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Computing differentially rotating general-relativistic polytropic models by a post-Newtonian hybrid approximative scheme

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Abstract: We use the so-called "hybrid approximative scheme", developed by the authors in the framework of the post-Newtonian approximation, for computing general-relativistic polytropic models simulating neutron stars in differential rotation. The differential equations governing the model are considered as a "complex initial value problem", solved by the "complex-plane strategy".

1 Introduction

This work is a continuation of [1] in which a "hybrid approximative scheme" (HAS) has been applied to general-relativistic polytropic models simulating critically rotating neutron stars. In order to extend our computations to differentially rotating models, we combine HAS with the so-called "complex-plane iterative technique" (CIT) [2]. We describe here both the theoretical and numerical basis for using this method in the computations of differentially rotating general-relativistic polytropic models.

2 Hybrid Approximative Scheme (HAS)

In [1], we have developed HAS on the basis of the "post-Newtonian approximation" (PNA) of [3] under the assumption that the relativistic distortion is involved by its complete solution as given in [4]. By using this complete solution, $\Theta_\sigma(\xi) = \theta_{00}(\xi) + \sigma\,\theta_{30}(\xi)$, we obtain

$$\begin{aligned}\Theta(\xi,\mu) = {} & \Theta_\sigma\,P_0(\mu) + \upsilon\,[\theta_{10}(\xi)\,P_0(\mu) + A_{12}\theta_{12}(\xi)P_2(\mu)] \\ & + \upsilon^2\,\{\theta_{20}(\xi)\,P_0(\mu) + [\theta_{22}(\xi) + A_{22}\theta_{12}(\xi)]\,P_2(\mu) + \;[\theta_{24}(\xi) + A_{24}\theta_{14}(\xi)]\,P_4(\mu)\}\,.\end{aligned} \tag{1}$$

The functions θ_{ij} obey Eqs. (37) and (38) of [3]. The parameters A_{ij} ([3], Eq. (59)) multiply properly the homogeneous solutions of θ_{ij} ([3], Eqs. (42) and (43)), so that certain boundary conditions be satisfied.

To compute the function Θ_σ we use the Oppenheimer–Volkoff equations of hydrostatic equilibrium ([4], Eqs. (19) and (20)), $\frac{d\Theta_\sigma}{d\xi} = -\frac{1}{\xi^2}\left(\Upsilon_\sigma + \sigma\xi^3\Theta_\sigma^{n+1}\right)\frac{[1+(n+1)\,\sigma\,\Theta_\sigma]}{1-2\,(n+1)\,\sigma\,(\Upsilon_\sigma/\xi)}$, $\Upsilon'_\sigma = \xi^2\,\Theta_\sigma^n\,(1+\sigma\,n\,\Theta_\sigma)$, where the function Υ_σ is defined in [4] (Eq. (18)). In the relativistic case $\sigma > 0$, Θ_σ is the total distortion owing to relativity and can be written as $\Theta_\sigma = \theta_{00} + \sum_{i=1}^{\infty}\sigma^i\,\theta_{3(i-1)}$. The PNA of [3] includes terms of first order in σ; thus the sum has the single term $\sigma\,\theta_{30}$. When with infinite terms, the sum should be equal to $\Theta_\sigma - \theta_{00}$. The basis of HAS consists in using (i) the complete solution of the relativistic distortion, and (ii) perturbation terms of up to second order in υ with respect to the rotational distortion.

3 Complex-Plane Iterative Technique (CIT)

CIT is an iterative method, aiming basically to the computation of the Lane–Emden function θ_{ij}^n and the gravitational potential U_{ij}^i on a predefined grid of ν and ξ points ([5], Sec 3). The basic idea of this study is to use in CIT the rest-mass density Θ^n which results from Eq. (1); details on CIT can be found in [2] (Sec. 3).

We take from Eq. (1) the term $\Theta = \Theta_{ij} = \Theta(\xi_j, \nu_i)$ for the interior of the star $\xi < \Xi_\mathrm{e}$ (Ξ_e is the equatorial radius), and we then use this array as initial guess of the generalized Lane–Emden function Θ^n_{ij}. We begin the first iteration by localizing an absolute minimun Θ_M in $j \in [1, \mathtt{KRP}]$ and then setting $\Theta_j = \Theta_M$, $M \le j \le \mathtt{KRP}$, in order to avoid possible positive values beyond Θ_M. We proceed with the computation of the gravitational potential inside the star, U^i, which obeys the equation

$$U^i(\xi_j, \nu_i) = \Theta(\xi_j, \nu_i) - \frac{\upsilon}{2} \int_0^s \omega^2(s_{ij}) s_{ij} ds + U_0, \tag{2}$$

where $s_{ij} = \xi_j (1 - \nu_i^2)^{1/2}$ is the cylindrical coordinate on the grid. To complete the computation of the function (2), we need to evaluate the integration constant U_0. To do so, we take into account all the elements of the arrays U^i_{ij}, Φ_{ij}, Θ_{ij} inside the star and compute the average value of $U_{0ij} = U_0(\xi_j, \nu_i)$ as $U_0 = \langle U_{0ij} \rangle = \langle U^i_{ij} + \upsilon \Phi_{ij} + \Theta_{ij} \rangle$ for all i and j such that $\Theta_{ij} \ge 0$. The array Φ_{ij} contains the differential rotation integral $\Phi_{ij} = \frac{\upsilon}{2} \int_0^s \omega^2(s_{ij}) s_{ij} ds$ at each grid point. The differential rotation used is the generalized Clement's model (see e.g. [5], Eq. (13)). The parameters a_i and b_i involved in this model can be computed by the well-known Levenberg–Marquard method, so that to simulate the differential rotation law under consideration (e.g. the law used in [6], Eq. (9)). A new iteration begins at this point, using the values computed in the previous one, in order to compute a proper correction of the values Θ_{ij} via Eq. (2). We write the new array Ψ_{ij} as $\Psi_{ij} = U^i_{ij} + \upsilon \Phi_{ij} + U_0$. We then proceed with certain significant corrections discussed in detail in [2] (Sec. 3).

4 Physical Characteristics

We proceed with the computation of several physical characteristics. Briefly, the most important points are: (1) The use of an ansatz for the proper volume element $d\mathcal{V}$ in the place of the coordinate volume element dV, $dV \to d\mathcal{V}$ (a discussion on such an ansatz for nonrotating relativistic models can be found in [7], Sec. 2), which is equivalent to the substitution $d\xi \to \Lambda d\xi$, where $d\xi$ is the coordinate differential and the function Λ plays the role of the metric function $e^{\lambda/2}$ (see e.g. [8], Eqs. (1) and (5)). In the case that a configuration suffers from both relativistic and rotational distortions, the function $\Lambda(\sigma, \upsilon, \xi)$ can be expressed as ([1], Eq. (36)) $\Lambda(\sigma, \upsilon, \xi) = \left[1 - \frac{2Gm(\sigma,\upsilon,\xi)}{c^2 \alpha \xi_\mathrm{avr}(\xi)}\right]$, where $\xi_\mathrm{avr}(\xi)$ is the average radius of the particular configuration. (2) The Newtonian relations for the physical characteristics of interest in the framework of PNA, are modified as in [1] (Sec. 5). For the rotational kinetic energy, T, we write $T = \frac{1}{2} \int_0^1 \int_0^{\xi_\mathrm{t}} \Psi^n \, \xi^4 \omega^2 \, (1 - \mu^2) \, \Lambda \, d\xi \, d\mu$, where ω is the differential rotation, ξ_t is the upper limit of the integration chosen so that $\Xi_\mathrm{e} < \xi_\mathrm{t} \le \xi_\mathrm{end}$. Details regarding units can be found in [1] (Secs. 3 and 5, Eqs. (29)–(32)).

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