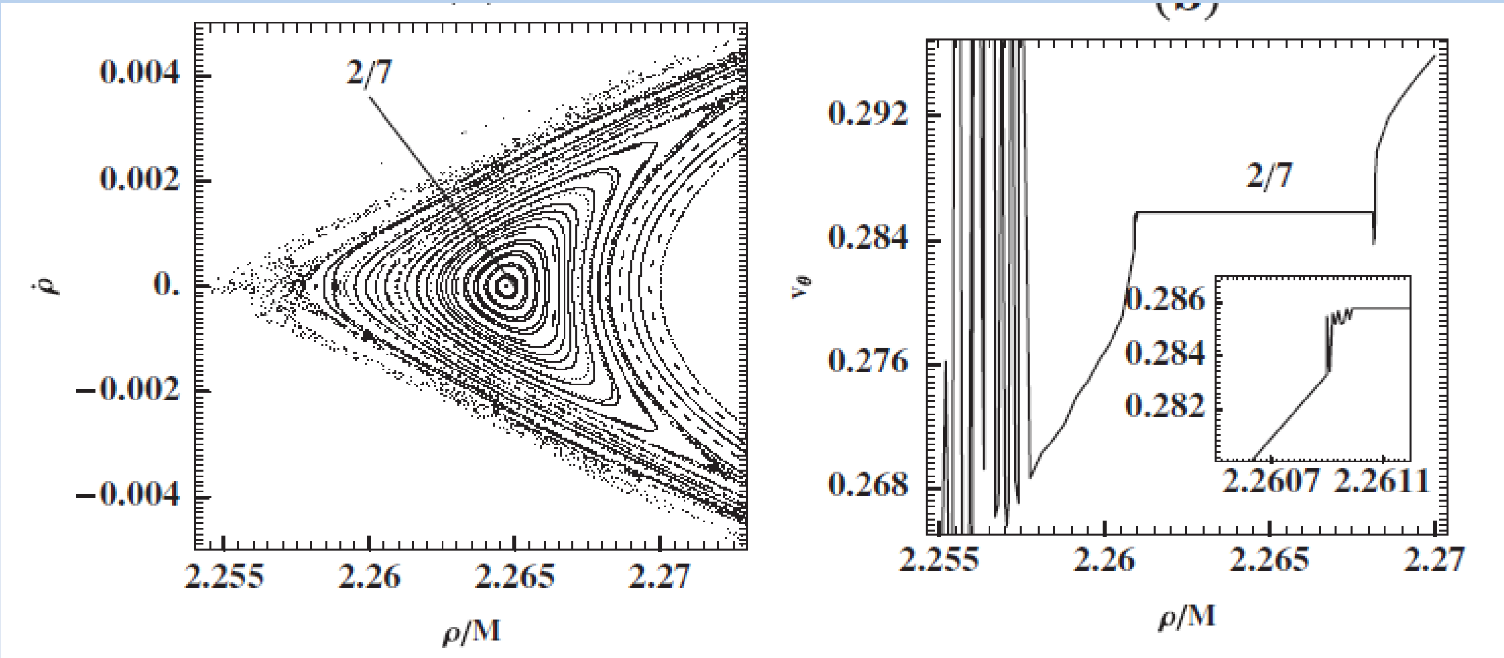
$$\sum_{i=1}^{n} k_i \omega_i = 0,\ k_i \in \mathbb{Z}, \quad |k| = \sum_{i=1}^{n} |k_i| \neq 0 \quad .$$

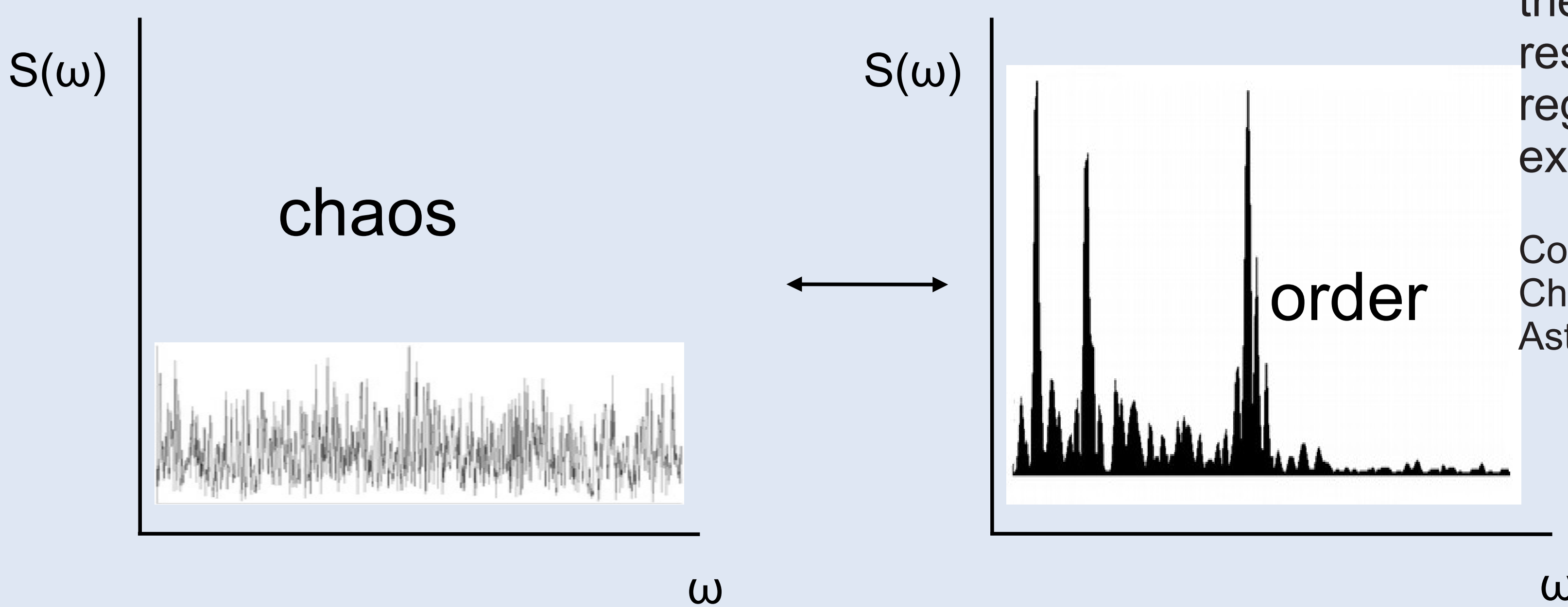
n=2





Beacon effect of stickiness





The stickiness concerns chaotic orbits which for various reasons stick for a long time interval in a region, close to an invariant curve, so that their behaviour may resemble that of regular orbits, before extending further away.

Contopoulos, Order and Chaos in Dynamical Astronomy, Springer (2002)

L-G, Apostolatos & Contopoulos, PRD 81:124005 (2010)

Effect of resonances

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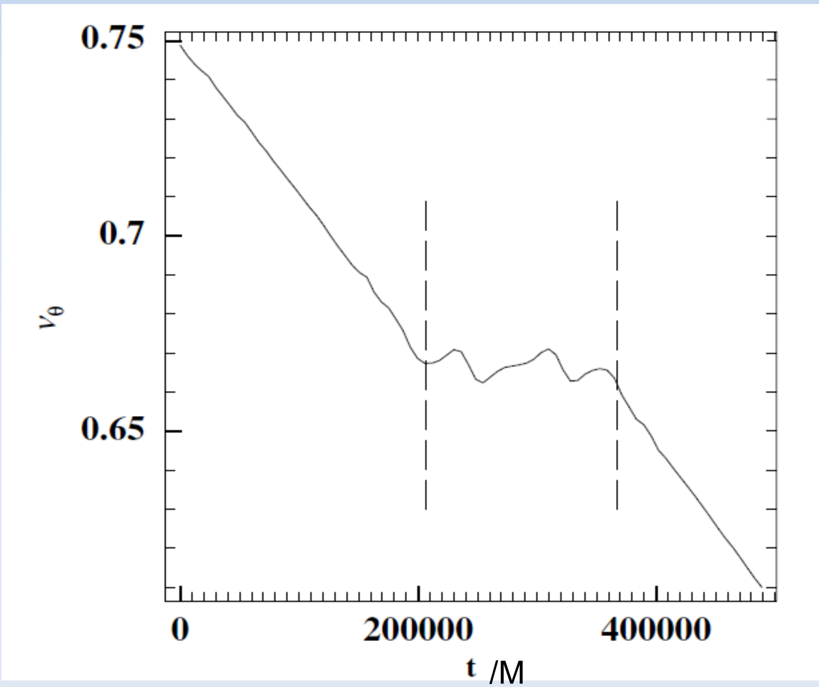

We modified the rates of energy and angular momentum loss proposed by Gair & Glampedakis (PRD, 2006) for a Kerr black hole according to the prescription given in Gair, Li & Mandel, PRD 77:024035 (2008)

$$E(t) = E(0) + \frac{dE}{dt}\bigg|_0^t$$

$$L_z(t) = L_z(0) + \frac{dL_z}{dt}\bigg|_0^t$$

The unit of time t is 5 M/M_{Sun} μs.

For $M=10^6 M_{Sun}$ a resonance crossing that lasts $\Delta t = 5\ 10^4$ M corresponds approximately to a week.

Apostolatos, L-G & Contopoulos, PRL 103:111101(2009)
L-G, Apostolatos & Contopoulos, PRD 81:124005 (2010)
Contopoulos, L-G & Apostolatos, IJBC 21:2261 (2011)

So...

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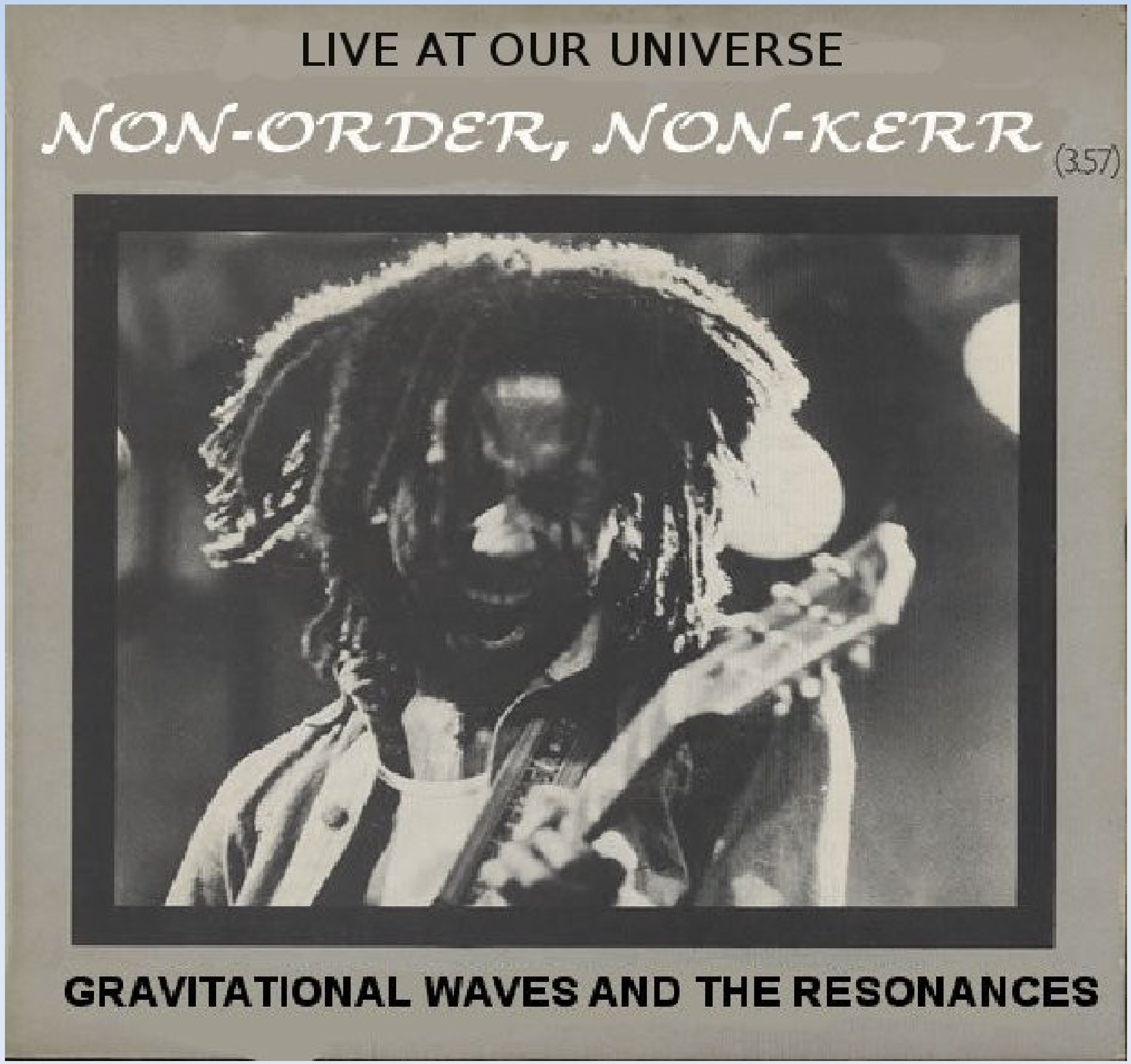
LIVE AT OUR UNIVERSE
NON-ORDER, NON-KERR
(3.57)
GRAVITATIONAL WAVES AND THE RESONANCES

Interesting questions

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Is every axisymmetric, and stationary perturbation of the Kerr spacetime non-integrable?
Can Carter-like constants (or constants coming from higher order Killing tensors) survive?

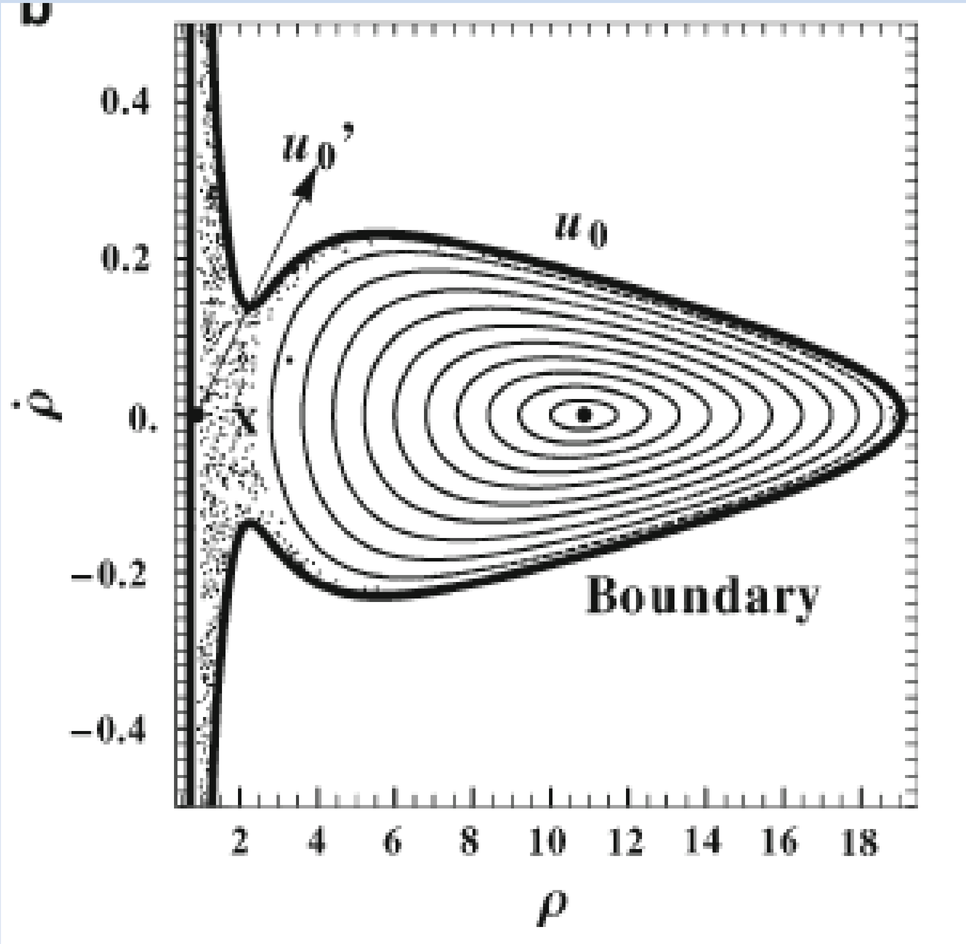
Jeandrew Brink, PRD 78:102002 (2008); PRD 81:022001 (2010); PRD 81:022002 (2010); PRD 84:104015 (2011)

Claimed that Manko-Novikov is partially (?) integrable

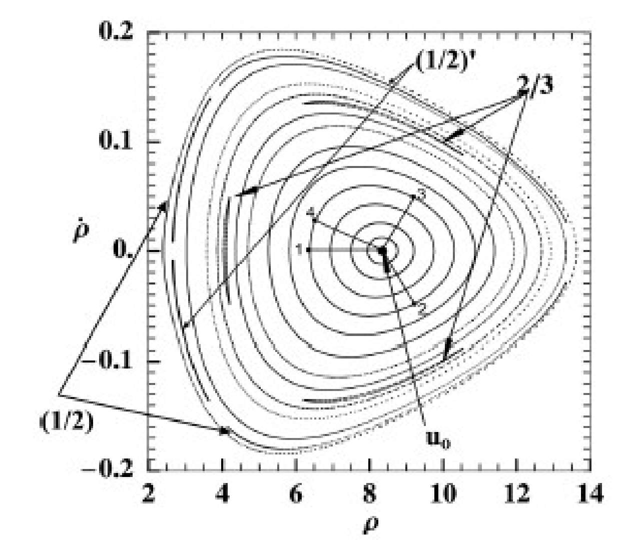
and Zipoy-Voorhees is integrable.

(Sota, Suzuki and Maeda, CQG 13:1241 (1996))

MN “fully” non-integrable



L-G, Apostolatos and Contopoulos, PRD 81:124005 (2010)
Contopoulos, L-G and Apostolatos, IJBC 21:2261 (2011)
Contopoulos, Harsoula and L-G, CMDA 113:255 (2012)



ZV non-integrable

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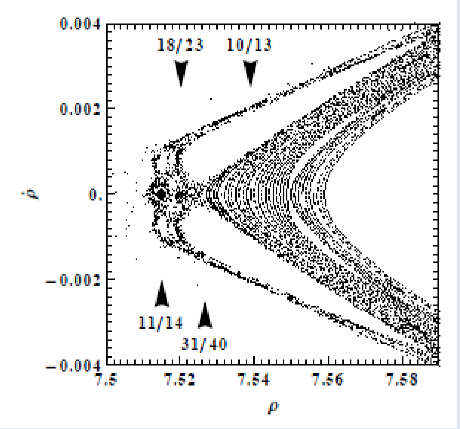
Theorem: There exists no smooth function $I: T^*U \to \mathbb{R}$, which is a polynomial in momenta of degree ≤ 6, such that H, I_1, I_2, I are functionally independent and Poisson commute.

$$I = \sum_{i+j+k+m=6} I_{ijkm} p_x^i p_y^j p_\phi^k p_t^m$$

Kruglikov and Matveev, PRD 85:124057 (2012)

Numerical evidence that ZV contains chaotic motion

L-G, PRD 86:044013 (2012)



Theorem 3 *There does not exist an additional, meromorphic first integral of the geodesic motion in the Zipoy-Voorhees metric (1).*

Maciejewski, Przybylska and Stachowiak arXiv: 1302.4234

Get Carter

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Vigeland, Yunes and Stein, PRD 83:104027 (2011)

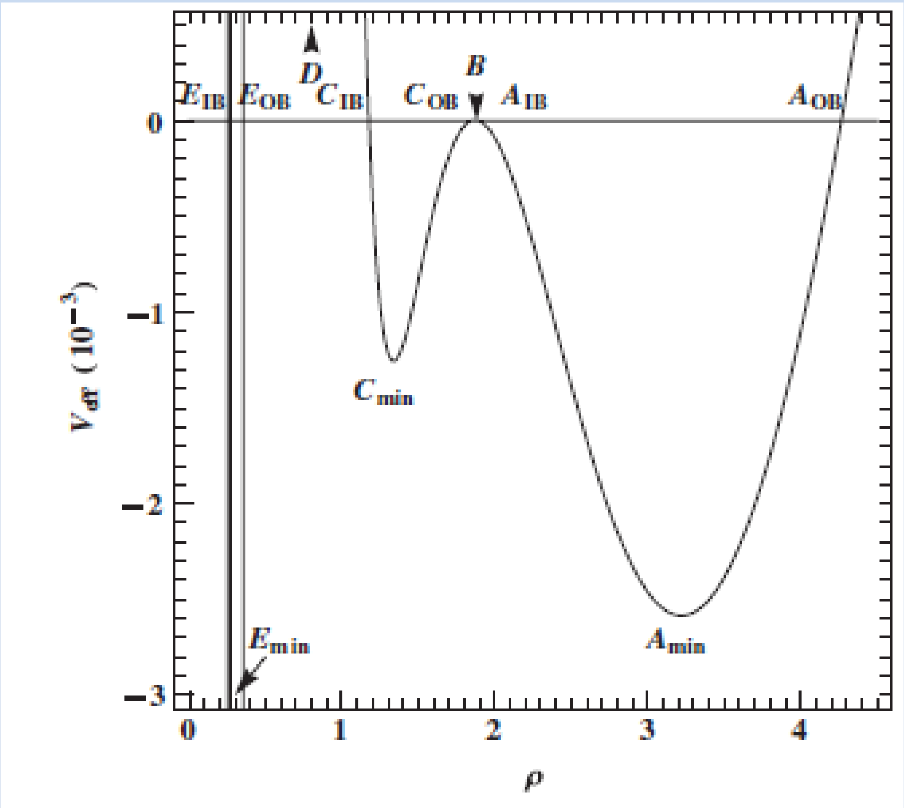
Perturbation of Kerr $g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu}$

Impose integrability,
preserve Carter-like constant

Alternative theories of gravity, spacetimes does not satisfy the Einstein equation.

Chirps and bursts

Bambi & L-G, PRD 87:083006 (2013)



Spacetimes with extra anomalous mass-multipole moments can exhibit multiple successive chirps and bursts.

$$\frac{1}{2}(\dot{\rho}^2 + \dot{z}^2) + V_{\rm eff}(\rho, z) = 0,$$

$$V_{\rm eff}(\rho, z) = \frac{1}{2} e^{-2\gamma}\left[f - E^2 + \left(\frac{f}{\rho}(L_z - \omega E)\right)^2\right]$$

Designed for chaos

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PHYSICAL REVIEW D **86,** 124013 (2012)

Symmetric integrator for nonintegrable Hamiltonian relativistic systems

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By combining a standard symmetric, symplectic integrator with a new step size controller, we provide an integration scheme that is symmetric, reversible and conserves the values of the constants of motion. This new scheme is appropriate for long-term numerical integrations of geodesic orbits in spacetime backgrounds, whose corresponding Hamiltonian system is nonintegrable, and, in general, for any non-integrable Hamiltonian system whose kinetic part depends on the position variables. We show by numerical examples that the new integrator is faster and more accurate (i) than the standard symplectic integration schemes with or without standard adaptive step size controllers and (ii) than an adaptive step Runge-Kutta scheme.

TABLE III. Results of the runs of the chaotic orbit with $\rho = 0.7$ for different integration schemes.

Integrator	ϵ	Result	$T_{\mathrm{calc}}[s]$ until abortion
RK5con	10^{-6}	Aborted after propagation time of $\tau = 11254.0$ because $\Delta H > 10^{-6}$	40739.7
RK5var	10^{-4}	Aborted after propagation time of $\tau = 1087.3$ because $\Delta H > 10^{-6}$	149469.9
CCM	0.01	Aborted after propagation time of $\tau = 9559.9$ because $h(l, \mathbf{y}) < 0$	34.2
CCM2	0.01	Aborted after propagation time of $\tau = 4180.1$ because $\Delta H > 10^{-6}$	1353.9
IGEM	0.1	No abortion in $\tau = [0, 50000]$, $T_{\mathrm{calc}} = 995.3$ s.	...

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Thank you!