# Particle acceleration and heating in regions of magnetic flux emergence: a statistical approach using test-particle- and MHD-simulations

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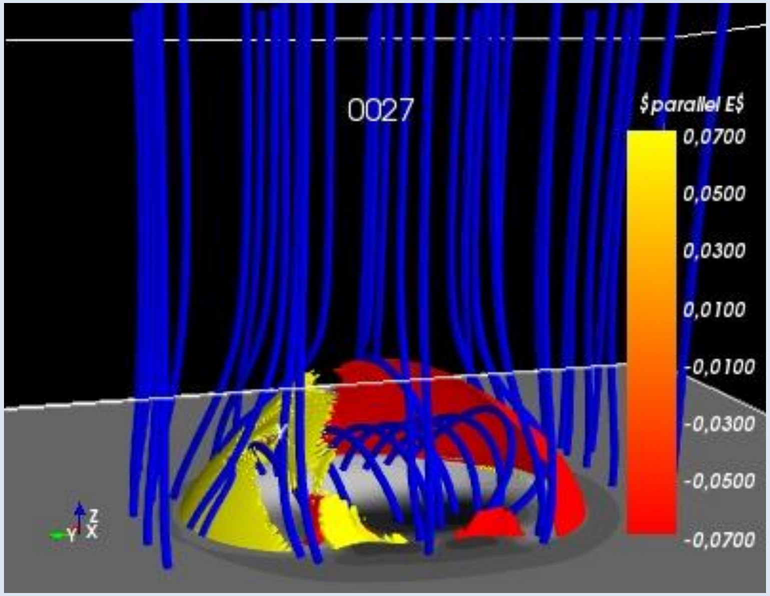
# Outline




3D nonlinear MHD simulations of an emerging flux tube,
from convection zone into the corona
(Vasilis Archontis):

In this study:

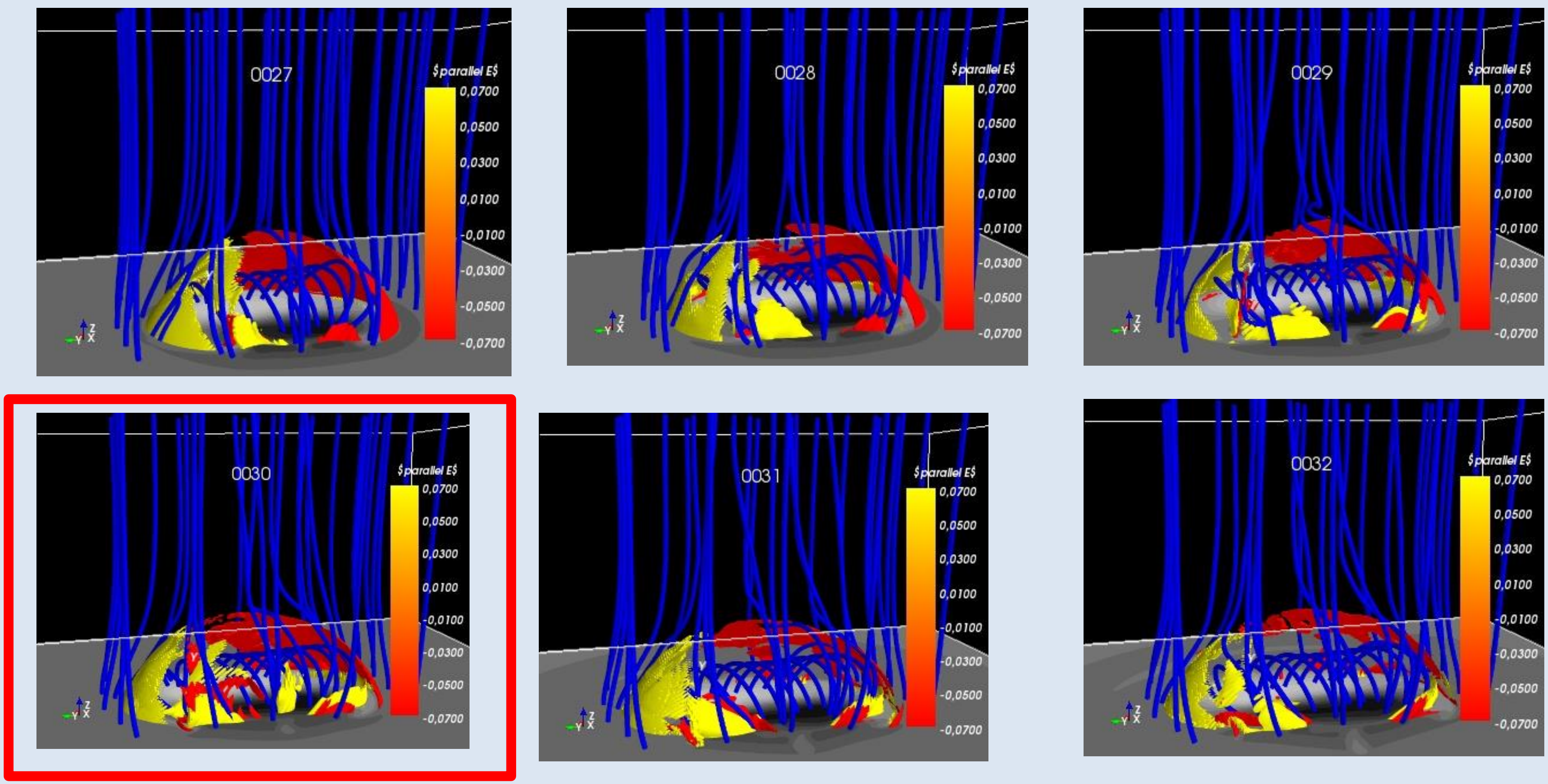
- Focus on the coronal part
- Spatial structure and statistics of the electric field and other MHD quantities

Test-particle simulations (electrons):
- heating
- acceleration

- Particular aim of this project also is to investigate the relation of the acceleration mechanism to Fermi acceleration and Fokker-Planck modeling → determine transport coefficients

# MHD simulations: reconnection and standard jet

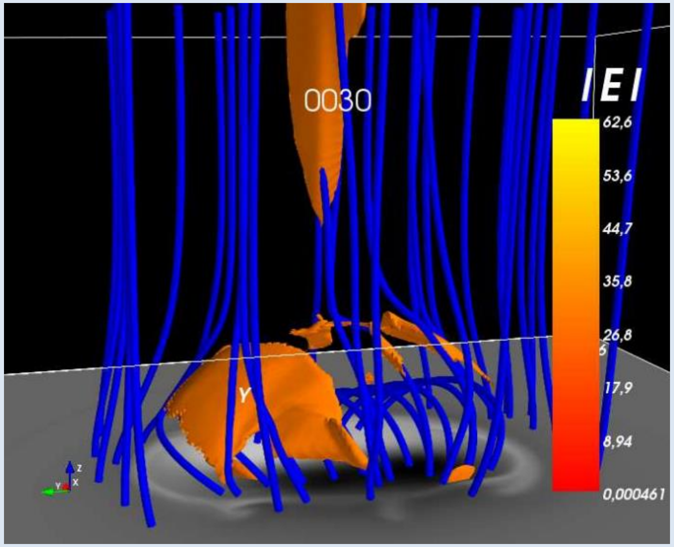
- MHD snapshots of emerging flux tube, reconnecting with pre-existing ambient coronal field:
  - 'standard' jet is formed, which is triggered by the eruption of dense and cool plasma from the emerging flux region
  - turbulence on small scales



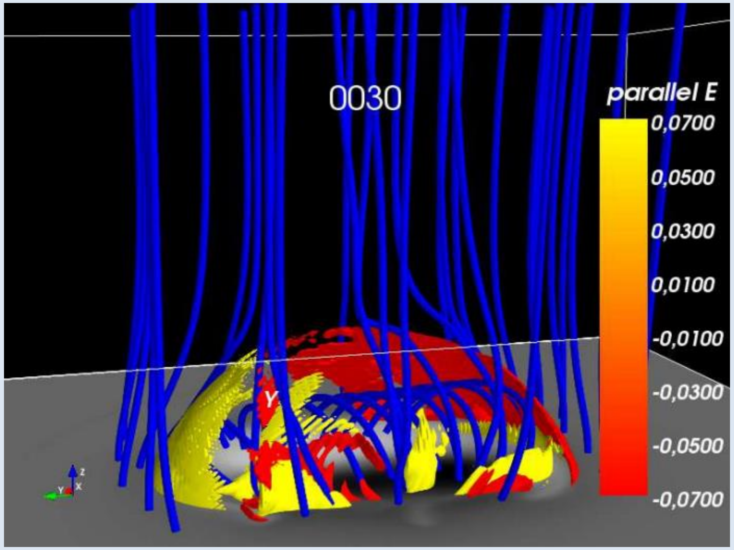
# Spatial structure of the magnetic and electric field

- Snapshot 30, standard outflow jet:
- We concentrate on parallel el. field since it is dominantly responsible for acceleration, see later
- Parallel electric field shows fragmented structures, and has preferred regions of pos. and neg. sign

magnitude of total el. field




parallel el. field

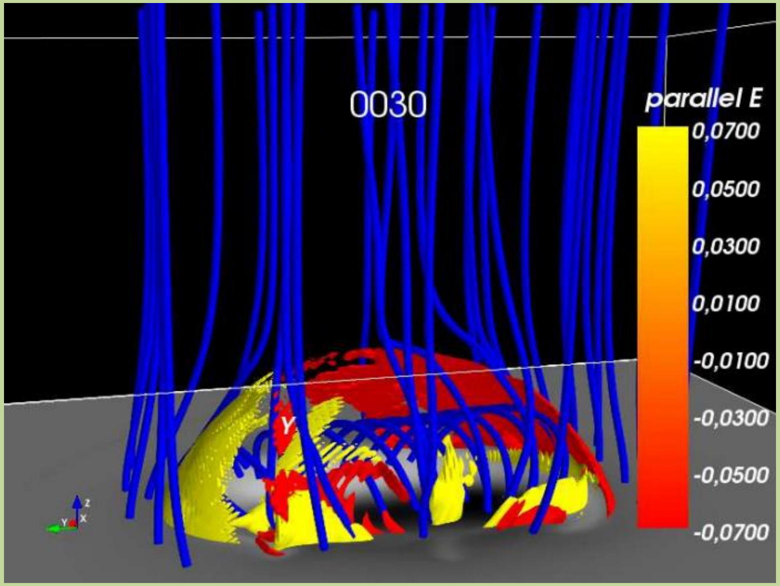



- Fragmentation needs to be quantified: cluster analysis, fractal dimension

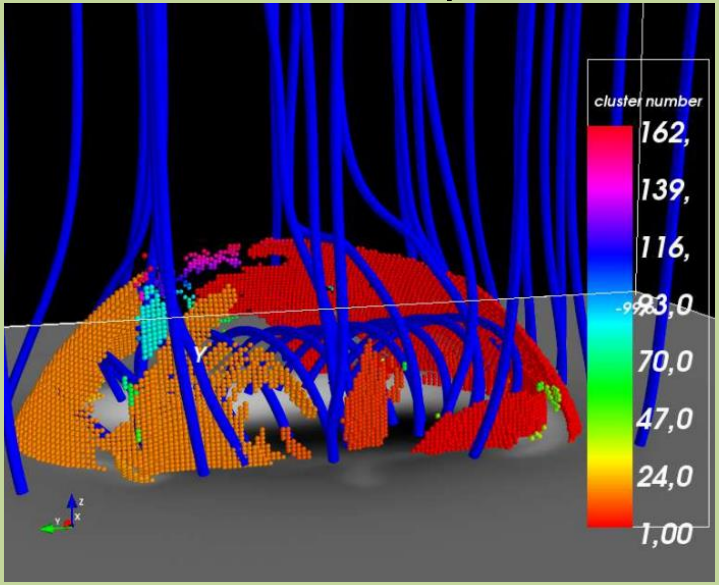
# Cluster analysis of parallel electric field

Cluster defined as grid-sites with above-threshold electric field, connected through nearest neighborhood
Similar to 3D iso-contour-plot, but identifies not connected regions
Free parameter: threshold 0.07, will be justified later

iso-contours



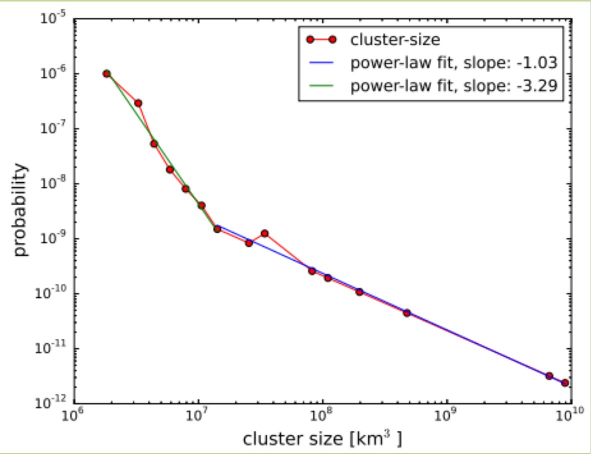

cluster analysis




→ Cluster-size distribution
→ fractal dimension of cluster distribution

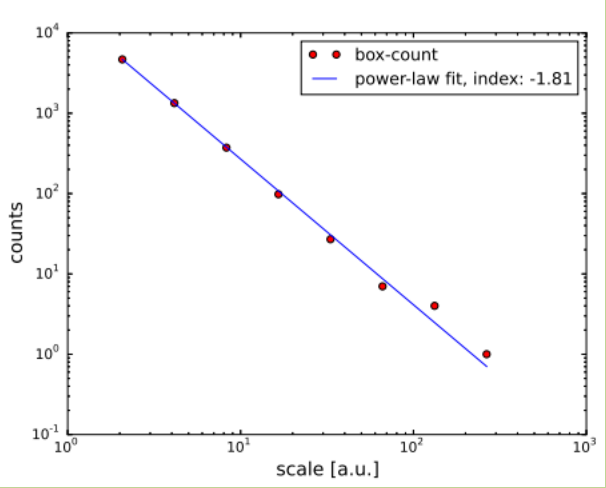
# Cluster-size distribution, fractal dimension

- Cluster-size distribution,
  here size = number of
  grid-points a cluster consists of,
  times elementary grid-volume:
  double power-law



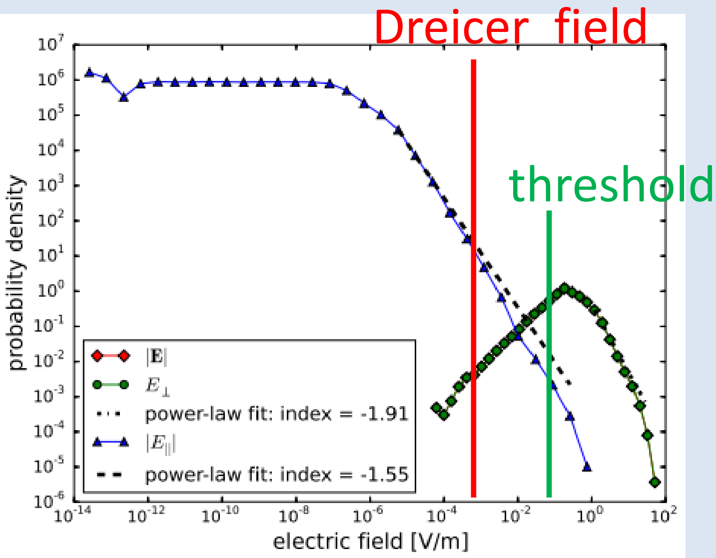

cluster-size [# grid-points*ΔV]

- Fractal dimension of clusters,
  box-counting method in 3D:
  dimension 1.8,
  rarified sheet-like structures
  → fragmented current sheets



# Statistics of the electric field

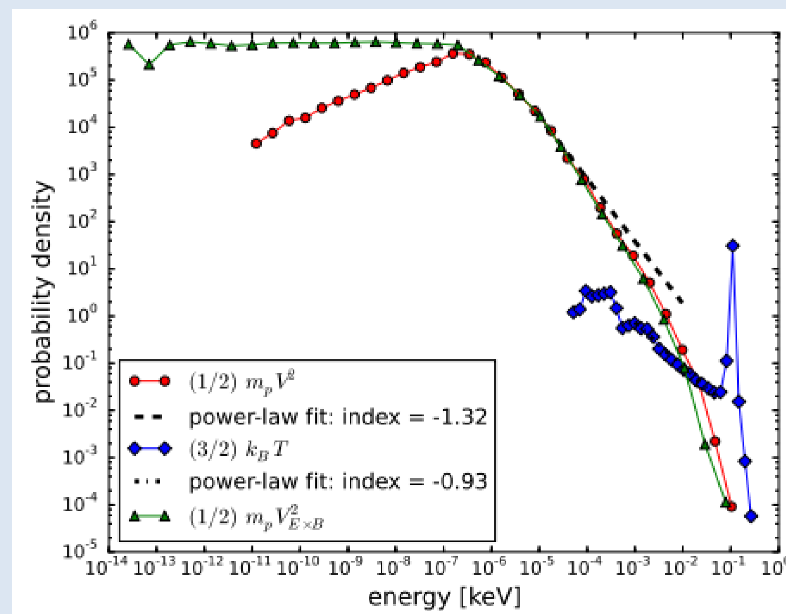
- Histogram of the magnitude of the total, parallel, and perp. electric field
- Parallel el. field 100 times smaller than perpendicular one,
  power-law tail, index  1.6
- Mean Dreicer field $E_D = 5 \times 10^{-4}$ V/m
- Perp. el. field clearly super-Dreicer, parallel el. field super-Dreicer only at a fraction of the grid-points




- Threshold for iso-contours, clusters, and fractal dimesion: 140 $E_D$
  → region of clearly super-Dreicer parallel el. Field.

# Statistical analysis of MHD data: Energies

- Perpendicular dynamics expected to be dominated by E cross B drift: expected energetics
  $E_{\text{kin, EcB}} = (1/2)\, m_e\, (\mathbf{E} \text{ cross } \mathbf{B}\text{-velocity})^2$
  histogram of $E_{kin}$ from all grid-sites:
  → power-law, index 1.3, yet maximum energy very small: 0.1 keV
  → we would expect perpendicular energization to be negligible
  (note: no test particles were run for this result)
- Distribution of MHD flow kinetic energy
  $E_{\text{kin, }\mathbf{B}} = (1/2)\, m_e\, \mathbf{V}^2$
  → similar to E cross B energy
- Distribution of MHD temperature $T$
  $(3/2)\, k_B T$
  power-law and peak at 0.1 keV
- Maximum energies in any case 0.2 keV

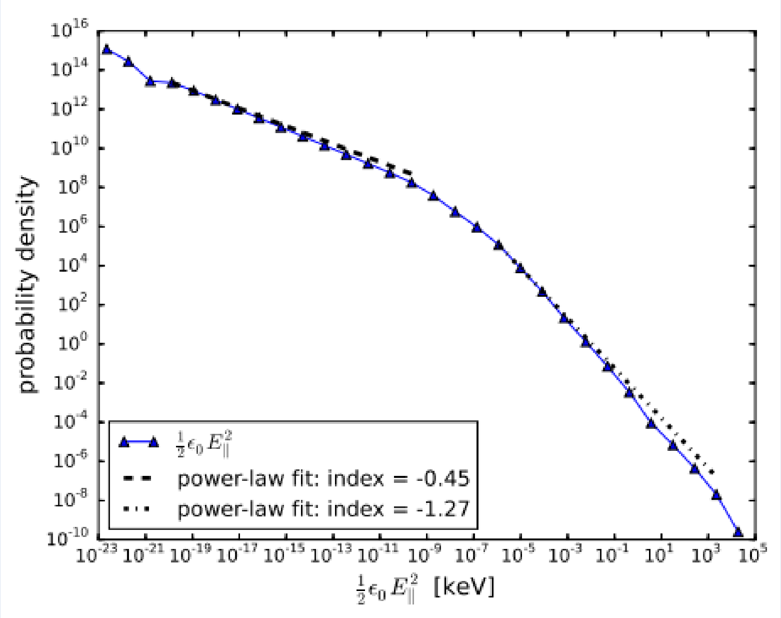


# Statistical analysis of MHD data: Energies

- Distribution of the parallel electric energy:

  $W_{E||} = (1/2)\ \varepsilon_0\ E_{||}^2$

  Very extended double power-law, index 1.3 at high energies




- highest energy 20 MeV

→ various power-law distributions already in MHD data !

→ MHD simulation highly non-linear, far from equilibrium

# Test-particle simulations: the equations of motions

- Direct, non-linear test-particle simulations, in 3D geometry:
  MHD fields are interpolated locally with 3D, continuous cubic polynomials
- 1st order relativistic guiding center approximation

$$\frac{d\mathbf{r}}{dt} = \frac{1}{B_{||}^*}\left[\frac{u_{||}}{\gamma}\mathbf{B}^* + \hat{\mathbf{b}} \times \left(\frac{\mu}{q\gamma}\nabla B - \mathbf{E}^*\right)\right]$$

$$\frac{du_{||}}{dt} = -\frac{q}{m_0 B_{||}^*}\mathbf{B}^* \cdot \left(\frac{\mu}{q\gamma}\nabla B - \mathbf{E}^*\right)$$

$$\mathbf{B}^* = \mathbf{B} + \frac{m_0}{q}u_{||}\nabla \times \hat{\mathbf{b}}$$

$$\mathbf{E}^* = \mathbf{E} - \frac{m_0}{q}u_{||}\frac{\partial \hat{\mathbf{b}}}{\partial t}$$

$$\mu = m v_\perp^2/(2B)$$

$$\hat{\mathbf{b}} = \mathbf{B}/B$$

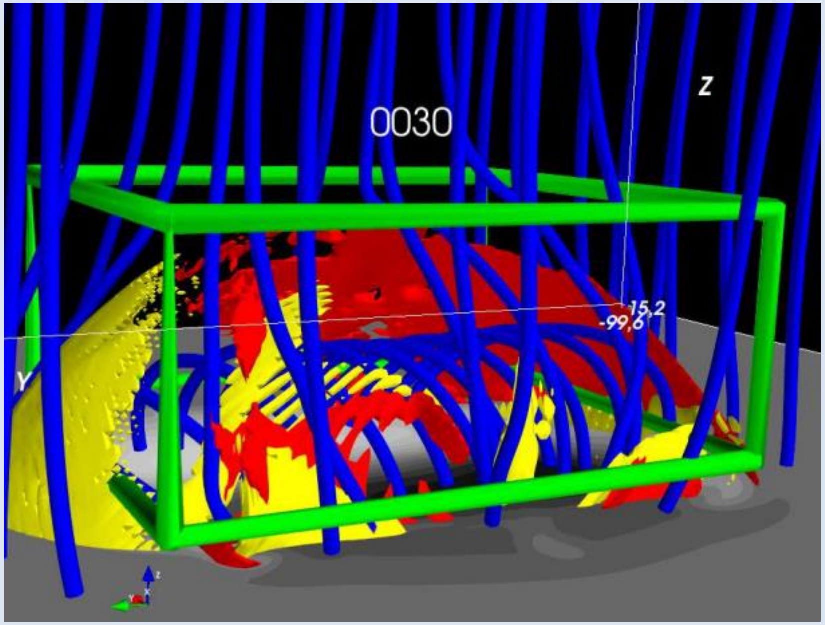
$$\mathbf{u} = \gamma \mathbf{v}$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

  (Tao, Chan,& Brizard (2007), Grebogi & Littlejohn (1984)
- Benchmarked by comparing to integrating Lorentz force
  d**r**/d*t* = **v**, d**u**/d*t* = (*q*/*m*)(**v** x **B** + **E**)
  ➔ Very good coincidence is found
- Important: parallel electric field has to be interpolated explicitly, else artificial fields are generated !
- In any case, numerical integration with 4th order Runge Kutta, adaptive time-step scheme (version of Dormand Prince)
- The code has been parallelized with OpenMP and with MPI

# Test-particle simulations: the set-up

- The electric and magnetic field are de-normalized to SI units and not further scaled.
- Mostly 100’000 to 500’000 electrons are traced, and intermediate and the final kinetic energy distribution is calculated
- Final integration time mostly is 0.1 sec:
  MHD time-step = 25 sec, so particles evolve in fixed MHD snap-shots
- the initial velocity is random Maxwellian, with temperature ≈ $9 \times 10^5$ K
- initial position is uniformly random in a box in the central region around the main reconnection regions

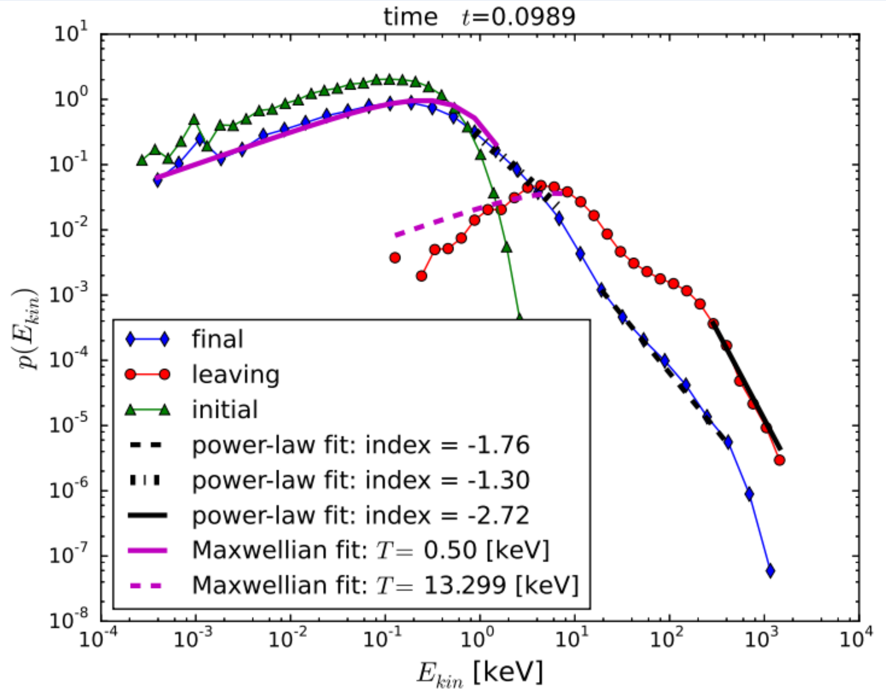


# Standard jet: results at 0.1s

- The final energy distribution at 0.1s is of Maxwellian shape at the low energies, and exhibits a double power-law tail

Acceleration:

- The maximum energy reached is about 2 MeV, and a power-law fit to the tail yields an index of about 1.30 at the lower energies and 1.76 at the higher energies.
- 13% of the 100’000 particle have left leave at 0.1 s, and they have energies in the same range than those that stay inside,
with a modulated power-law tail that is steeper though,
with index 2.72 at the highest energies.



# Standard jet: results at 0.1s

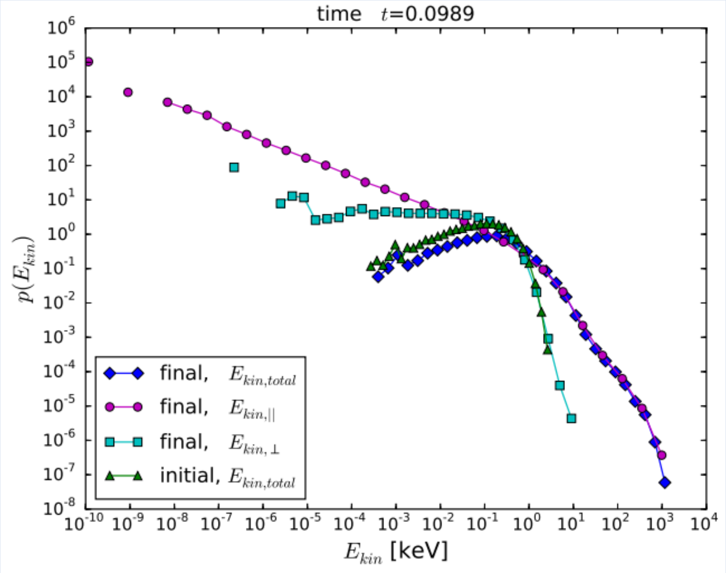
Acceleration, cont'd:

- final total, parallel, and perpendicular kinetic energy at 0.1 s:
  - the power-law tail in the total kinetic energy stems from the parallel kinetic energy,
  - essentially no energization in the perpendicular direction,

important conclusion:

acceleration is acting
exclusively in the parallel direction.

- simulation with 10 times more particles (1'000'000): no real changes in the results, statistics is good enough

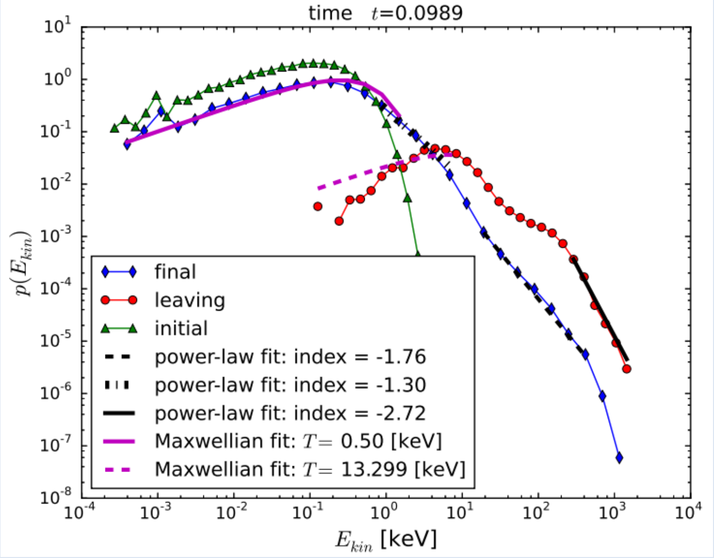


# Standard jet: results at 0.1s

## Heating:

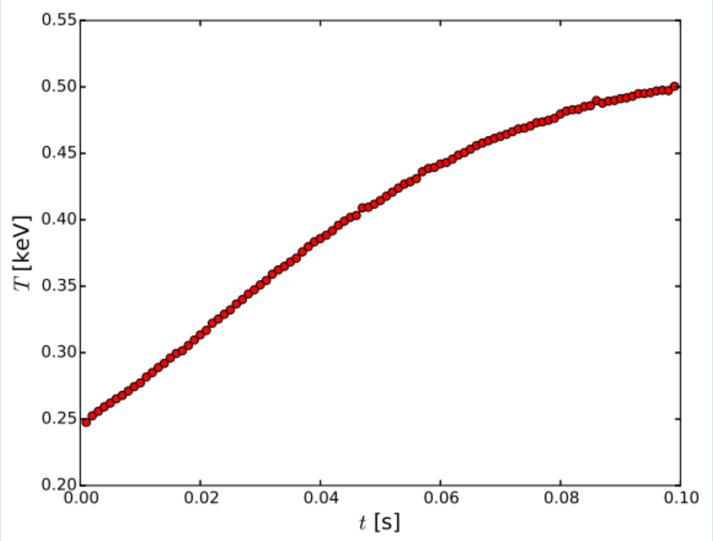
### particles that stay inside:

- Maxwellian shape of the energy distribution at low energies: heating from the initial 0.24 keV to 0.50 keV (1.8MK)
- Temperature as a function of time increases linearly until 0.05 s and then starts to turn over, reaches a peak value of 0.50 keV at 0.1 s




### leaving particles at low energies:

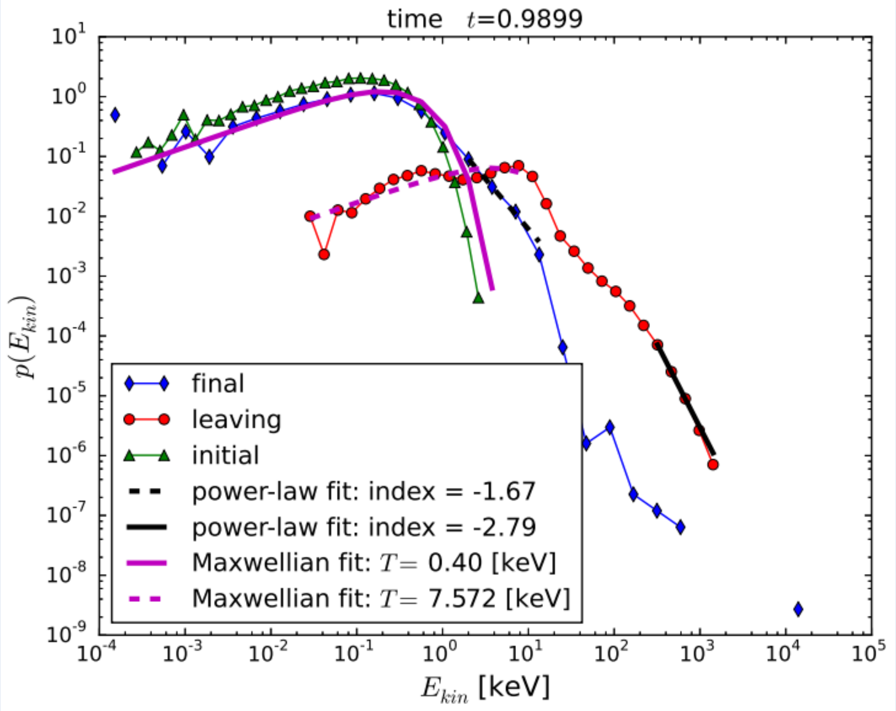
- distribution reminiscent of a Maxwellian, temperature of about 13.3 keV:
  → super-hot population of 50 MK.
  (the statistics is not very good)
- It is to note though that the energies are collected at different times for each particle.



# Standard jet: results at 1.0s

Longer times: 1.0 s

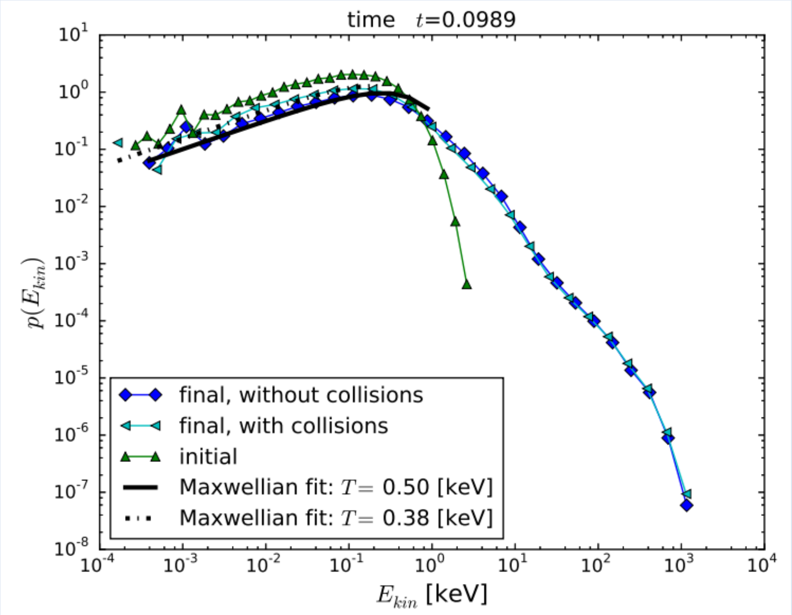
- 57% of the particles have left the system.
- Still power-law tail, no clear scaling anymore at higher energies
  (poor statistics due leaving particles)
  highest energy reached  20 MeV (2MeV at 0.1s).
  Heating to a temperature of 0.4 keV (0.5 keV at 0.1s),
- leaving particles: power-law tail similar as at 0.1 s,
  low energy part is closer to Maxwellian shape (T = 7.5 keV (28 MK))



# Standard jet: results at 0.1s

The effect of collisions:

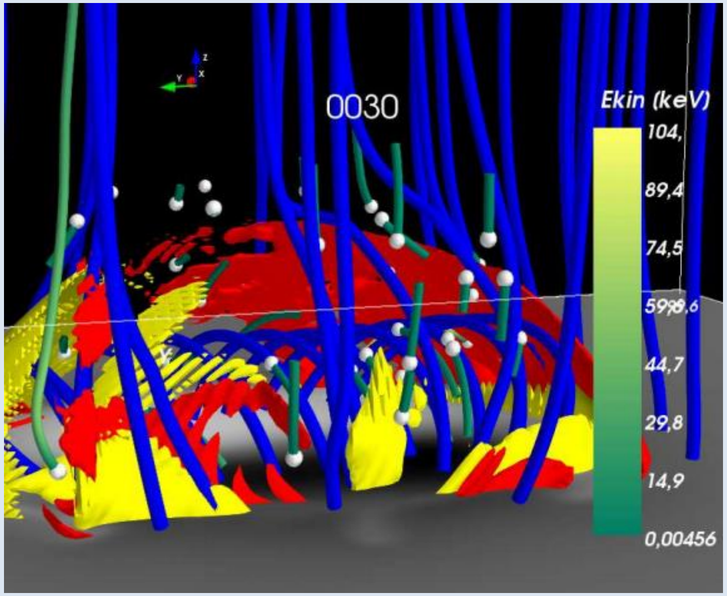
- collisions with background electrons of the same T as the initial T of the test-particles.
  Collisions play a role at low energies only,
  as expected, cooling down the electrons
- With collisions: T = 0.38 keV,
  without collisions: T = 0.50 keV.
  cooling down → heating of the background



# Standard jet: results at 0.1s

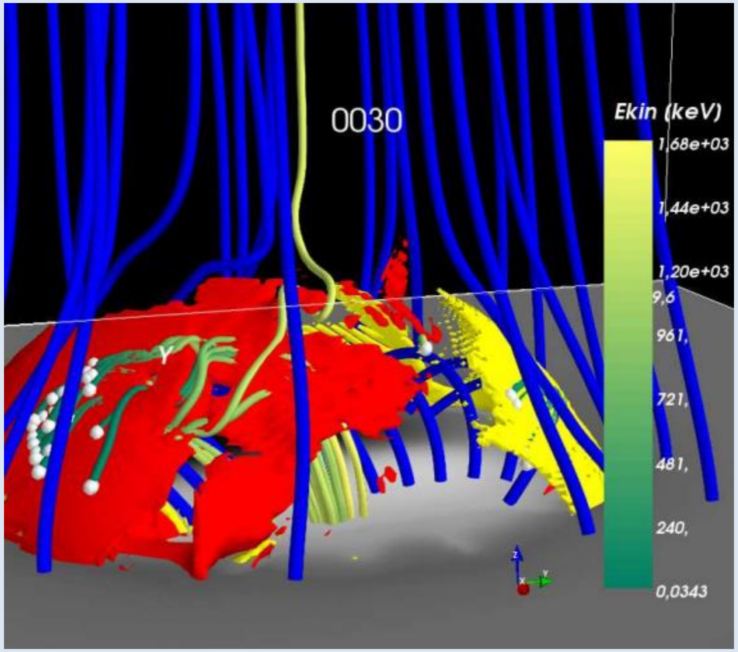
Orbits of 40 randomly chosen particles:

- shows population that is heated or moderately accelerated




orbits of the 40 most energetic particles:

- clear preference in initial conditions: close to strong pos. and neg. parallel el. field.
- particles move some distance along E-field pass then through it at some point, whereby their energy increases strongly




- Chosen threshold in parallel electric field out-lines region of most efficient acceleration

# Diffusion coefficients: the connection to Fermi acceleration and Fokker-Planck description

- The Fokker-Planck equation (FPE) in velocity space writes as
  $$\partial f/\partial t = \nabla_{\mathbf{u}} \cdot [-\mathbf{D} \cdot \nabla_{\mathbf{u}} f + \mathbf{F} f]$$
  and in cylindrical coordinates $\mathbf{u} = (u_\perp, u_{||})$ it takes the form

$$\begin{aligned} \frac{\partial f}{\partial t} &= \frac{1}{u_\perp}\frac{\partial}{\partial u_\perp} u_\perp \left[ -D_{\perp\perp}\frac{\partial f}{\partial u_\perp} - D_{\perp ||}\frac{\partial f}{\partial u_{||}} + F_\perp f \right] \\ &+ \frac{\partial}{\partial u_{||}} \left[ -D_{||\perp}\frac{\partial f}{\partial u_\perp} - D_{||||}\frac{\partial f}{\partial u_{||}} + F_{||} f \right] \end{aligned}$$

- Time-dependent, velocity-space averaged transport coefficients, where the average is over the sample of the test-particles,
- Note, we use running estimate, at predefined monitoring times $t_k$ (Ragwitz and Kantz, PRL, 2001)

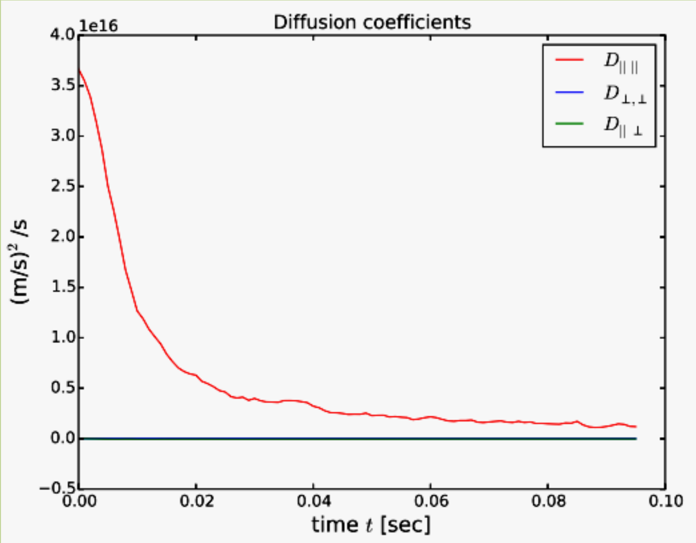
$$D_{||||}(t_k) = \frac{1}{2(t_{k+h} - t_k)} \left\langle (u_{||}(t_{k+h}) - u_{||}(t_k)))^2 \right\rangle$$

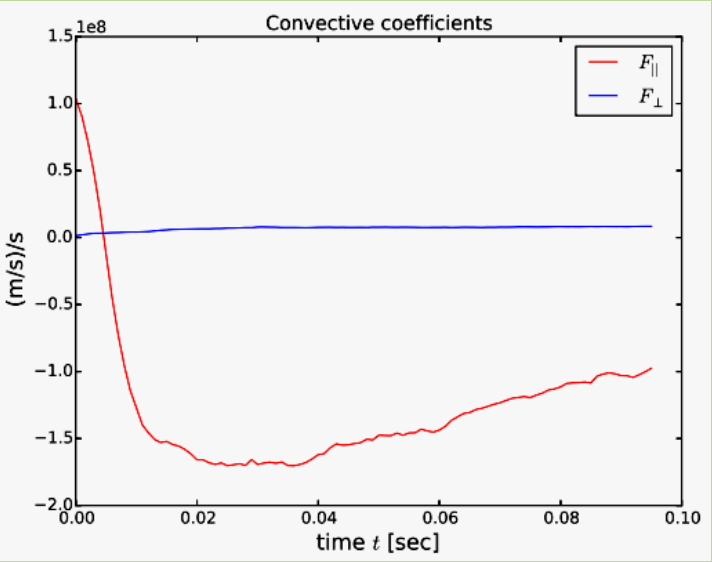
$$F_{||}(t_k) = \frac{1}{(t_{k+h} - t_k)} \left\langle (u_{||}(t_{k+h}) - u_{||}(t_k))) \right\rangle$$

→ seems a detail, but turned out to be important
(e.g. when trying to reproduce the results of a classical random walk with the numerical solution of the FPE)

# Time-dependent, velocity-space averaged transport coefficients

- Parallel diffusion/convection clearly dominates




- Yet, this picture is too simple, we have to consider the velocity dependence

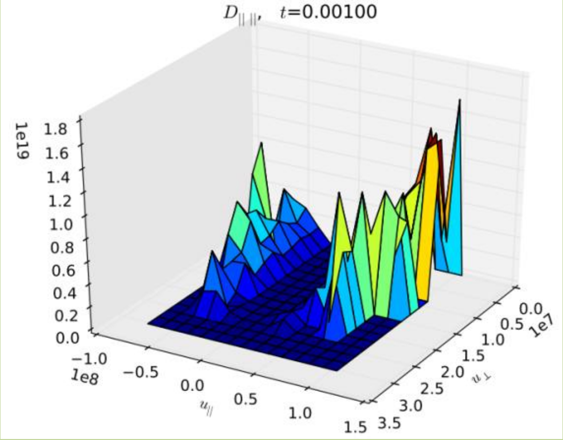
# Time- and velocity-space-dependent transport coefficients

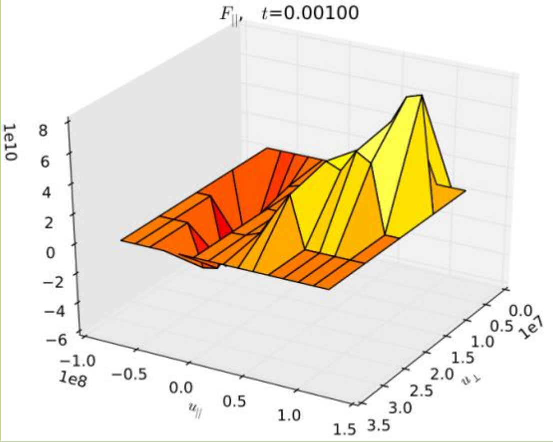
- An estimate of the velocity-space- and time-dependence of the transport coefficients, for given time $t_k$, can be made by
  - first prescribing 2D bins in the $u_\perp$ - $u_{||}$ - plane, with mid-points $(u_{\perp,i}, u_{||,j})$,
  - and then,, considering $[(u^{(l)}_{||}(t_{k+h}) - u^{(l)}_{||}(t_k))^2]$ a function of $(u_{\perp,i}, u_{||,j})$ if $(u^{(l)}_{\perp}(t_k), u^{(l)}_{||}(t_k))$ lies in the respective bin (i,j) for each particle, indexed with $l$,
  - and do binned statistics to find the mean values such as

$$D_{||\,||}\left(t_k, u_{\perp,i}, u_{||,j}\right) = \frac{1}{2(t_{k+h} - t_k)} \left\langle \left( u_{||}^{(l)}(t_{k+h}) - u_{||}^{(l)}(t_k) \right)^2 \right\rangle_l \left(t_k, u_{\perp,i}, u_{||,j}\right)$$

  and the like for the other transport coefficients

- Quite noisy, nonetheless with clear structure in parallel direction:
  → we can neglect the dependence on $u_\perp$ !



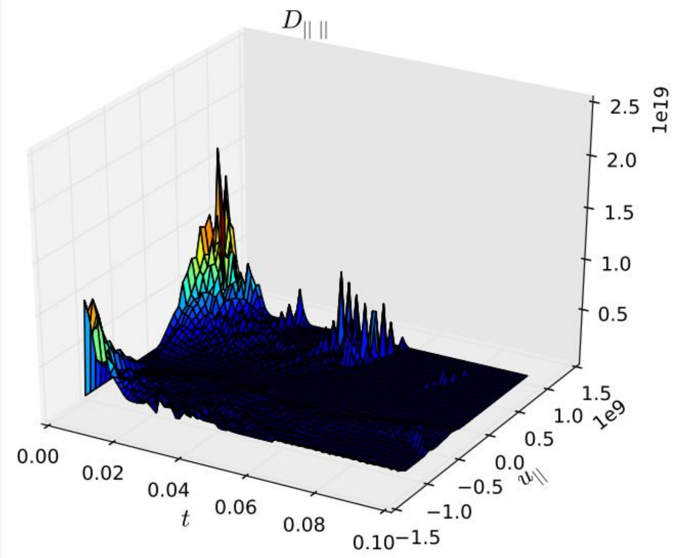



# Time- and $v_{||}$-dependent transport coefficients

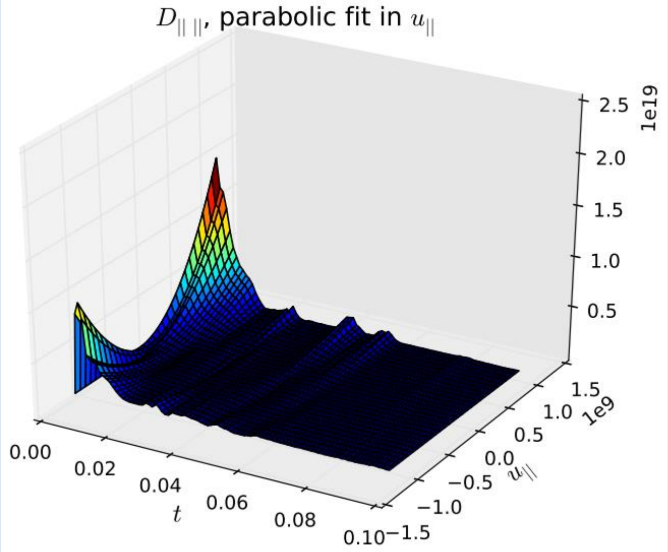
- Diffusion coefficient: We ignore the dependence on $u_\perp$

$$D_{||\,||}\left(t_k, u_{||,j}\right) = \frac{1}{2(t_{k+h} - t_k)} \left\langle \left( u_{||}^{(l)}(t_{k+h}) - u_{||}^{(l)}(t_k) \right)^2 \right\rangle_l \left(t_k, u_{||,j}\right)$$

binned statistics




parabolic fit




- Some noise, yet parabolic fit seems reasonable, above all at small times

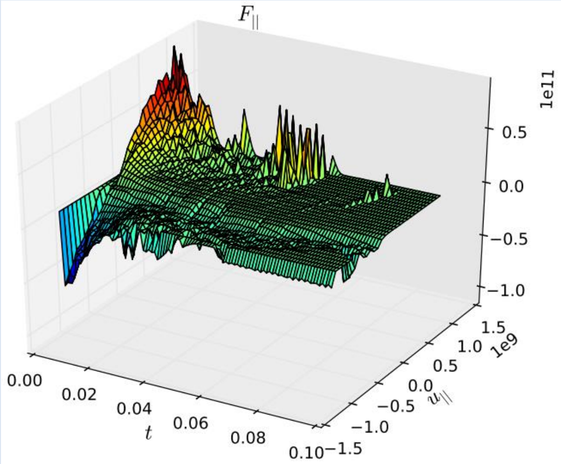
$$D_{||\,||}(t, u_\perp, u_{||}) \propto u_{||}^2\, g(t)$$

# Time- and $v_{||}$-dependent transport coefficients

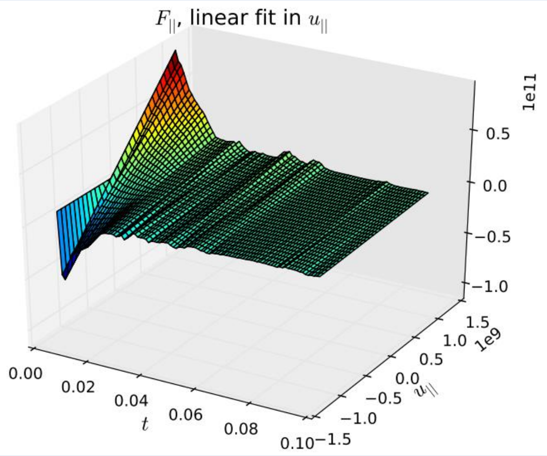
- Convective coefficient: We again ignore the dependence on $u_\perp$

$$F_{||\,||}\left(t_k, u_{||,j}\right) = \frac{1}{\left(t_{k+h} - t_k\right)} \left\langle \left( u_{||}^{(l)}\left(t_{k+h}\right) - u_{||}^{(l)}\left(t_k\right) \right) \right\rangle_l \left(t_k, u_{||,j}\right)$$

binned statistics




linear fit




- Again, there is a lot of noise, yet the linear fit seems more or less reasonable at small times

$$F_{||}(t, u_\perp, u_{||}) = u_{||}\, g(t)$$

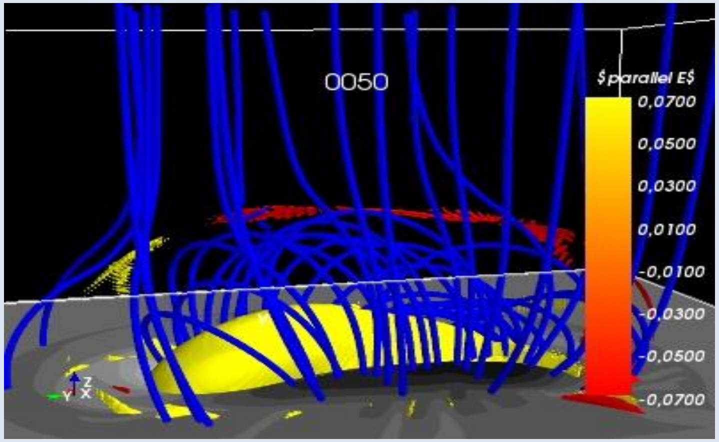
- $D_{||\,||}$, $F_{||}$: Basic dynamic acceleration takes place up to 0.02sec,
  ➔ particles have taken the energy that is available for them
  ➔ there is a kind of saturation effect

# Summary

- The MHD simulations yield various power-law distributions, and fractal structures, they are far from equilibrium
- For the test-particles, we find acceleration and heating.
  Both are a transient phenomenon, there is a kind of saturation effect
- Leaving particles form super-hot population, with power-law tail in energy
- The parallel dynamics clearly dominate the energetics
- transport coefficient are velocity-dependent, and seem to have simple functional form

Future steps:

- How are the various power-law indices related ?
- Analyze different MHD snapshots, e.g. with ‘blowout jet’
- Spatial structure of E-field needs still to be further analyzed: e.g. Eulerian vs Lagrangian correlation.

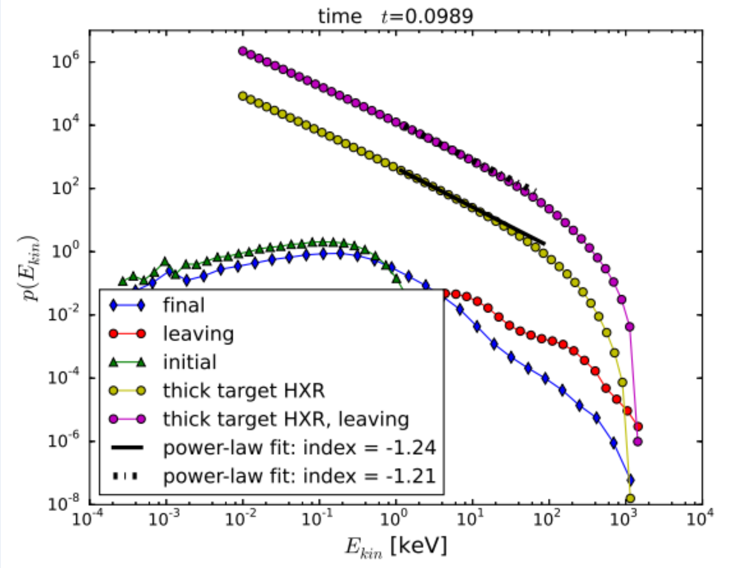


THE PRESENTATION ENDS HERE

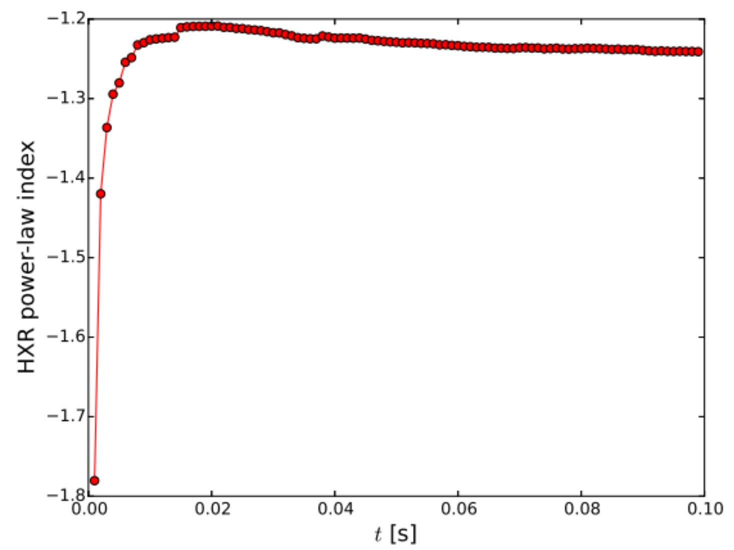
# Standard jet: results at 0.1s

thick target HXR emission spectrum

- The HXR spectrum is rather flat, with index mostly close to 1.2.
  HXR spectrum of the leaving particles is very similar in shape
  (asynchronous distribution !)

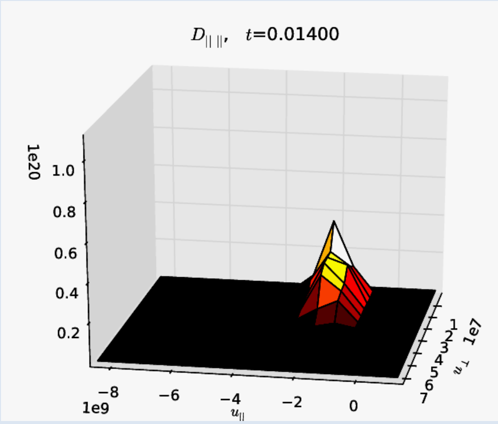


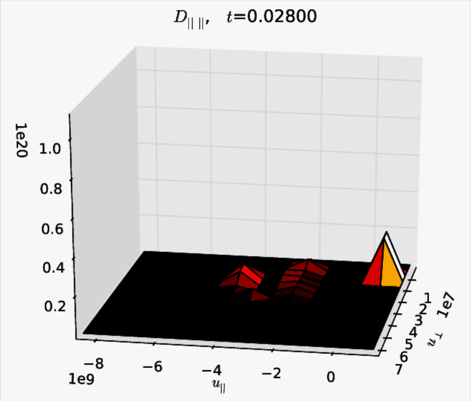

- the time evolution of the power-law index of the spectrum.

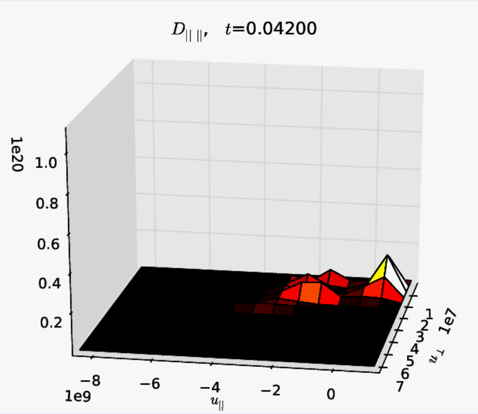


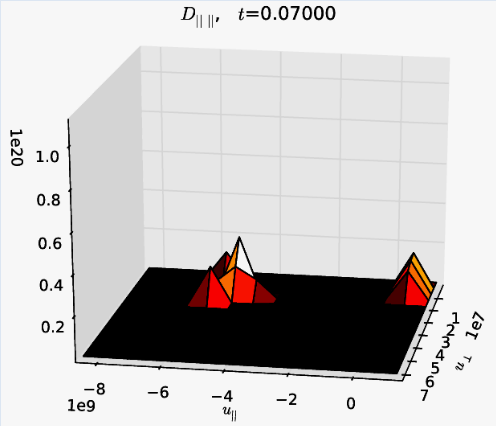
# Time- and velocity-space-dependent transport coefficients

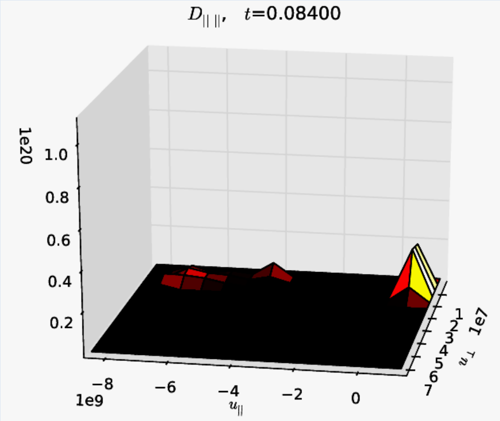
- Parallel diffusion coefficient $D_{||\,||}$
  → old result: $D_{||\,||}$ prop. to $v_{||}^2$
  ➔ clearly velocity dependent $D$, no obvious functional form, though

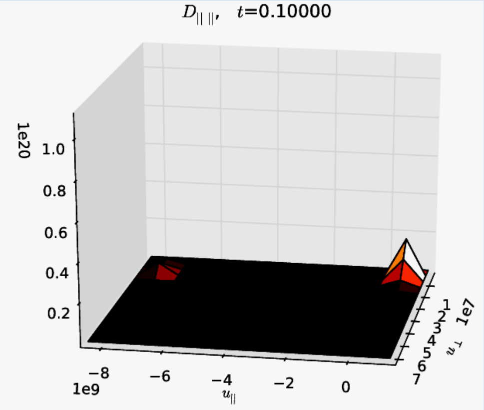


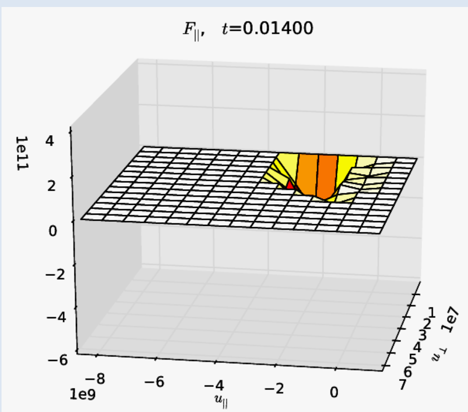



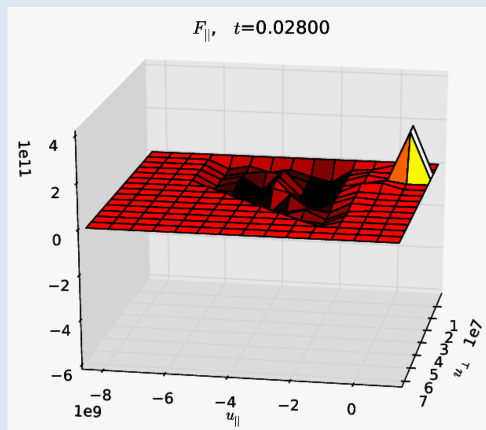



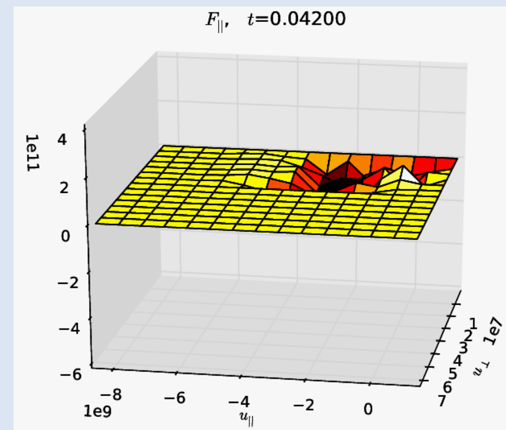



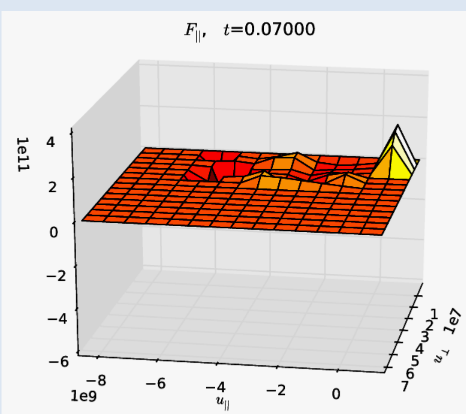
# Time- and velocity-space-dependent transport coefficients

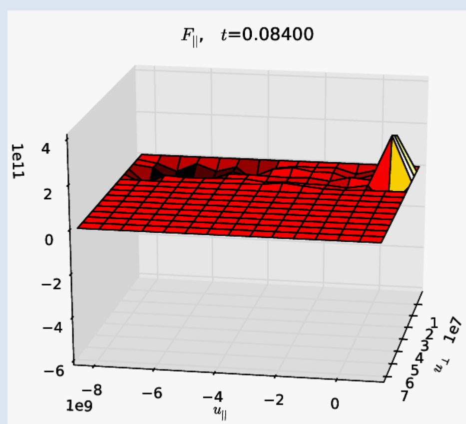
- Parallel convective coefficient $F_{||}$
  → old result $F_{||}$ prop to $v_{||}$
  ➔ again: much less clear functional form now

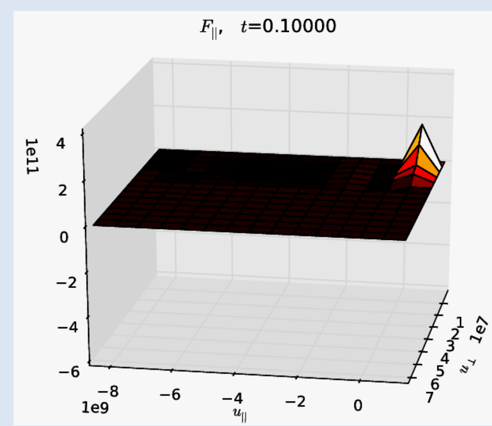


# Correct form of transport coefficients

- is the form really correct,

  i.e. will – in well behaved cases – the FPE yield the same as do the test-particle simulations ?
- We set up a simple random walk problem in 1D, with given step-size distribution for velocity and time, where $D_{||\,||}$ can be derived analytically , and we estimate
  $D_{||\,||}$ also numerically from the random walk:
  The numerical $D_{||\,||}$ gave distributions clearly different from the random walk
- Corrected form of estimator for $D_{||\,||}$ (and the other coefficients):
  use running estimate, at predefined monitoring times $t_k$, and define

$$D_{||\,||}(t_k) = \frac{1}{2(t_{k+h} - t_k)} \left\langle \left( v_{||}(t_{k+h}) - v_{||}(t_k) \right)^2 \right\rangle$$

→ numerical solution of FPE and random walk coincide very well !
→ seems a detail, but turned out to be important

# The most explosive phase: collisions, relativistic effects

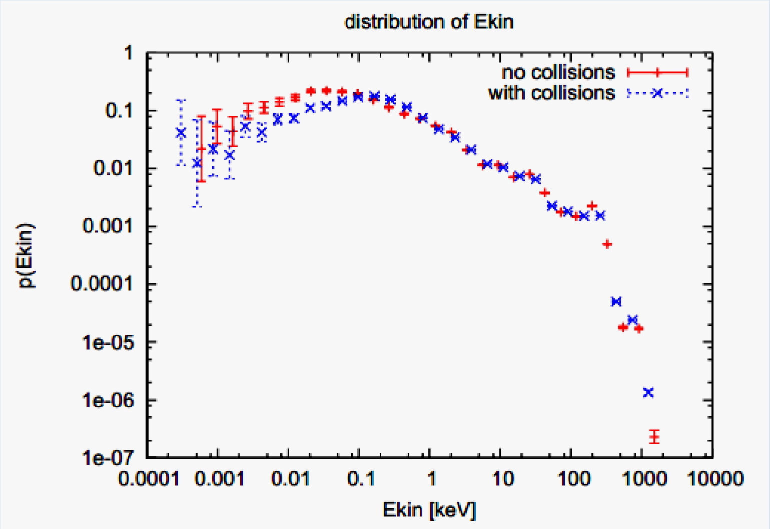
- Collisions (with background electrons) play a minor role at low energies

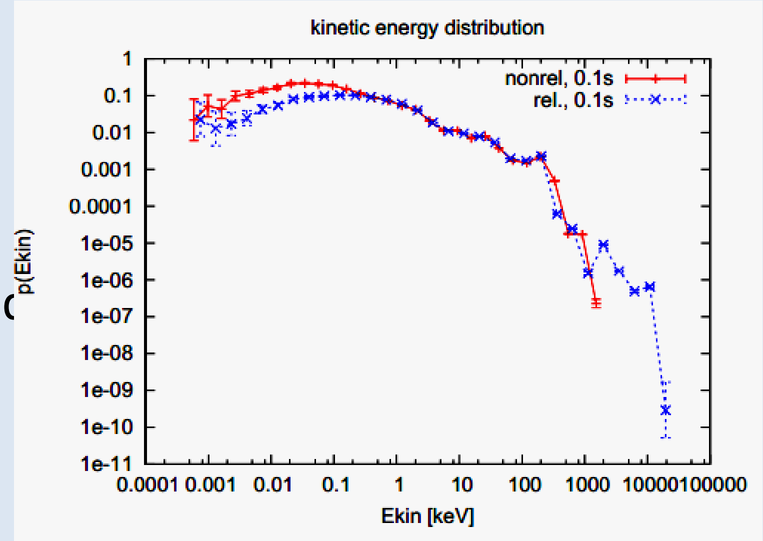
  ➔ collisions can be neglected

  (Monte Carlo collision operator,
  Hamamatsu, K et al.,
  Plasma Phys Contr Fus **49**,
  1955 (2007))

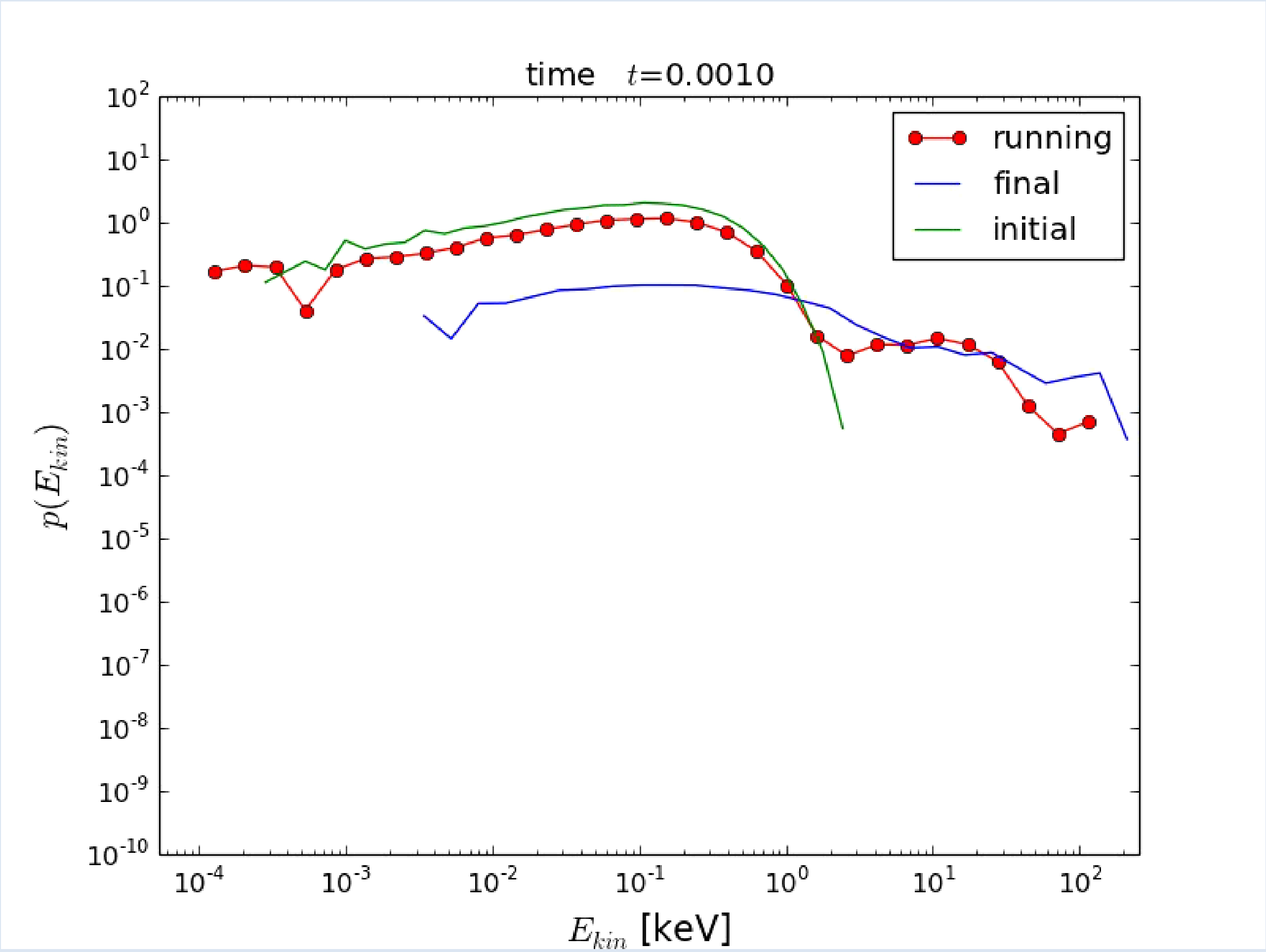
- We consider the relativistic equations of guiding center motion, and compare with the non-relativistic case.
  In the relativistic case,
  the particles reach higher energies,
  in the intermediate to high energy range
  though, where a clear power-law is formed
  distribution is unaltered.

  ➔ to be on the safe side, use relat. eqs.



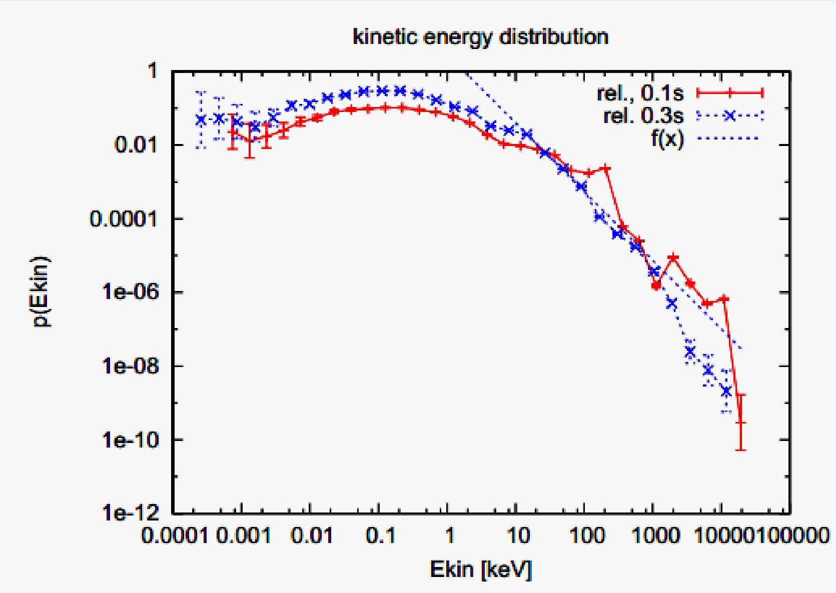
# The most explosive phase



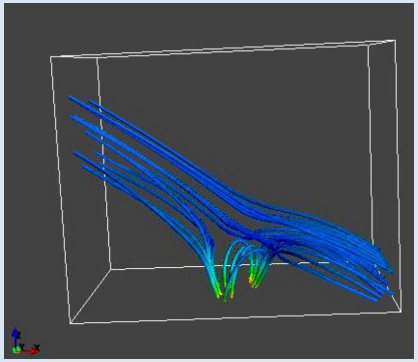
# The most explosive phase: longer times

- We consider the relativistic equations of guiding center motion, for final times 0.1s and 0.3
  → The distributions are similar in shape,
  → at 0.3s the high energy part is less noisy and shows now a clear power-law

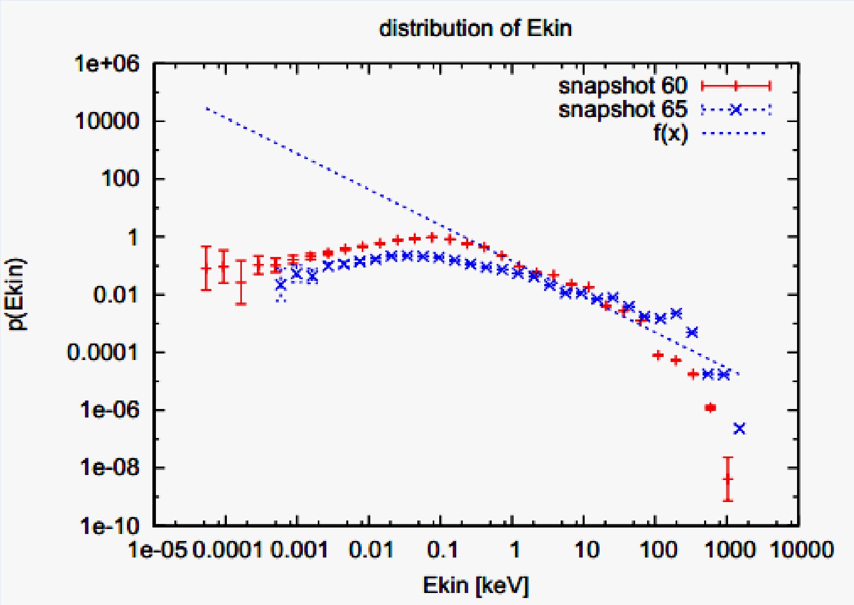
- A fit in the range [10,1000] yields an index -1.9
  (the fit is though not very good at the higher energies)



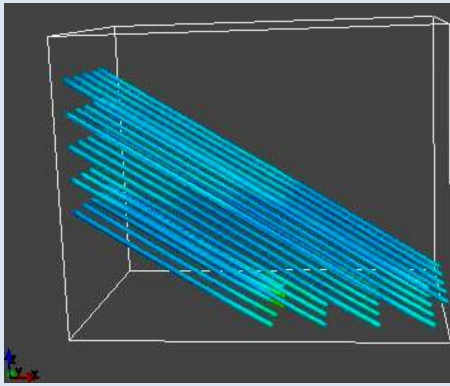
# Before the most explosive phase



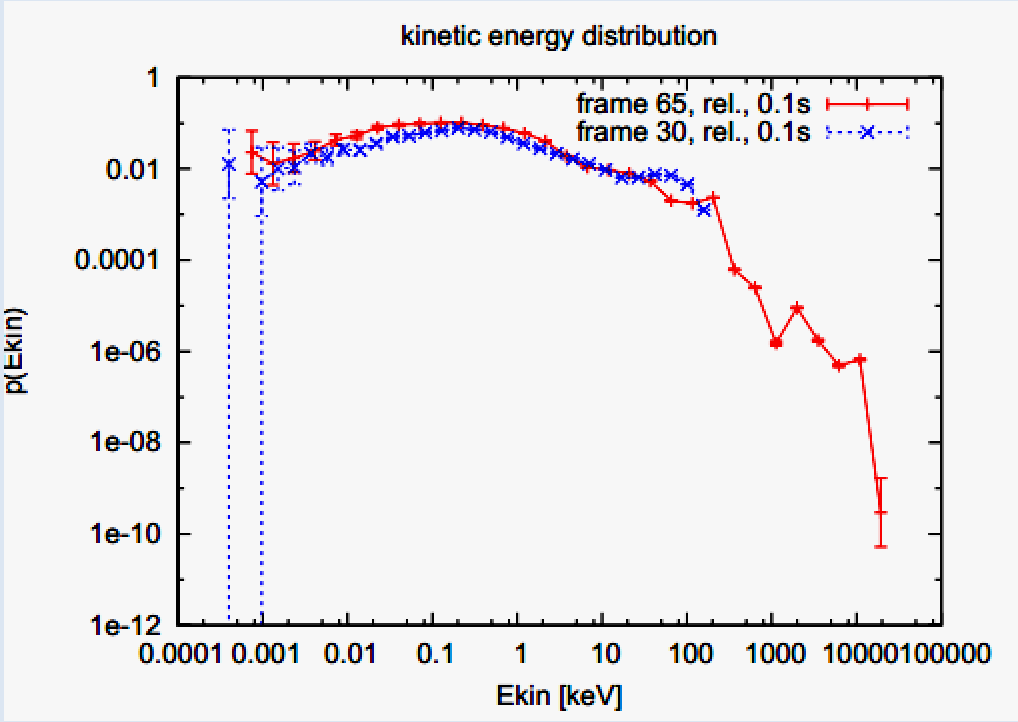
- We consider the start of the blob formation
- a fit in the range [0.4,80] yields a slope 1.2,
  i.e. the distribution is steeper (1.2 vs 0.6) in lesser developed turbulence,
  there seems to be less heating taking place at low energies



# Initial, quiet phase

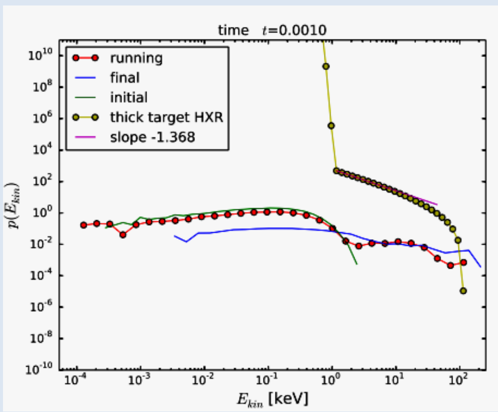


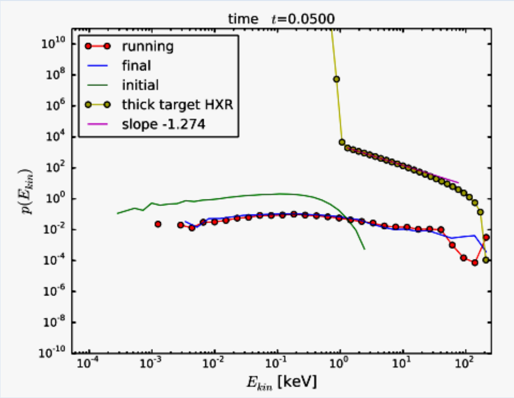
- snapshot 30 is far away from the blob formation, the coronal part still is rather quiet
- The highest energy reached, 150 keV, is much smaller than in developed turbulence
- up to 150keV the distributions are very similar, heating is similar in both MHD time instances

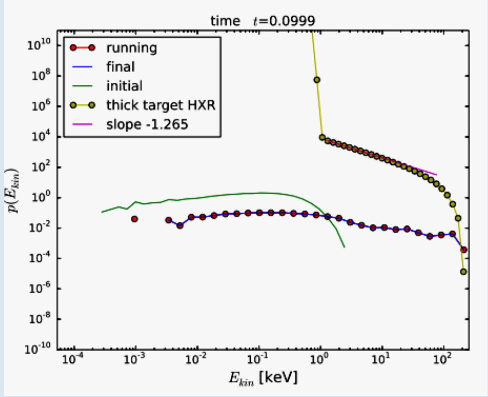


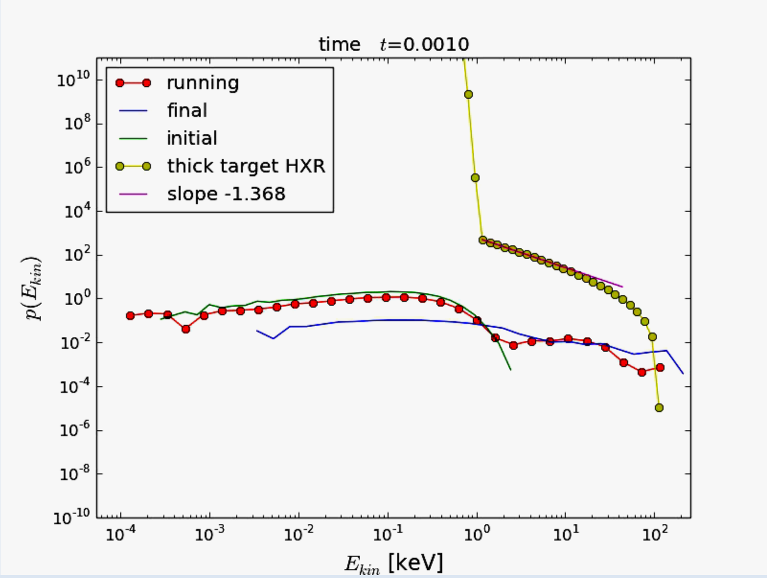
# The most explosive phase: HXR emission

- Assuming that the particles instantaneously would precipitate onto the lower corona, we calculate the thick target HXR spectrum from the energy distribution (Brown, Holman)
- HXR emission at three different times:
  The HXR spectrum is rather flat (slope 1.2 to 1.4).



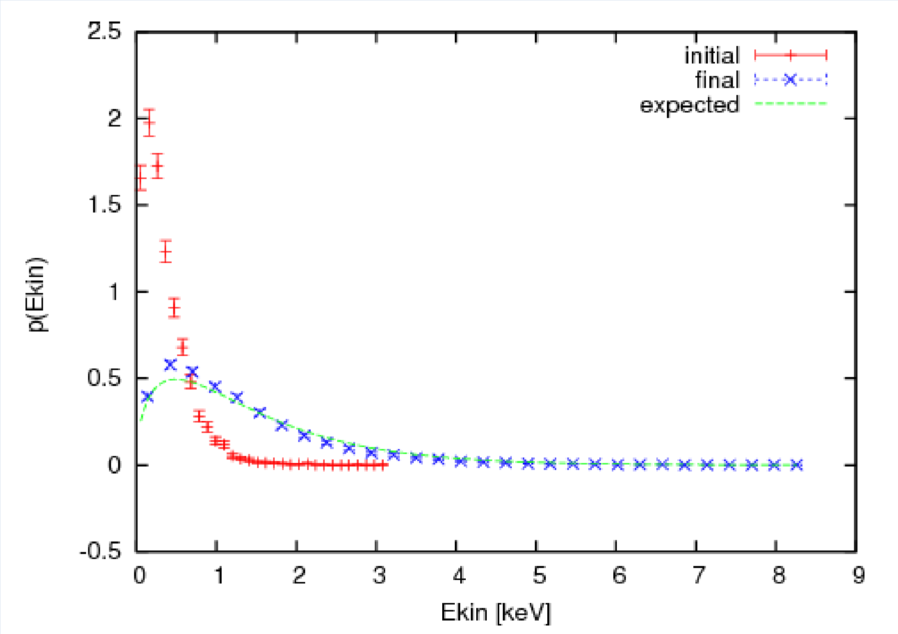



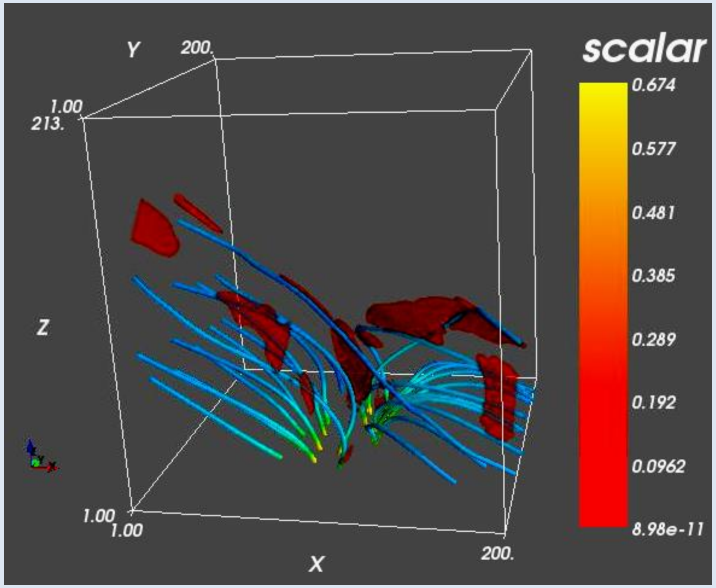
# Collisions: benchmark

- We use the Monte Carlo collision operator as described in Hamamatsu, K et al., Plasma Phys Contr Fus **49**, 1955 (2007):
  random walk steps super-imposed on deterministic motion,
  after time intervals related to the collision time
- Particles collide with a Maxwellian background
- Benchmark: ions colliding with background electrons that have 4 times larger temperature than the ions initially:
  Final ion distribution well coincides with expected distribution, the ions are heated
  (after 300 sec,
  with collision time = 0.3 sec)



# Spatial structure of the magnetic and electric field (2/2)

- Magnitude of parallel electric field also fragmented  (dominantly responsible for acceleratiom, see later)




- Fragmentation needs to be quantified: cluster analysis, fractal dimension: work in progress

# Summary of older results, presented in Ioannina

- Collisional effects  are not import
- Relativistic equations of motion should be used (the particles reach higher energies)
- Longer integration times lead to more well formed power-laws
- During explosive MHD phases, there is heating and acceleration, during quiet phases, there is only heating
- calculating the thick target HXR spectrum from the we find so-far that the HXR spectrum is rather flat.
- Scaling factor for **E** is necessary and strongly affects the results