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3D asymmetric periodic orbits in the Sun-Jupiter-Trojan Asteroid-Spacecraft system

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Abstract: We consider the Sun, the Jupiter and an hypothetical Trojan Asteroid to lie at the apices of an equilateral triangle while a Spacecraft is moving under the Newtonian gravitational attraction of these three primary bodies. This problem has planar families of non-symmetric periodic orbits which we have already found in previous work [1]. The planar family which emanates from the equilibrium point L_3 has two vertical-critical periodic orbits. From these two critical points we found two families of three-dimensional asymmetric periodic solutions. Then we focused our research to find 3D periodic orbits close to Asteroid. We found a family of non-symmetric three-dimensional periodic orbits around the Asteroid and the stable equilibrium points L_6 and L_7 of the problem as well. Characteristic curves of all these families are presented. The stability of each periodic solution we found is also studied.

1 Introduction

We consider that the dominant primary body m_1 (Sun), is on the negative x-axis at the origin of time and the three point masses moving in circular periodic orbits around their center of mass. The equations of motion of the massless fourth body (Spacecraft) referred to a synodic rotating coordinate system with the same origin as the primaries are [1],

$$\ddot{x} - 2\dot{y} = x - \sum_{i=1}^{3} \frac{m_i(x - x_i)}{r_i^3}, \ddot{y} + 2\dot{x} = y - \sum_{i=1}^{3} \frac{m_i(y - y_i)}{r_i^3}, \ddot{z} = -\sum_{i=1}^{3} \frac{m_i(z - z_i)}{r_i^3}$$

when $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$, $i = 1, 2, 3$ and x_i, y_i are the coordinates of the primaries (for details see [1]). The gravitational potential in synodic coordinates is given by the equation $\Omega = \frac{1}{2}(x^2 + y^2) + \frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}$ and a Jacobian type of integral of the problem is $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - C$ where C is the Jacobian constant.

2 Asymmetric 3D periodic orbits

In Fig. 1 (left) we present the positions of the three primary bodies m_1, m_2, m_3 and the eight equilibrium points of the problem as well, for $m_1 = m_{Sun}/M$, $m_2 = m_{Jupiter}/M$ and $m_3 = m_{Asteroid}/M$ when $M = m_{Sun} + m_{Jupiter} + m_{Asteroid}$, $m_{Sun} = 1.98892 \times 10^{30}$, $m_{Jupiter} = 1.8986 \times 10^{27}$ and $m_{Asteroid} = 1.4 \times 10^{19}$ or $m_1 \simeq 0.999046$, $m_2 \simeq 0.000953$ and $m_3 \simeq 0.7032 \times 10^{-11}$. We have considered as $m_{Asteroid}$ the mass of an actual asteroid of the Trojan group, 624 Hektor, a main one of the group. Because the very small mass of the Asteroid, with respect to the other primaries, all calculations reported in this paper were performed in quadruple precision. Using a standard corrector-predictor procedure we calculated the family f_{L_3} which emanates from the equilibrium point L_3 and consists of retrograde non-symmetric periodic orbits around it. This family is entirely unstable and it has two vertical-critical periodic solutions. It is well known that these vertical-critical orbits are starting points for the determination of the families of three-dimensional periodic orbits. So, we calculated the two 3D families emanate from them ($f_{L_3}^{1v}$ and $f_{L_3}^{2v}$ correspondingly, see Fig. 1 right). In Fig. 2 (left-up) a sample

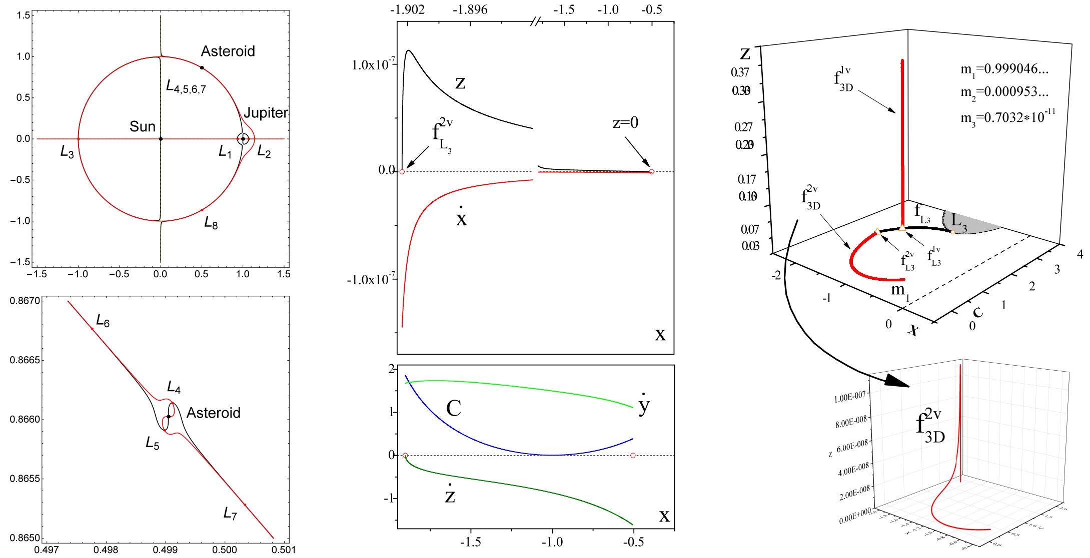

Figure 1: Left: The positions of the primary bodies and equilibrium points of the problem (up) and zoom area close to Asteroid (down). Middle: Characteristic curves of the three-dimensional family f_{3D}^{2v}. Right: The two three-dimensional families which emanate from the planar vertical critical periodic orbits of family f_{L_3}.

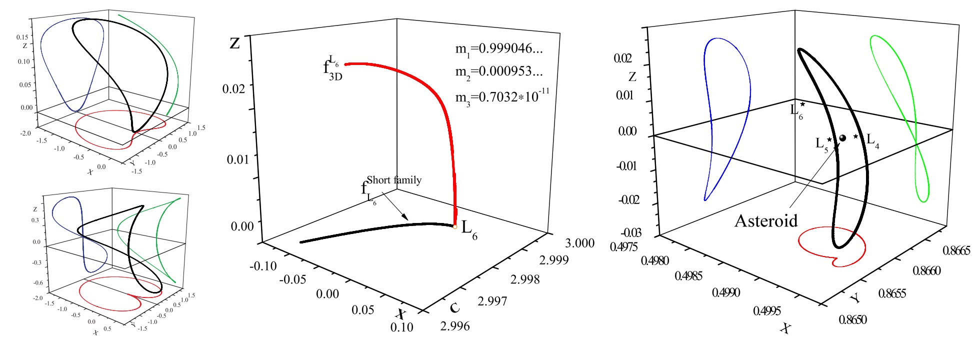

Figure 2: Left: Three-dimensional non-symmetric periodic orbit of family f_{3D}^{1v} (up) and f_{3D}^{2v} (down). Middle: The three-dimensional family $f_{3D}^{L_6}$ which emanates from a point close to L_6. Right: Three-dimensional non-symmetric periodic orbit of family $f_{3D}^{L_6}$ around the Asteroid

of a 3D orbit of this family is presented. The second family f_{3D}^{2v} goes up until the parameter z becomes maximum ($z \simeq 1 \times 10^{-7}$) and then goes down, very slowly almost asymptotically to the plain, until z becomes again zero (Fig. 1 middle and right). This is not the end of the 3D family because the “last” orbit is not a planar periodic orbit, since $\dot{z} \neq 0$ (Fig. 1 middle, green line), but just an orbit which begins its 3D path from the plane $x - y$ (so $z = 0$). We stopped in this orbit and we didn't continue for $z < 0$. An other characteristic of this family is that its periodic orbits begin with 2 intersections with the $x-$axis and then they change multiplicity and become with 4 intersections. Because of the practical interest we searched close to Asteroid for families of three-dimensional periodic orbits. So we found an other 3D family of non-symmetric periodic orbits ($f_{3D}^{L_6}$) which emanates from a point very close to equilibrium point L_6. In Fig. 2 (middle) we present this family which consists of periodic orbits around the Asteroid. In the same figure (in the right panel) a sample of a non-symmetric of 3D periodic orbit of this family is illustrated. Families f_{3D}^{1v} and $f_{3D}^{L_6}$ grow up as z tends to large values. Family $f_{3D}^{L_6}$ has stable and unstable 3D periodic orbits while families f_{3D}^{1v} and f_{3D}^{2v} are entirely unstable.

References

[1] Baltagiannis, A. N. and Papadakis, K. E., 2013, “Periodic solutions in the Sun-Jupiter-Trojan Asteroid-Spacecraft system”, *Planetary and Space Science*, **75**, pp. 148–157.