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Families of 3D periodic orbits in the photogravitational restricted four-body problem

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Abstract: Let m_1, m_2 and m_3 be the masses of three bodies, called primaries, with $m_1 >> m_2 = m_3$ moving in circular periodic orbits around their center of mass fixed at the origin of the coordinates. These masses always lie at the vertices of equilateral triangle with the dominant body m_1 (Sun) being on the negative $x-$axis at the origin of time. The Sun is a radiation source. A massless particle is moving under the Newtonian gravitational attraction of the primaries and does not affect the motion of the three bodies. Using the vertical-critical orbits of planar families of symmetric periodic orbits as starting points, we determine and present in this paper, families of three-dimensional periodic solutions of the problem as the dominant body m_1 radiates. The stability of every three-dimensional periodic orbit which numerically calculated is also studied.

1 The photogravitational four-body problem

Here we consider the photogravitational restricted four-body where three point masses, called primaries, moving in circular periodic orbits around their center of mass under the mutual Newtonian gravitational attraction, forming an equilateral triangle configuration, while the dominant primary (say m_1) radiates. A fourth massless body is moving under the gravitational attraction of the primaries. The equations of motion of the problem, in the usual dimensionless rectangular rotating coordinate system are written as [1],

$$\ddot{x} - 2\dot{y} = x - \sum_{i=1}^{3} \frac{q_i m_i (x - x_i)}{r_i^3}, \ddot{y} + 2\dot{x} = y - \sum_{i=1}^{3} \frac{q_i m_i (y - y_i)}{r_i^3}, \ddot{z} = -\sum_{i=1}^{3} \frac{q_i m_i (z - z_i)}{r_i^3}$$

when q_i are the radiation pressure parameters, the distance of the fourth particle from each of the three primaries is $r_i^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2$, $i = 1, 2, 3$ where x_i, y_i and $z_1 = z_2 = 0$ (plane case), are the coordinates of the primaries

$$x_1 = -\frac{|K|\sqrt{m_2^2 + m_2 m_3 + m_3^2}}{K}, \qquad y_1 = 0, \qquad x_2 = \frac{|K|[(m_2 - m_3)m_3 + m_1(2m_2 + m_3)]}{2K\sqrt{m_2^2 + m_2 m_3 + m_3^2}},$$

$$y_2 = -\frac{m_3}{m_2^{3/2}} M, \qquad x_3 = \frac{|K|}{2\sqrt{m_2^2 + m_2 m_3 + m_3^2}}, \qquad y_3 = \frac{1}{m_2^{1/2}} M$$

where we have abbreviated $K = m_2(m_3 - m_2) + m_1(m_2 + 2m_3)$ and $M = \frac{\sqrt{3}}{2}\left(\frac{m_2^3}{m_2^2 + m_2 m_3 + m_3^2}\right)^{1/2}$. The equations of motion admit a Jacobian type of integral $\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - C$ where $\Omega = \frac{1}{2}(x^2 + y^2) + \frac{q_1 m_1}{r_1} + \frac{q_2 m_2}{r_2} + \frac{q_3 m_3}{r_3}$ and C is the Jacobian constant.

2 Numerical results

In this research we found the families of three dimensional periodic orbits which emanate from the planar vertical-critical periodic orbits of the basic families of the restricted four-body problem. We

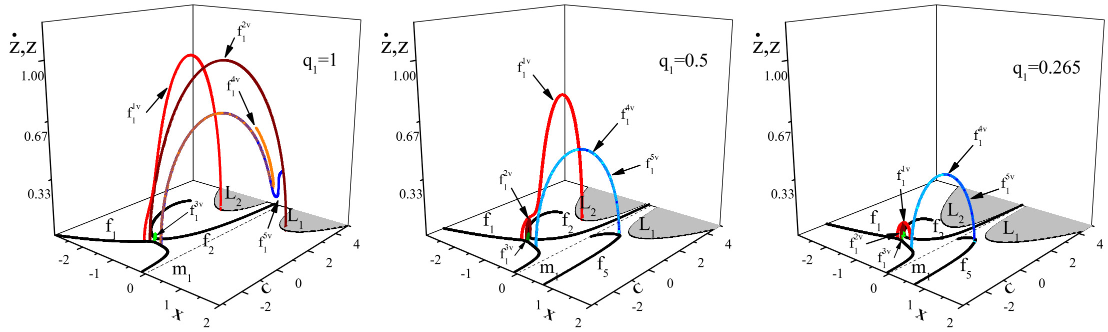

Figure 1: Left: Families of 3D periodic orbits for $m_1 = 0.99$, $m_2 = m_3 = 0.005$. Middle - Right: The same families when the dominant primary body (Sun) radiates

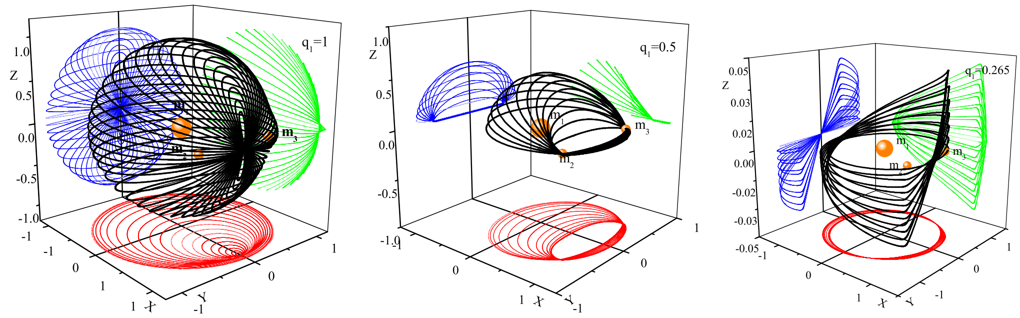

Figure 2: Left: The family f_1^{2v} of three-dimensional doubly-symmetric periodic orbits around the primary bodies. Middle: The family f_1^{4v} of three-dimensional plane-symmetric periodic orbits when the Sun radiates ($q_1 = 0.5$). Right: The family f_1^{3v} of three-dimensional axis-symmetric periodic orbits when the Sun radiates ($q_1 = 0.265$)

consider that the dominant primary body m_1 (Sun) is a radiation source. So we calculated the vertical-critical periodic orbits of the basic families, as the Sun radiates, for $m_1 = 0.99$ and $m_2 = m_3 = 0.005$. We used these vertical-critical orbits as starting points for the determination of the families of three-dimensional periodic orbits. In Fig. 1 (left) we present all the 3D-families which emanate from the five critical-periodic orbits of the planar family f_1 which consists of retrograde periodic orbits around the primary bodies. In the same figure we illustrate the same families as the dominant primary radiates. So, in the middle panel we present the evolution of the five families when the radiation parameter is $q_1 = 0.5$ and in the right panel when $q_1 = 0.265$. The stability for each 3D-periodic orbit we found, is also calculated. From our numerical results we found out that the effect of the radiation pressure is very strong on the families of three-dimensional periodic orbits. For instance, the 3D family f_2^{1v}, which have orbits doubly symmetric with respect to xz plane and to $x-$axis, emanates from the second vertical-critical orbit of the planar family f_1 and then goes up in three dimensions until the vertical parameter z gets the value about $z \simeq 1.001$ and then goes down again to the plain on the equilibrium point L_1 of the problem. When the Sun radiates ($q_1 = 0.5$) the same family has not any more the same end (L_1) since it goes down to plane on a vertical-critical periodic orbit, namely on the f_2^{2v}. Similarly, when the radiation parameter is $q_1 = 0.265$ then the same family has a different end from the two previous cases and now ends on the vertical-critical periodic orbit f_2^{4v}. In Fig. 2 we plot 3D-periodic orbits of family f_1^{2v} (doubly-symmetric) when the Sun do not radiate (left), in the middle panel we plot the family f_1^{4v} (plane-symmetric) when the Sun radiates ($q_1 = 0.5$) and in the right panel we plot the family f_1^{3v} (axis-symmetric) when the Sun is a strong radiation source ($q_1 = 0.265$).

References

[1] Papadouris, J. P. and Papadakis, K. E., 2014, “Periodic solutions in the photogravitational resticted four-body problem”, *Monthly Notices of the Royal Astronomical Society (MNRAS)*, **442**, pp. 1628–1639.