

A search for stable orbits in triple-star systems

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Abstract: We model a triple star system with the planar general three-body problem (GTBP). The mutual gravitational interactions of the bodies are perturbations of the integrable Keplerian motion, which holds in a two-body framework. Such mutual interactions become very strong when all bodies are of masses and distances of the same order. When the two bodies are very small (planets), with respect to the third one (star), we get the GTBP of planetary type. In this model, and under the introduction of a rotating frame, we can obtain many families of periodic orbits, which can be linearly stable. Also, it has been shown that these periodic orbits are continued with respect to the planetary masses. In this work we apply the continuation technique and try to compute periodic orbits by increasing the planetary masses up to values which approach the mass of the star. Thus, we obtain families of periodic orbits in triple-star systems. The challenge is to find orbits, for masses of the same order, which are periodic and, also, linearly stable.

1 Model and Method

We consider the general three body problem in a rotating frame [1], which describes the dynamics of a planetary system consisting of a star and two planets with masses m_0 , m_1 and m_2 , respectively, where index 1 represents the inner planet. It is proved that such a system has families of *circular periodic orbits* for small planetary masses and for all mass ratios m_1/m_2 . These families are composed by various parts. There is one continued part (part I) for $R_2/R_1 > 1.58$, where R_1 and R_2 the radius of the planetary orbits. Such periodic orbits can be continued by changing the planetary masses [2]. Thus we can obtain new families with the parameters now being the masses. Also we can examine the linear stability of the periodic orbits by using Floquet's theory. So we classify orbits as *stable* or *unstable*. We remark that slightly stable periodic orbits show a regular evolution for long-term intervals. Instead, unstable orbits are surrounded by chaotic orbits. The width of the chaotic region increases as planetary masses increase. Stability margins with respect to the planetary mass are given in [3] for some particular extarsolar systems.

In this study we start from families of circular periodic orbits for small planetary masses and apply differential continuation with respect to the mass. We remark that all these periodic orbits are stable except for a segment around the 3:1 resonances, namely $R_2/R_1 \approx 2.1$. We increase the planetary masses such that their ratio remains constant and also we consider the normalization $m_1 + m_2 + m_0 = 1$. Thus we obtain families as the masses increase, we examine the linear stability and, then, we determine the upper limit of stability.

2 Results

In our computations we consider the families of periodic orbits with mass ratio $m_2/m_1=0.5, 1.0, 2.0$ and 5.0. We start from a periodic orbit with $m_1 = 0.001$ and increase this value and m_2 , too, along continuation preserving the particular mass ratio. The upper limit of stability obtained is presented in Fig. 1. The most important conclusion is that circular periodic orbits of $R_2/R_1 > 2.1$ are continued and are stable up to large masses that can describe the motion of a potential triple-star systems. We mention that at 3 : 1 resonance, orbits are unstable even for small planetary masses. Numerical

integrations show that linearly stable orbits are surrounded by orbits of long-term stability. An example is shown in Fig. 2. As masses increase, the system takes the shape of a hierarchical triple system, where two of the bodies form a binary pair, which in turn forms a bigger binary pair with the third body.

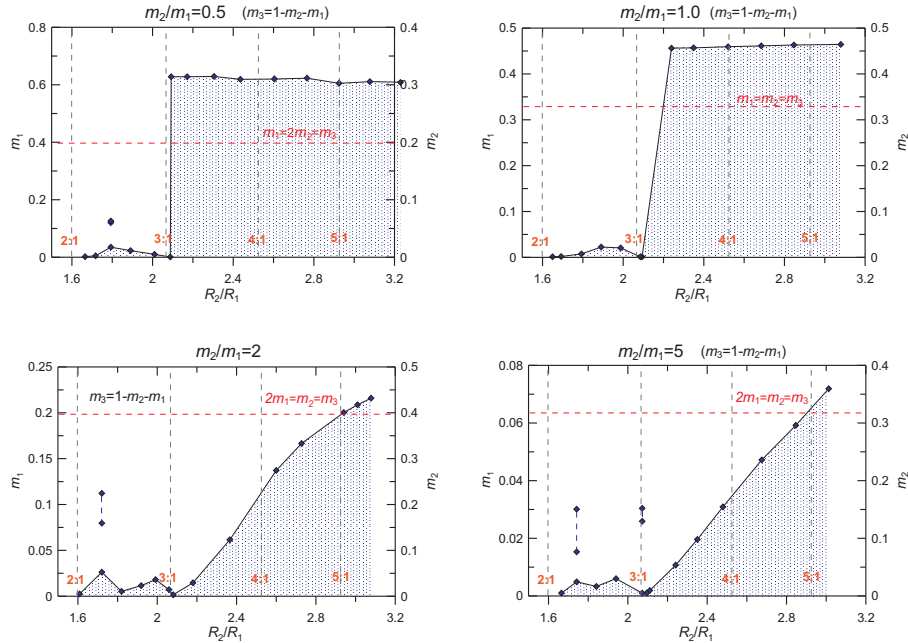


Figure 1: The critical (maximum) mass values up to which the continued circular families with respect to the mass, have linearly stable orbits

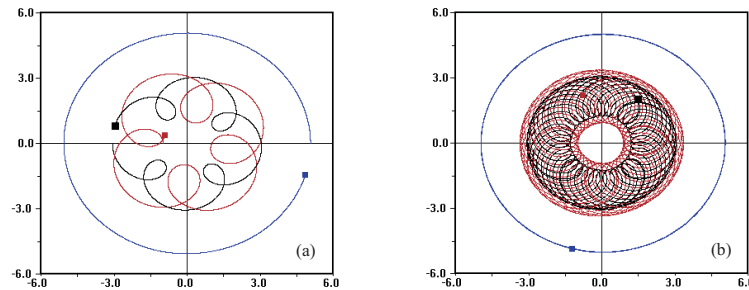


Figure 2: a) A stable orbit with $m_1 = m_2 = m_3 (= 0.33)$. The orbit has been obtained from the continuation of a circular periodic orbit with $R_2/R_1 = 2.24$ and is presented for one period. b) The previous orbit with slightly different initial conditions shown for 20 periods

References

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- [3] Kuznetsov E.D., Kholshevnikov K.V., 2009, *Solar System Research*, 43, 220.