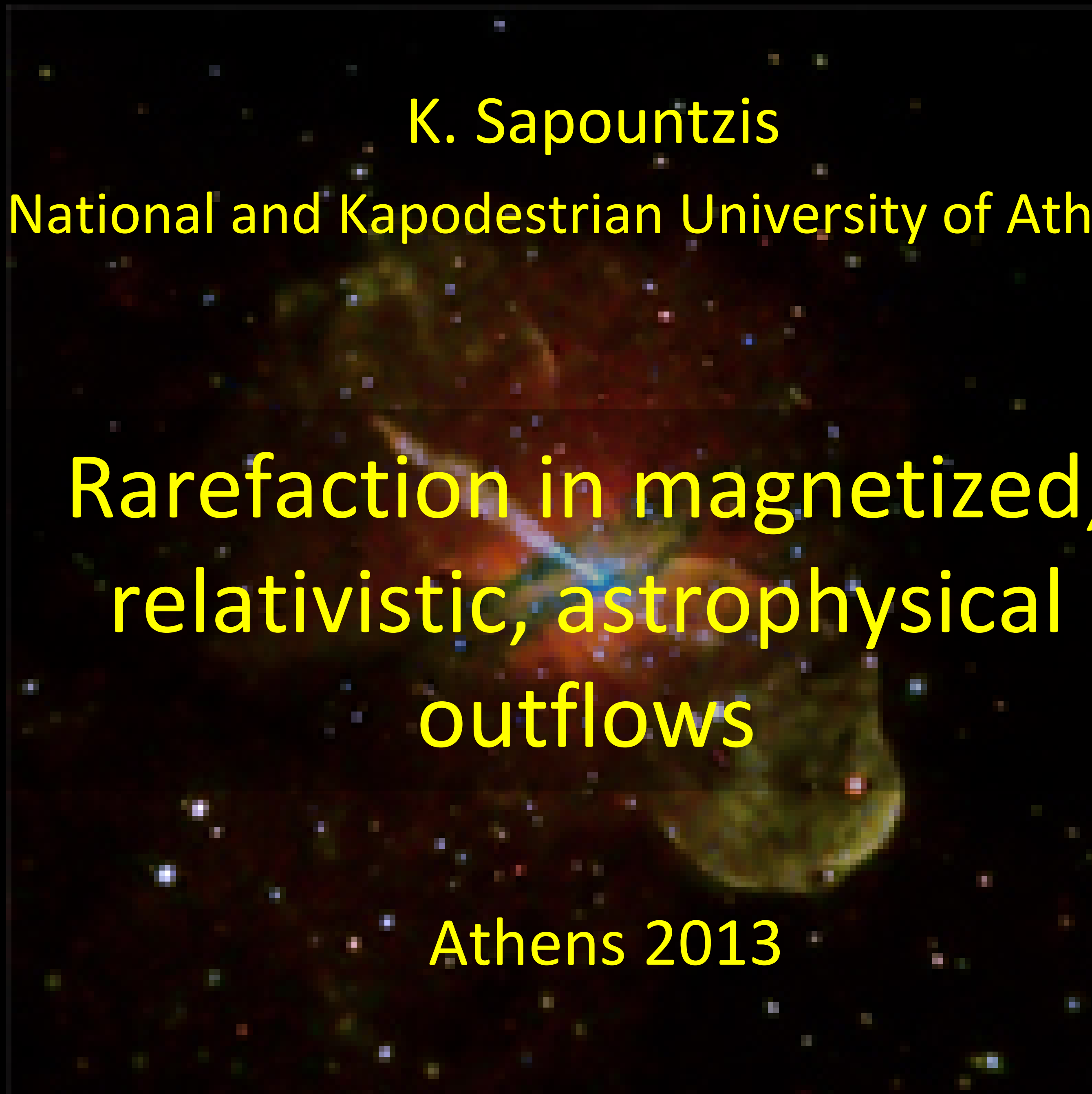

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Rarefaction in magnetized, relativistic, astrophysical outflows

Athens 2013

Contents

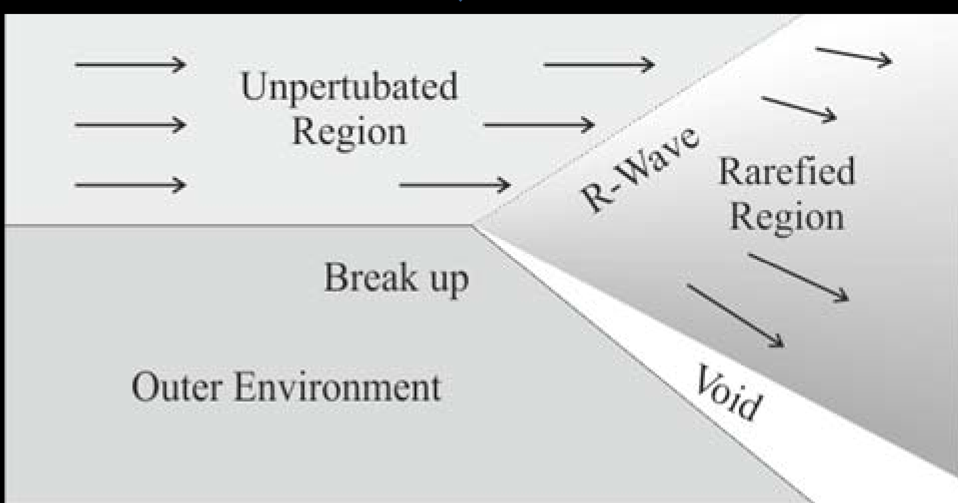
Motivation

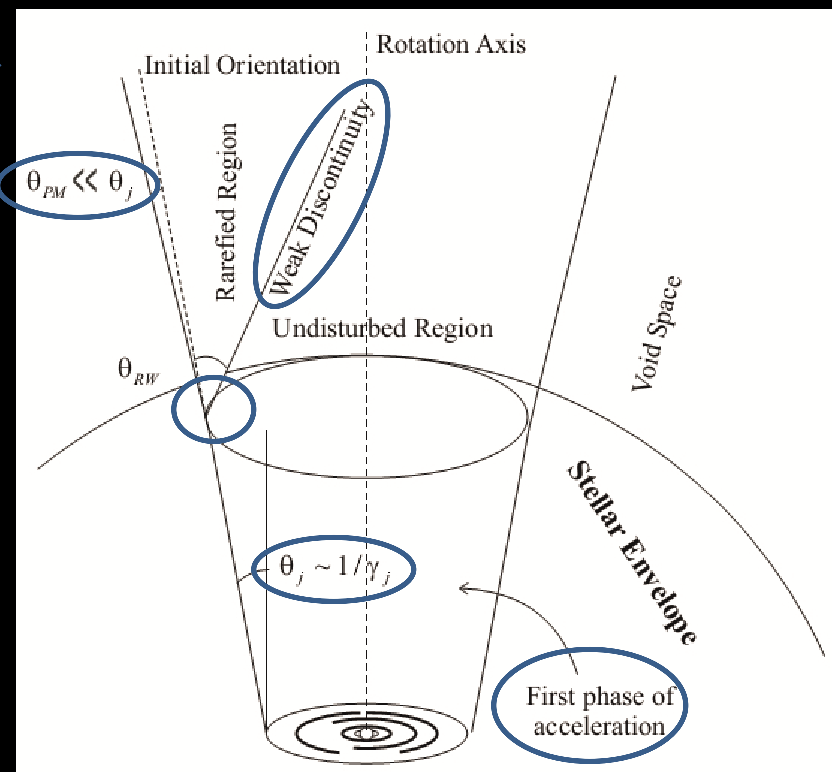
- Study of the super-fast magnetosonic jets applied to the Gamma Ray Bursts environment and especially in the Collapsar Model where rarefaction occurs

- General application in astrophysical jets under a specific external pressure profile (in progress)

Geometry of the Collapsar outflow

Geometry of Rarefaction Wave





Framework

In contrast with most **sophisticated numerical simulation** that solve the full **time dependent problem** we analyze the steady state case

Velocities are relativistic and the trivial assumption of an ideal conducting plasma was made.

Assumption of cold outflow (specific enthalpy unity) since any initial thermal content will have been converted to bulk kinetic long before the break up

Our model is axisymmetric

R MHD equations

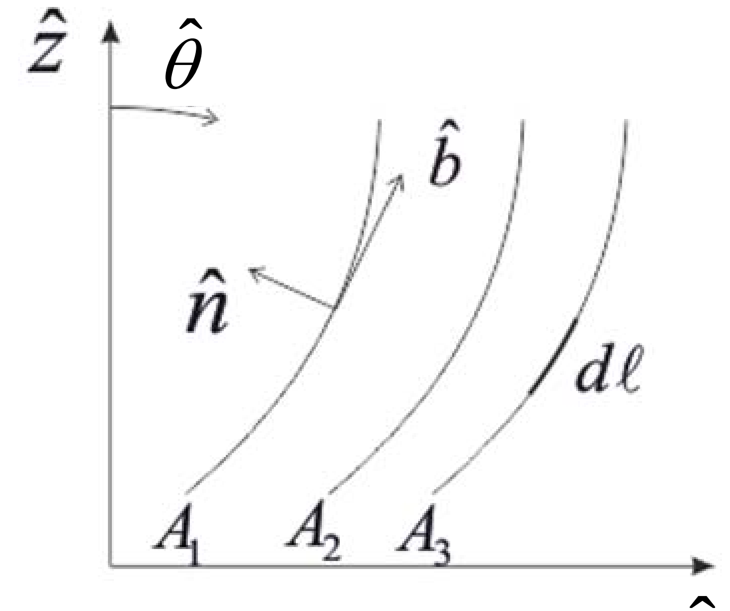
Steady State
$\partial/\partial t = 0$

Ideal
$\sigma \to \infty$

Cold Outflow
$\text{enthalpy per mass} \to 1$

Axisymmeric
$\partial/\partial\varphi$

Basic Annotation



labels each poloidal field line

$$A = \frac{1}{2\pi}\iint \vec{B}_p \cdot d\vec{S}$$

ℓ : physical parameter

Measure the length along a field line

$$\mu = \frac{\text{total energy flux (inertial + poynting)}}{\text{rest mass energy flux}}$$

$$\sigma = \frac{\text{poynting energy flux}}{\text{inertial energy flux}}$$ magnetization parameter

The Algorithm Schema I

Under analytical consideration we reach to a system of 5 first order pdes

$$F(A,\ell)\frac{\partial U}{\partial \ell} + G(A,\ell)\frac{\partial U}{\partial A} = H(A,\ell) \qquad U = \begin{bmatrix} \omega \\ z \\ \theta \\ S \\ \Lambda \end{bmatrix}$$

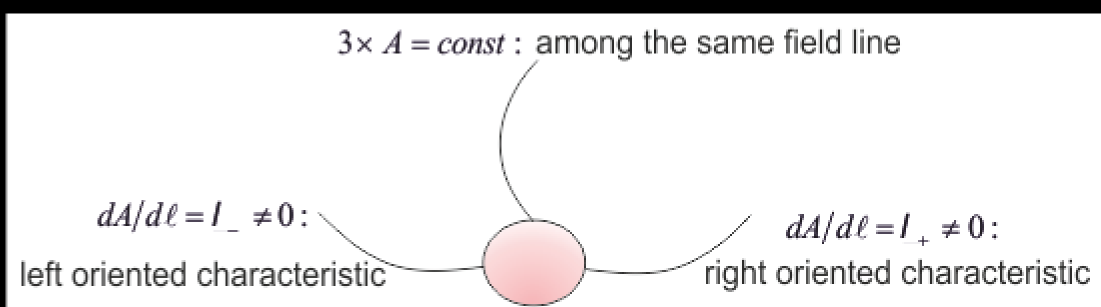
Coordinates of the poloidal plane

Line inclination

Quantities instead of

$\partial\omega/\partial A, \partial z/\partial A$

The system has 5 characteristics

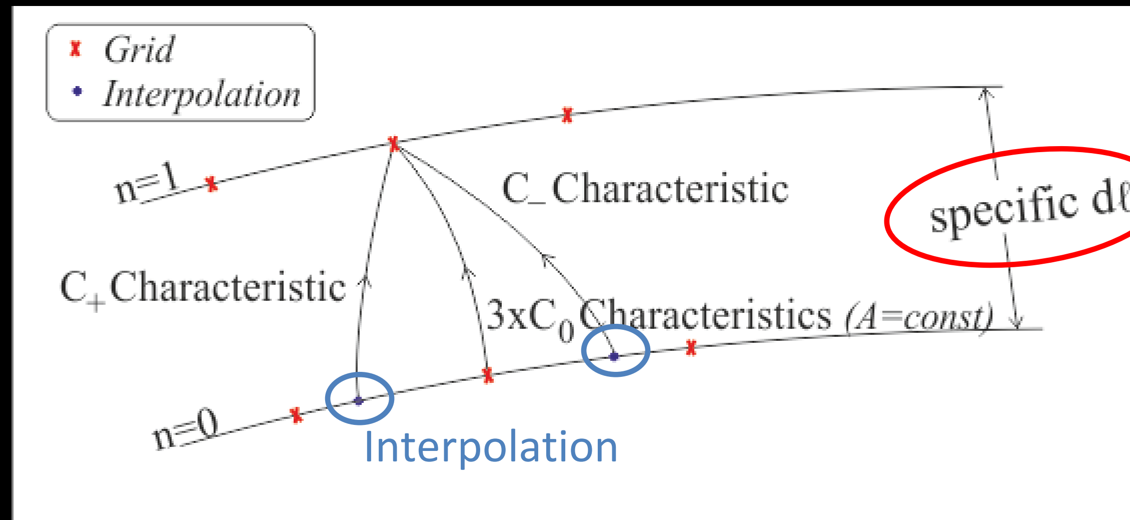


The Algorithm Schema II

Procedure

1)For a specific $d\ell$ we find the intermediate points by linear interpolation on the characteristics inclination

2)We solve the system of the 5 characteristics and determine the new values of the quantities

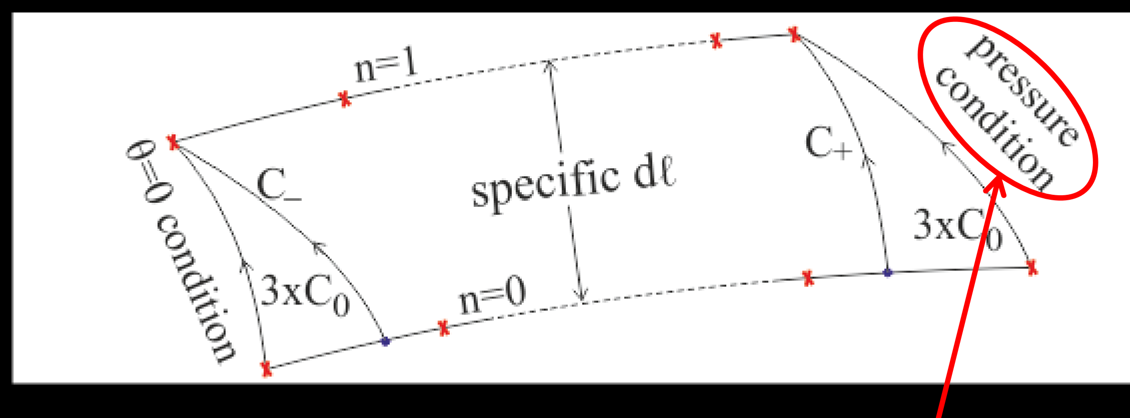


In the inner (axis) and outer boundary one characteristic is missing. We fill the missing information by the physical requirements

3) Inner the magnetic field must be continuous and due to axis-symmetry

4) For the wall $\theta_l = 0$ we set the pressure outflow to be equal with its environment . In rarefaction

$$P = P_{ext} = 0$$



The shape of the boundary line is obtained self consistently

The initial conditions

The initial configuration settled in a plane cross section $z_i = 10^5 \left(\ell_i = const = 0 \right)$

and consists of a homogenous velocity flow with no azimouthal velocity. $\gamma_i = 100 \quad v_{\varphi i} = 0$

Poloidal streamlines (and poloidal field lines) parallel to z-axis $\vartheta_i = 0$

Magnetic field has been chosen as to satisfy core like in the innermost structure the transfield component of momentum equation

$$B_{pi} = \frac{B_j}{\left[1 + \left(\omega / \omega_o\right)^2\right]^{\zeta}}$$

ζ Model parameter

ω_0 Core radius

B_j Scale of the magnetic field

$$B_{\varphi i} = -\frac{B_j \gamma_i}{\left(\omega / \omega_o\right)\left[1 + \left(\omega / \omega_o\right)^2\right]^{\zeta}} \sqrt{\frac{\left[1 + \left(\omega / \omega_o\right)^2\right]^{2\zeta} - 1 - 2\zeta \left(\omega / \omega_o\right)^2}{2\zeta - 1}}$$

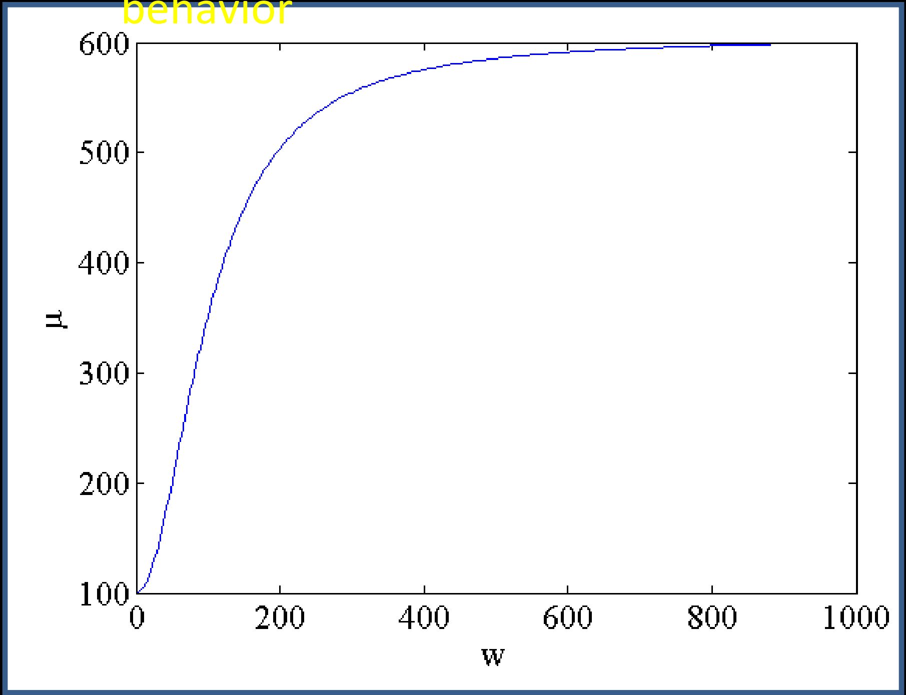
Using the above expressions we calculate $\omega = \omega(A)$ and the other two S_i, Λ_i

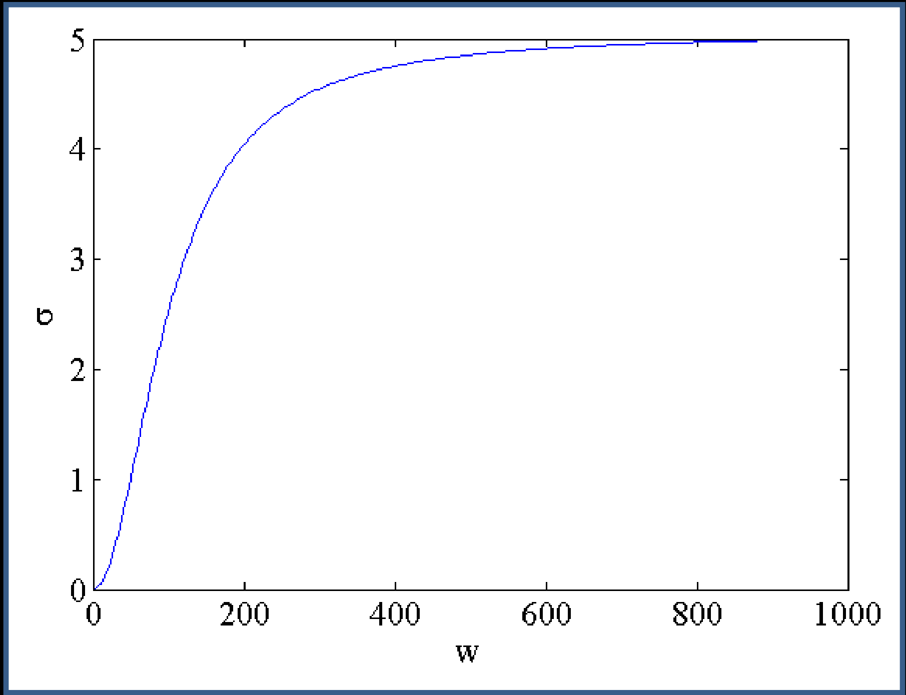
The initial conditions

One more quantity or integral has to be given

$\mu(\text{or } \sigma)$ Determines the maximum attainable Lorentz factor $\gamma_{mx} = \mu$

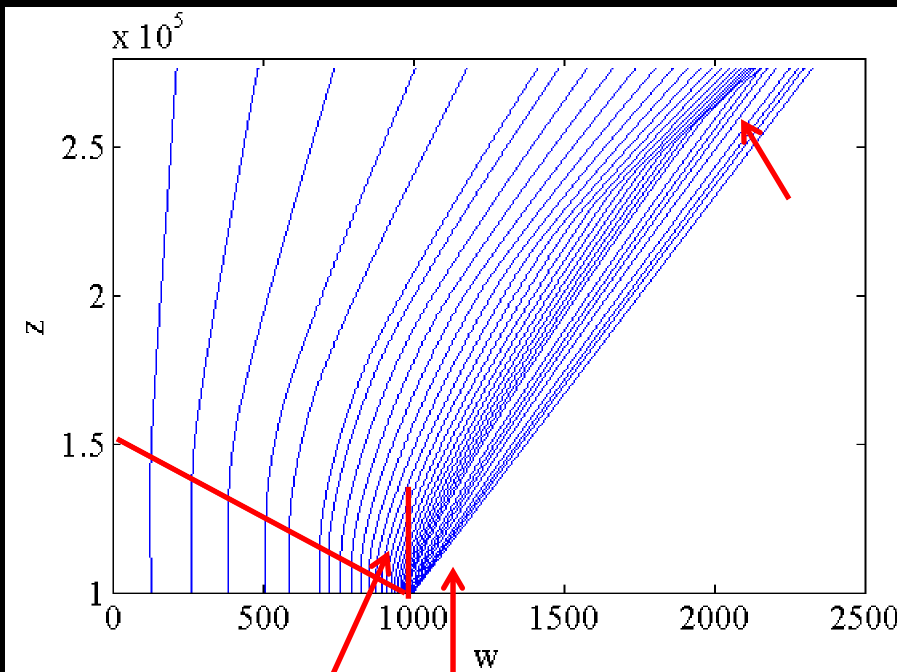
Close to axis expressions alters in order to simulate the core and proper behavior

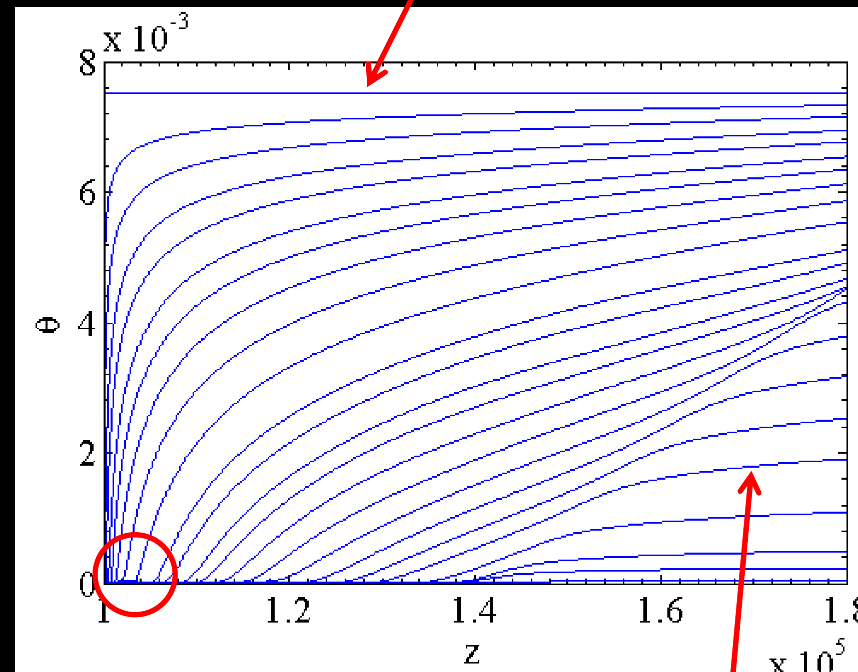




Results

$\theta_{PM} = 0.0075$





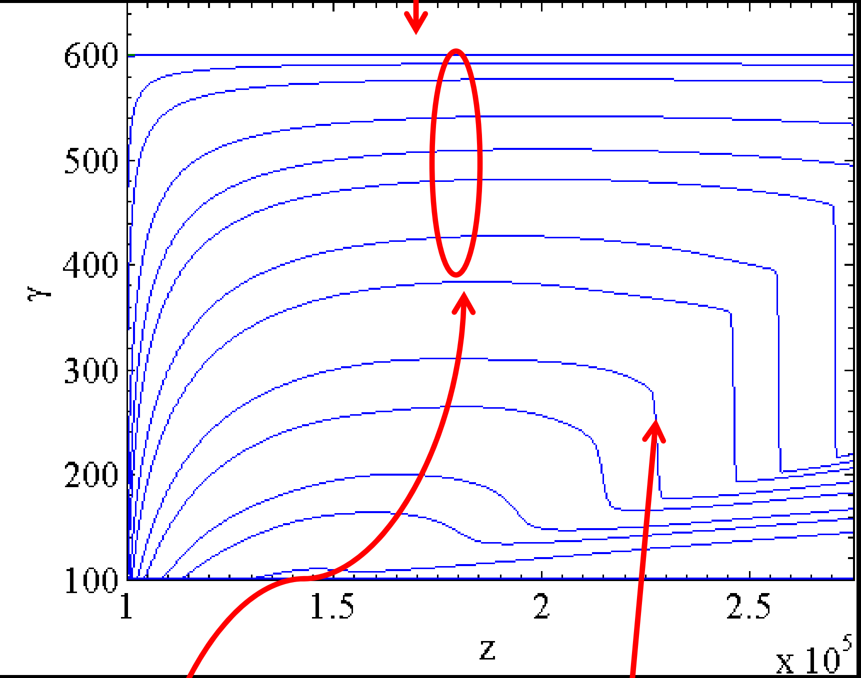
$\theta_{RW} = 0.023$

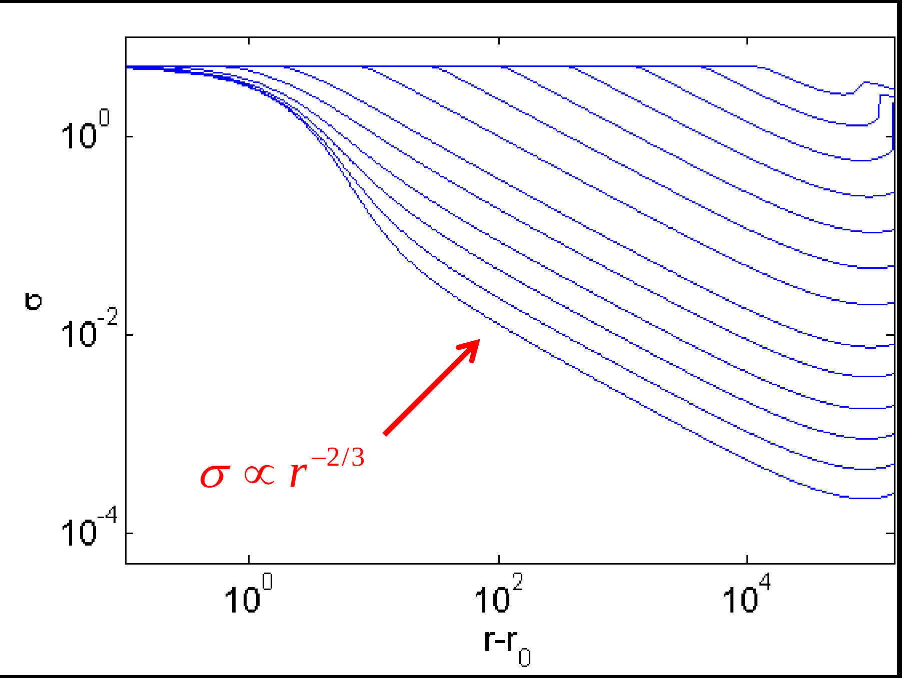
Physical Shape of the flow: The flow accelerates without further collimation, actually its opening-angle increases slightly

Reflection

Results

μ: maximum attainable Lorentz Factor





$\sigma \propto r^{-2/3}$

The reflected wave ceases acceleration

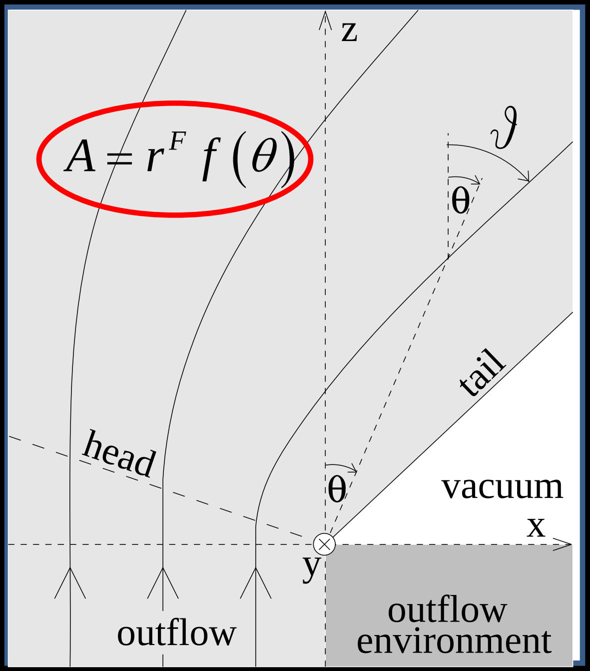
The efficiency is high at the initial "Rarefaction" stage

Previous Work - Comparison

We used the set of r-self similar solutions to solve the planar symmetric case

generalization of hydro (ex Landau) and hydro relativistic (Granik 1982)

K. Sapountzis, N. Vlahakis , MNRAS, 2013



Assumptions

- Relativistic, planar symmetric, ideal
- Thermal context included
- Poloidal magnetic field negligible $(B_p \ll B_y)$

  generalization available, to be submitted

Obtained

- Semi-analytical system, 2 ordinary differential equations
- Easier to handle analytically in specific limits (ex cold one)

Previous Works - Comparison

Conclusions

- Magnetic driven acceleration acts in shorter spatial scales
- The front is the envelope of the fastest moving disturbances
- The efficiency is very high
- Appearance in cold limit of Rarefaction Wave at $\sin\theta_{RW} = -\frac{\sqrt{\sigma_i}}{\gamma_i} = 0.023$
- Extension of rarefaction region $\theta_{PM} = \frac{2\sqrt{\sigma_i}}{\gamma_i\left(1+\sigma_i\right)} = 0.0075$
- It follows the scaling $\sigma \propto r^{-2/3}$

Application to GRB/Collapsar

According to the Collapsar scenario the most prominent candidate are the Wolf-Rayet stars

Typical values $> 20M_{\odot}$

Radius: $\sim 10R_{\odot} \sim \left(10^{11} - 10^{12}\right) cm$

Our parameter selection actually defines a characteristic length. In the results shown:

$$\hat{\ell} \leftrightarrow 10^{7}\, cm$$

In distances $10^{11}\, cm \to 10^{4}\hat{\ell}$ efficiency is high due to purely rarefaction

In distances $10^{12}\, cm \to 10^{5}\hat{\ell}$ efficiency decreases because reflection has occurred

Conclusions

- Rarefaction is an effective mechanism converting magnetic energy (poynting) to bulk kinetic
- It arises naturally at the point where the outflow crosses the envelop of the star and continues to the interstellar medium
- Allows potential panchromatic breaks to appear due to the geometry it provides
- Reflection occurs and the reflecting wave ceases acceleration and causes a significant deceleration
- Our model determines the shape of the last field line self-consistently
- The curvature seems not to play important role in the first phase of acceleration and so the simpler planar symmetric models can be used
- The spatial distances are in accordance with the ones that fireball - internal shock require

END