The applications of Complexity Theory and Tsallis Non-extensive Statistics at Space Plasma Dynamics

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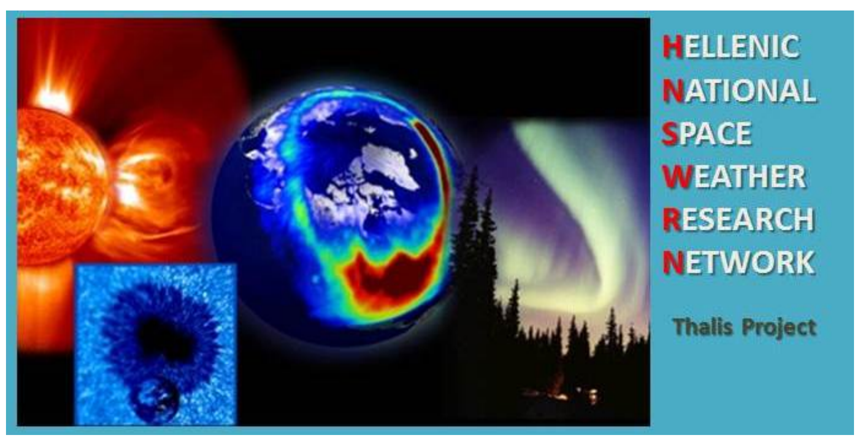
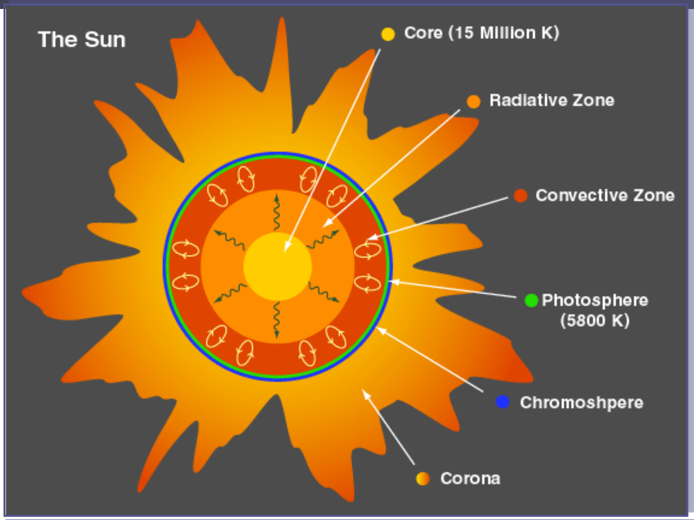
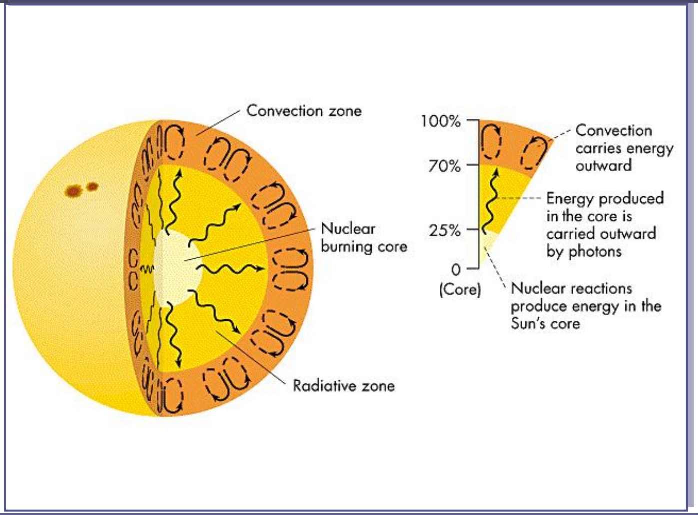


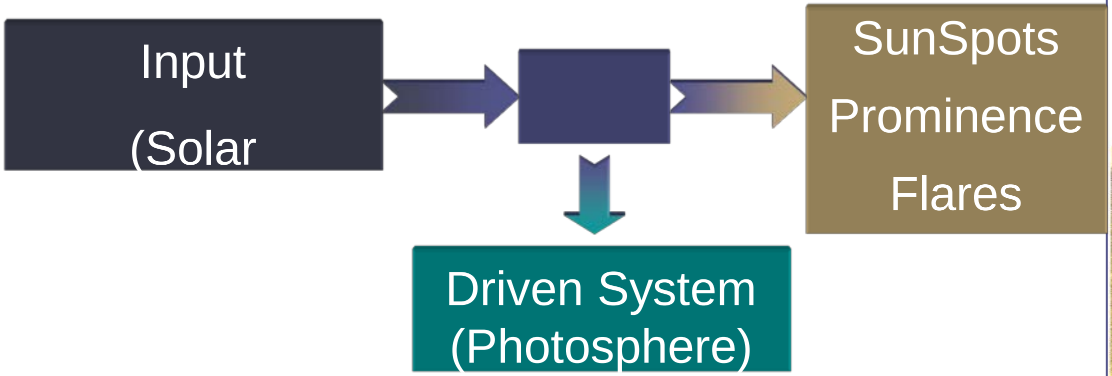
HELLENIC ASTRONOMICAL SOCIETY

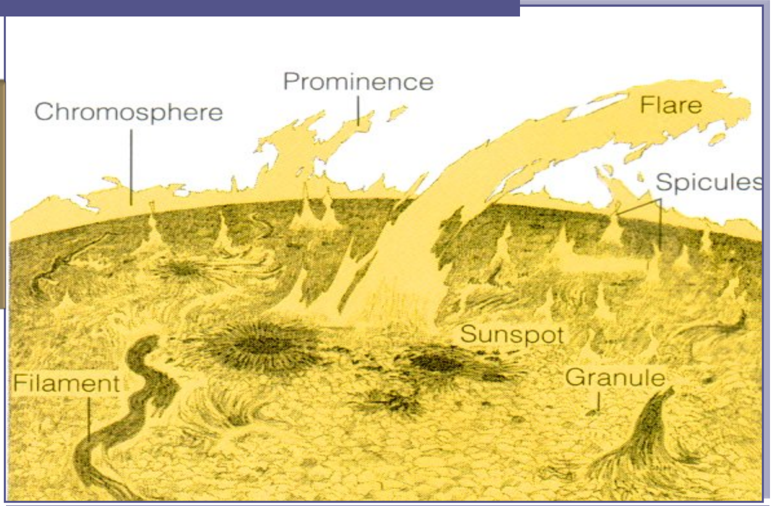
DRIVEN SYSTEMS (LOADING - UNLOADING)
Solar Dynamics





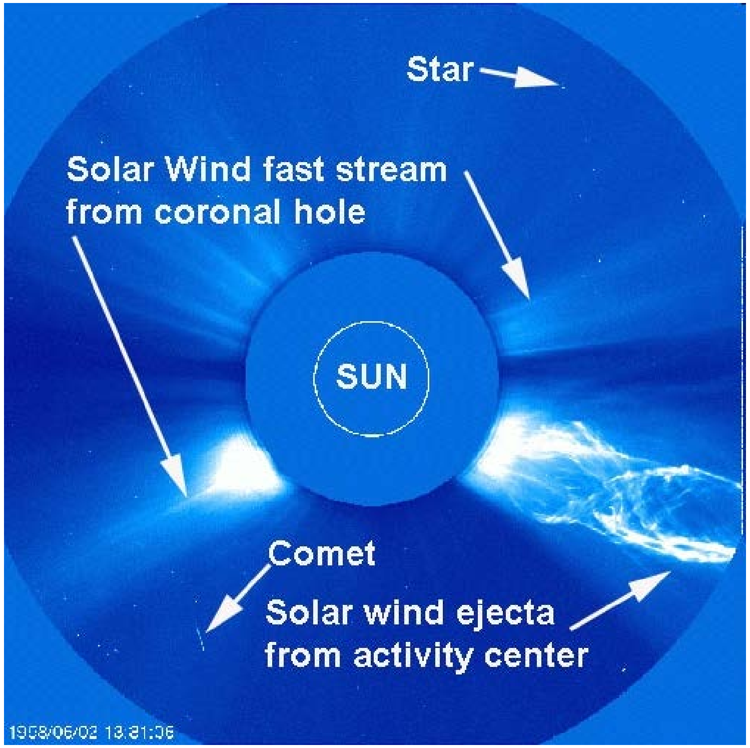
Description of Sun's Interior



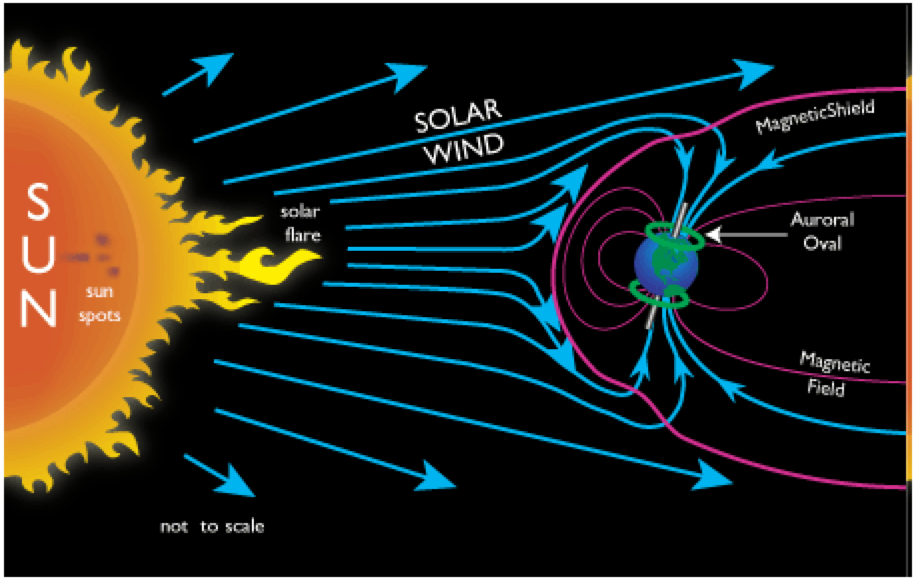


Description of Sun's Photosphere

Solar Wind



Star
Solar Wind fast stream from coronal hole
SUN
Comet
Solar wind ejecta from activity center
1998/06/02 13:31:05



SUN
sun spots
solar flare
SOLAR WIND
MagneticShield
Auroral Oval
Magnetic Field
not to scale

Complexity Theory ⟷ Entropy Principle

Nonequilibrium self-organization

$$S_q = k\ell n_q W = \max$$

$$F_q = U_q - TS_q = -\frac{1}{\beta}\ell n_q Z_q = \min$$

F_q (Free energy) = minimum => Nonequilibrium stationary states (NESS)

Tsallis Non-extensive Statistical Mechanics

$$S_q = -k\sum_i p_i^q \ell n_q p_i = k\frac{1-\sum p_i^q}{q-1}$$

$$P_{opt} = \frac{[1-(1-q)\beta \mathrm{E}]^{1/1-q}}{Z_q} = \frac{e_q^{-\beta \mathrm{E}}}{Z_q}$$

$$Z_q = \sum e_q^{-\beta \mathrm{E}}$$

q=1 => Boltzman – Gibbs Theory

THEORETICAL CONCEPTS

Near Thermodynamic Equilibrium	Far from Thermodynamic Equilibrium
Linear Dynamics	**Non-Linear Dynamics**
Euclidean Geometry-Topology Smooth Functions – Smooth differential Equations Normal derivatives-integrals	Fractal Geometry -singular functions Fractional differential-integral equations
MHD –Vlason-Blotzman theory Normal diffusion-Brownian motion Gaussian statistics-dynamics Normal Langevin-FP equations Extensive statistics – BG entropy Infinite dimensional noise White-colored noise Normal Central Limit Theorem (CLT)	Fractional MHD theory Anomalous diffusion – motion Strange Kinetics Non-Gaussian statistics-dynamics Fractional Langevin-FP equations Non-Extensive statistics – Extremization of Tsallis entropy q-extended CLT Intermittent turbulence
Normal Liouville theory Locality in space and time Separation of time-spatial scales Microscopic-macroscopic locality Equilibrium RG	Fractional Liouville theory (multi)Fractal Topology Power laws – mulitscale processes Memory – long range correlations Nonlocality in space and time Non-Equilibrium RG

Renormalization Group Theory (RGT) (Chang, 1992)

Non-Equilibrium – Non-linear Space Plasma Physics

$$\frac{\partial \varphi_i}{\partial t} = f_i\left(\boldsymbol{\varphi}, \mathbf{x}, t\right) + n_i\left(\mathbf{x}, t\right) \qquad i = 1, 2, \ldots$$

Generalized Langevin Stochastic Equation

$$P\left(\boldsymbol{\varphi}\left(\mathbf{x}, t\right)\right) = \int D\left(\mathbf{x}\right) \exp\left\{-i \cdot \int L\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{\varphi}, \mathbf{x}\right) d\mathbf{x}\right\} dt$$

Random Field Distribution Function

Plasma System Stochastic Lagrangian

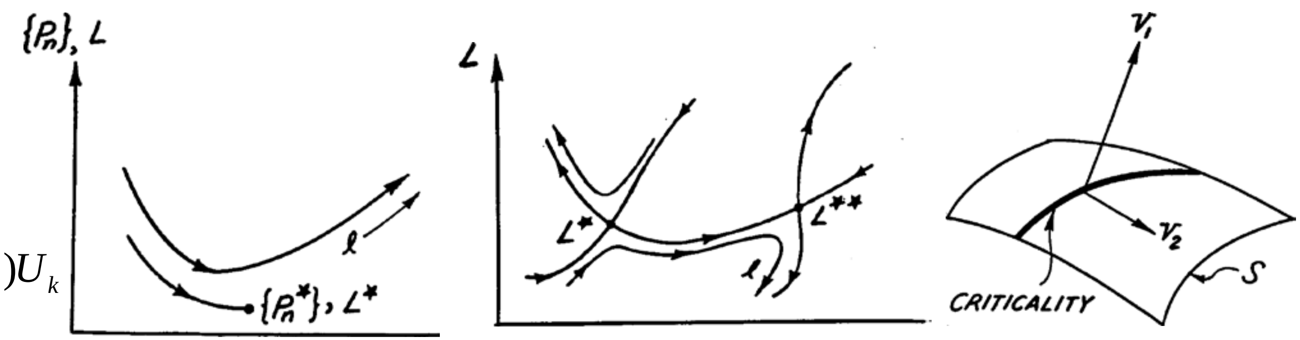
Scale Invariance - RGT - Fixed Points (SOC, Chaos, Intermittency etc.)

$$\partial L / \partial l = RL$$

$$\partial L' / dl = R_L L' \quad \dot{\eta}$$

$$dP'_m / dl = \Sigma (R_L)_{mn} P'_n$$

$$L'(l) = \Sigma V_k(l) U_k = \Sigma V_{k0} \exp(\lambda_k l) U_k$$



Non-equilibrium Non-extensive Random Field Theory (Partition Function Theory)

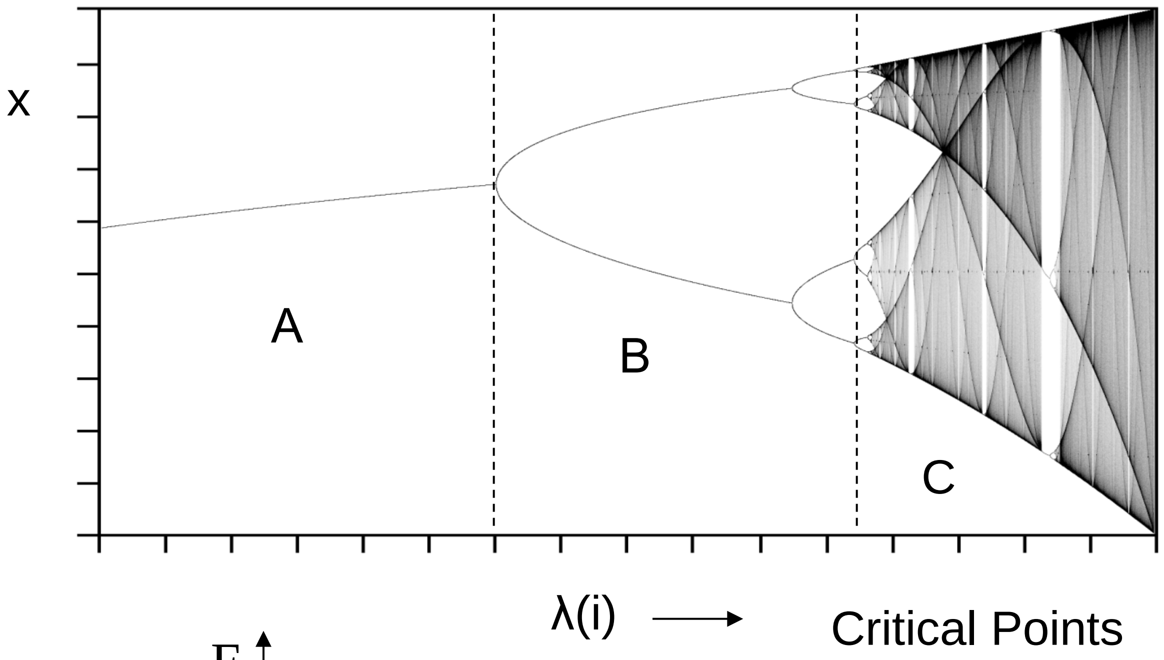
$$G_N^q\left(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N\right) = \frac{1}{Z} \frac{\delta^N Z_q\left(J\left(\vec{x}\right)\right)}{\delta J\left(\vec{x}_1\right) \cdot \delta J\left(\vec{x}_2\right) \cdot \cdots \cdot \delta J\left(\vec{x}_N\right)}$$

$$Z_q = \lim_{J \to 0} Z\left(J\left(\vec{x}\right)\right) \qquad Z\left(J\left(\vec{x}\right)\right) = \int D\left[\Psi\right] \cdot e^{-\int F_q(J) d^D x}$$

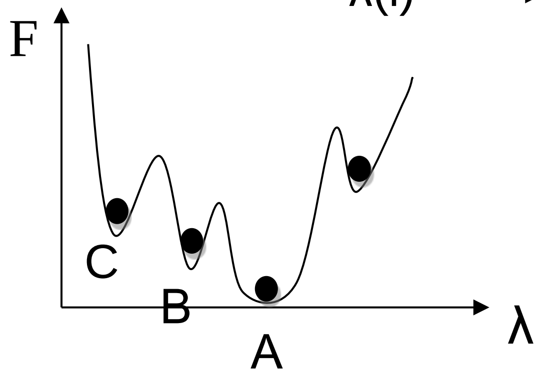
Dynamical Bifurcation

$$dx/dt = f(x, \lambda)$$

Non – Linear Dynamics



Critical Points
Bifurcation Points



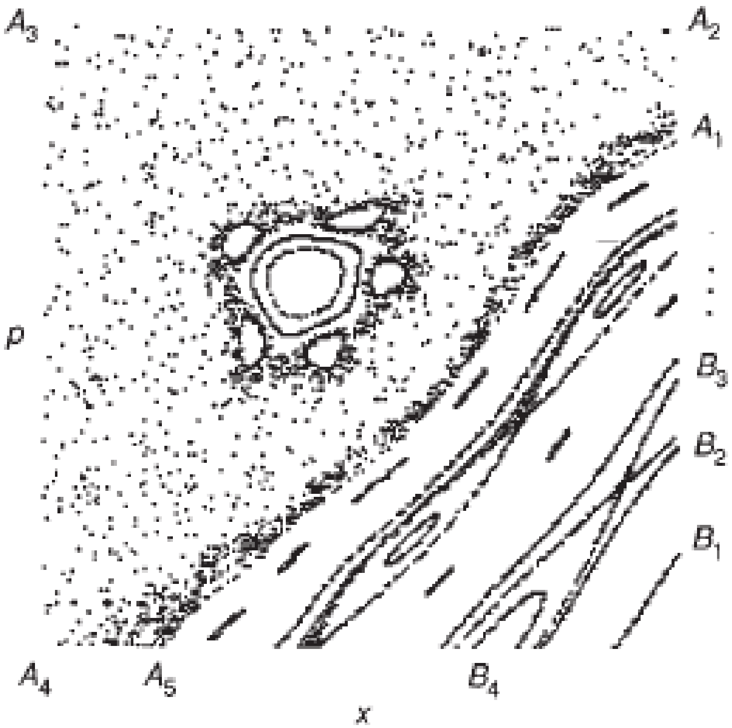
(A)
Gaussian Equilibrium
Critical States
Equilibrium Phase Transition
Power Laws

(B)
Self-Organization Structure
Dissipative Structures
Long Range Correlations

(C)
Spatiotemporal Chaos
Strange Dynamics (Attractors)
Levy Processes
Scale Invariance
Intermittent Turbulence
Tsallis Entropy
Fractal Topology
Anomalous Diffussion

Strange kinetics

Michael F. Shlesinger, George M. Zaslavsky & Joseph Klafter NATURE · VOL 363 · 6 MAY 1993



Poincare Section of Phase Space
Intermittency in Phase Space

- Stochastic Sea
- Fractal Set (Island, Cantori)
- Trapping – Stickiness – Levy flights
- Lyapunov Exponents ≥ 0
- Strange Topology – Strange Dynamics
- Scale Invariance (RGT)

Levy Distribution – Anomalous Diffusion

$$\langle R^2(t)\rangle \sim t^{\gamma} \qquad p_n(k) = \exp(-\text{constant} \times n|k|^{\alpha})$$

$$\langle |R|\rangle \sim t^{\mu} \qquad (t \leftarrow \infty) \qquad p_n(x) \sim \text{constant} \times n/x^{1+\alpha}$$

$$\mathscr{B} = \lim_{\Delta t \to 0, \Delta x \to 0} |\Delta x|^{2\alpha}/|\Delta t|^{\beta} = \text{constant}$$

Basic Space Plasma Theory

BBGKY Hierarchy → **Boltzmann - Vlassov, MHD Theory**

Scale Invariance - Singularities

$$\vec{\tau}' = \lambda\,\vec{\tau} \qquad t' = \lambda^{1-\alpha/3} t = \lambda^{1-h_t} \qquad \vec{u}' = \lambda^{\alpha/3}\vec{u} = \lambda^{h} u \qquad \vec{b}' = \lambda^{\alpha/3}\vec{b}'$$

Fractional Extension

Fractional Real - Space Derivative

$$\frac{\partial^{\beta}}{\partial x_i^{\beta}} f(t,\vec{r}) = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial x_i} \int_{i-\infty}^{x} \frac{dx_i}{(x_i - x_i')^{\beta}} f(t,\vec{r})$$

$$\nabla_{\vec{r}}^{\beta} \equiv \frac{\partial^{\beta}}{\partial \vec{r}^{\beta}} \equiv \sum_i \hat{e}^i \frac{\partial^{\beta}}{\partial x_i^{\beta}}$$

$$\nabla_{\vec{r}}^{2\beta} \equiv \frac{\partial^{2\beta}}{\partial \vec{r}^{2\beta}} \equiv \sum_{i,k} \delta^{i,k} \frac{\partial^{2\beta}}{\partial x_i^{\beta} \partial x_k^{\beta}}$$

Riesz – Weyl operator

Levy flights – Levy walks

Power Law distribution

$$\varphi(\ell) = \frac{1}{\ell^{1+2\beta}}$$

Fractional Time Derivative

$$\frac{\partial}{\partial t^{a}} f(t,\vec{r}) = \frac{1}{\Gamma(m-a)} \frac{\partial^{m}}{\partial t^{m}} \int_{0}^{t} \frac{d\theta}{(t-\theta)^{1+\alpha-m}} f(\theta,\vec{r})$$

Riemann – Liouville operator

Fractal – time random walks (FTRW)

Fractal active time (Cantor set)

$1 \leq a \leq r$ persistent (super-diffusive) process

$0 \leq a \leq 1$ anti-persistent (sub-diffusive) process

Waiting time Power Law distribution

$$\phi(r) \sim \frac{1}{r^{1+a}}$$

FRACTIONAL KINETIC EQUATION
(FRACTIONAL FOKKER–PLANCK–KOLMOGOROV EQUATION)

Zaslavsky G.M., Chaos, fractional kinetics, and anomalous transport, *Physics Reports 371, 461-580*, 2002.

$$\frac{\partial^\beta P(x,t)}{\partial t^\beta} = \frac{\partial^\alpha}{\partial(-x)^\alpha}(\mathscr{A}(x)P(x,y)) + \frac{\partial^{\alpha_1}}{\partial(-x)^{\alpha_1}}(\mathscr{B}(x)P(x,y))$$

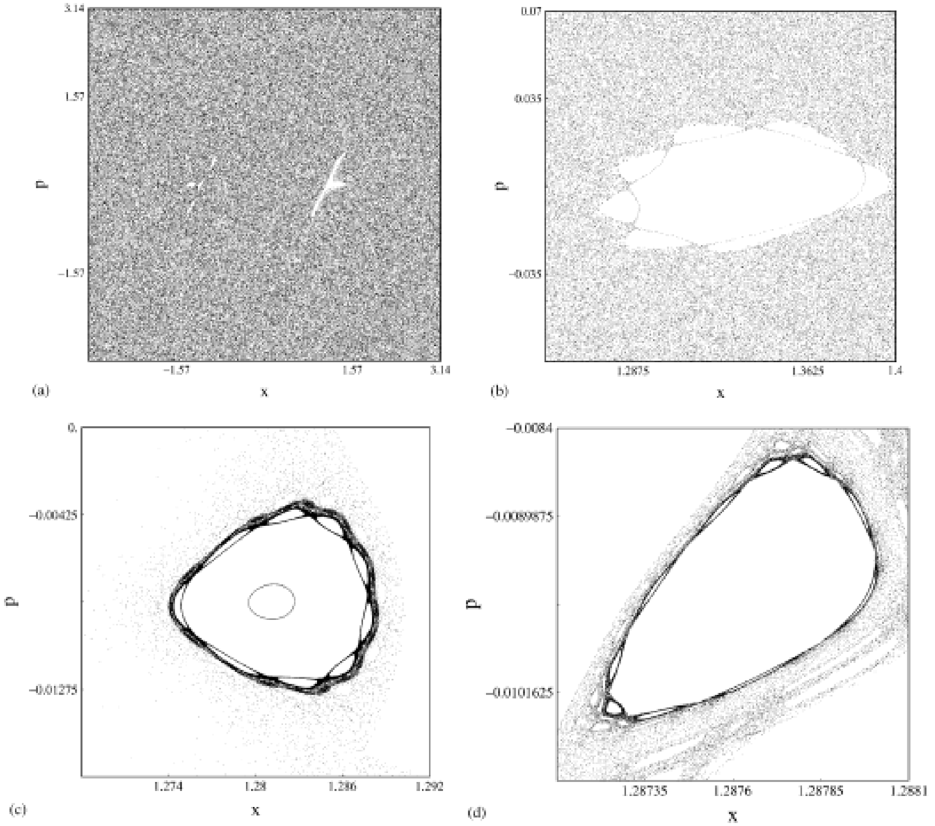
$$\mathscr{B}(x) = \frac{1}{\Gamma(2+\alpha)} \lim_{\Delta t\to 0} \frac{\langle\langle|\Delta x|^{\alpha+1}\rangle\rangle}{(\Delta t)^\beta},$$

$$\mathscr{A}(x) = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t\to 0} \frac{\langle\langle|\Delta x|^{\alpha}\rangle\rangle}{(\Delta t)^\beta}$$

$$\langle R^2(t)\rangle \sim t^\gamma$$

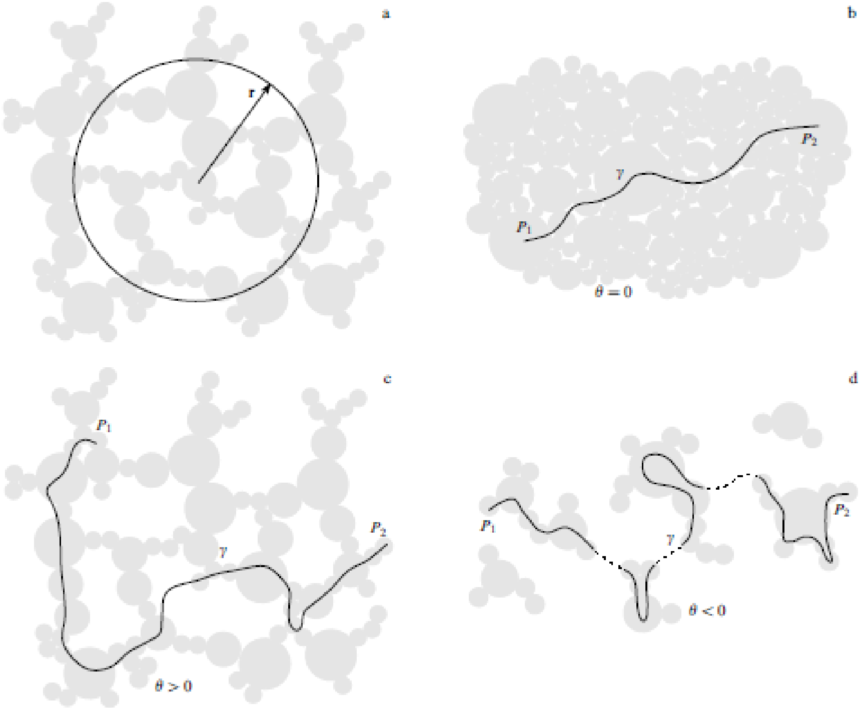
$$p_n(k) = \exp(-\text{constant} \times n|k|^\alpha)$$

$$p_n(x) \sim \text{constant} \times n/x^{1+\alpha}$$



Fractal topology and strange kinetics: from percolation theory to problems in cosmic electrodynamics

L M Zelenyĭ, A V Milovanov *Physics – Uspekhi* **47** (8) 749 – 788 (2004)



Mean – square displacement
Normal – Anomalous Diffusion

$$< x^2(t) > \cong Dt^{\mu} = Dt^{2H}, 0 \le \mathrm{H} \le 1$$

$$\mu = \frac{\beta}{\alpha} = \frac{ds}{df} = \frac{2}{2+\theta}, \; 1 < \mu \le 2$$

(Non Gaussian dynamics)

$$\mu \ne 1, \begin{Bmatrix} \theta > 0, \mu < 1 \leftrightarrow \text{subdiffusion} \\ \theta < 0, \mu > 1 \leftrightarrow \text{superdiffusion} \end{Bmatrix}$$

$\mu = 2H$, d_s=spectral fractal dimension
θ=connectivity index
d_f= Hausdorff fractal dimension of space
H=Hurst exponent
d_w=1/H=fractal dimension of trajectories
$\mu = 1, \mathrm{H} = 1/2, \mathrm{dw} = 2 \leftrightarrow$ Normal diffusion (Gaussian dynamics)
μ={competition between FTRW – Levy process}

Fractional Solar Plasma

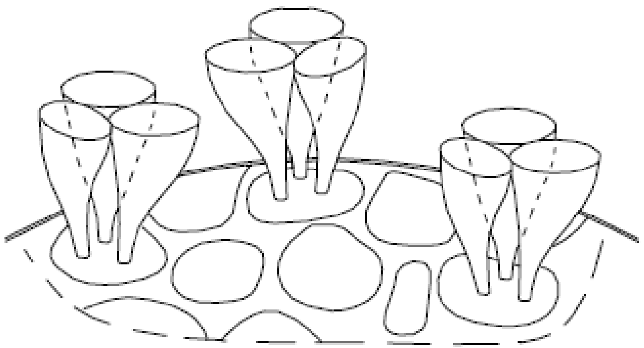
L M Zelenyĭ, A V Milovanov (1999,2004)

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} = \nabla_{\mathbf{r}}^{2\beta}(\mathcal{B}\psi).$$

$$\frac{\partial^\alpha}{\partial t^\alpha}\psi(t,\mathbf{r}) = \frac{1}{\Gamma(m-\alpha)}\frac{\partial^m}{\partial t^m}\int_0^t \frac{d\vartheta}{(t-\vartheta)^{1+\alpha-m}}\psi(\vartheta,\mathbf{r}),$$

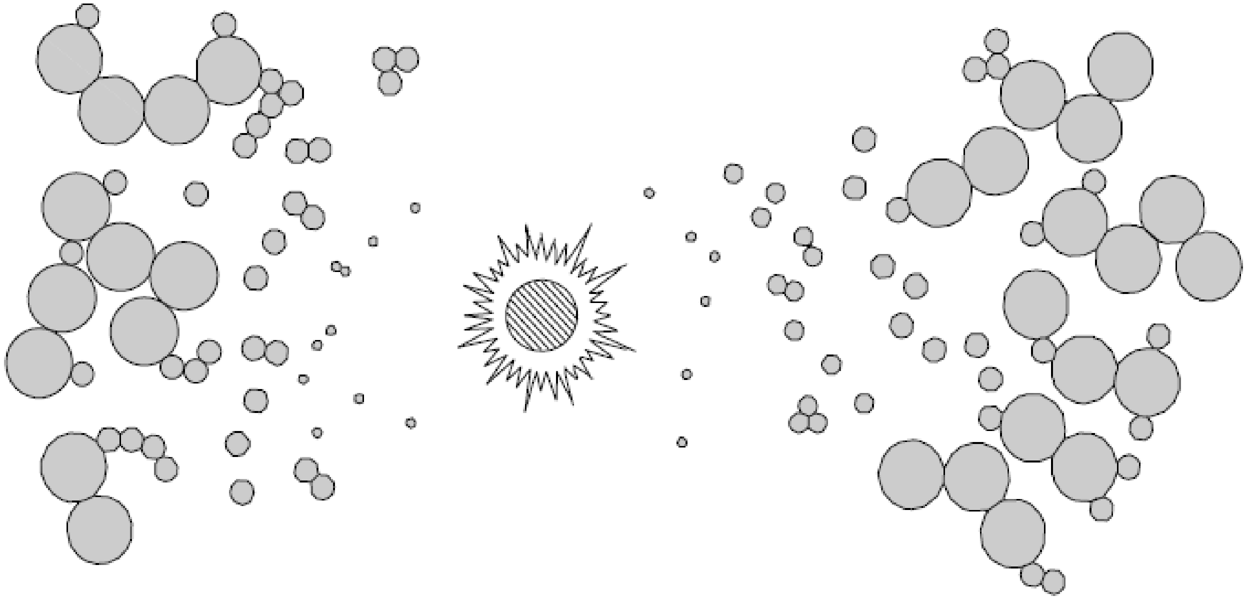
$$\frac{\partial^\beta}{\partial x_i^\beta}\psi(t,\mathbf{r}) = \frac{1}{\Gamma(1-\beta)}\frac{\partial}{\partial x_i}\int_{-\infty}^{x_i}\frac{dx_i'}{(x_i-x_i')^\beta}\psi(t,\mathbf{r}').$$

Solar Photosphere



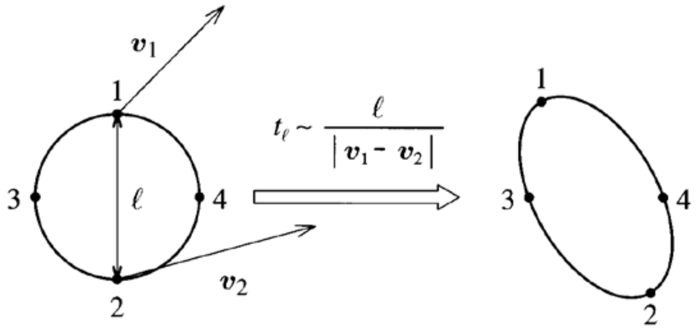
Magnetic Flux - Fractal Distribution

Solar Wind - Fracton Theory - Intermittent Turbulence



Turbulence – Intermittent Turbulence

Uriel Frisch – Turbulence (The Legacy of A.N. Kolmogorov)



Eddies Transformation

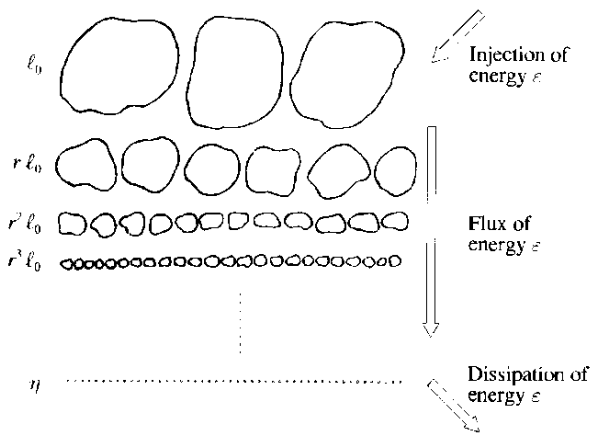
$$S_p(\ell) \equiv \langle (\delta v_{\|}(\ell))^p \rangle.$$

$$S_p(x) \equiv \left\langle \left[\left(\boldsymbol{v}(\boldsymbol{r} + x\boldsymbol{\ell}^0) - \boldsymbol{v}(\boldsymbol{r}) \right) \cdot \boldsymbol{\ell}^0 \right]^p \right\rangle$$

$$S_p(\ell) \propto \ell^{p/3}.$$

$$S_p(\ell) = C_p \varepsilon^{p/3} \ell^{p/3};$$

‘mother – eddy’ ➔ ‘daughters



Space filling Cascade over Energy dissipation (K41)

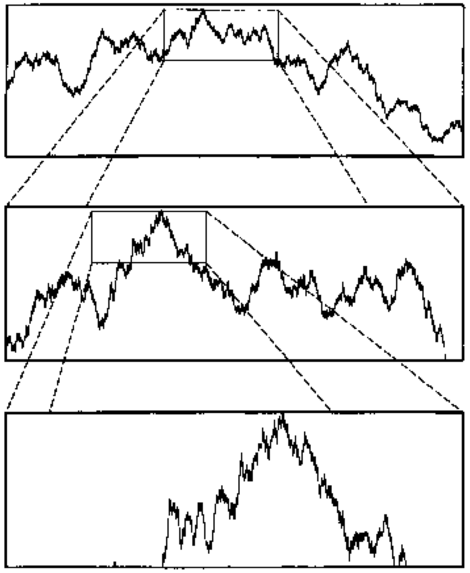
$$\Pi'_\ell \sim \frac{v_\ell^3}{\ell} \sim \varepsilon.$$

$$v_\ell \sim \varepsilon^{1/3} \ell^{1/3},$$

$$v_\ell \sim \sqrt{\langle \delta v_{\|}^2(\ell) \rangle},$$

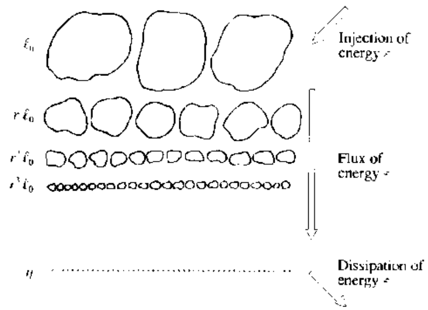
$$t_\ell \sim \frac{\ell}{v_\ell}.$$

Uriel Frisch – Turbulence (The Legacy of A.N. Kolmogorov)



Brownian self similarity

'mother – eddy' → 'daughters



β-model cascade (eddies without space filling)

(β-model)

$$\beta \ (0 < \beta < 1).$$

$$\ell = r^n \ell_0$$

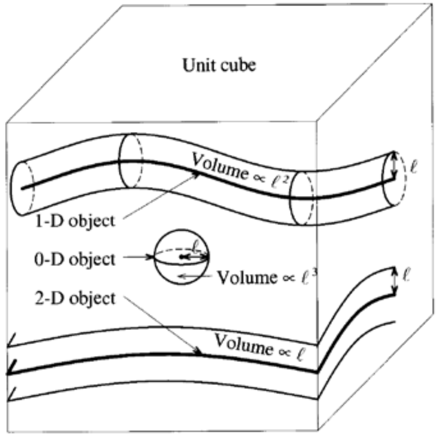


Fig. 8.10. The probability that a sphere of radius ℓ encounters an object of dimension D behaves as ℓ^{3-D} as $\ell \to 0$.

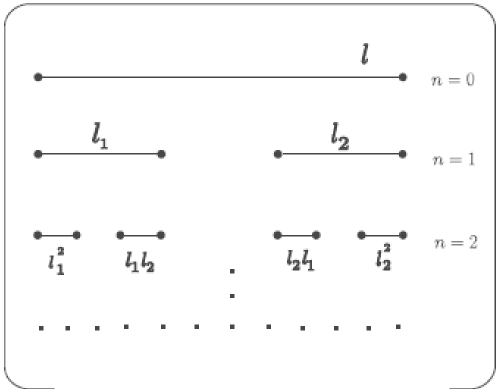
(probability density)

$$p_\ell = \beta^n = \beta^{\frac{\ln(\ell/\ell_0)}{\ln r}} = \left(\frac{\ell}{\ell_0}\right)^{3-D}$$

$$p_\ell \propto \ell^{3-D}, \qquad \ell \to 0.$$

$$p_\ell \propto \ell^{d-D}, \qquad \ell \to 0.$$

'p-model



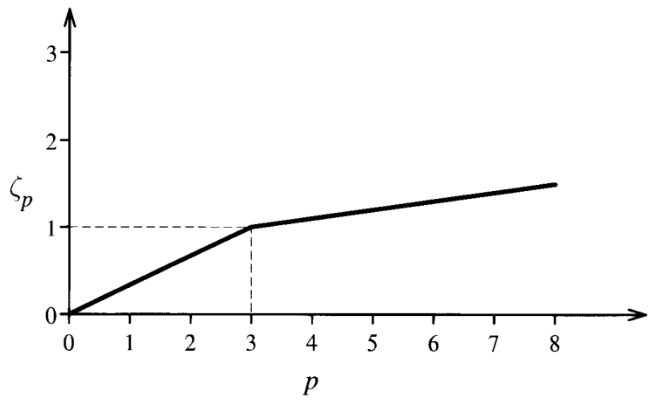
Two-scale Cantor set

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1$$

$$f(\alpha) = q\alpha(q) - \tau(q)$$

Multifractal intermittency velocity structure

Uriel Frisch – Turbulence (The Legacy of A.N. Kolmogorov)



bifractal model

$$v_\ell \sim v_0 \left(\frac{\ell}{\ell_0}\right)^{\frac{1}{3}-\frac{3-D}{3}},$$

singular exponent $h = \frac{1}{3} - \frac{3-D}{3}$

$$S_p(\ell) = \langle \delta v_\ell^p \rangle \sim v_0^p \left(\frac{\ell}{\ell_0}\right)^{\zeta_p}$$

Bifractal β Model

$$\frac{\delta v_\ell(r)}{v_0} \sim \begin{cases} \left(\frac{\ell}{\ell_0}\right)^{h_1}, & r \in \mathscr{S}_1,\ \dim \mathscr{S}_1 = D_1 \\ \left(\frac{\ell}{\ell_0}\right)^{h_2}, & r \in \mathscr{S}_2,\ \dim \mathscr{S}_2 = D_2. \end{cases}$$

singularity manifold fractal set-fractal dimension

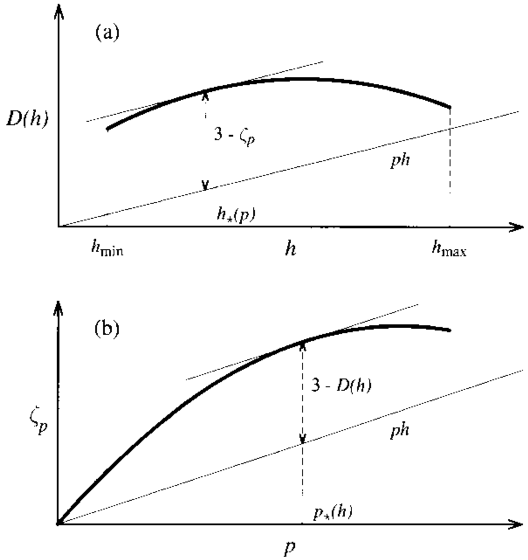
$$\langle \delta v_\ell^p \rangle \propto \ell^{\zeta_p}, \qquad \zeta_p = \min\left(ph_1 + 3 - D_1,\ ph_2 + 3 - D_2\right).$$

$$\zeta_p = \begin{cases} p/3 & 0 \le p \le 3 \\ p/3 + (3 - D_2)(1 - p/3) & p \ge 3. \end{cases}$$

multiscaling exponents spectrum

Multifractal β model

Uriel Frisch – Turbulence (The Legacy of A.N. Kolmogorov)



Legendre transform

$$\frac{\delta v_\ell(\boldsymbol{r})}{v_0} \sim \left(\frac{\ell}{\ell_0}\right)^h, \qquad \boldsymbol{r} \in \mathscr{S}_h.$$

$$\frac{S_p(\ell)}{v_0^p} \equiv \frac{\langle \delta v_\ell^p \rangle}{v_0^p} \sim \int_I d\mu(h) \left(\frac{\ell}{\ell_0}\right)^{ph+3-D(h)}.$$

Legendre transformation

$$\lim_{\ell \to 0} \frac{\ln S_p(\ell)}{\ln \ell} = \zeta_p,$$

$$\zeta_p = \inf_h \left[ph + 3 - D(h)\right].$$

$$\frac{S_p(\ell)}{v_0^p} \sim \left(\frac{\ell}{\ell_0}\right)^{\zeta_p}, \qquad \ell \to 0.$$

$$\zeta_p = ph_\star(p) + 3 - D\left(h_\star(p)\right).$$

$$D'\left(h_\star(p)\right) = p,$$

$$D(h) = \inf_p \left(ph + 3 - \zeta_p\right).$$

$$\frac{d\zeta_p}{dp} = h_\star(p) + \left[p - D'\left(h_\star(p)\right)\right] \frac{dh_\star(p)}{dp} = h_\star(p)$$

H$_{\text{pmf}}$ *Under the same assumptions as in H1, there is a universal function $D(h)$ which maps real scaling exponents h to scaling dimensions $D \le 3$ (including negative values and the value $-\infty$), such that for any h, the probability of velocity increments satisfies*

$$\lim_{\ell \to 0} \frac{\ln \bar{P}_\ell^{\text{inc}}\left(\pm \ell^h\right)}{\ln \ell} = 3 - D(h). \qquad (8.50)$$

Multifractal Intermittent Energy Dissipation

$$\varepsilon_\ell(\mathbf{r}) = \frac{1}{(4/3)\pi\ell^3} \int_{|\mathbf{r}'-\mathbf{r}|<\ell} d^3r' \frac{1}{2}\nu \sum_{ij} \left[\partial_j v_i(\mathbf{r}') + \partial_i v_j(\mathbf{r}')\right]^2$$

Definition. *The dissipation is said to be* multifractal *if there is a function* $F(\alpha)$ *which maps real scaling exponents* α *to scaling dimensions* $F \leq 3$ *(including negative values and the value* $-\infty$*), such that for any* α

$$\lim_{\ell \to 0} \frac{\ln \bar{P}_\ell^{\mathrm{diss}}\left(\ell^{\alpha-1}\right)}{\ln \ell} = 3 - F(\alpha), \qquad (8.79)$$

$$\left.\begin{array}{l} \dfrac{\varepsilon_\ell(\mathbf{r})}{v_0^3/\ell_0} \sim \left(\dfrac{\ell}{\ell_0}\right)^{\alpha-1} \quad \text{as } \ell \to 0, \\ \text{for} \quad \mathbf{r} \in \mathscr{D}_\alpha \subset \mathbb{R}^3; \quad \dim \mathscr{D}_\alpha = F(\alpha). \end{array}\right\}$$

$$\langle \varepsilon_\ell^q \rangle \sim \left(\frac{v_0^3}{\ell_0}\right)^q \left(\frac{\ell}{\ell_0}\right)^{\tau_q}, \qquad \tau_q = \min_\alpha \left[q(\alpha-1) + 3 - F(\alpha)\right]$$

$$\varepsilon_\ell(x) \equiv \frac{1}{2\ell} \int_{|x'-x|<\ell} dx' \frac{1}{2}\nu \sum_{ij} \left[\partial_j v_i(x') + \partial_i v_j(x')\right]^2$$

$$\frac{\varepsilon_\ell(x)}{v_0^3/\ell_0} \sim \left(\frac{\ell}{\ell_0}\right)^{\alpha-1} \quad \text{as } \ell \to 0 \quad \text{for} \quad x \in \mathscr{D}'_\alpha; \quad \dim \mathscr{D}'_\alpha = f(\alpha),$$

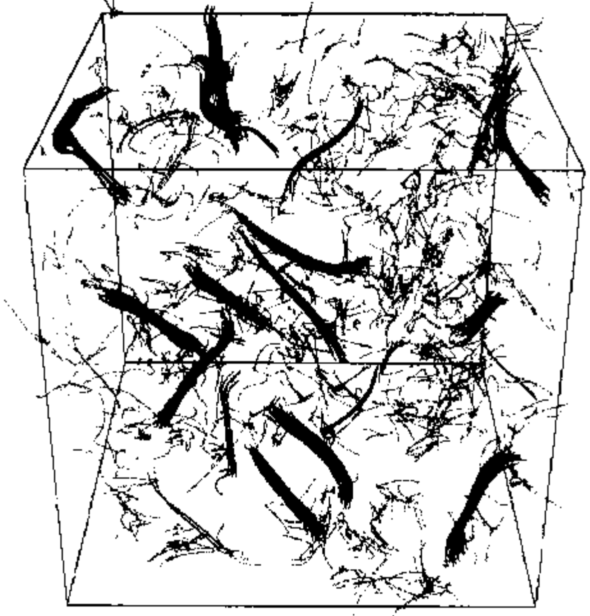
$$f(\alpha) \equiv F(\alpha) - 2.$$

Velocity – Energy Dissipation Multifractal Structures

$$h = \frac{\alpha}{3}, \qquad D(h) = F(\alpha) = f(\alpha) + 2, \qquad \zeta_p = \frac{p}{3} + \tau_{p/3}.$$

Space time scale envariance

$$\mathbf{r}' = \lambda \mathbf{r}, \quad \mathbf{u}' = \lambda^{\alpha/3}\mathbf{u}, \quad t' = \lambda^{1-\alpha/3} t, \quad (p/\rho)' = \lambda^{2\alpha/3}(p/\rho).$$



Intermittent vortex filaments.
Three dimensional turbulence simulation

Multifractal Theory

Theiler J., Vol. 7, No. 6/June 1990/J. Opt. Soc. Am. A, 1055

generalized dimensions

$$D_q = \frac{1}{q-1} \lim_{r \to 0} \frac{\log \sum_i P_i^q}{\log r}.$$

most-dence points

$$D_\infty = \lim_{r \to 0} \frac{\log\left(\max_i P_i\right)}{\log r},$$

least-dence points

$$D_{-\infty} = \lim_{r \to 0} \frac{\log\left(\min_i P_i\right)}{\log r}.$$

Information entropy

$$S(r) = -\sum_i P_i \log_2 P_i,$$

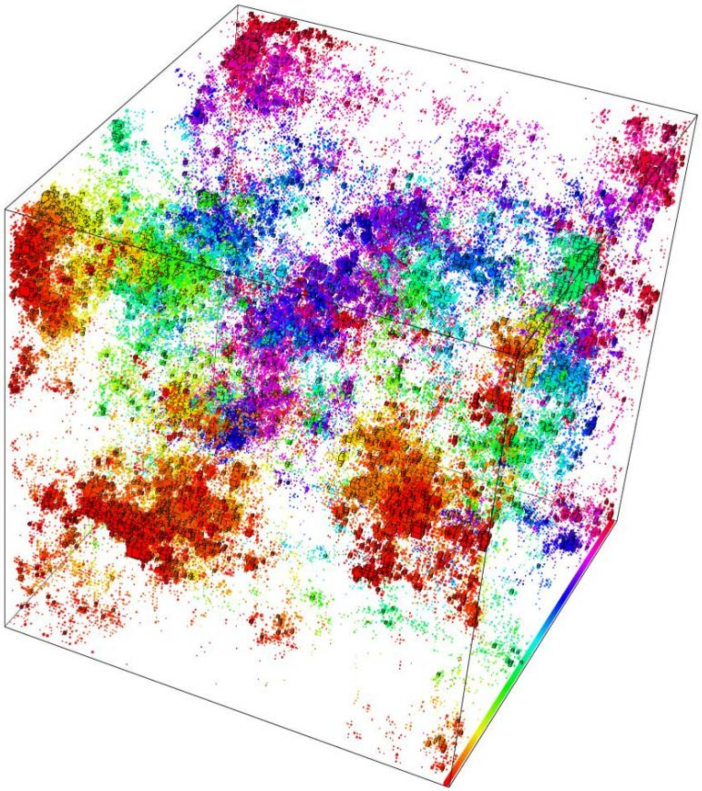
$$D_I = \lim_{r \to 0} \frac{-S(r)}{\log_2 r}$$

Information dimension

$$= \lim_{r \to 0} \frac{\sum_i P_i \log_2 P_i}{\log_2 r}.$$

Rényi entropy

$$S_q(r) = \frac{1}{q-1} \log \sum_i P_i^q,$$



$$D_q = \lim_{r \to 0} \frac{-S_q(r)}{\log r} = \frac{1}{q-1} \lim_{r \to 0} \frac{\log \sum_i P_i^q}{\log r},$$

Multifractal Theory

Theiler J., Vol. 7, No. 6/June 1990/J. Opt. Soc. Am. A, 1055

$$P_i = r^{\alpha}$$

$$n(\alpha, r) \sim r^{-f(\alpha)} \Delta\alpha.$$

$$\sum_i P_i^q = \int n(\alpha, r) r^{q\alpha} \mathrm{d}\alpha$$

$$\sim \int r^{-f(\alpha)} r^{q\alpha} \mathrm{d}\alpha \sim r^{\theta},$$

$$\theta = \min_{\alpha}\{q\alpha - f(\alpha)\}$$

$$\sum_i P_i^q \sim r^{(q-1)D_q},$$

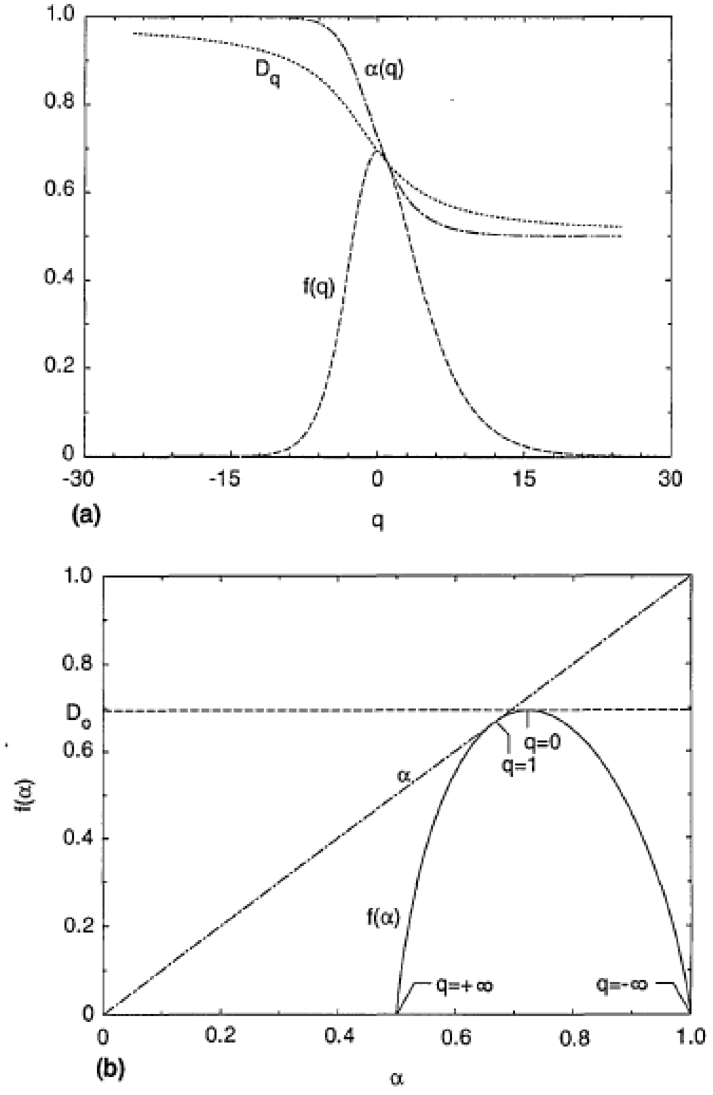
$$\alpha = \frac{\partial\tau}{\partial q} \qquad f = \alpha q - \tau$$

and

$$q = \frac{\partial f}{\partial\alpha} \qquad \tau = \alpha q - f.$$

$$f(\alpha) = \min_q\{q\alpha - \tau(q)\},$$

$$\tau(q) = \min_{\alpha}\{q\alpha - f(\alpha)\}.$$



Tsallis Extension of Statistics

Nonextensive Statistical Mechanics

Microscopic Level → Macroscopic Level

Quantum Complexity
Quantum Phase Transition
(Q.P.T.)

Equilibrium Phase Transition
(E.P.T.)

Non – Equilibrium
Phase Transition
(N.E.P.T.)

Tsallis Theory
Nonextensive Statistical Mechanics

$$S_q = k \ln_q W \quad (S_1 = S_{BG}).$$

$$S_q = k \langle \ln_q (1/p_i) \rangle .$$

$$S_q = k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} .$$

$$y = [1 + (1-q)x]^{1/(1-q)} \equiv e_q^x \quad (e_1^x = e^x).$$

$$\frac{dy}{dx} = y^q$$

$$y = \frac{x^{1-q} - 1}{1-q} \equiv \ln_q x \quad (x > 0;\ \ln_1 x = \ln x),$$

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k} .$$

$$S_q[A+B] = S_q[A] + S_q[B|A] + (1-q) S_q[A] S_q[B|A],$$

Generalized Fokker-Planck Equations

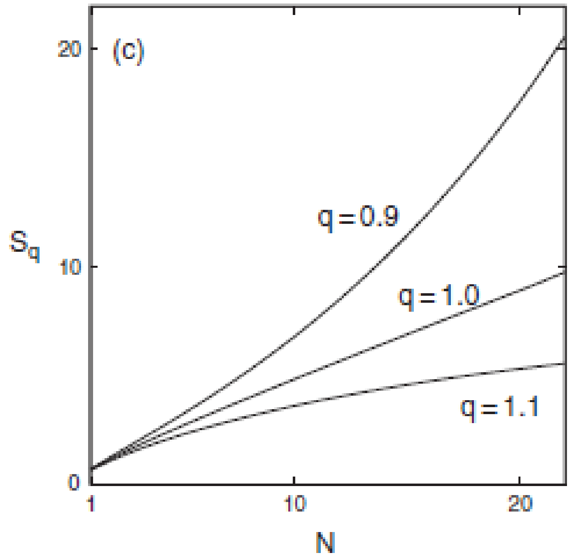
$$\frac{\partial^\beta p(x,t)}{\partial |t|^\beta} = D_{\beta,\gamma,q} \frac{\partial^\gamma [p(x,t)]^{2-q}}{\partial |x|^\gamma} \ (0 < \beta \le 1; 0 < \gamma \le 2).$$

$$\frac{\partial p(x,t)}{\partial t} = D_{\gamma,q} \frac{\partial^\gamma [p(x,t)]^{2-q}}{\partial |x|^\gamma} \ (0 < \gamma \le 2; q < 3).$$

$$L_\gamma(x) \propto \frac{1}{|x|^{1+\gamma}} \quad (|x| \to \infty; 0 < \gamma < 2),$$ Levy Distribution

$$p_q(x) \propto \frac{1}{|x|^{2/(q-1}} \quad (|x| \to \infty;\ 1 < q < 3).$$ q-Gaussian Distribution

$$\gamma = \begin{cases} 2 & \text{if } q \le 5/3, \\ \frac{3-q}{q-1} & \text{if } 5/3 < q < 3, \end{cases}$$



q-extension of thermodynamics

$$Z_q = \sum_{conf} e_q^{-\beta q(E_i - V_q)}$$

$$\beta_q = \beta / \sum_{conf} p_i^q \qquad \beta = 1/KT$$

$$\langle E \rangle_q \equiv \sum_{conf} p_i^q E_i / \sum_{conf} p_i^q = U_q$$

$$F_q \equiv U_q - TS_q = -\frac{1}{\beta} \ln qZ_q$$

$$U_q = \frac{\partial}{\partial \beta} \ln qZ_q, \frac{1}{T} = \frac{\partial S_q}{\partial U_q}$$

$$C_q \equiv T \frac{\partial \delta_q}{\partial T} = \frac{\partial U_q}{\partial T} = -T \frac{\partial^2 F_q}{\partial T^2}$$

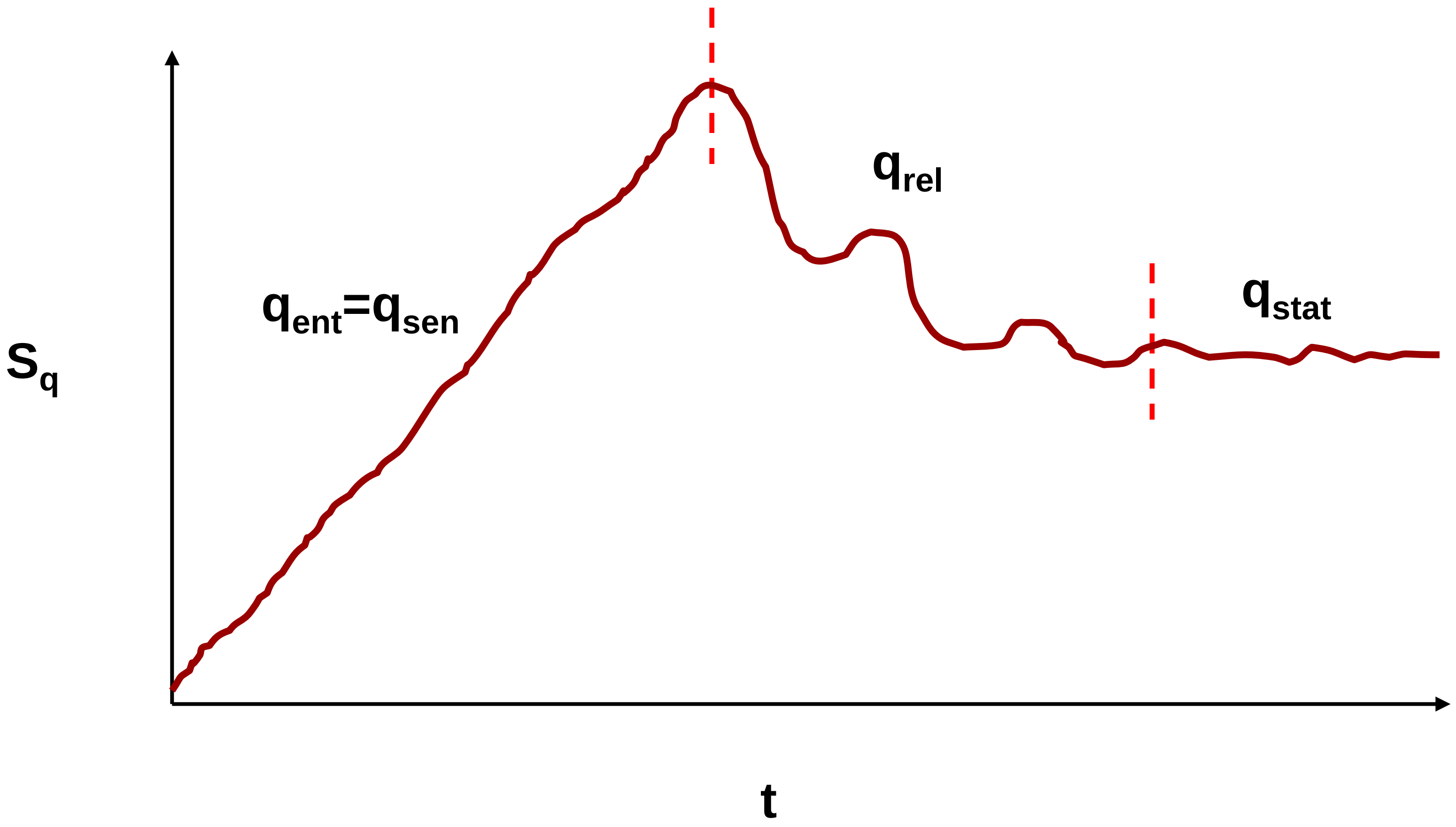
q-extension of central limit theorem

q-independent random variables

$$F_q[X+Y](\xi) = F_q[X](\xi) \otimes_{\frac{1+q}{3-q}} F_q[Y](\xi), \qquad F_q[f](\xi) \equiv \int dx\, e_q^{i\xi x} \otimes_q f(x).$$

where $h(x, y)$ is the joint distribution. Therefore, q*-independence* means *independence* for $q = 1$ (i.e., $h(x, y) = f_X(x) f_Y(y)$), and it means *strong correlation* (of a certain class) for $q \neq 1$ (i.e., $h(x, y) \neq f_X(x) f_Y(y)$).

q-triplet Tsallis
One-Dimensional Systems (Timeseries)



The q-triplet of Tsallis

q - CLT → (q_{k-1}, q_k, q_{k+1})= qstat, qrel, qsen)

Gaussian-BG equilibrium (qstat=qsen=qrel=1) → Nonequilibrium (qstat, qsen, qrel)

$\frac{dy}{dx} = y^q$, ($y(0)=1, q \in \Re$) → (qstat, qsen, qrel)

Equilibrium PDF → Metaequilibrium PDF

(qstat) $\frac{d(p_i Z_{stat})}{dE_i} = -\beta q_{stat} (p_i Z_{stat})^{q_{stat}}$ → $p(x) \propto \left[1-(1-q)\beta_{q_{stat}} x^2\right]^{1/1-q_{stat}}$

Equilibrium BG entropy production → Metaequilibrium q-entropy production

(qsen)

$K_q \equiv \lim_{t\to\infty} \lim_{W\to\infty} \lim_{N\to\infty} \frac{< S_q > (t)}{t}$ → $\frac{1}{1-q_{sen}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}}$ $\quad f(a_{min}) = f(a_{max}) = 0$

$\frac{d\xi}{dt} = \lambda_{q_{sen}} \xi^{q_{sen}}$, → $\xi = e_{q_{sen}}^{\lambda_{q_{sen}} t}$, $\quad \xi \equiv \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)}$,

Equilibrium relaxation process → Metaequilibrium nonextensive relaxation process

(qrel) $\Omega(t) \equiv [O(t) - O(\infty)] / [O(0) - O(\infty)]$ $\quad \frac{d\Omega}{dt} = -\frac{1}{T_{q_{rel}}} \Omega^{q_{rel}}$ → $\Omega(t) \simeq e_{q_{rel}}^{-t/\tau_{rel}}$

Intermittent Turbulence – Tsallis Theory

Fractal dissipation of Energy

$$\varepsilon_n \sim \varepsilon_0 \left(l_n / l_0\right)^{\alpha-1}$$

$$\sum_n \varepsilon_n^q l_n^d \sim l_n^{(q-1)D_n} = l_n^{\tau(q)}$$

$$dn(\alpha) \sim l_n^{-f_d(\alpha)} d\alpha$$

$$\left.\begin{array}{l} f_d(a) = a\bar{q} - (\bar{q}-1)(D_{\bar{q}} - d + 1) + d - 1 \\ a = \dfrac{d}{d\bar{q}}[(\bar{q}-1)(D_{\bar{q}} - d + 1)] \end{array}\right\}$$

$$f(a) = a\bar{q} - \tau(\bar{q}),$$

$$a = \frac{d}{d\bar{q}}[(q-1)D_q] = \frac{d}{d\bar{q}}\tau(\bar{q})$$

$$\bar{q} = \frac{df(a)}{da}$$

Tsallis Entropy Extremization - Singular Spectrum

$$P(a) = Z_q^{-1}[1-(1-q)\frac{(a-a_0)^2}{2X/\ln 2}]^{\frac{1}{1-q}}$$

$$f(a) = D_0 + \log_2[1-(1-q)\frac{(a-a_o)^2}{2X/\ln 2}]/(1-q)^{-1}$$

$$\tau(\bar{q}) = \bar{q}a_0 - 1 - \frac{2X\bar{q}^2}{1+\sqrt{C_{\bar{q}}}} - \frac{1}{1-q}[1-\log_2(1+\sqrt{C_{\bar{q}}})]$$

$$a_{\bar{q}} - a_0 = (1-\sqrt{C_{\bar{q}}})/[\bar{q}(1-q)\ln 2]$$

$$\frac{1}{1-q} = \frac{1}{a_-} - \frac{1}{a_+}$$

Intermittence Turbulence Spectrum

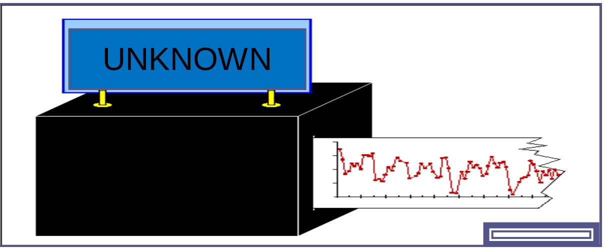
$$Sp(l^n) = <|\delta u_n|^p>$$

$$J(p) = 1 + \tau(\bar{q} = \frac{p}{3})$$

$$J(p) = \frac{a_0 p}{3} - \frac{2Xp^2}{q(1+\sqrt{C_{p/3}}} - \frac{1}{1-q}[1-\log_2(1+\sqrt{C_{p/3}})]$$

$$J(p) = \frac{p}{3} + T^{(u)}(p) + T^{(F)}(p)$$

FLOW CHART OF NON LINEAR ANALYSIS



Local Data = Timeseries

Reconstructed Phase Space

– SVD analysis

Phenomenological Characteristics

Time – Frequency Domain

Dynamic Degrees of Freedom
Dynamic Instability – Strange Dynamics – Fractal
Topology - Percolation
Chaos – SOC - Turbulence

Law of Internal Dynamics

$$\frac{d\mathbf{X}}{dt} = \mathbf{F}(\mathbf{X}(t), \mathbf{u}(t), \lambda, w(t))$$

Non-extensive Statistics (q-triplet Tsallis)

Linearity – Non-Linearity - Noise

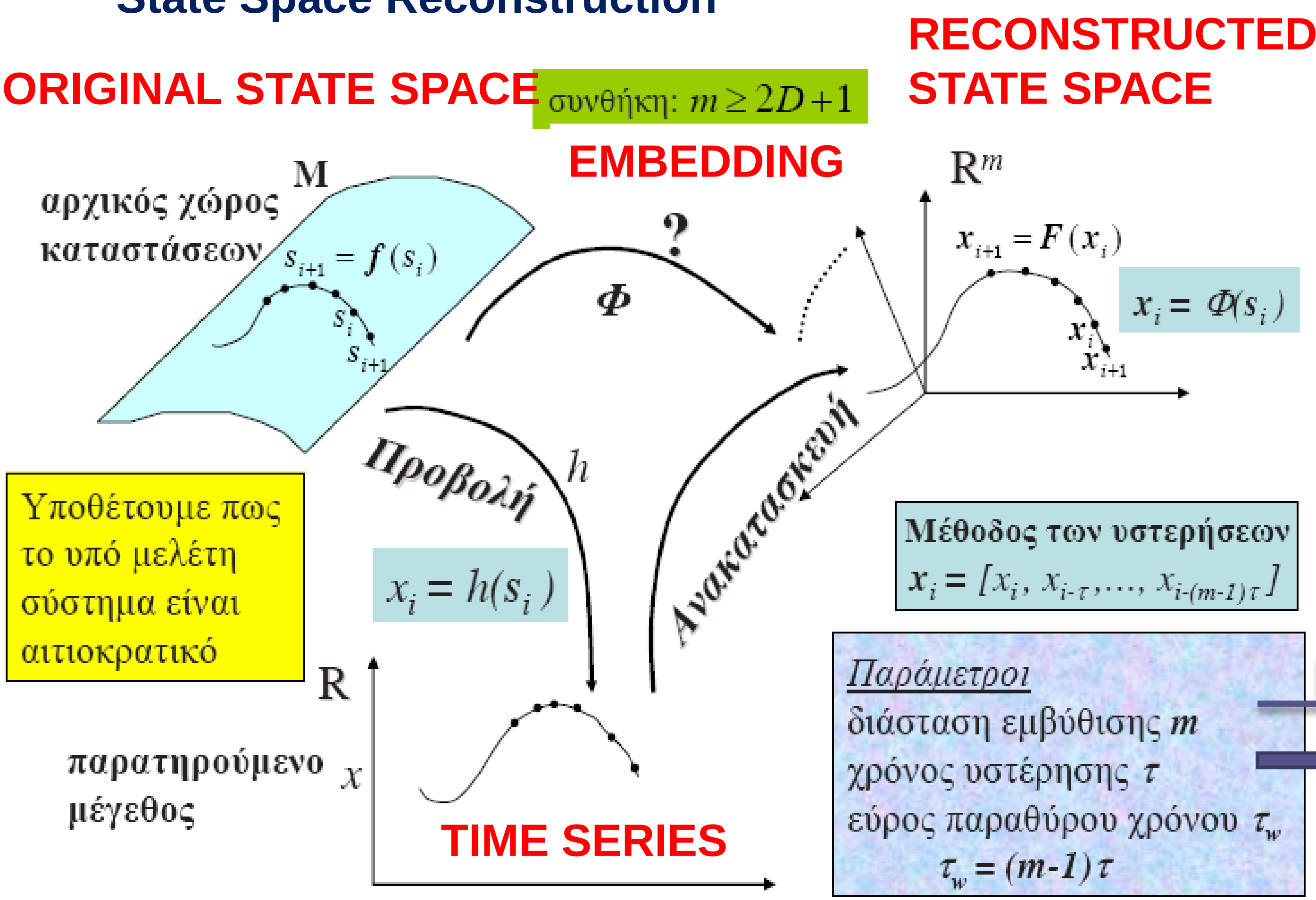
Non-Linear Prediction Models

Spatiotemporal Modeling

CHAOTIC ALGORITHM

Takens Theorem (1981)

State Space Reconstruction



Notes: ***D. Kugiumtzis, AUTH***

The Chaotic Algorithm is based in proficient mathematical concepts: **Embedding theory, metrics, fractals, multiple manifolds, probability – information theory, dynamical systems and maps**

- ✓*m* small, self-cutting
- ✓*m* big, noise

- ✓Autocorrelatrion
- ✓Mutual Information

COMPLEXITY AND TIME SERIES ANALYSIS

I. Analysis in the time-frequency domain

• Autocorrelation and Power Spectrum (noise-chaoticity)
• Mutual Information (Nonlinear Correlations)
• Flatness Coefficient (F) (Gaussian – non-Gaussian proccess)
• Hurst Exponent (temporal fractality – singularities, persistence – anti persistence, normal – anomalous diffusion
• structure function (normal – intermittent turbulence, singularity spectrum (h, $D(h)$)
• q-entropy production (S_q)

II. Fractal geometry and fractional dynamics

• Singularity spectrum (a, $f(a)$)
• Generalized Spectrum $(\overline{q}, D(\overline{q}))$
• P –model (multiplicative process)
• Probability distribution function (Tsallis distributions)
• q-extension of CLT and Tsallis q-triplet parameters (q_{sen}, q_{rel}, q_{stat})
• Tsallis modeling of non-equilibrium stationary states (NESS)

III. Low dimensionality and Nonlinearity

• Correlation Dimension of reconstructed phase space and existence of strange attractor.
• Strange Dynamics and sensitivity of initial conditions (Lyapunov exponents spectrum)
• Discrimination of stochasticity and nonlinear randomnicity - chaoticity
• Discrimination of dynamical components

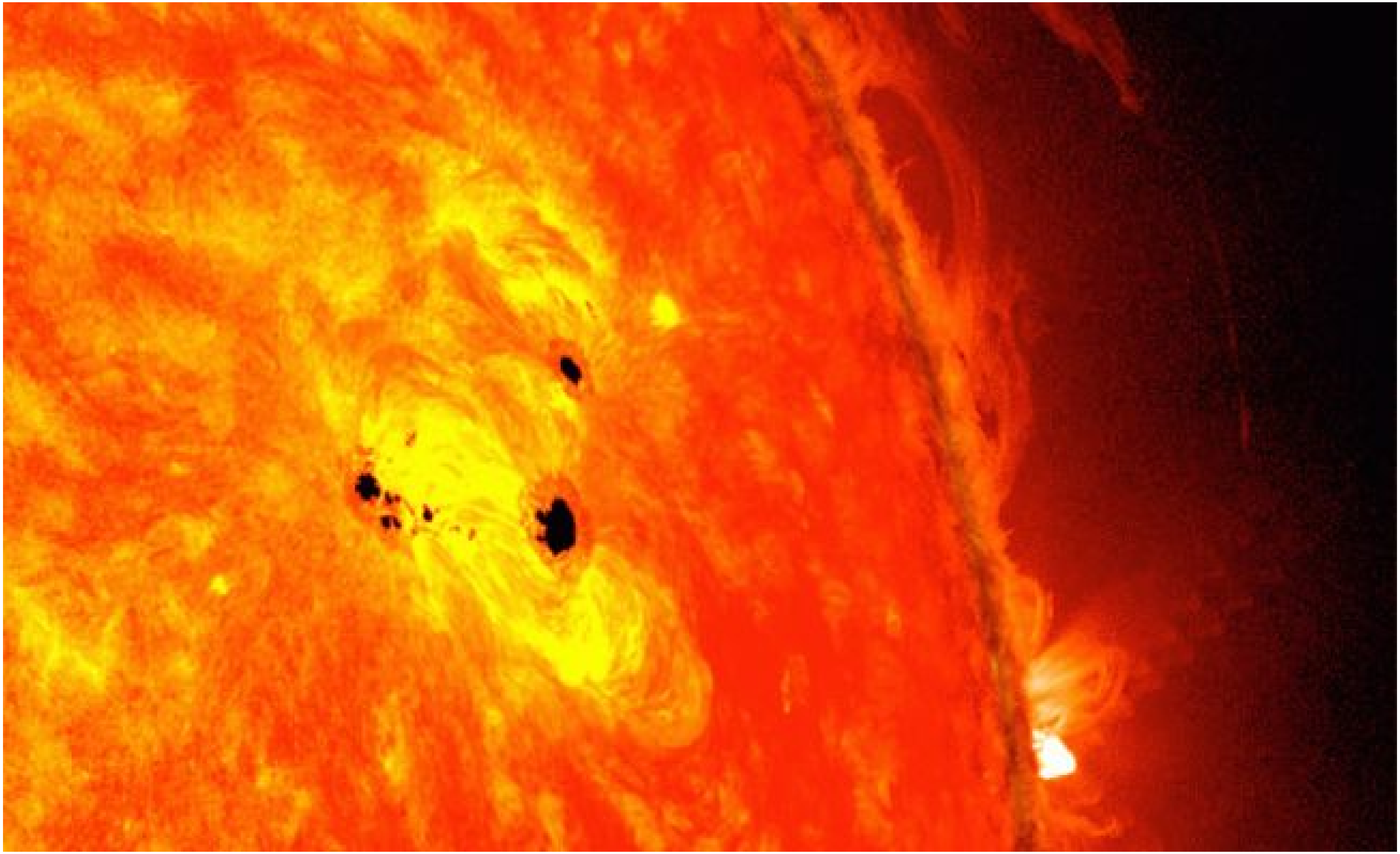
Synopsis

Chaotic Analysis
Dimensional Analysis

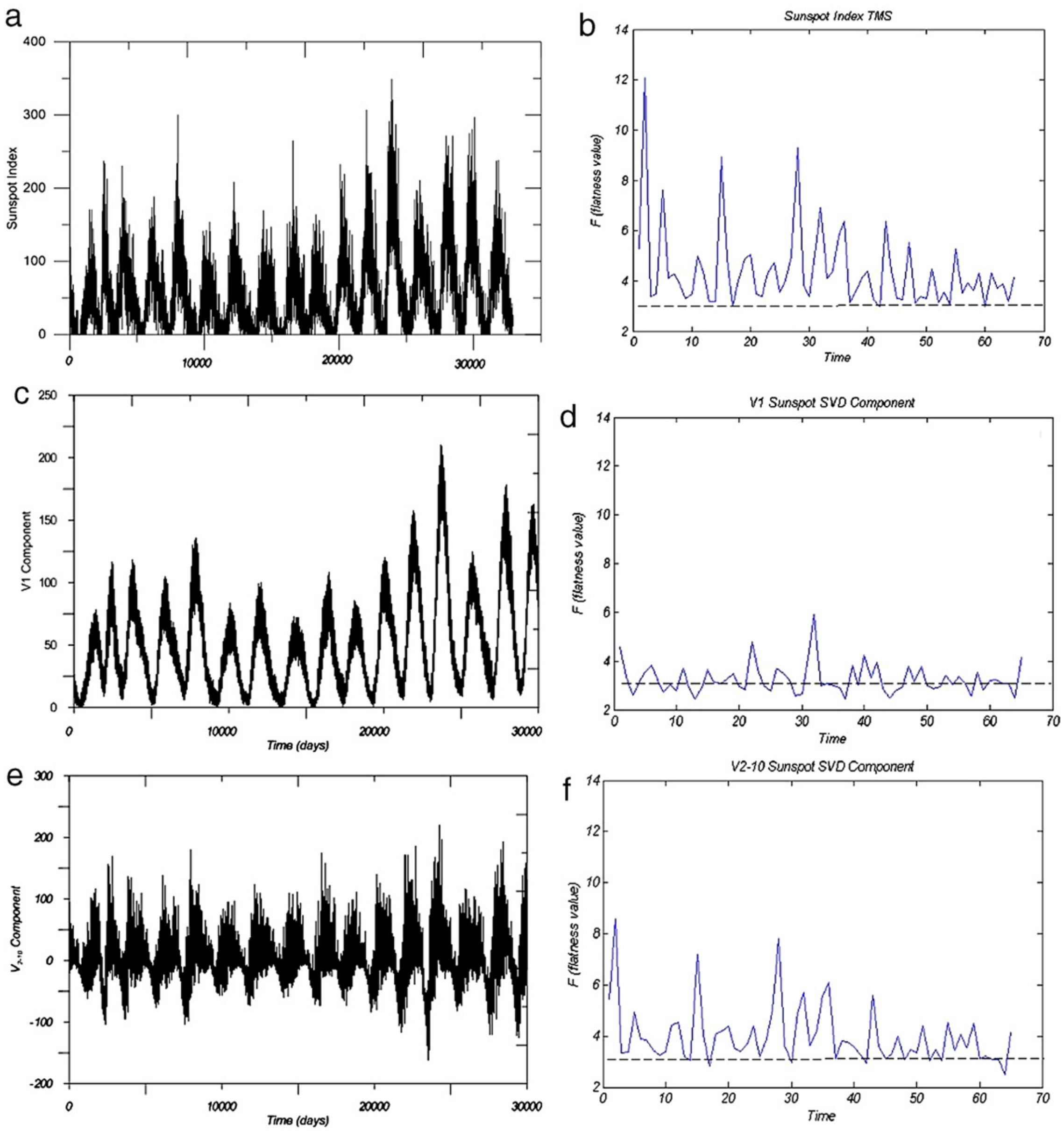
Statistical Analysis

Turbulence Analysis

Sunspot

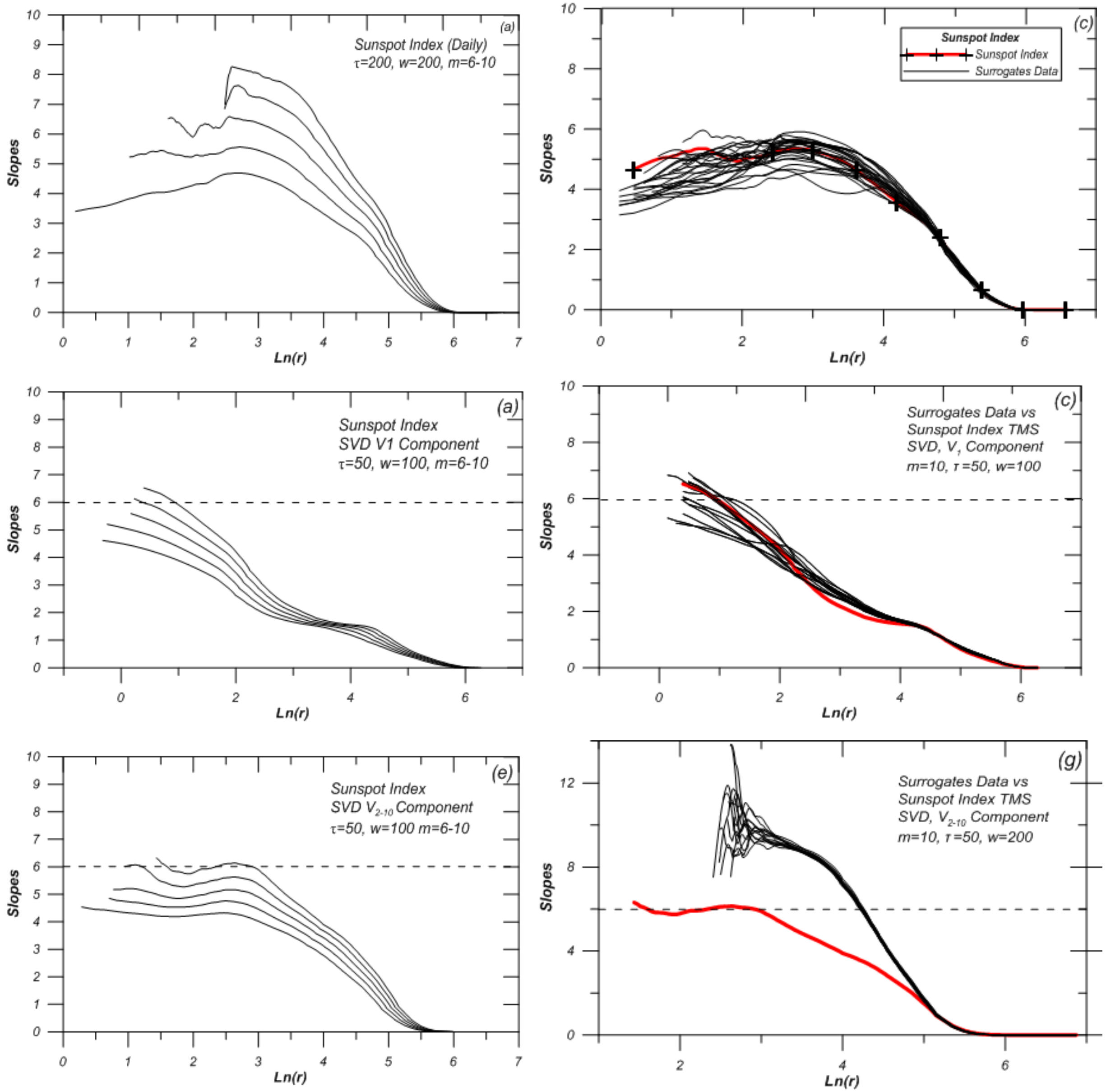


Sunspot Index (Original – SVD Component)

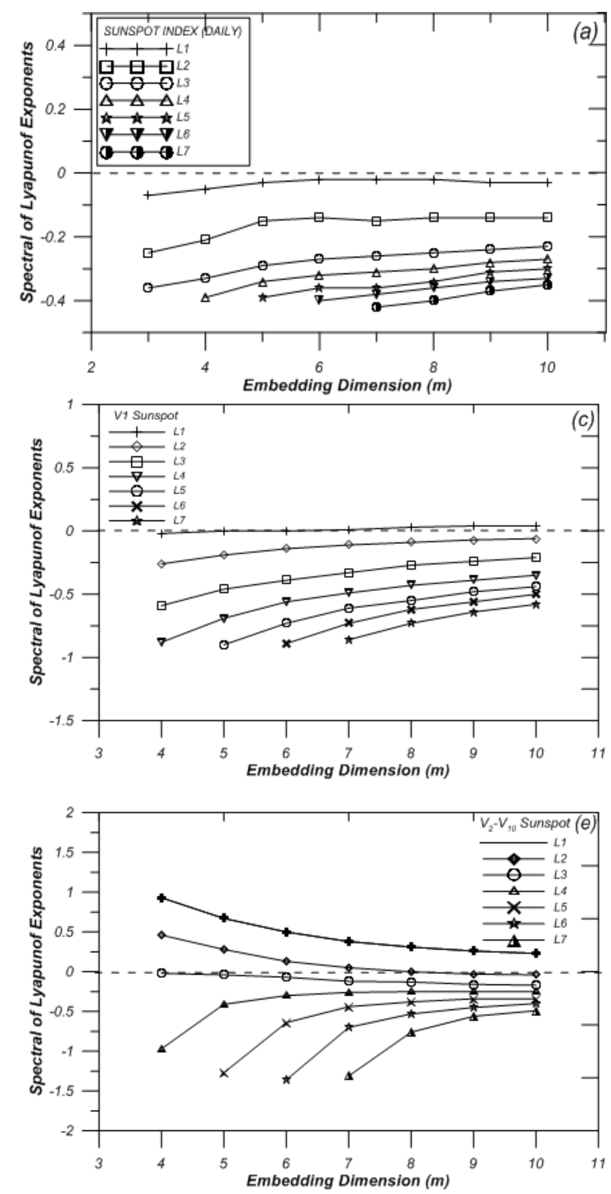


Sunspot Index (Correlation Dimension & Lyapunov Exponent)

Correlation Dimension

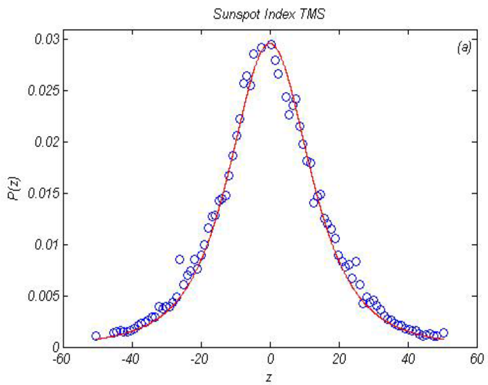


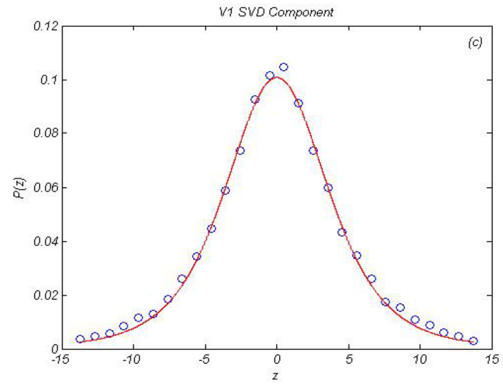
Lyapunov Spectrum Exponents

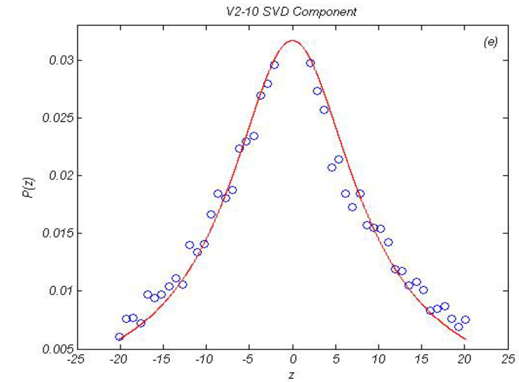


Sunspot Index (Tsallis Statistics)

Sunspot TMS - q_{stat}=1.53 ±0.04 | **V1 SVD Component - q_{stat}=1.40 ±0.08** | **V2-10 SVD Component - q_{stat}=2.12 ±0.20**



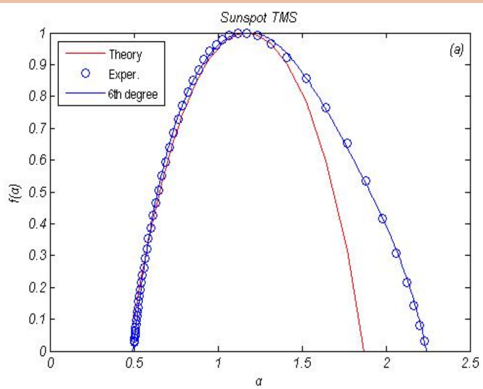


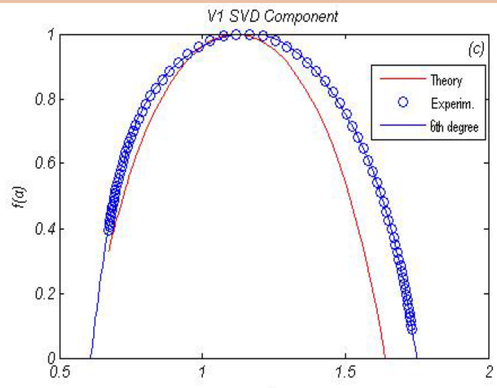


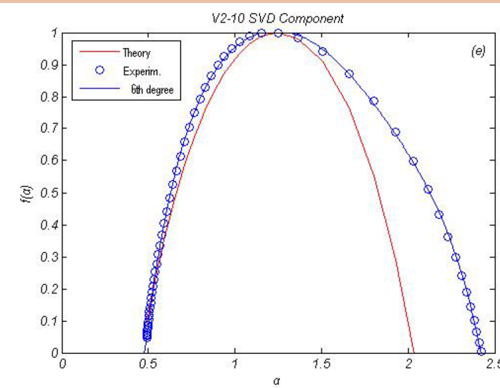
Sunspot TMS TMS - q_{sen}=0.368
$\Delta\alpha$= 1.752

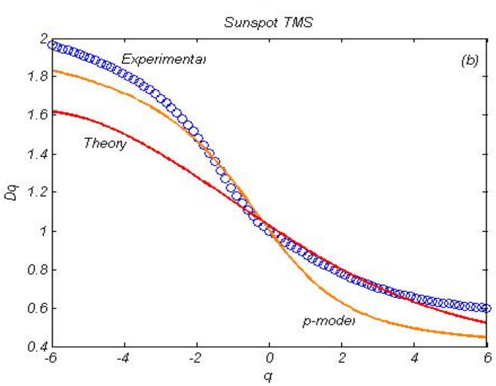
V1 SVD Component - q_{sen}=-0.055
$\Delta\alpha$= 1.133

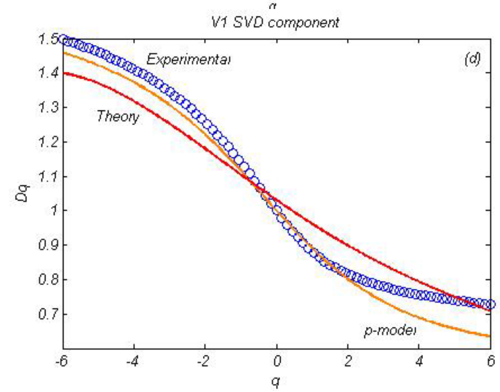
V2-10 SVD Component – q_{sen}=0.407
$\Delta\alpha$= 1.940

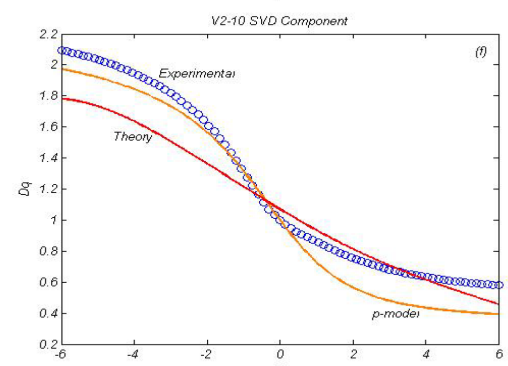




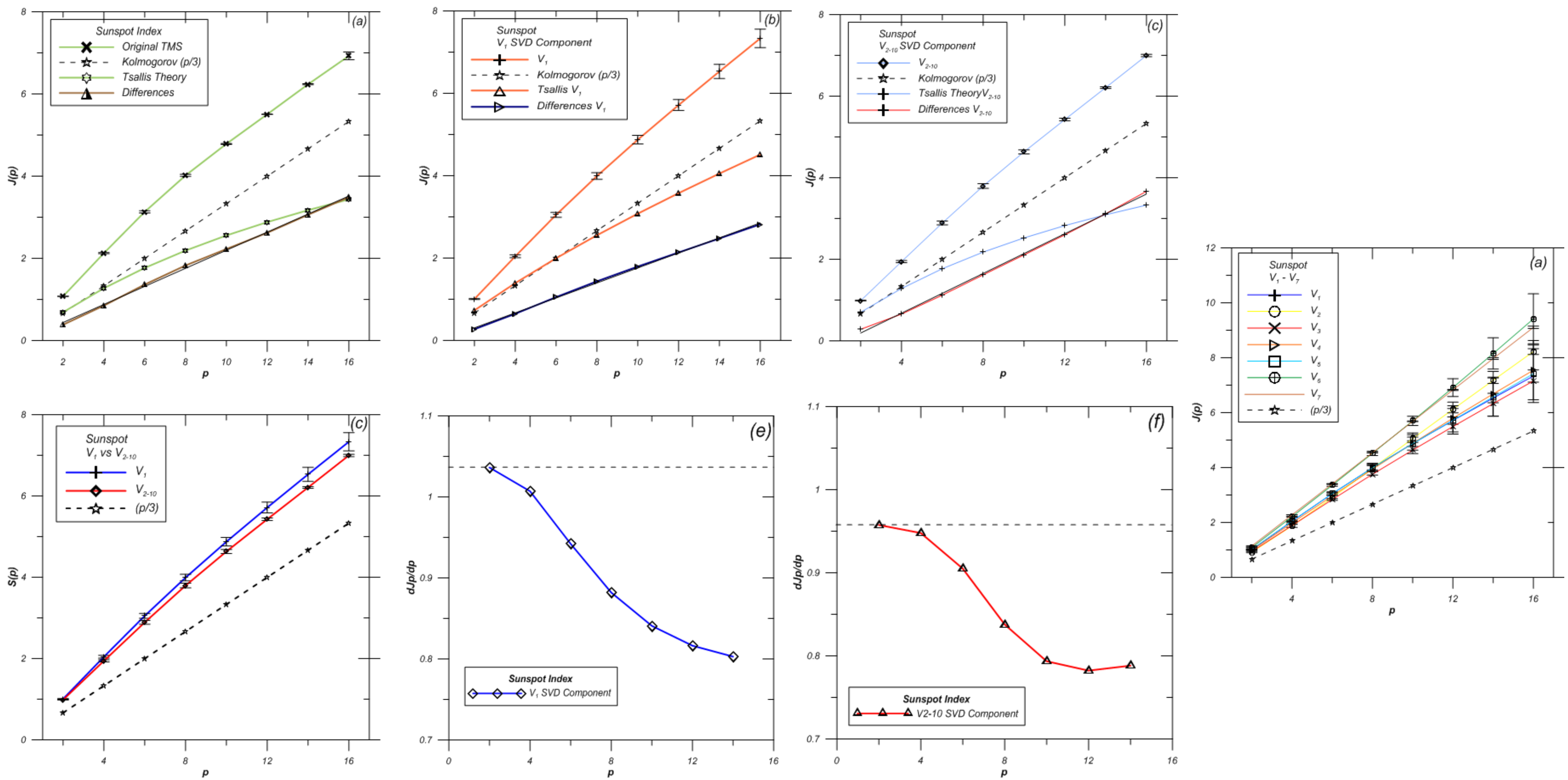




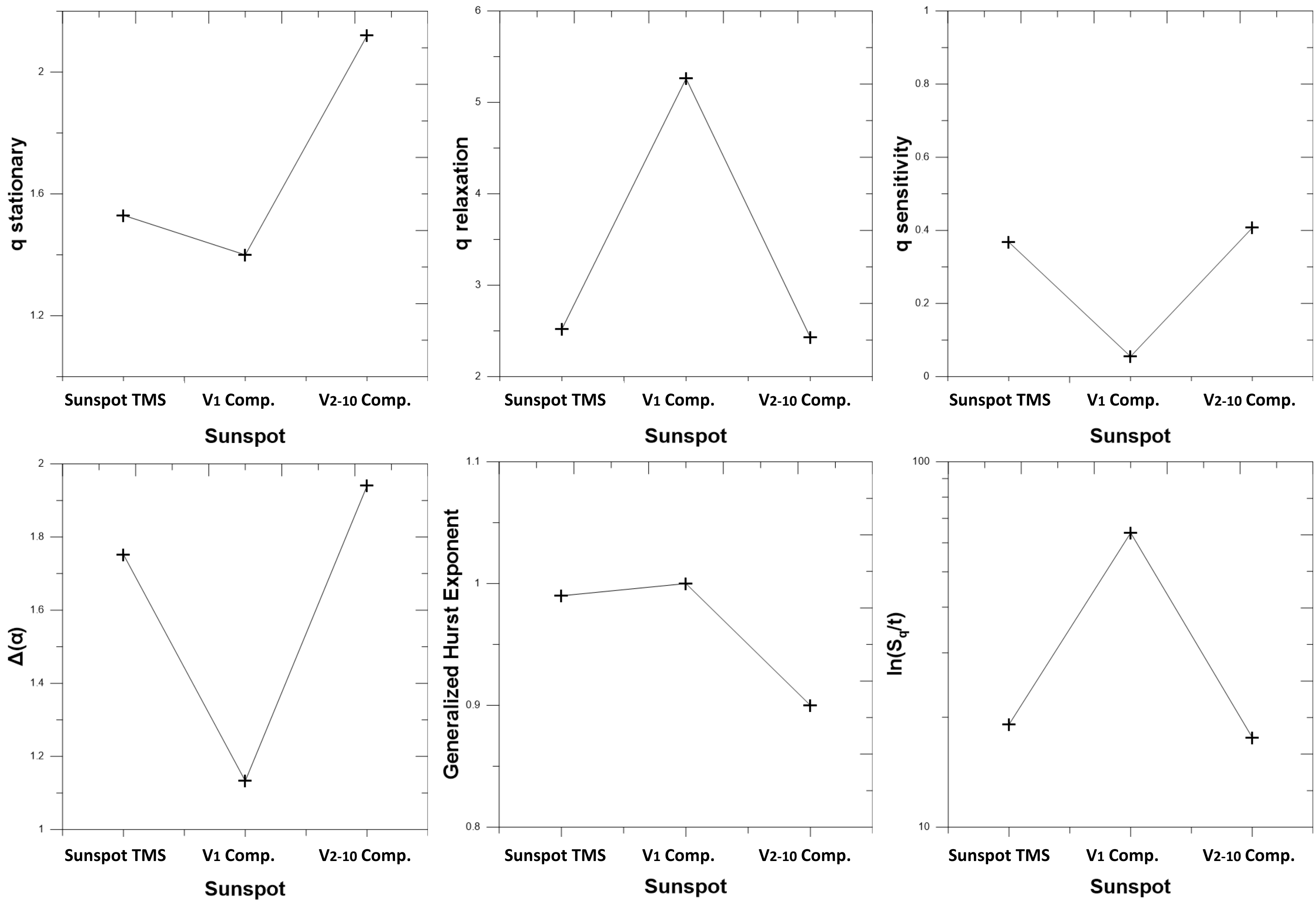




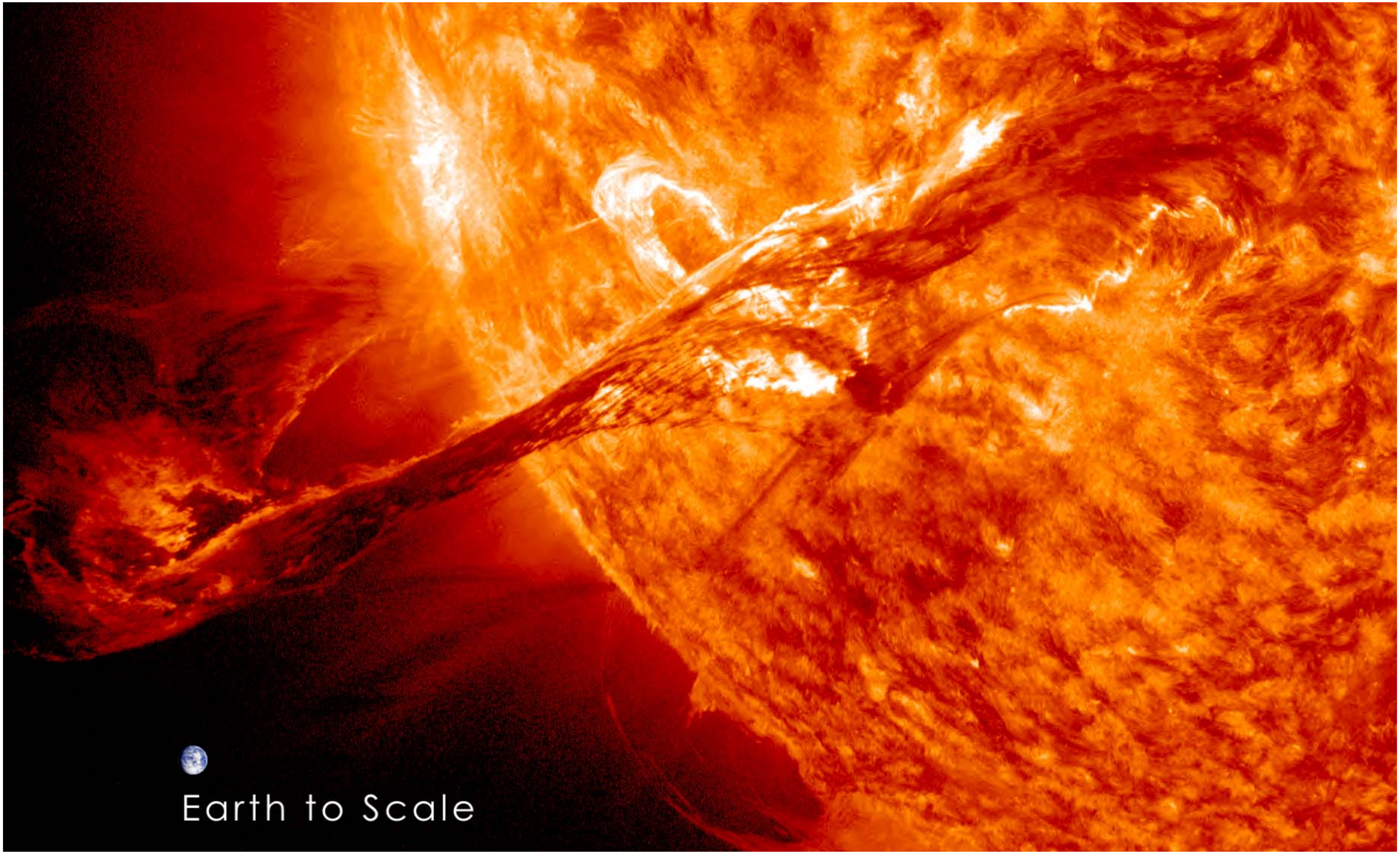
Sunspot Index (Turbulence Analysis)



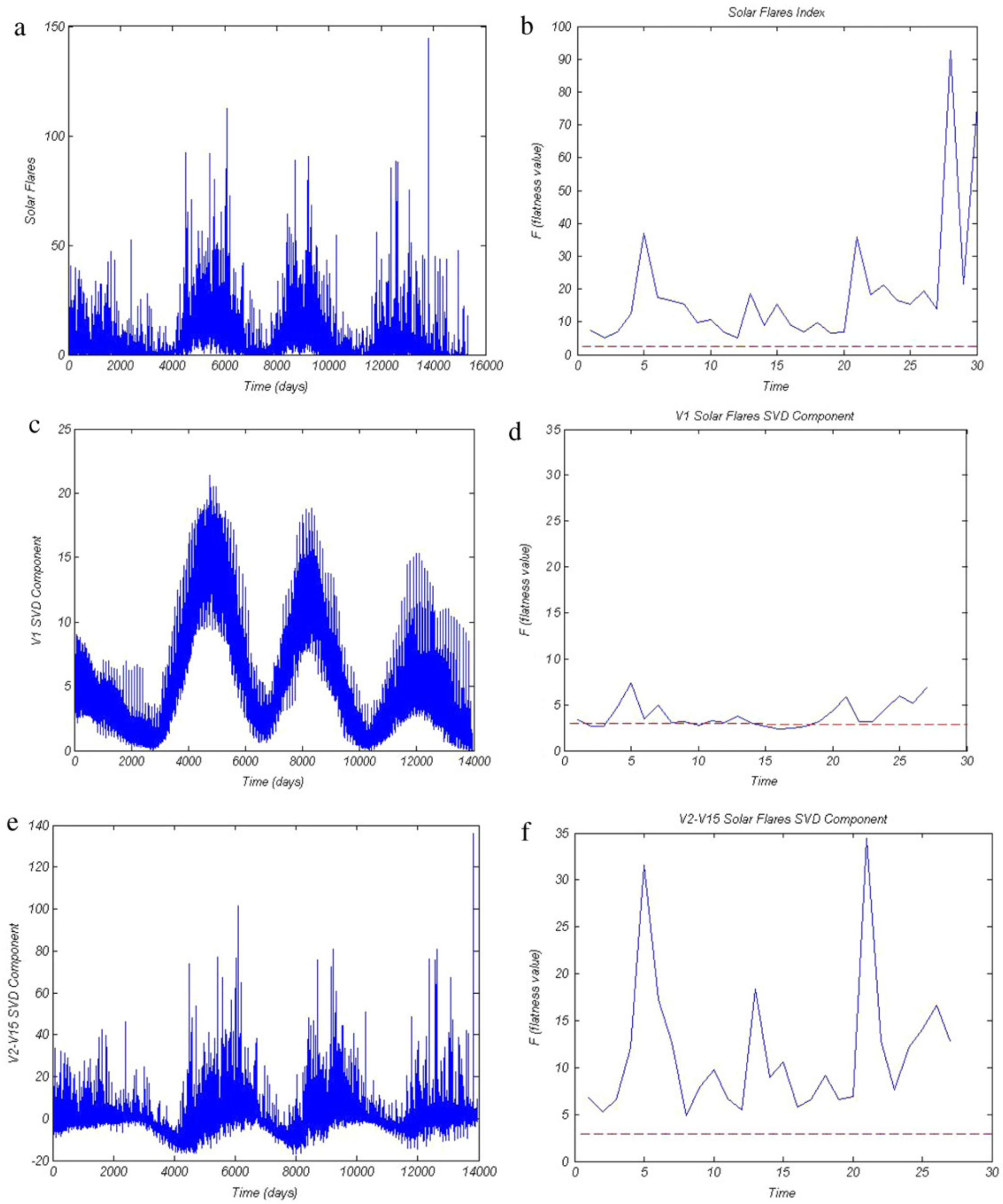
Sunspot Index (Parameters, Generalized Hurst Exponent & Entropy Production)



Solar Flares

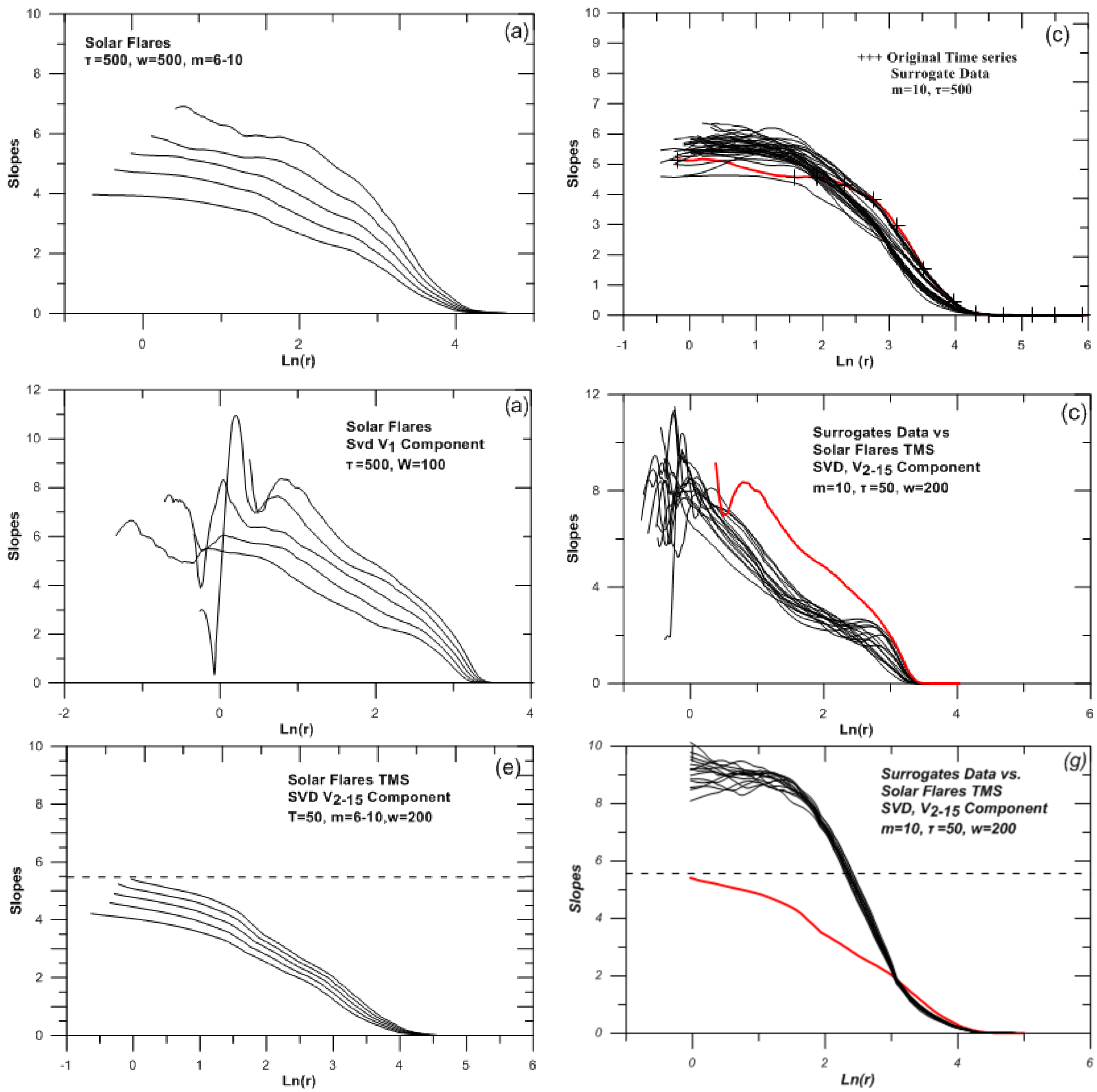


Solar Flares Index (Original – SVD Components)

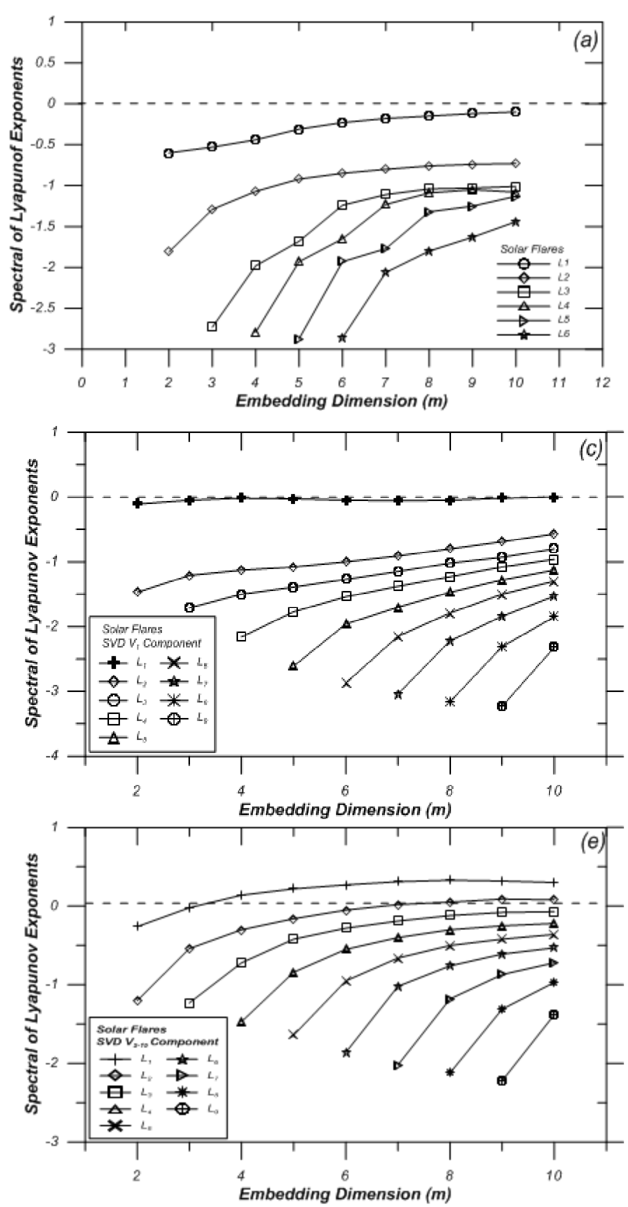


Solar Flares Index (Correlation Dimension & Lyapunov Exponent)

Correlation Dimension

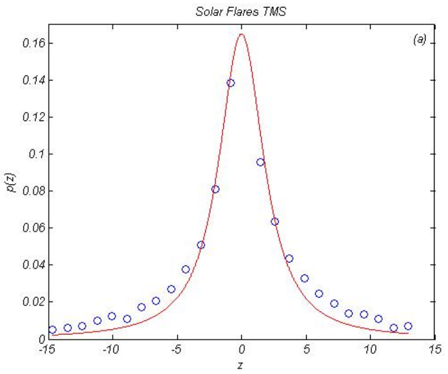


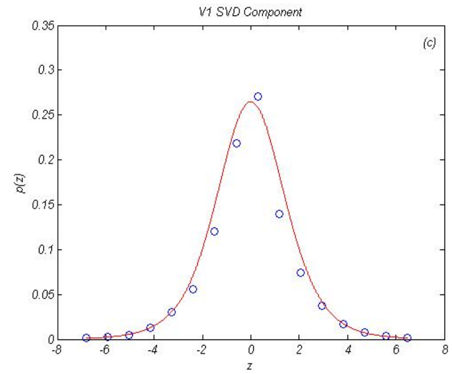
Lyapunov Spectrum Exponents

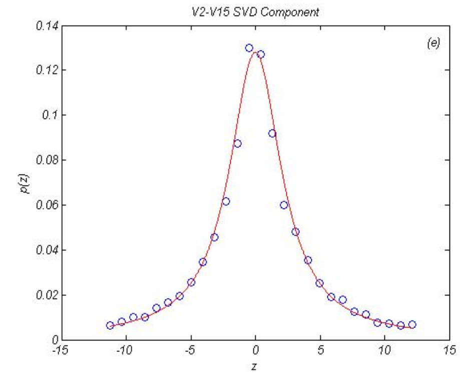


Solar Flares Index (Tsallis Statistics)

Solar Flare TMS **- q_{stat}=1.87 ±0.05** | **V1 SVD Component - q_{stat}=1.28 ±0.04** | **V2-15 SVD Component - q_{stat}=2.02 ±0.15**



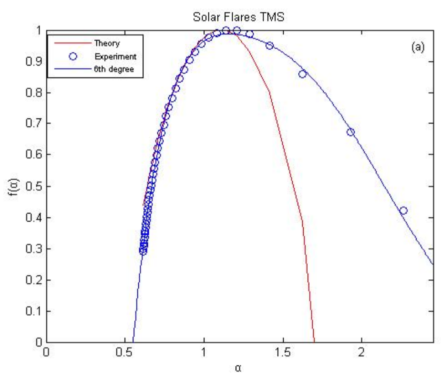


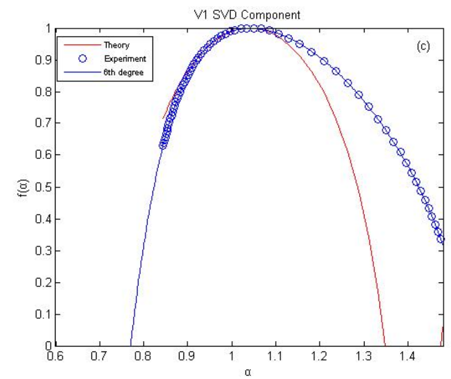


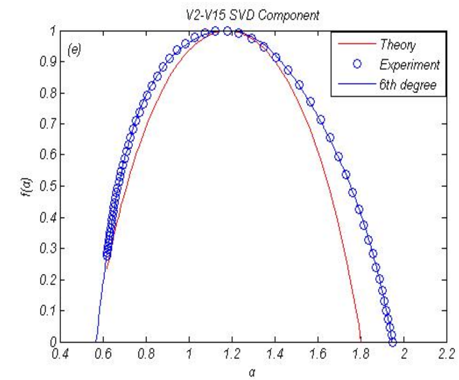
Solar Flare TMS **- q_{sen}=0.308**
Δα= 2.15

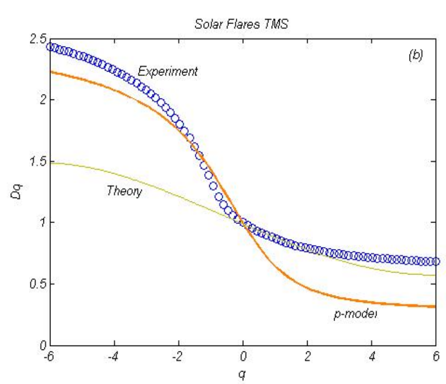
V1 SVD Component - q_{sen}=-0.540
Δα= 0.77

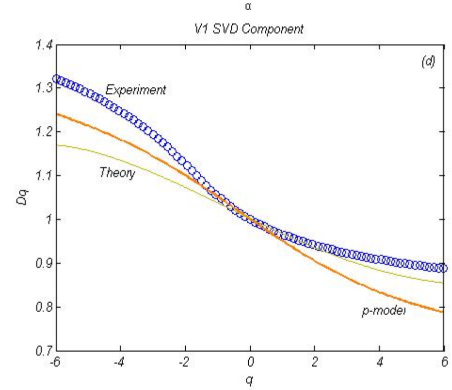
V2-15 SVD Component – q_{sen}=0.192
Δα= 1.37

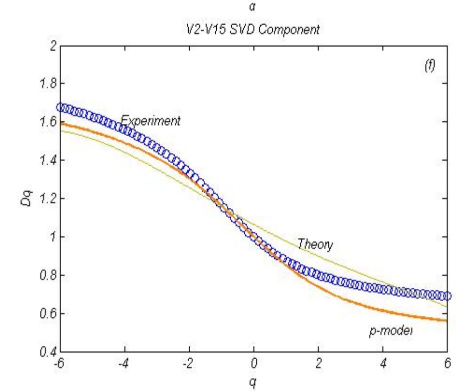




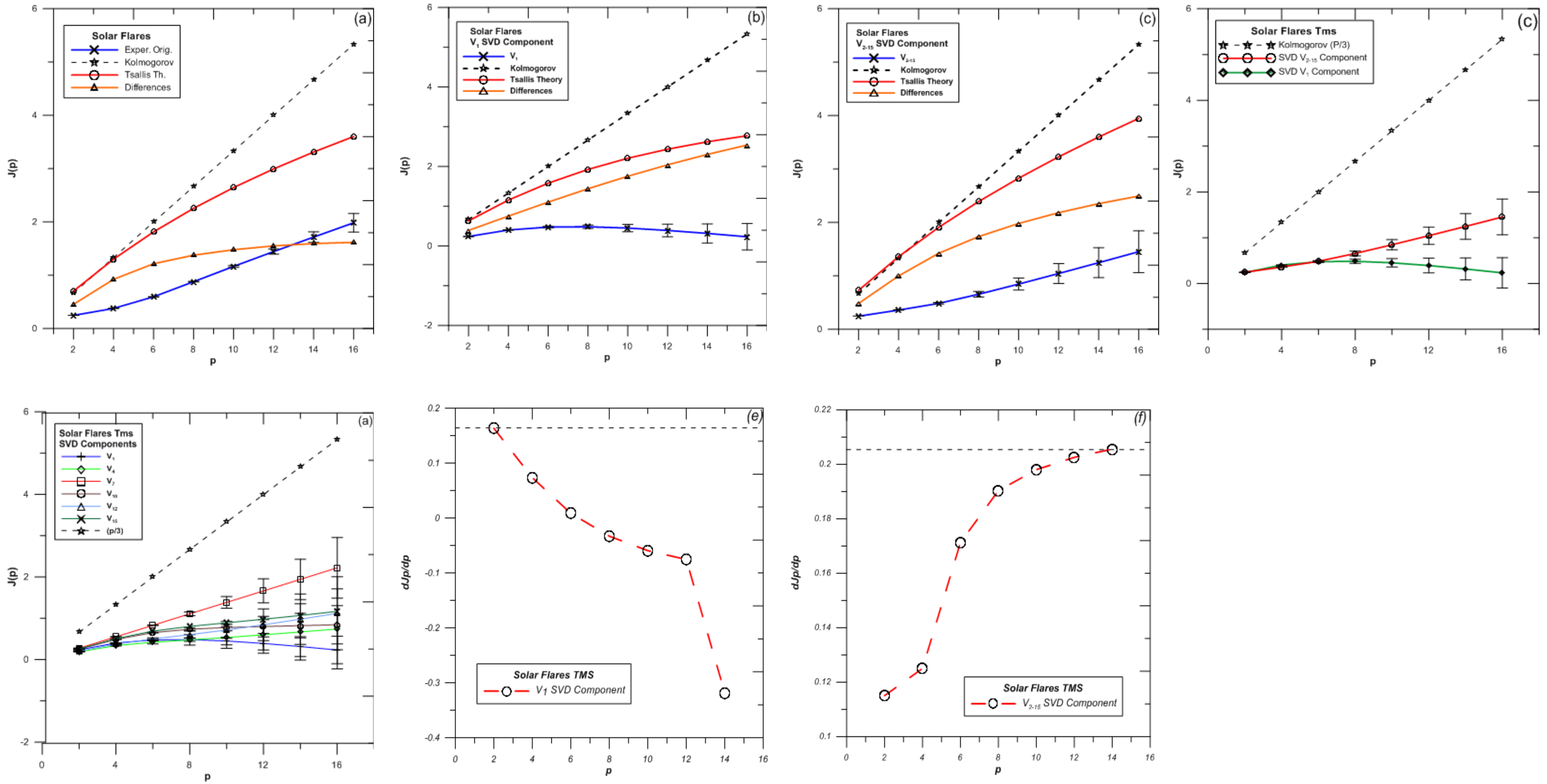




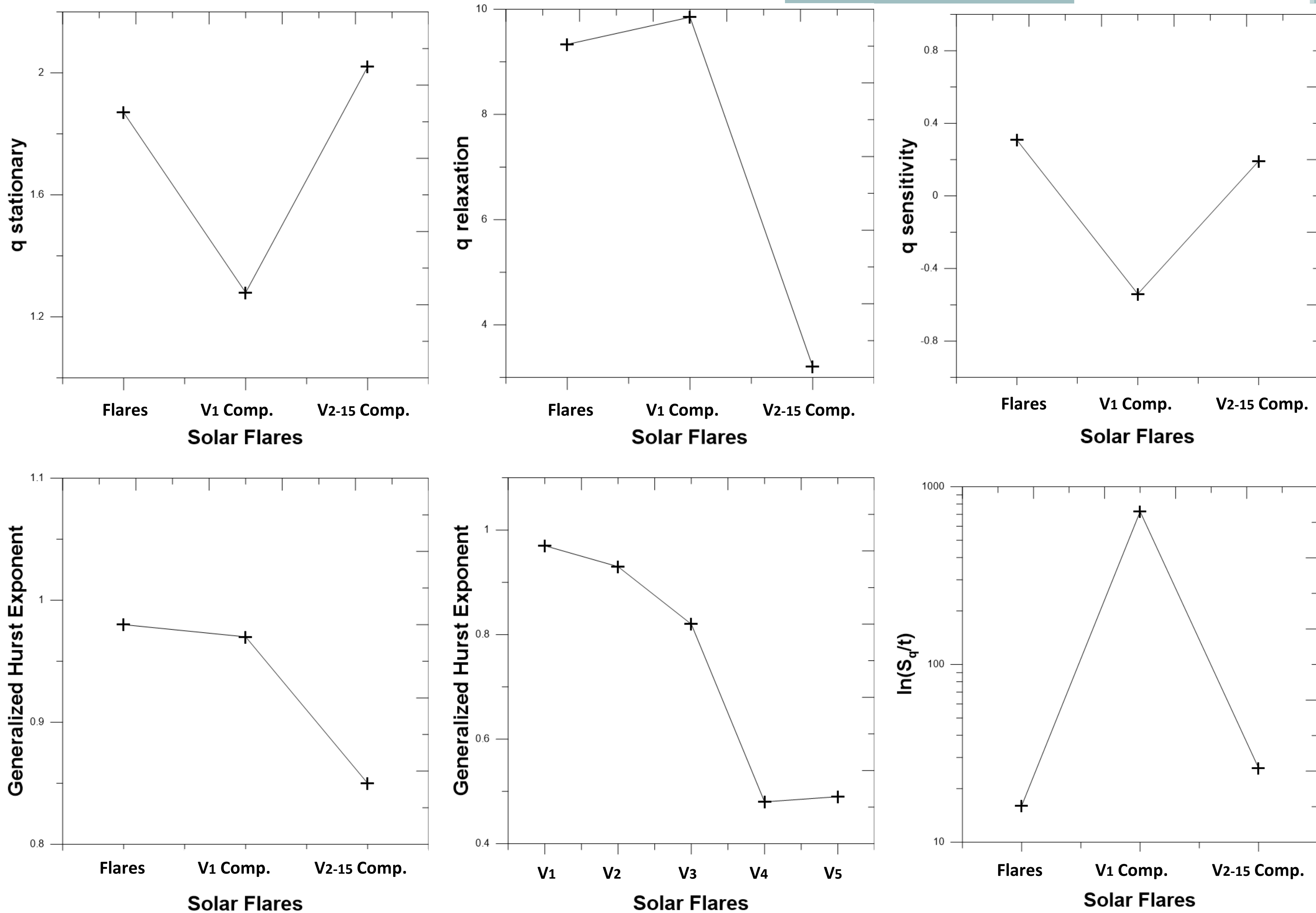




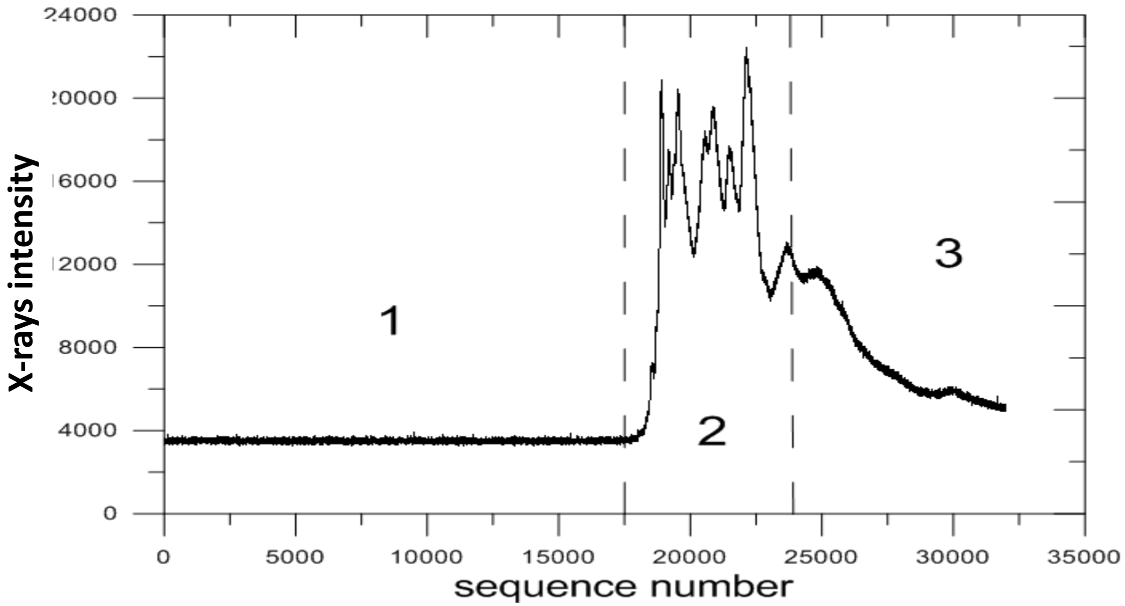
Solar Flares Index (Turbulence Analysis)



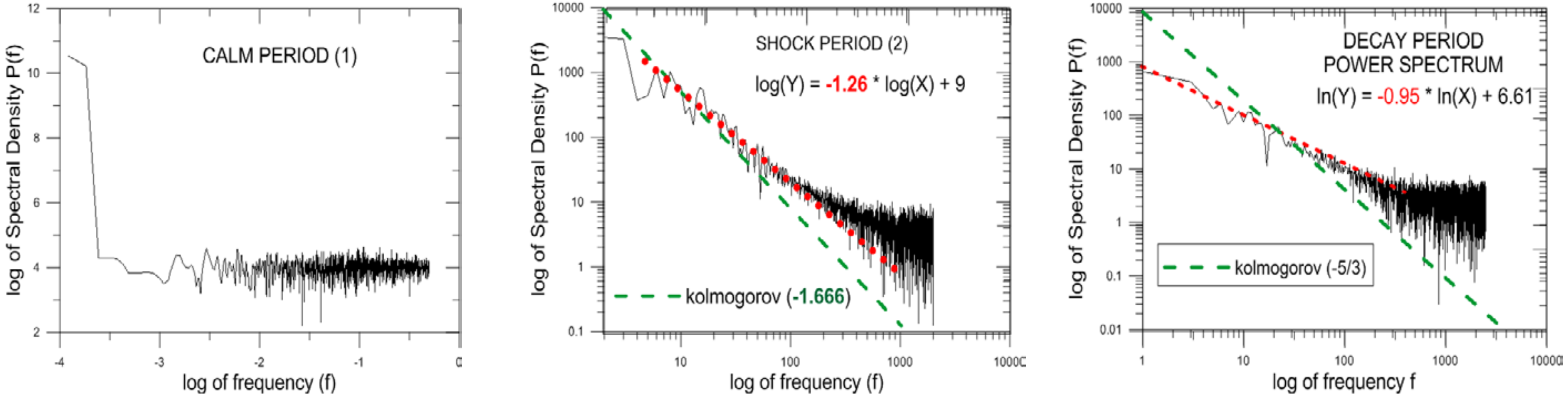
Solar Flares Index (Parameters, Generalized Hurst Exponent & Entropy Production)



Solar Flares Event 1

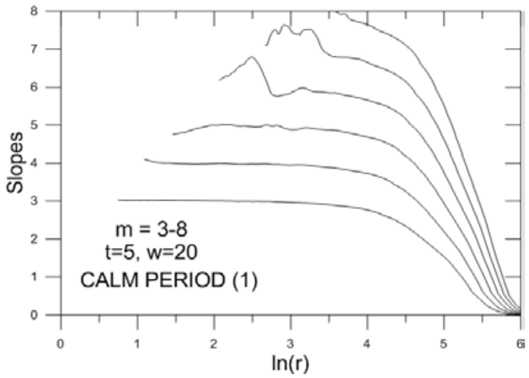


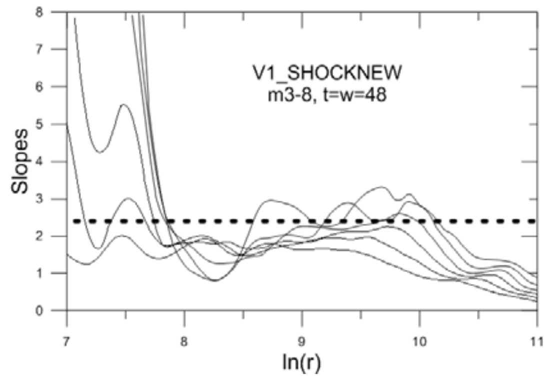
Power Spectrum

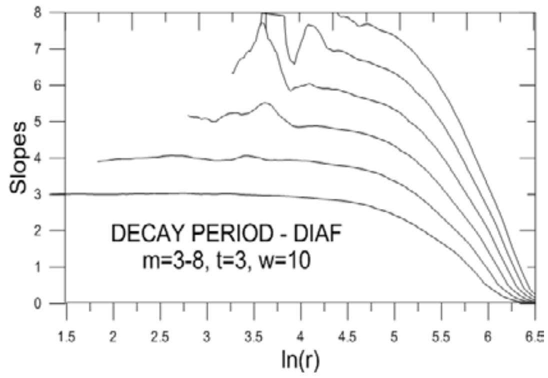


Solar Flares Event 1 (Chaotic Analysis)

Correlation Dimension

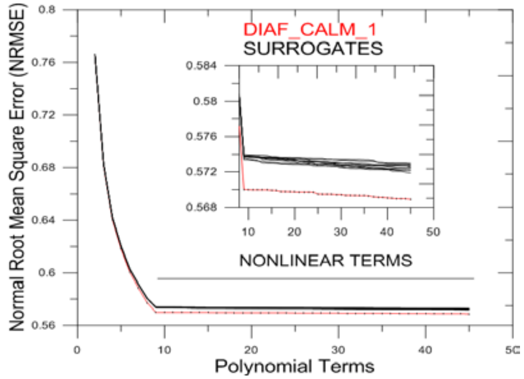


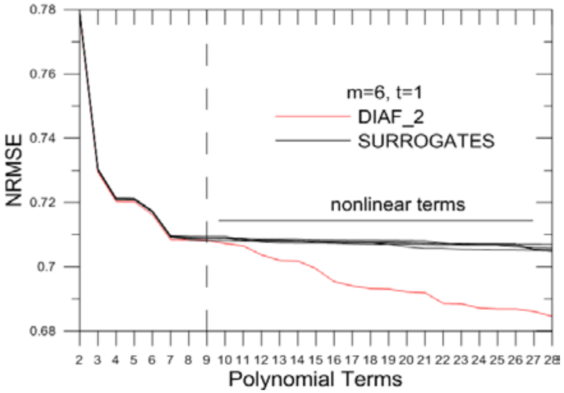


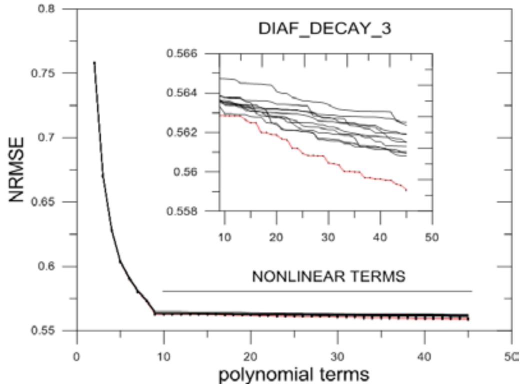


Lyapunov Spectrum Exponents			
	CALM (m=10)	SHOCK (m=7)	DECAY(m=10)
L1 maximum	-0.3	0.02 > 0	-0.01
L2	-0.32	-0.04	-1.19
L3	-0.34	-0.04	-2.42
L4	-0.37	-0.05	-2.48
L5	-0.4	-0.35	-2.57

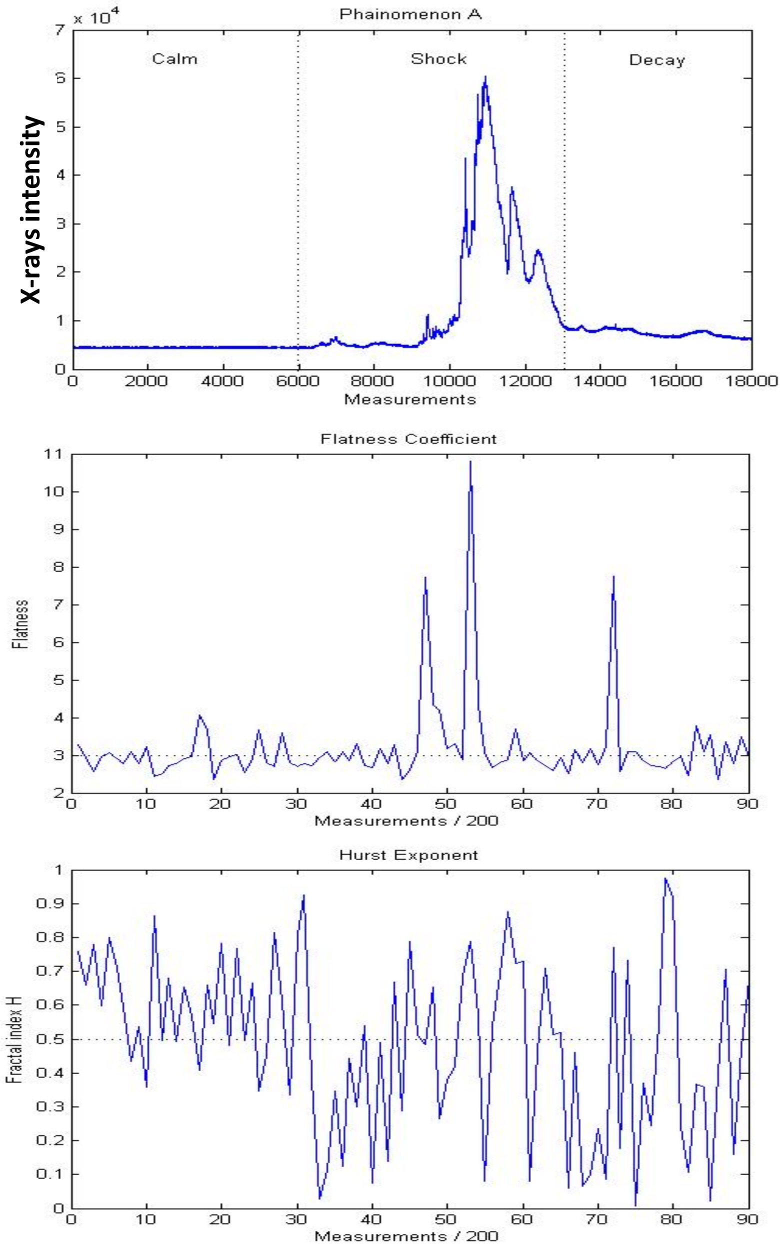
Voltera – Weiner Model





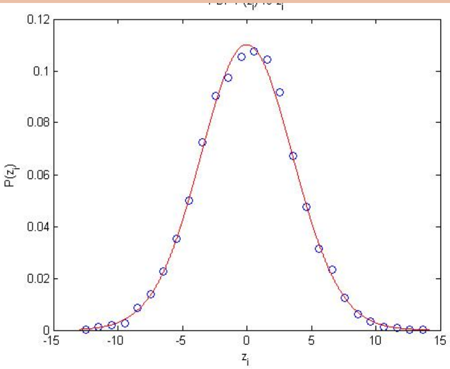


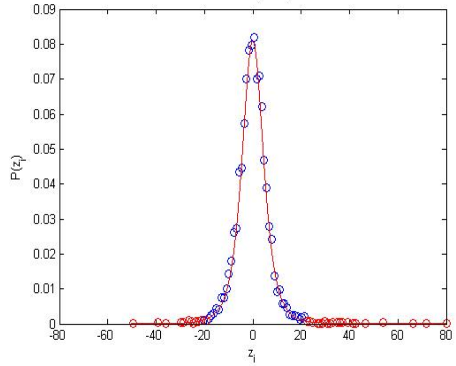
Solar Flares Event 2 (Statistical Analysis)

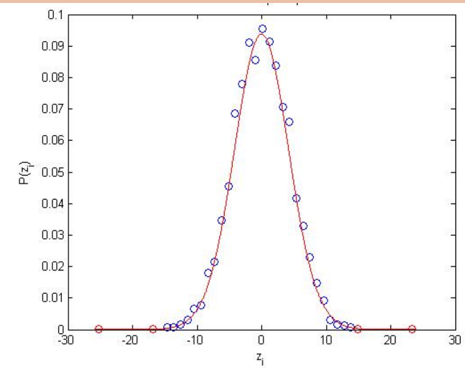


Solar Flares Event 2 (Tsallis Statistics)

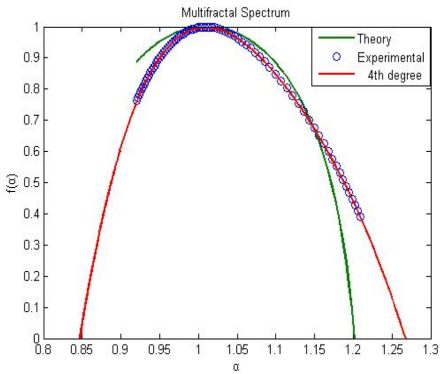
Calm- q_{stat}=1.04±0.02	Shock- q_{stat}=1.43±0.05	Decay- q_{stat}=1.08±0.02

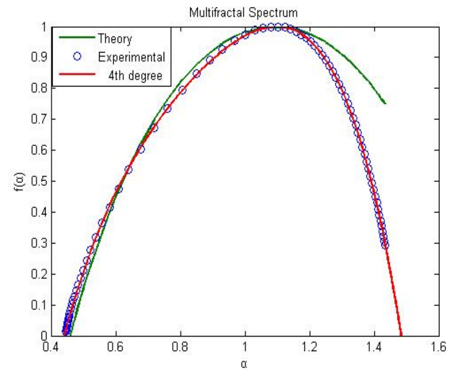


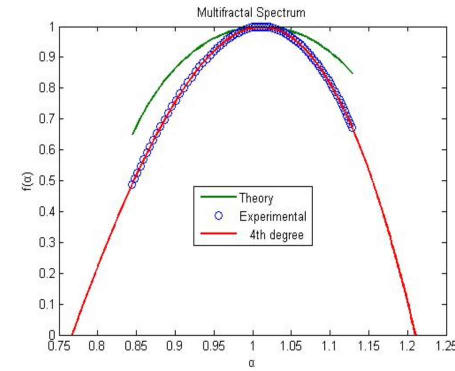




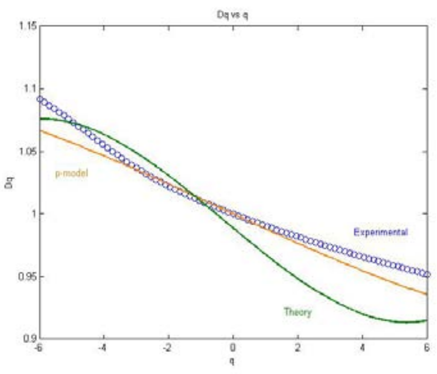
Calm - q_{sen}=-1.547 $\Delta(\alpha)$= 0.421	Shock - q_{sen}=0.376 $\Delta(\alpha)$= 1.045	Decay - q_{sen}=-1.089 $\Delta(\alpha)$= 0.441

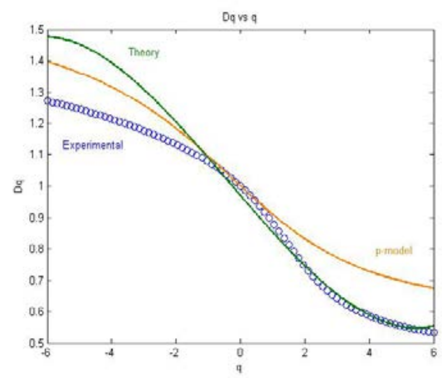


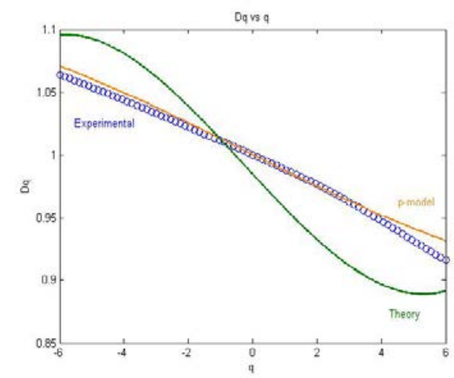




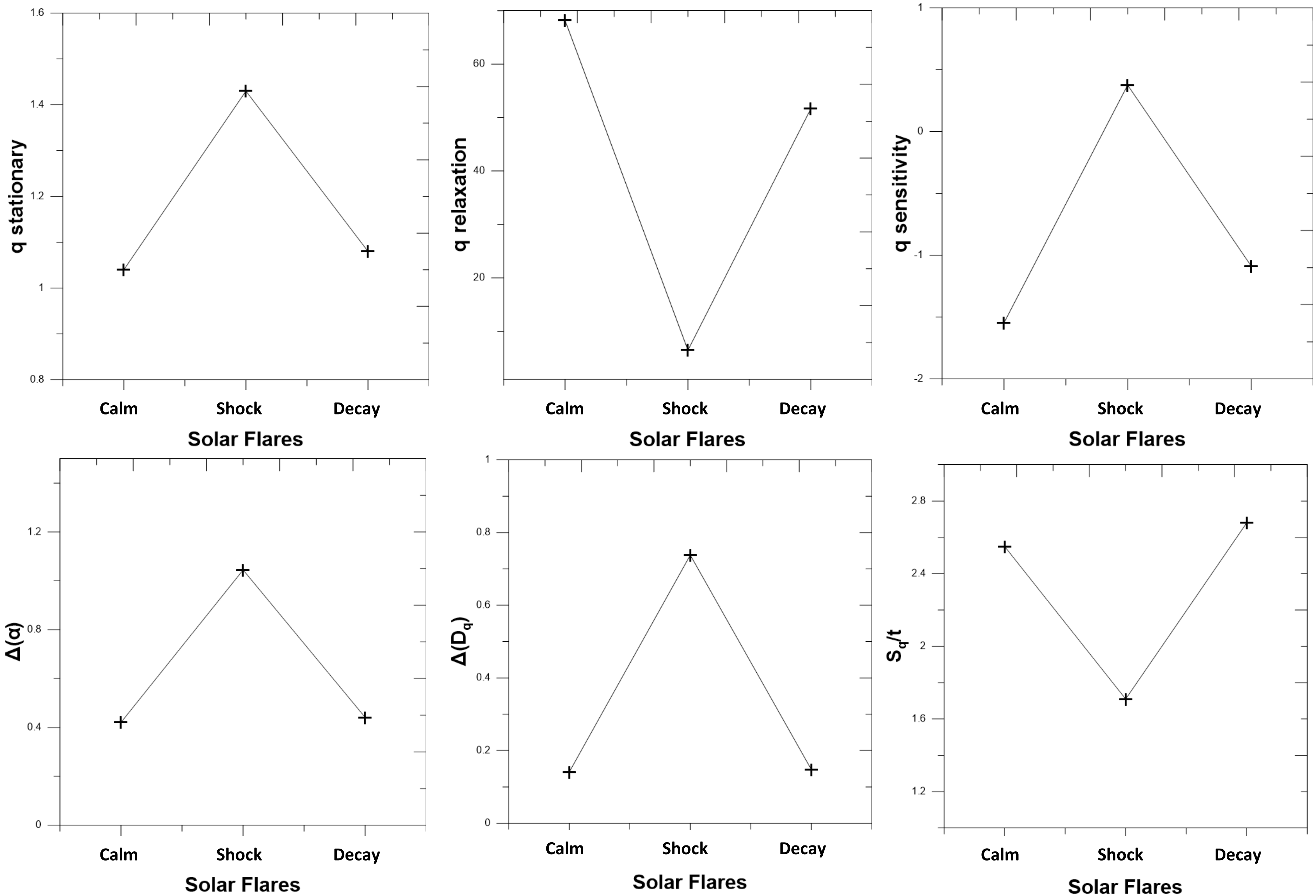
Calm – $\Delta(Dq)$=0.141	Shock – $\Delta(Dq)$ =0.738	Decay – $\Delta(Dq)$ =0.148



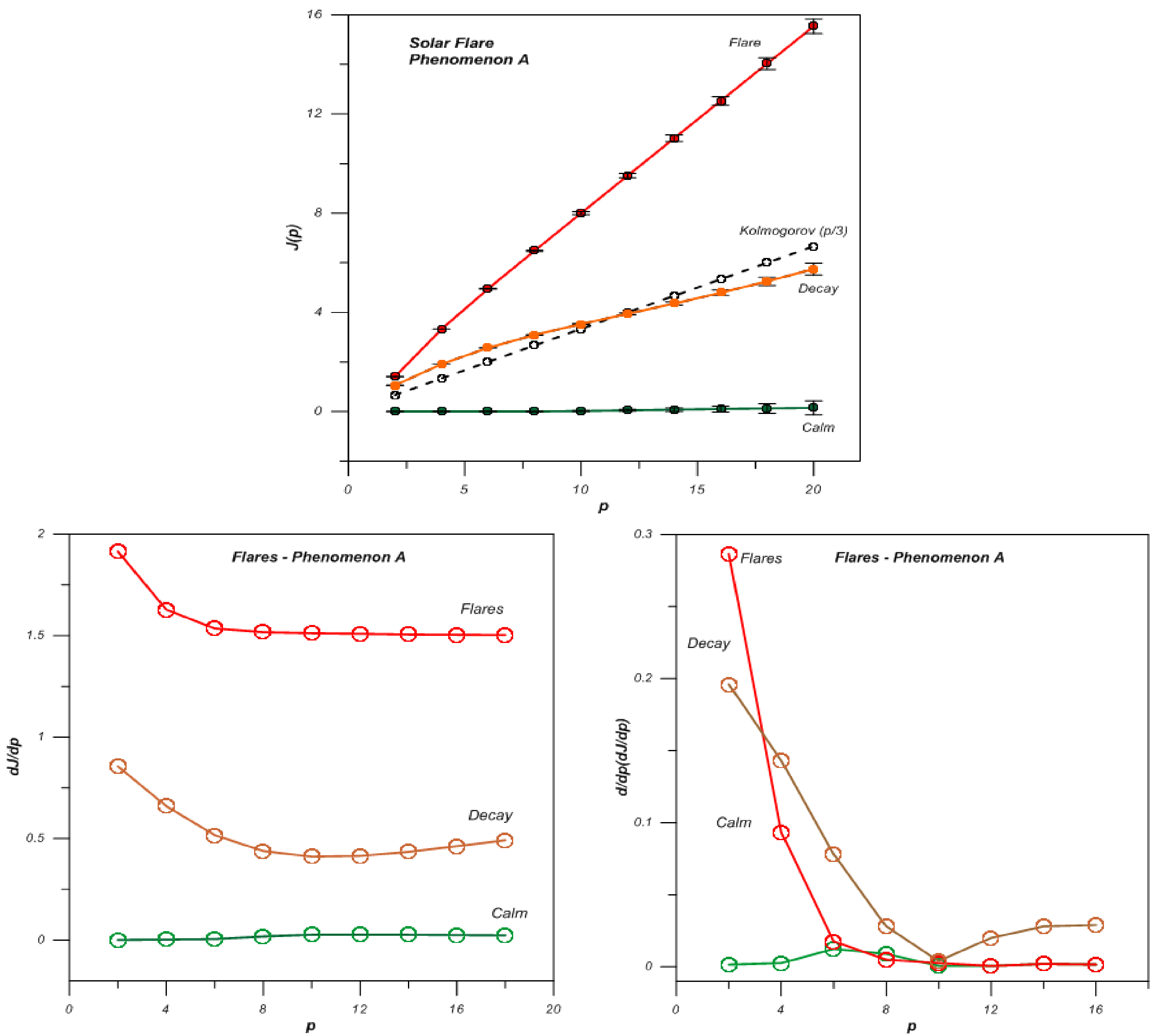




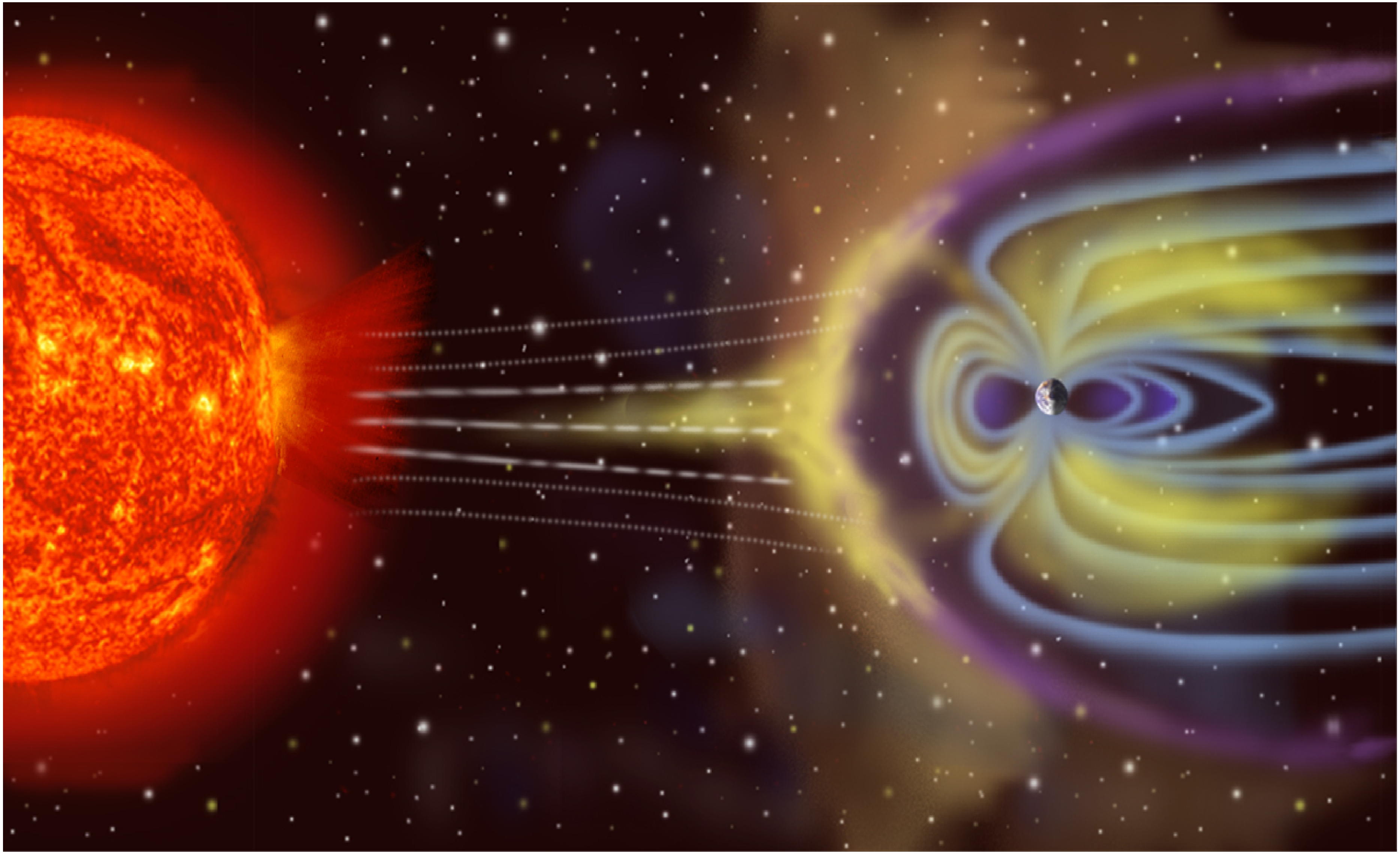
Solar Flares Event 2 (Parameters)



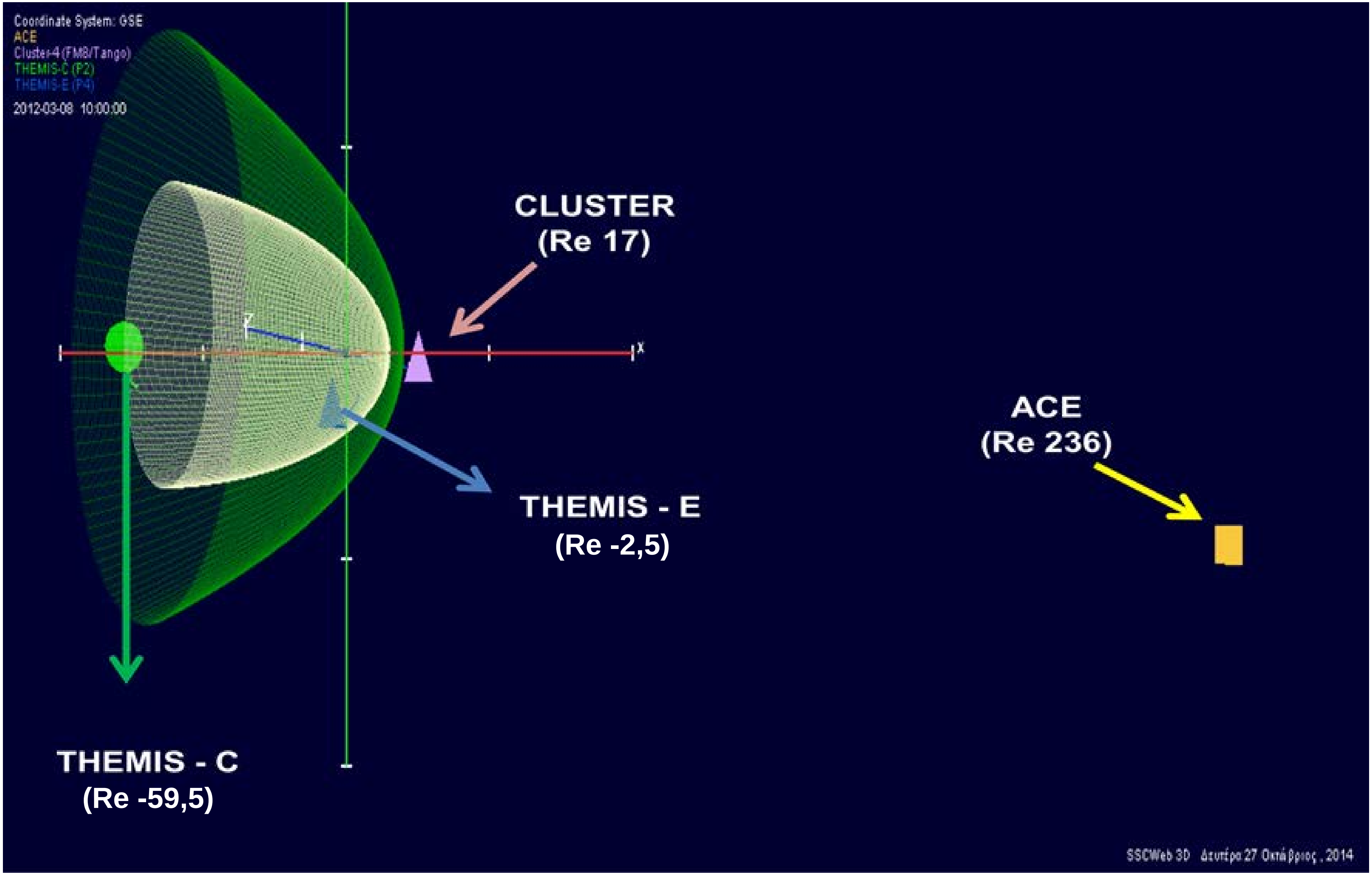
Solar Flares Event 2 (Turbulence Analysis)



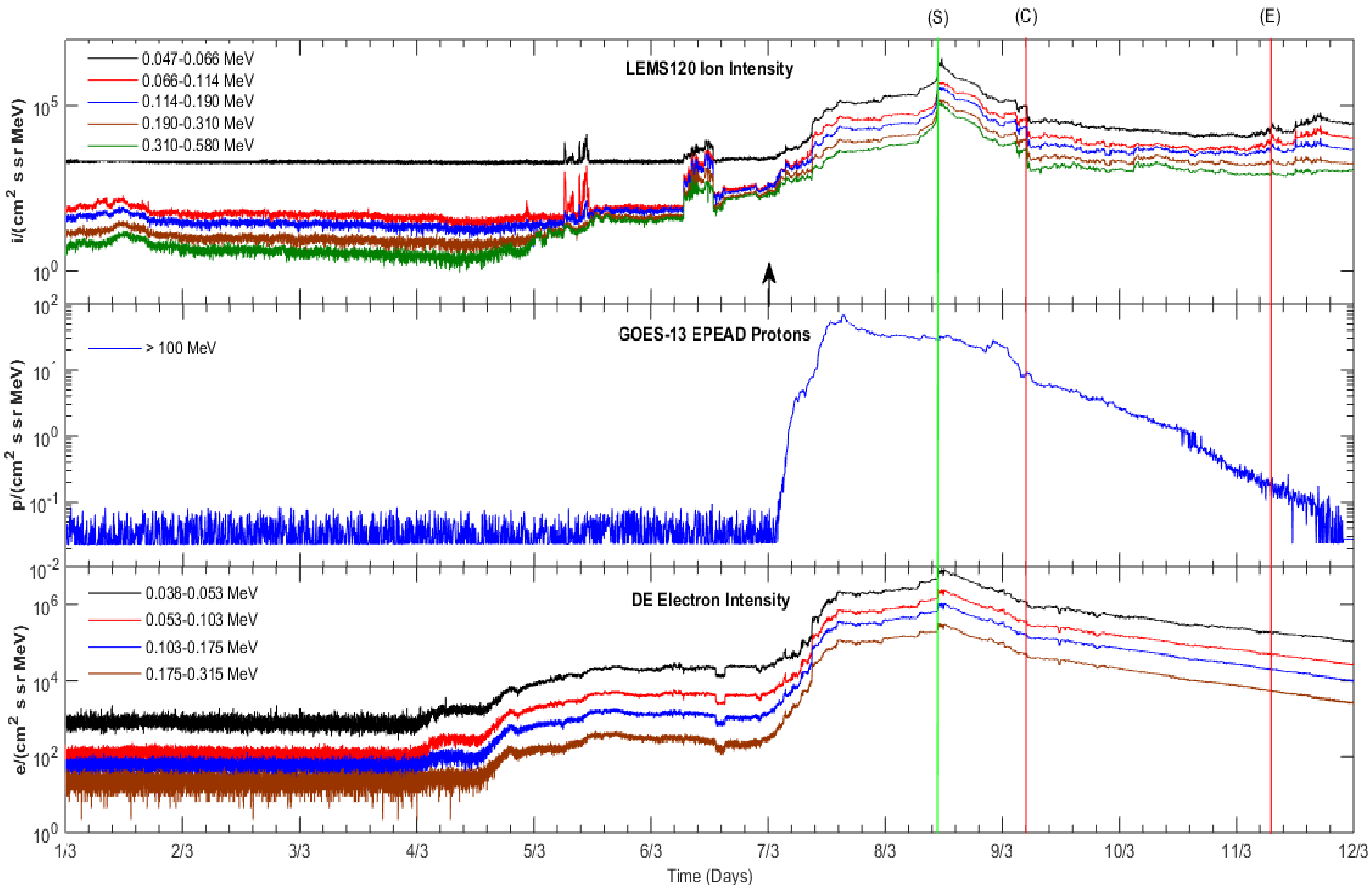
Solar Energetic Particle Interplanetary Space



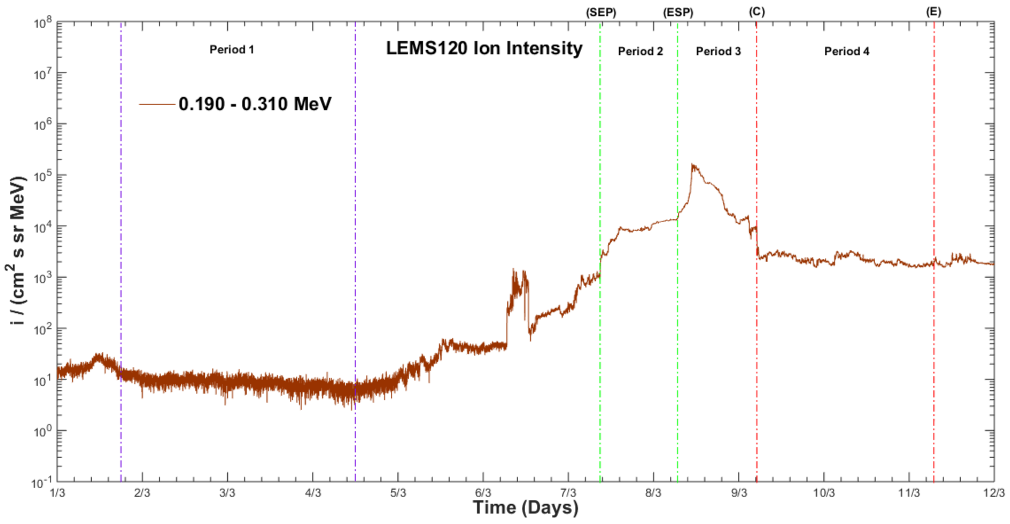
Interplanetary Observations (CME 7 March 2012)

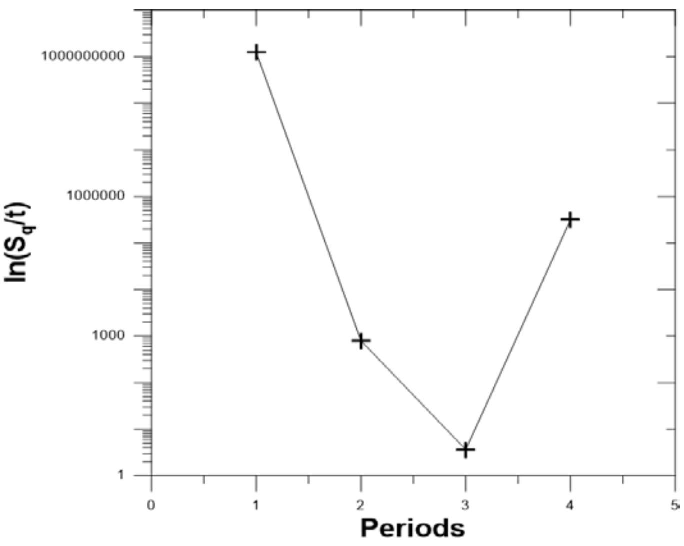


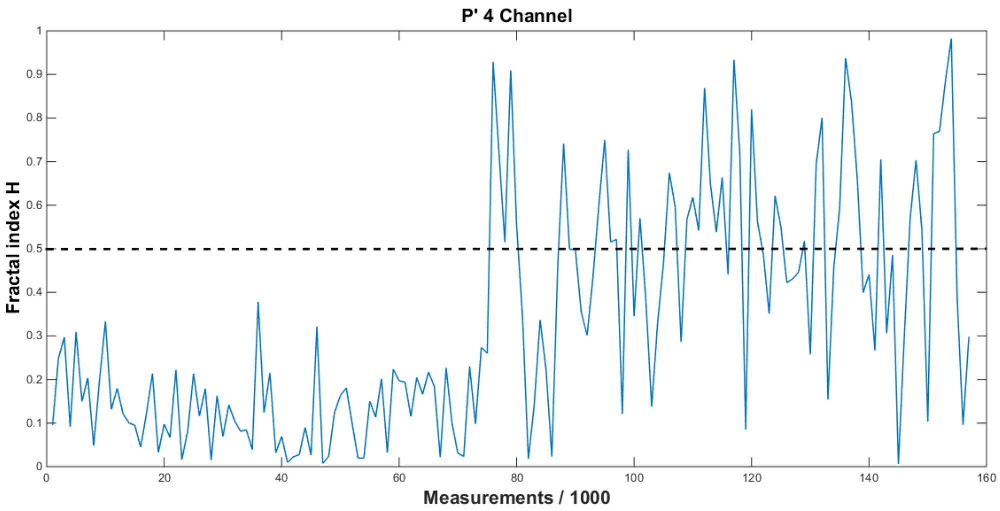
Solar Energetic Particles

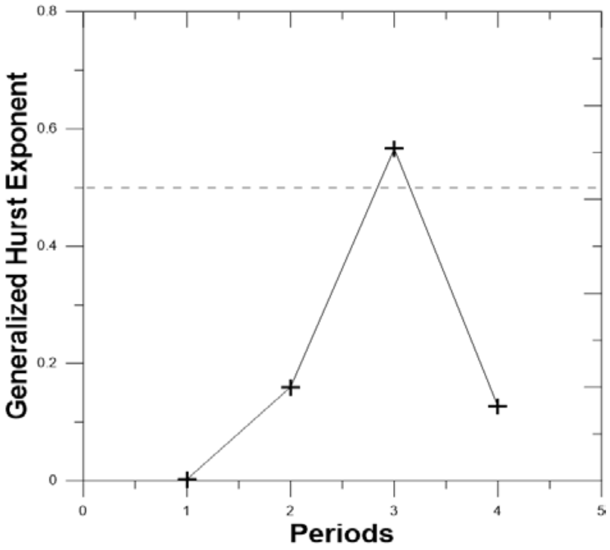


Solar Energetic Particle (Generalized Hurst Exponent & Entropy Production)

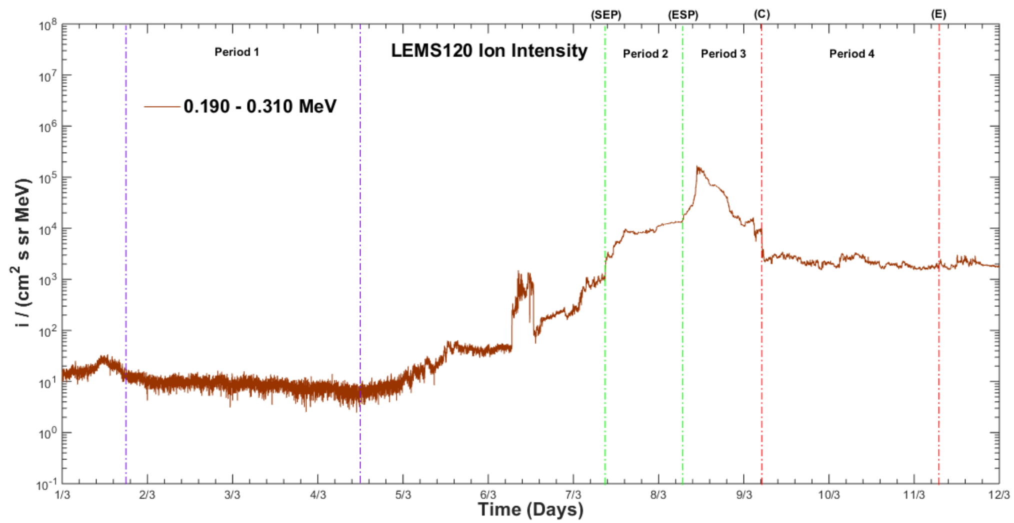




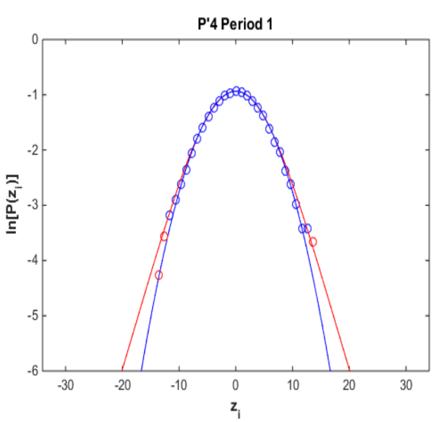


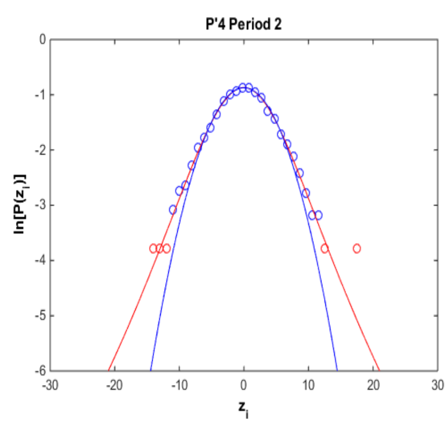


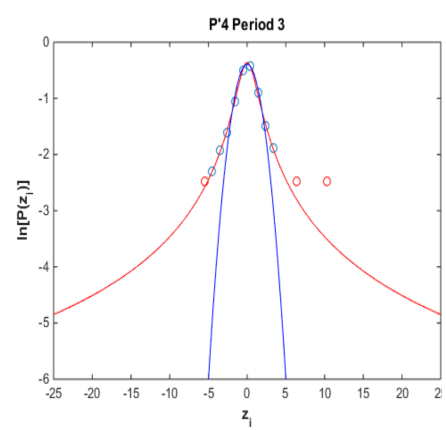
Solar Energetic Particle (Tsallis Statistics)

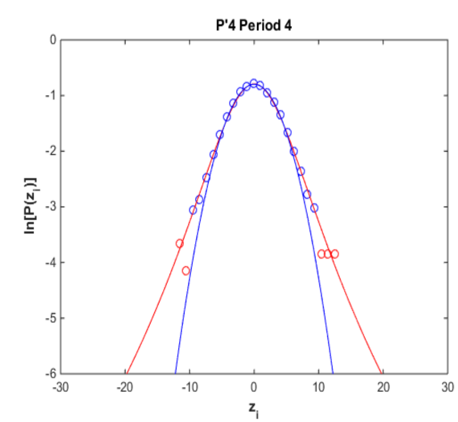


Period 1 **- q_{stat}=1.08 ±0.03** **Period 2- q_{stat}=1.18 ±0.03** **Period 3 - q_{stat}=1.57 ±0.08** **Period 4 - q_{stat}=1.11 ±0.05**



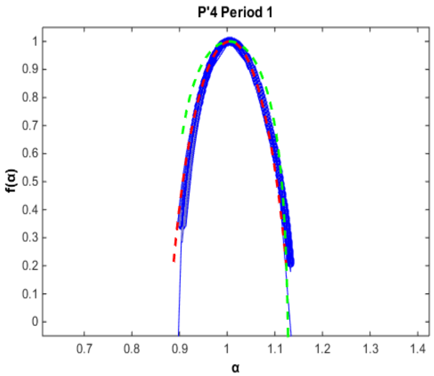


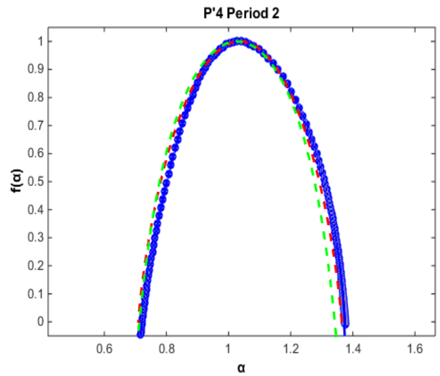


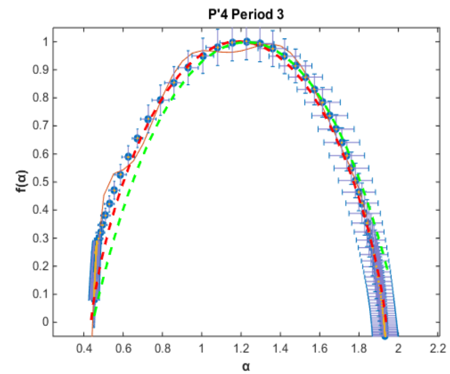


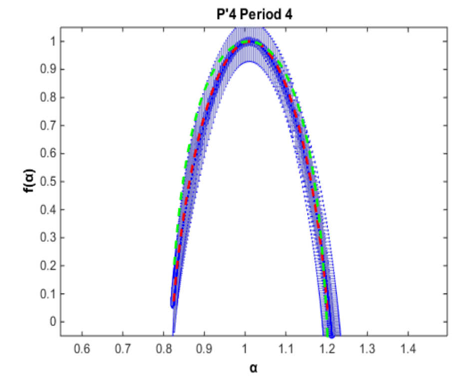
Solar Energetic Particle (Multifractal Spectrum)

Period 1 **- q_{sen}=-3.049 Δα=0.251**	**Period 2 - q_{sen}=-0.521 Δα=0.653**	**Period 3 - q_{sen}=0.427 Δα=1.483**	**Period 4 - q_{sen}=-1.563 Δα=0.388**

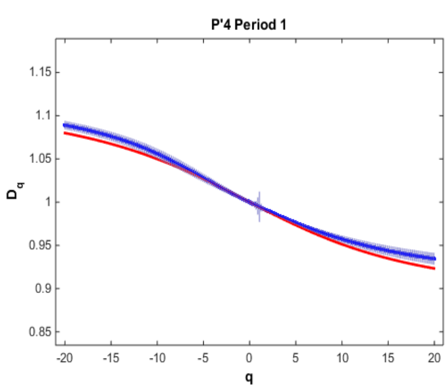


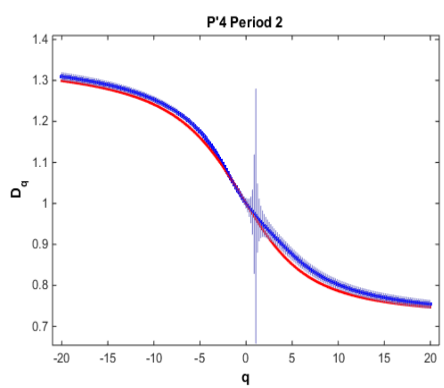


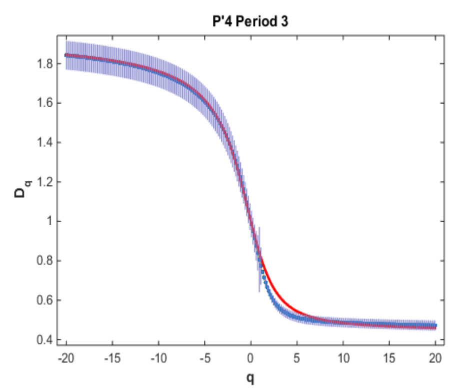


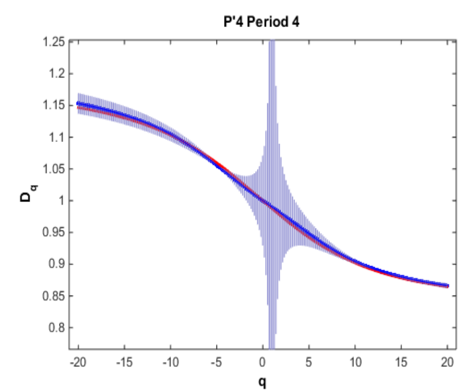


Period 1 **- ΔD_q=0.155 p-model=0.546**	**Period 2 - ΔD_q =0.556 p-model=0.612**	**Period 3 - ΔD_q =1.371 p-model=0.753**	**Period 4 - ΔD_q =0.287 p-model=0.566**



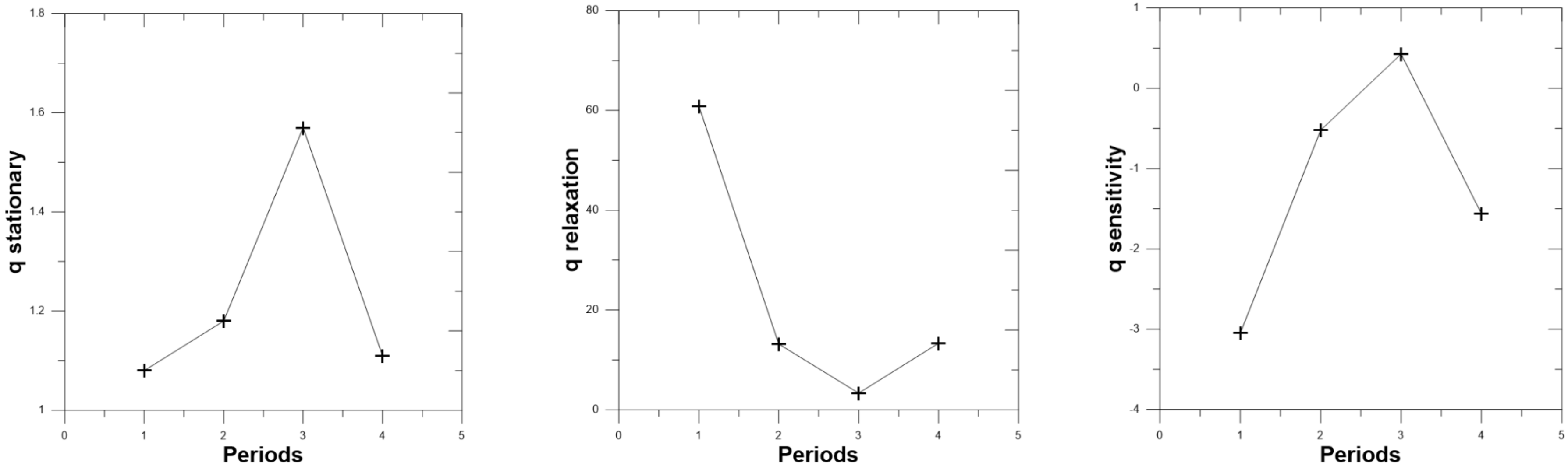




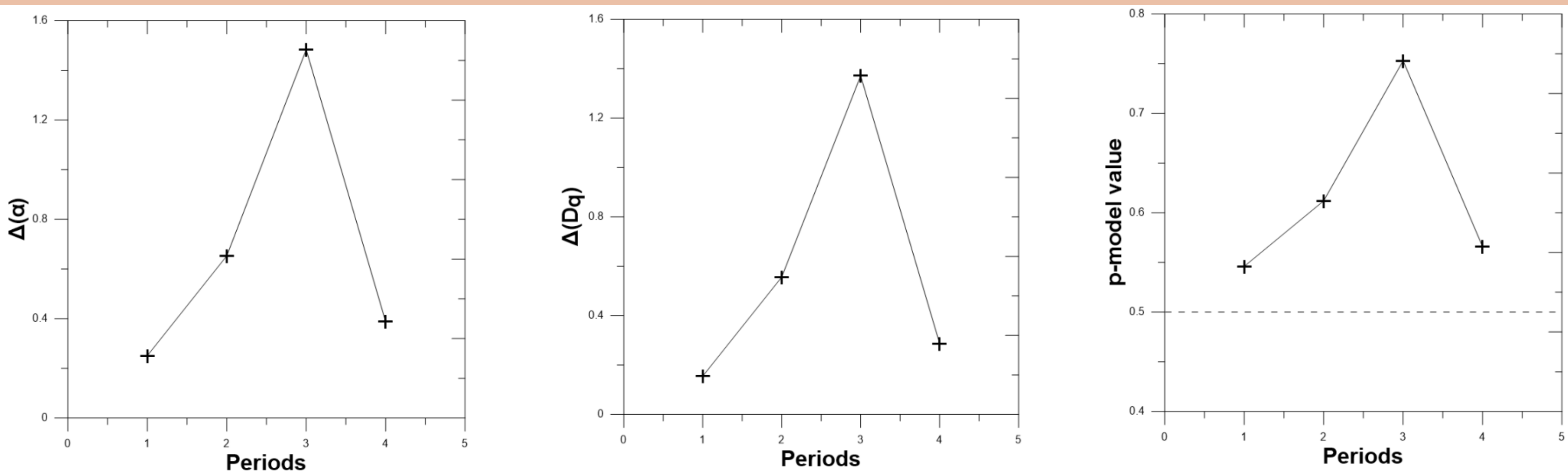


Solar Energetic Particle (Parameters)

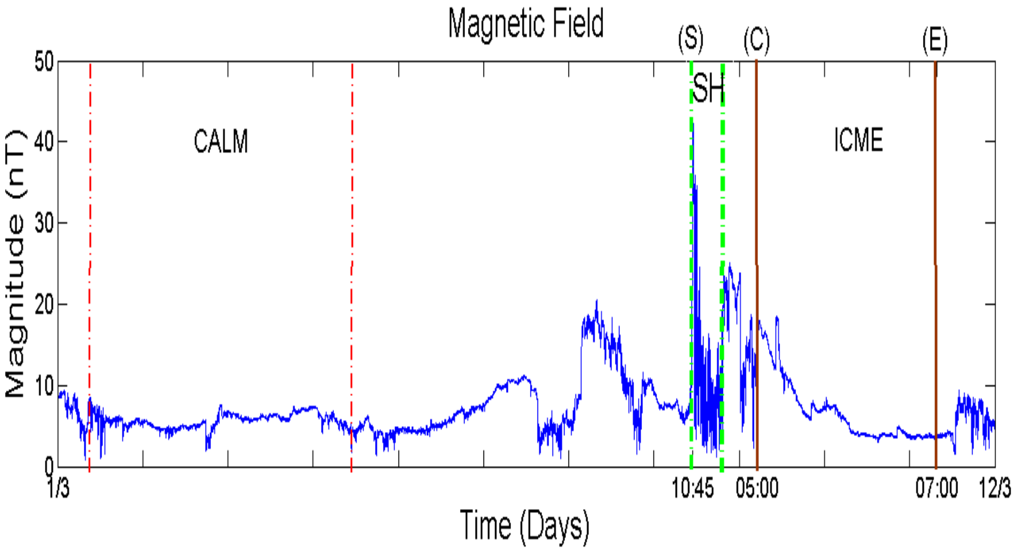
Phase Transition through Tsallis q-triplet

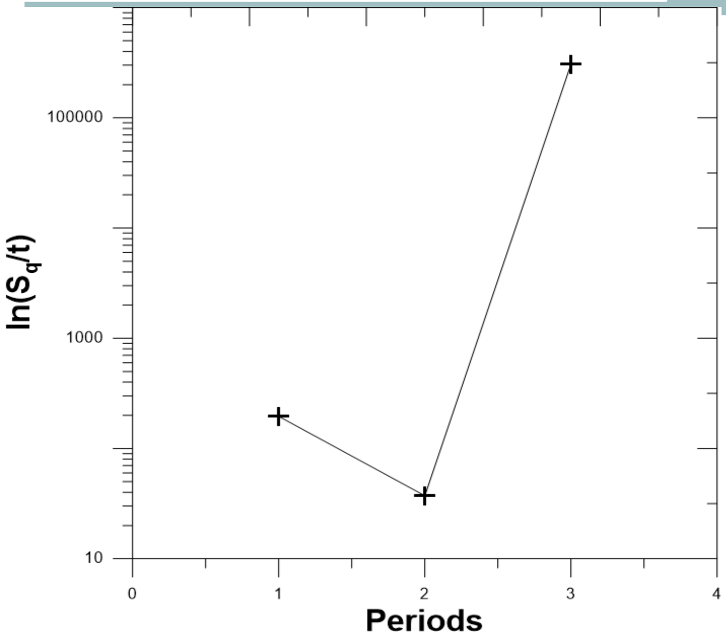


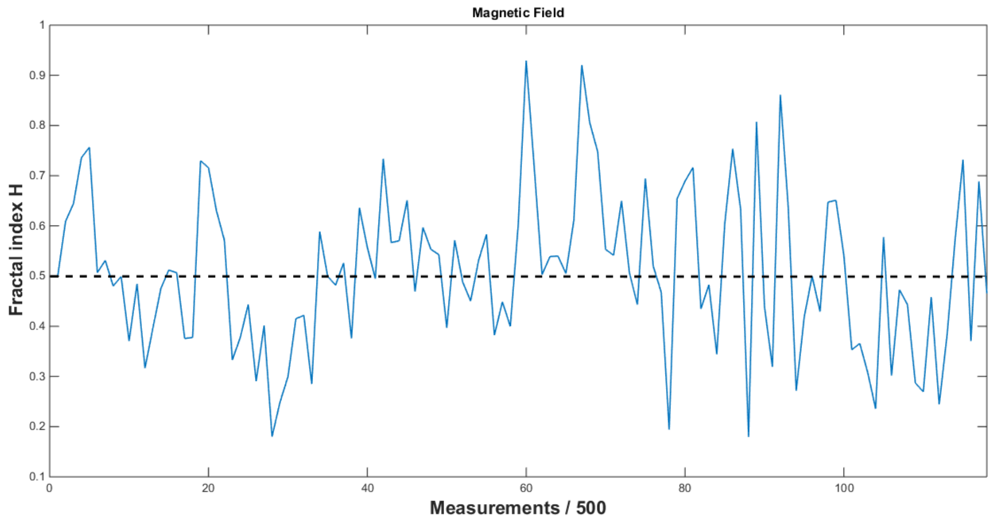
Phase Transition through Multifractal Structure and p-model values

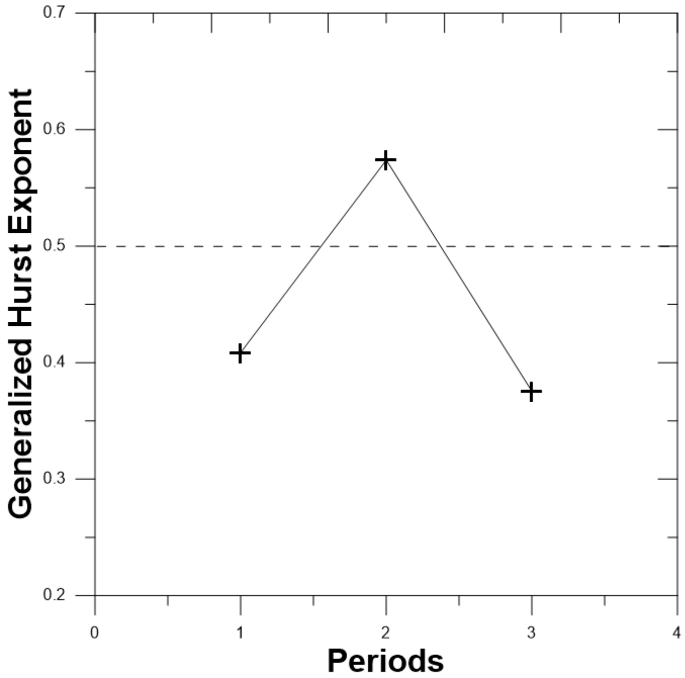


ACE/MAG Project - Solar Wind (Magnetic Field)

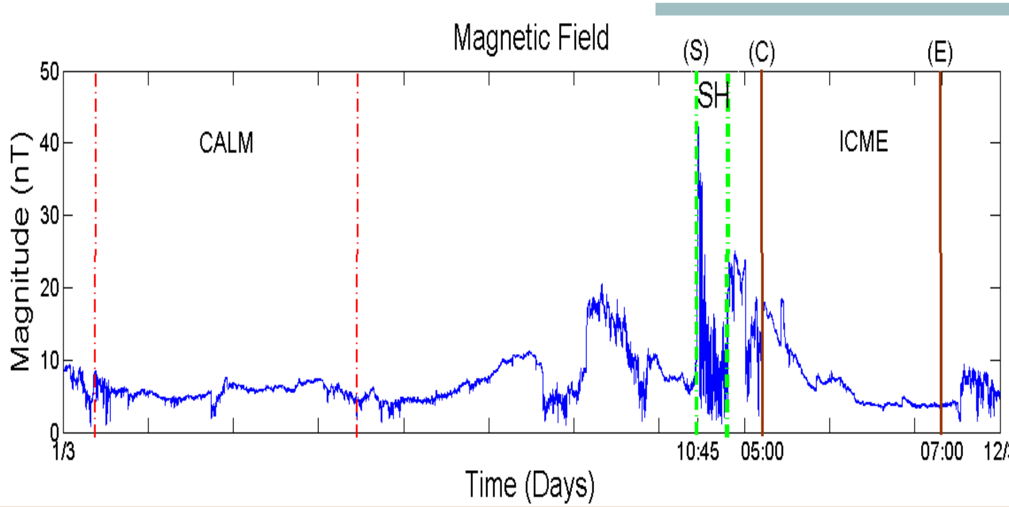




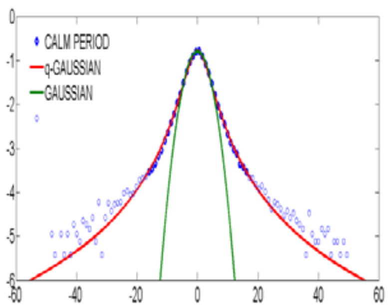


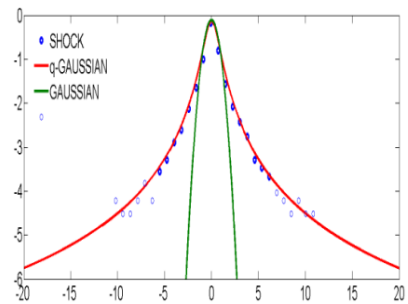


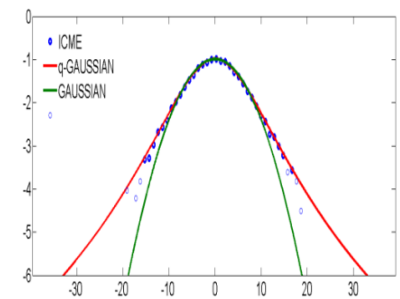
Solar Wind Magnetic Field (q-stationary & q-relaxation)



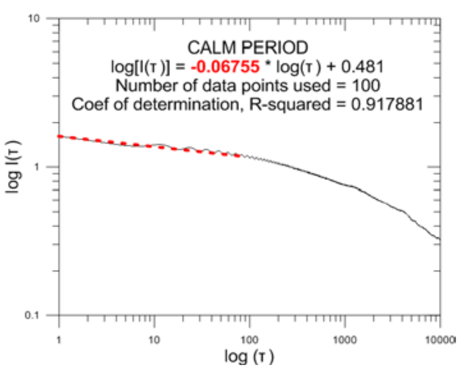
Calm - qstat= 1.38 ±0.04 | **Shock - qstat=1.74 ±0.05** | **ICME - qstat = 1.17 ±0.02**

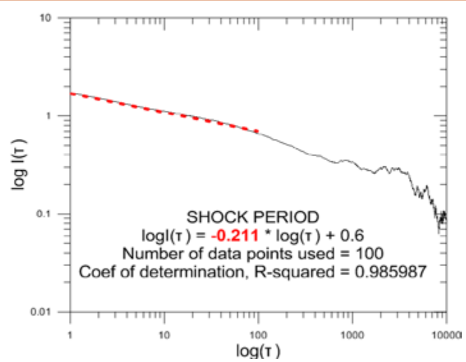


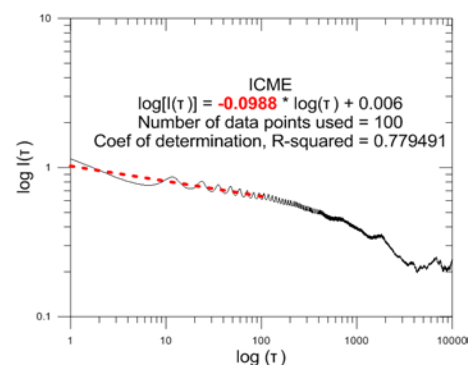




Calm **- qrel = 15.804** | **Shock - qrel=5.74** | **ICME - qrel = 11.12**





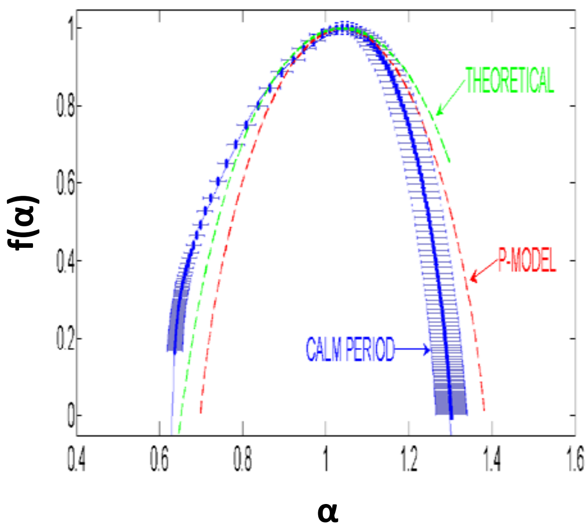


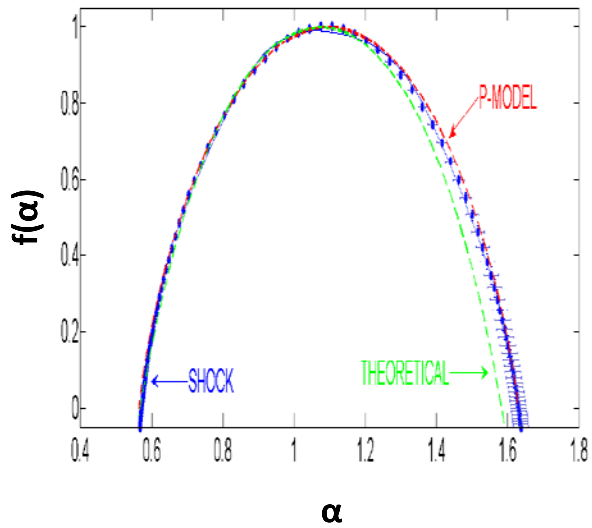
Solar Wind Magnetic Field (Multifractal Spectrum)

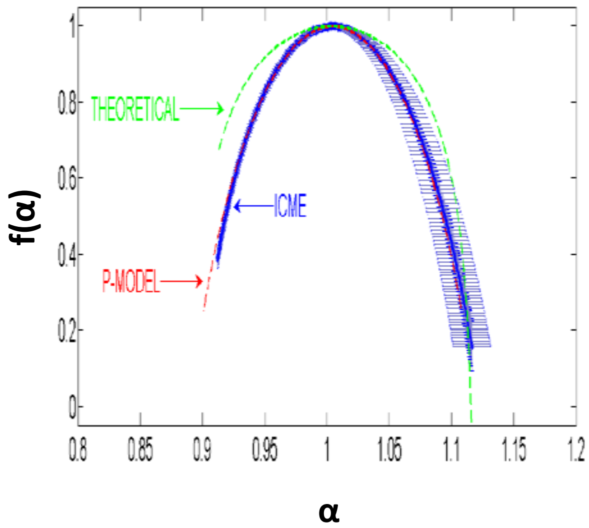
Calm **- qsen=-0.198**
Δα=o.679

Shock - qsen=0.117
Δα=1.057

ICME - qsen=-3.456
Δα=0.227



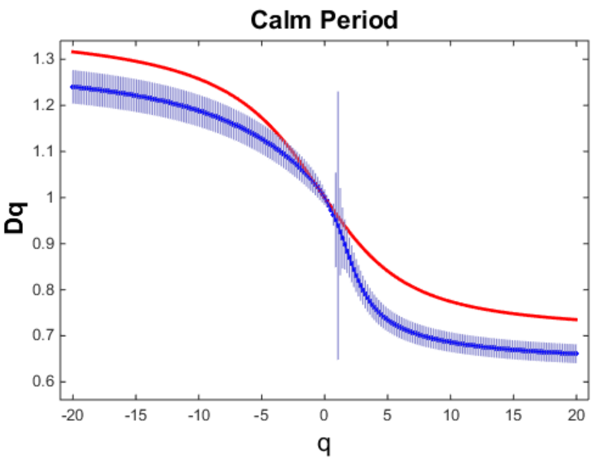


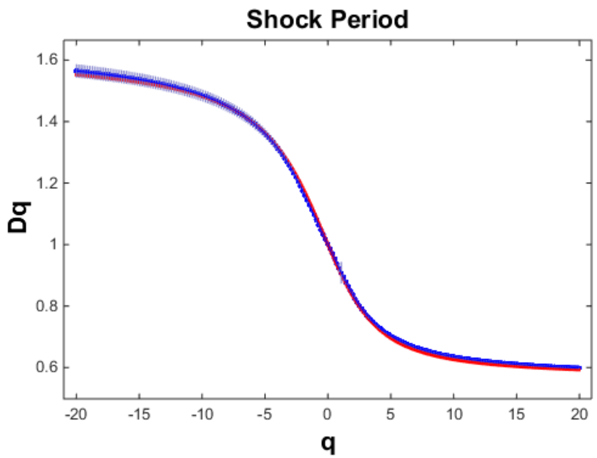


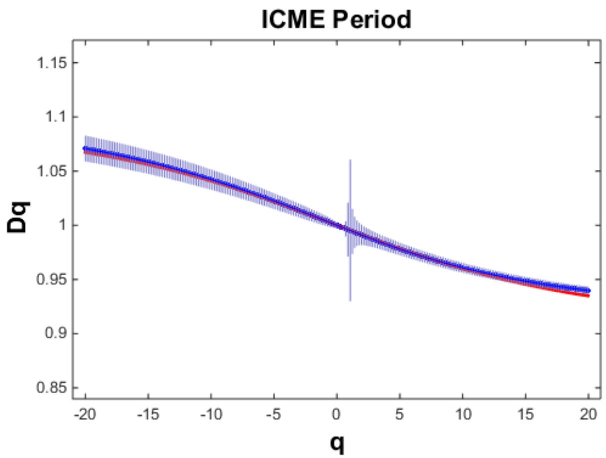
Calm **- ΔDq=0.579**
p-model=0.616

Shock - ΔDq =0.967
p-model=0.677

ICME - ΔDq =0.131
p-model=0.539

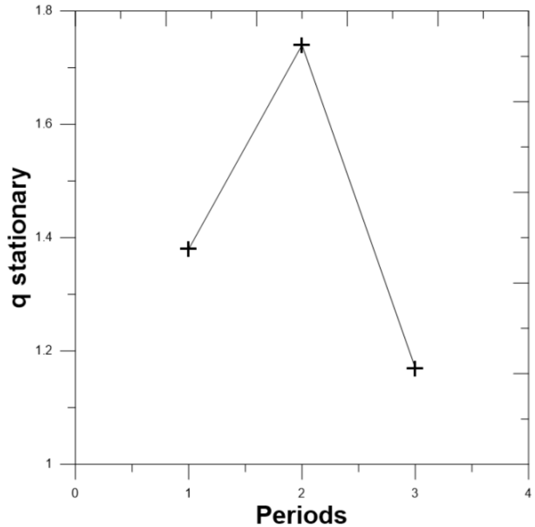


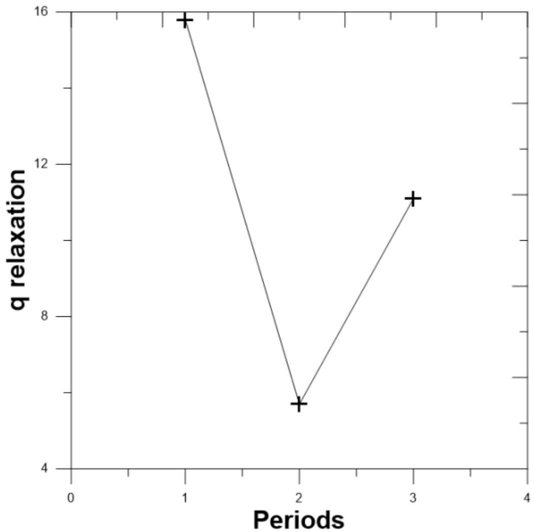


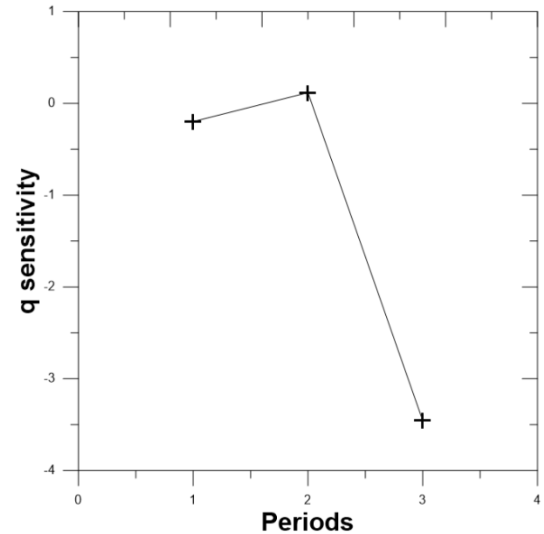


Sola Wind Magnetic Field (Parameters)

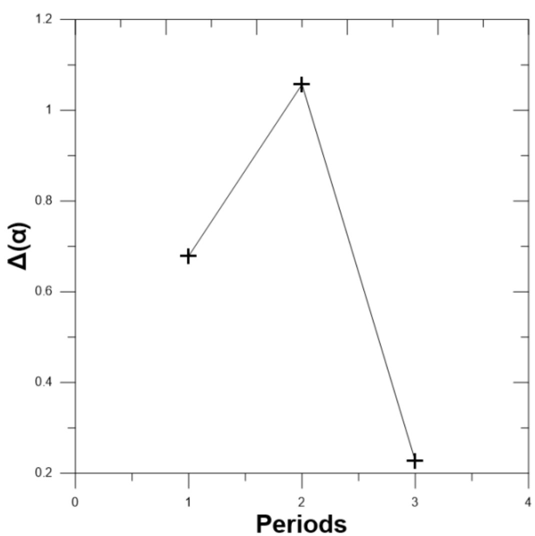
Phase Transition through Tsallis q-triplet

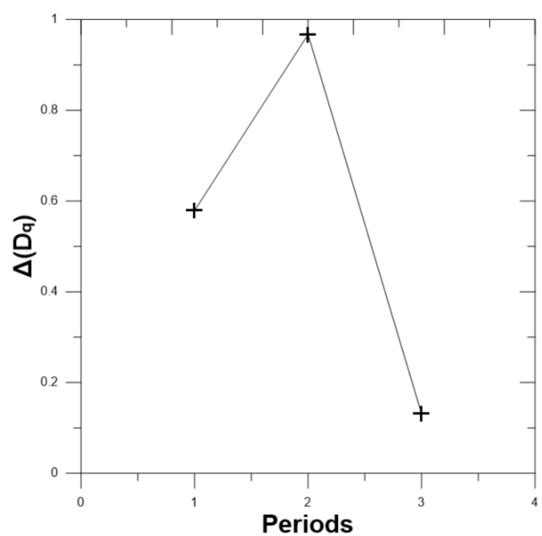


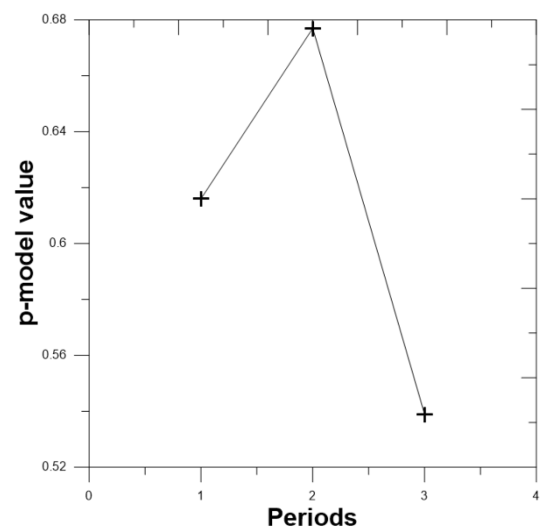




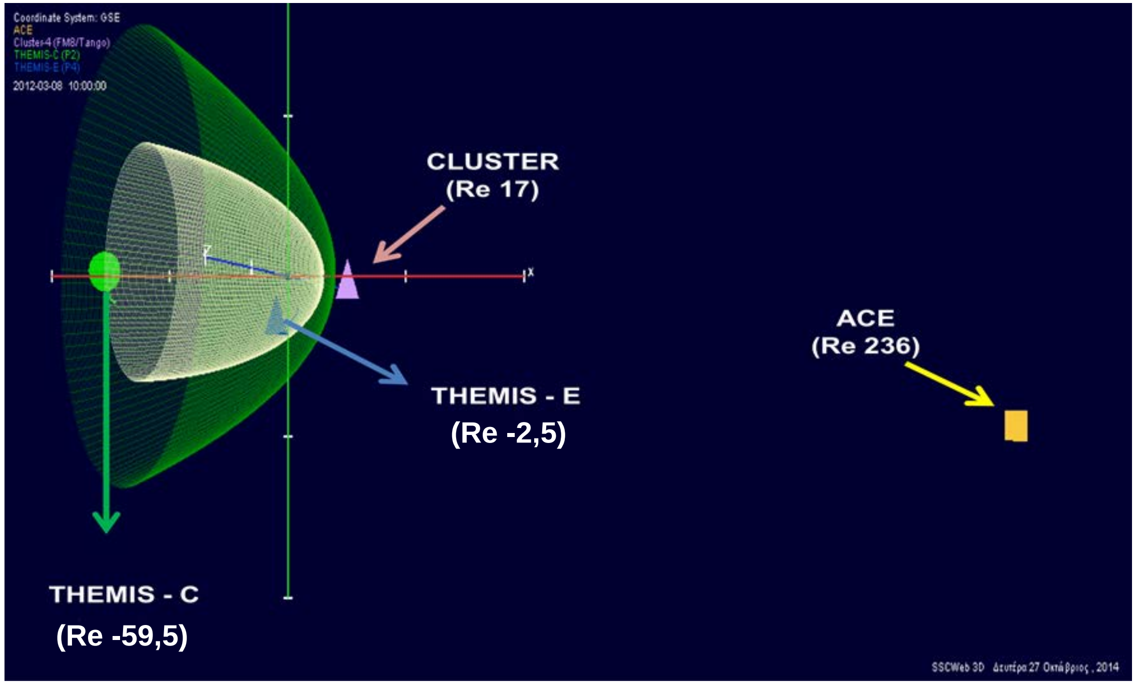
Phase Transition through Multifractal Structure and p-model values







Interplanetary Solar Magnetic Field



	ACE (B_{total})	CLUSTER (B_{total})	THEMIS-E (B_{total})	THEMIS-C (B_{total})
$q_{stationary}$ (CALM)	1.37 ± 0.01	1.26 ± 0.05	1.26 ± 0.05	1.27 ± 0.06
$q_{stationary}$ (SHOCK)	1.74 ± 0.03	1.53 ± 0.07	1.53 ± 0.07	1.73 ± 0.06

Solar Wind (High Time Resolution Ion Fluxes)

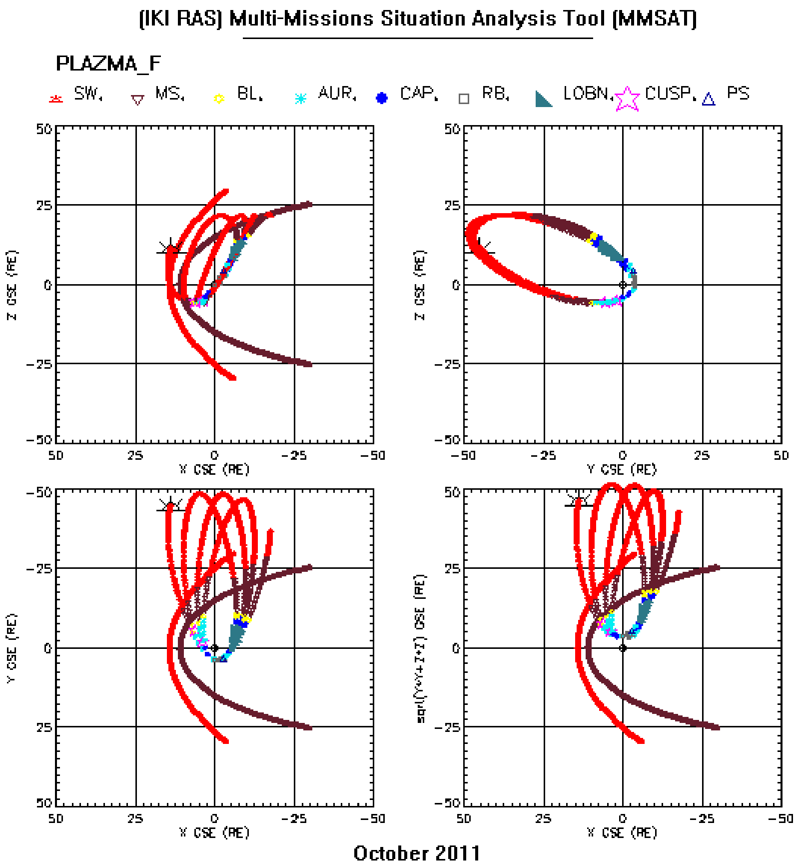
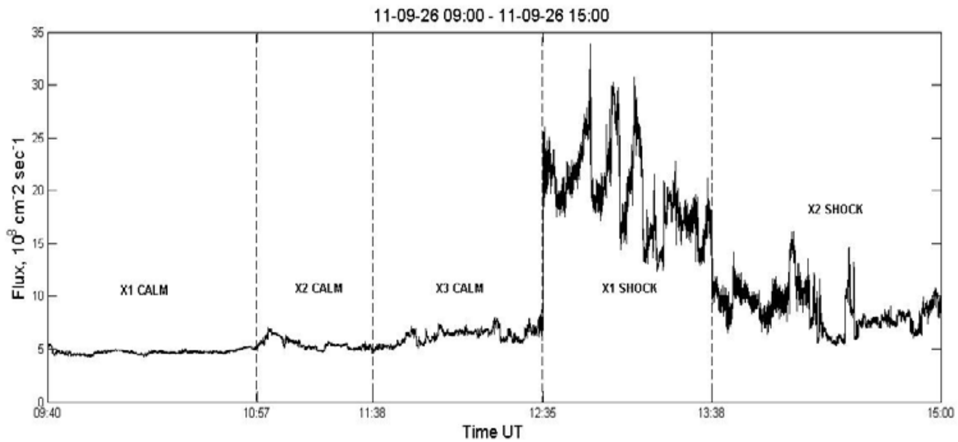
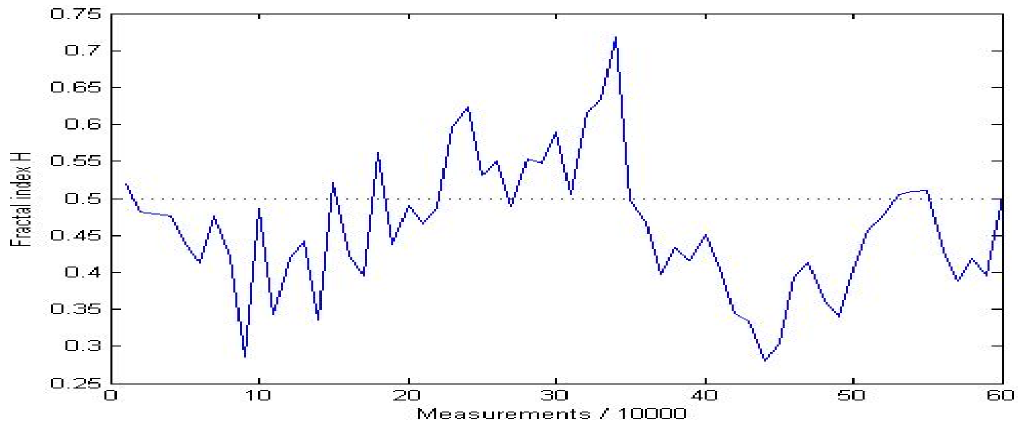
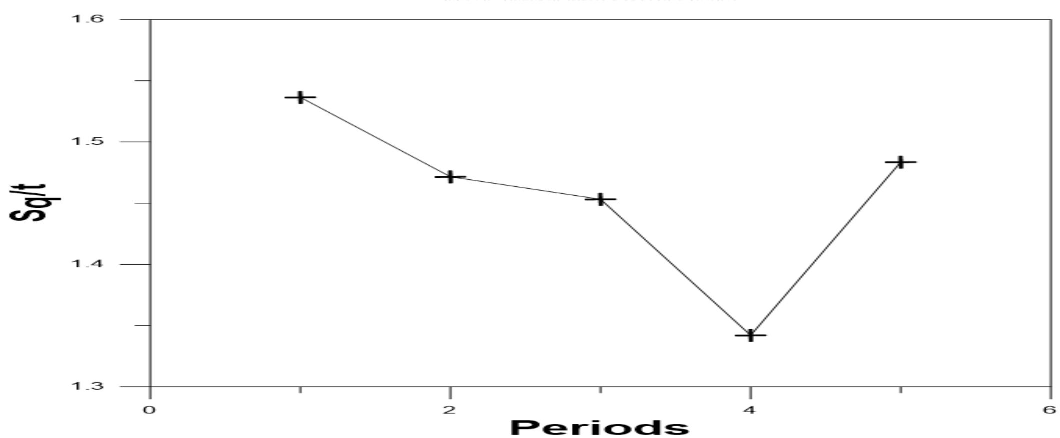


Photo of BMSW instrument (Fast Monitor of the Solar Wind) and Typical SPECTR-R orbits evolution relative the Earth's magnetosphere in October 2011.

Solar Wind Ion Fluxes

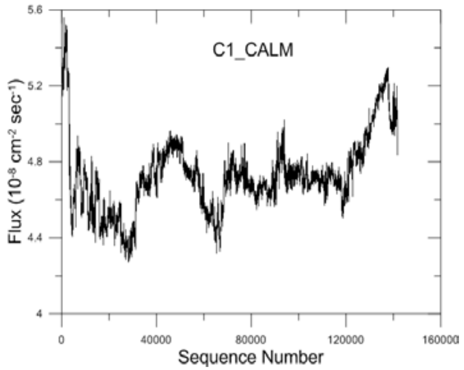


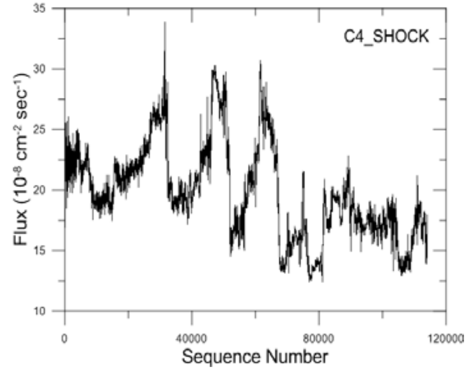




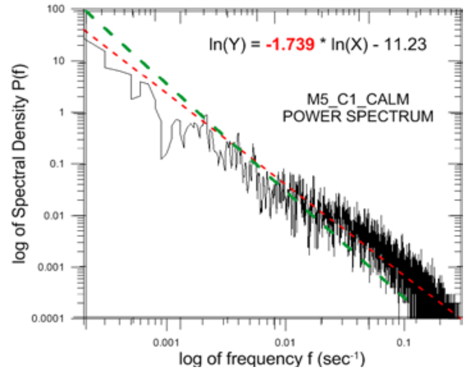
Solar Wind Ion Fluxes (Chaotic Analysis)

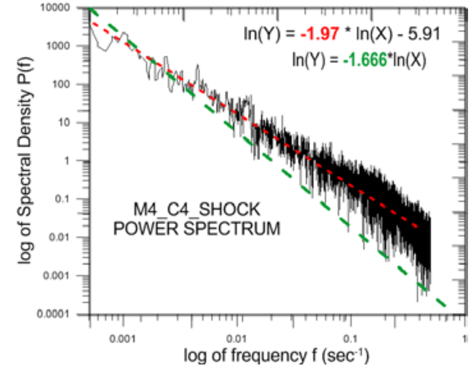
Time Series



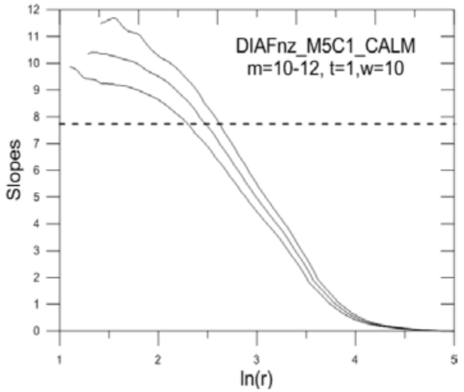


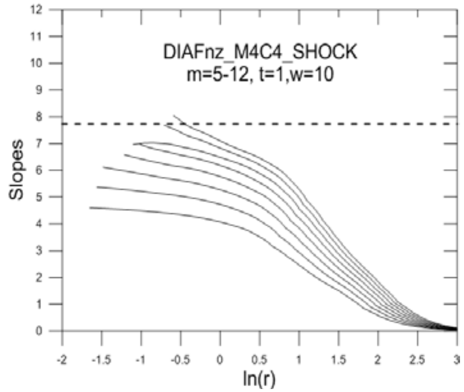
Power Spectrum



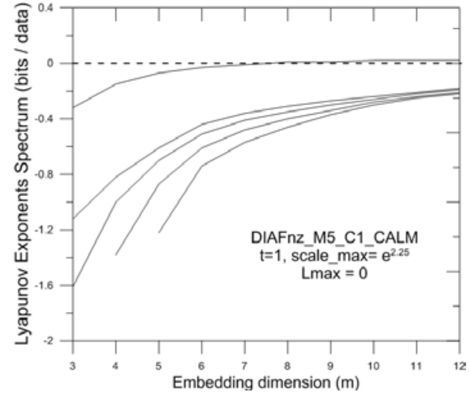


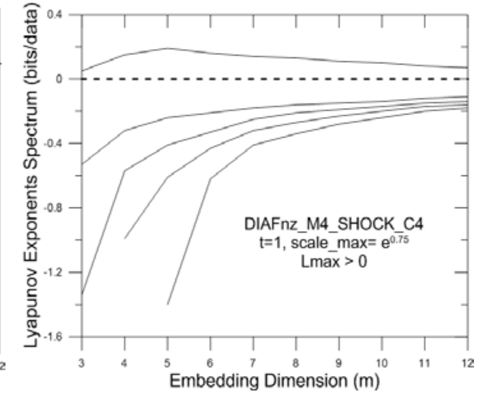
Correlation Dimension

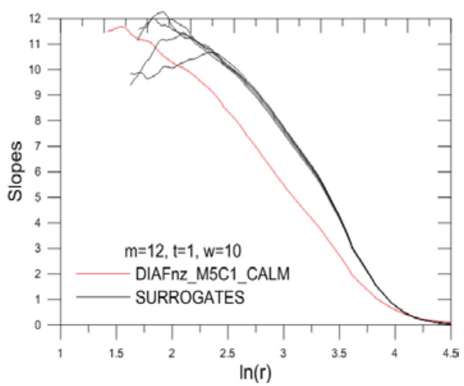


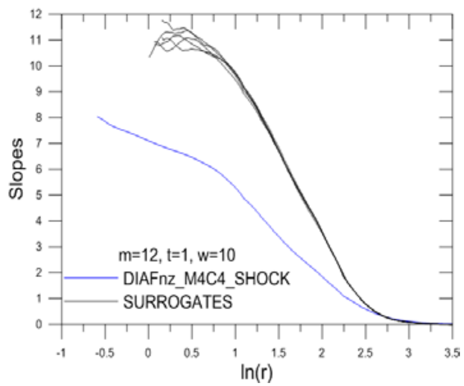


Lyapunov Spectrum Exponents



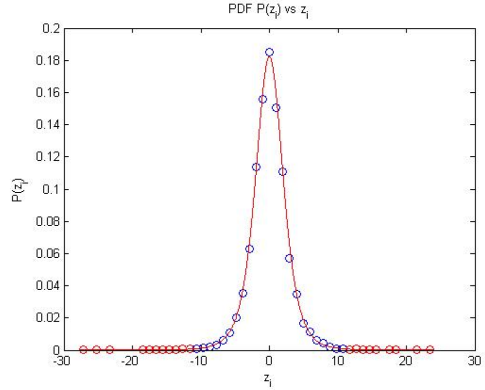


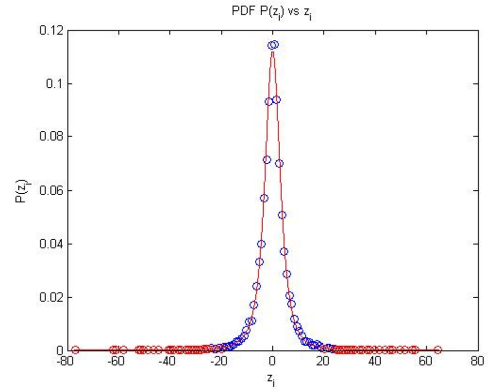




Solar Wind Ion Fluxes (Tsallis Statistics)

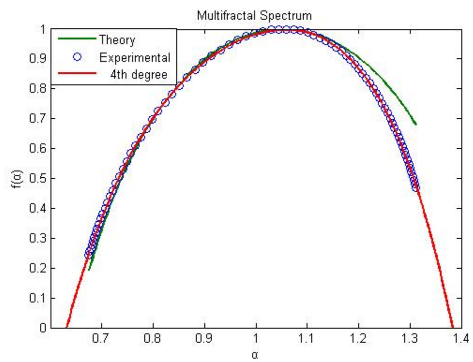
Calm - qstat= 1.37 ±0.05 | **Shock - qstat = 1.61±0.03**

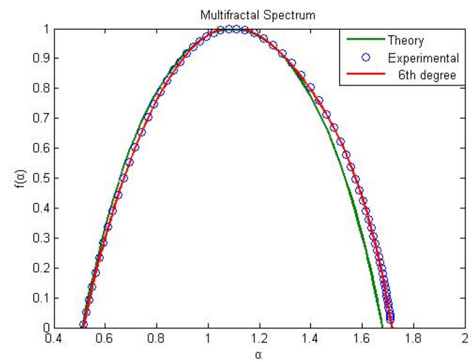




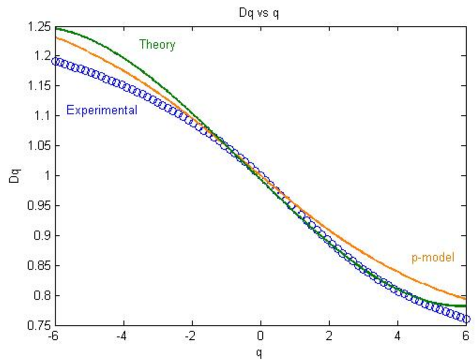
Calm **- qsen=-0.161**
Δα=0.753

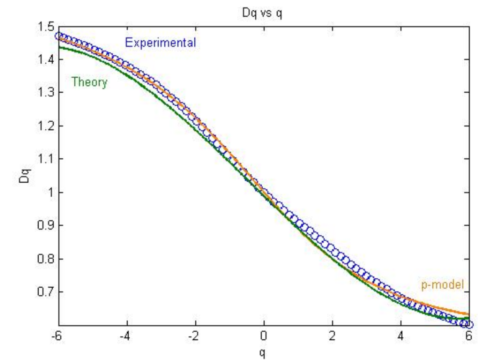
Shock - qsen=0.268
Δα=1.205



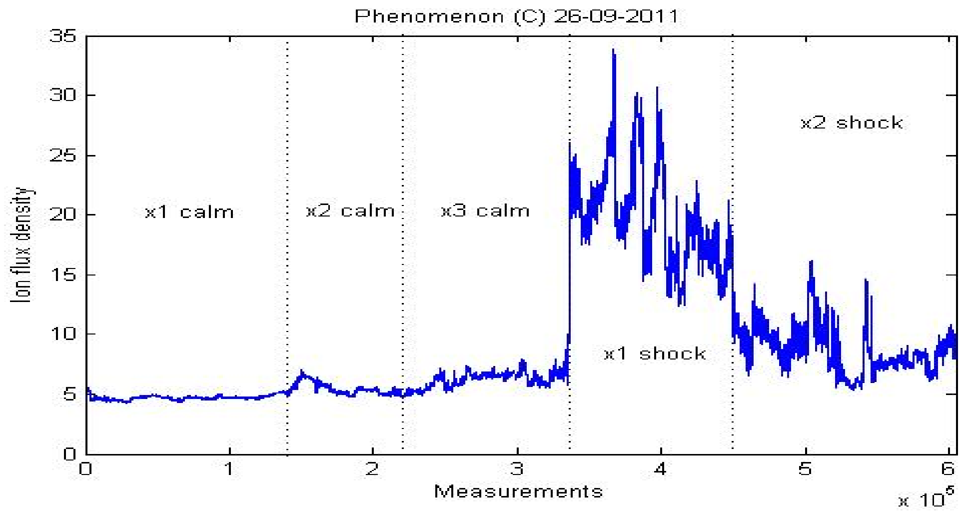


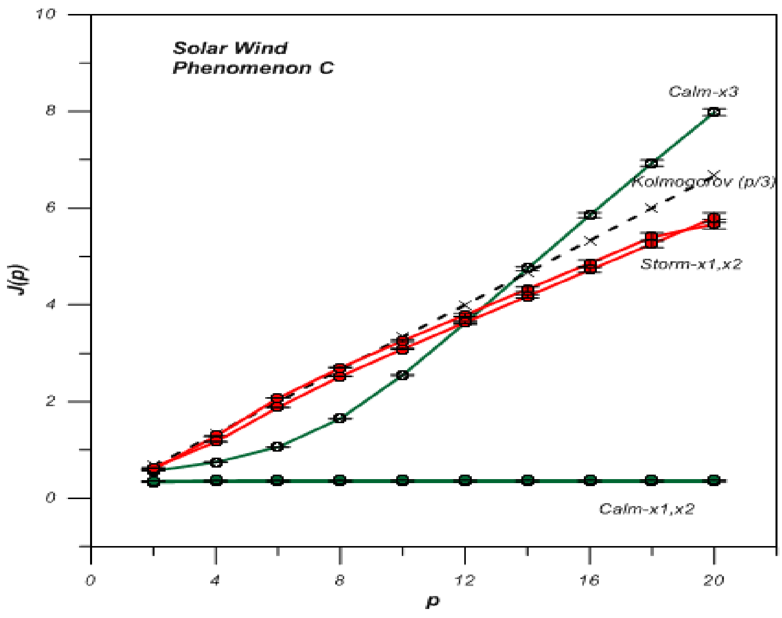
Calm **- ΔDq=0.431** | **Shock - ΔDq =0.871**

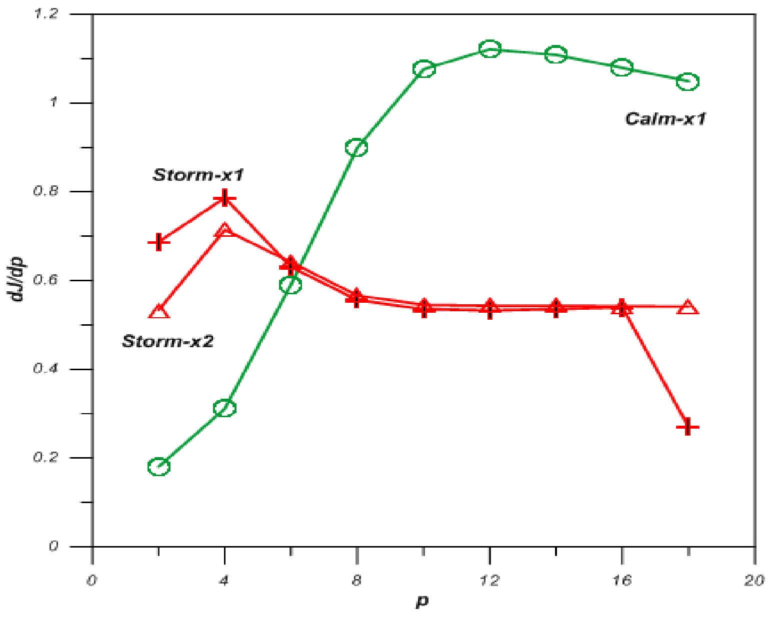




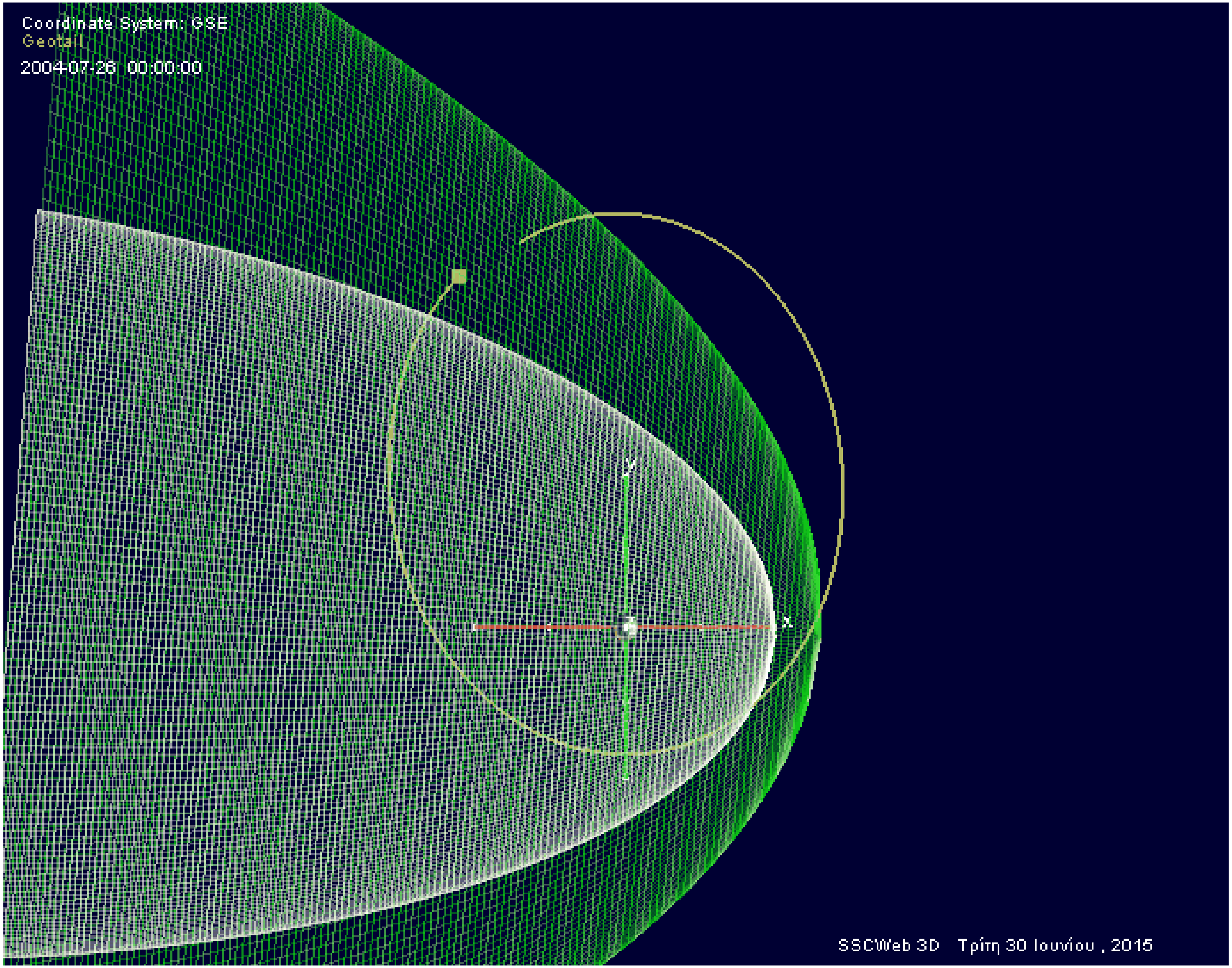
Solar Wind Ion Fluxes (Turbulence Analysis)





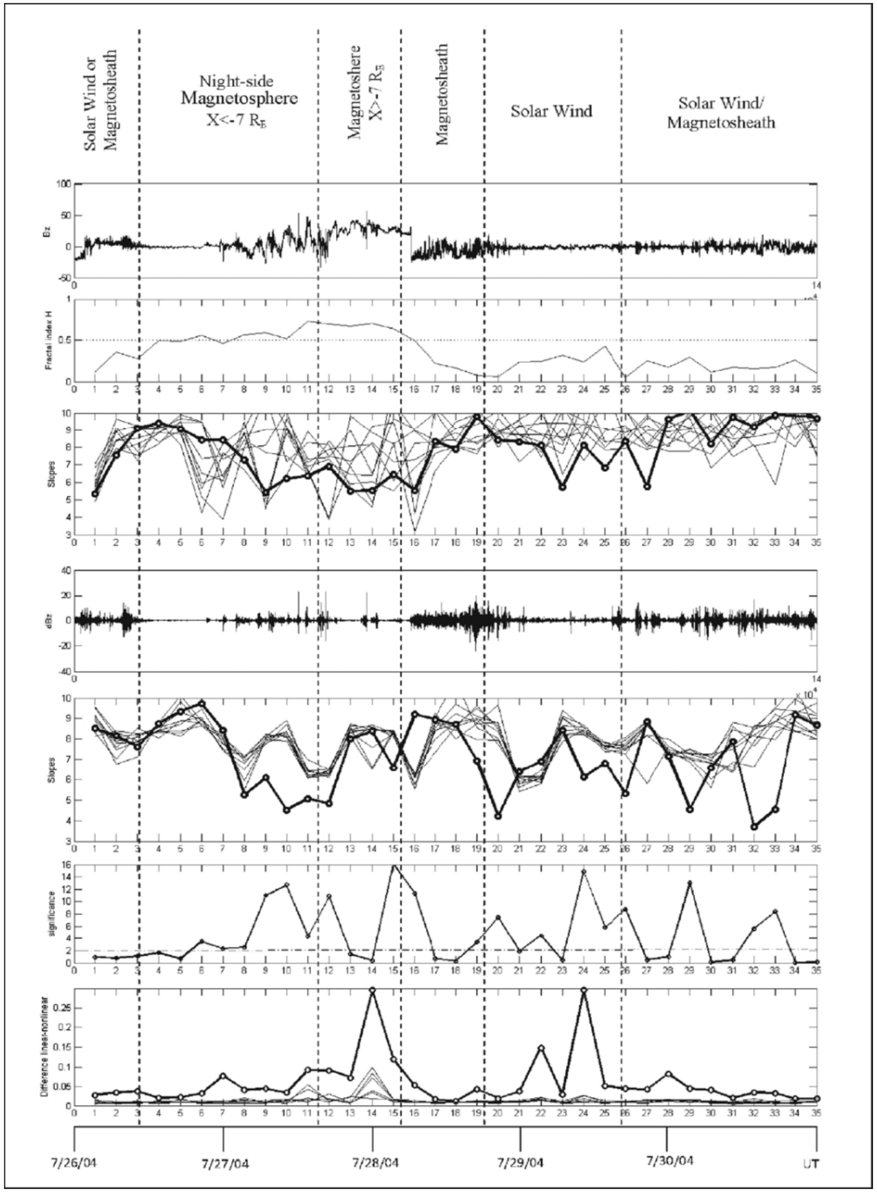


Magnetosphere 26-30 July 2004 – GEOTAIL Orbit

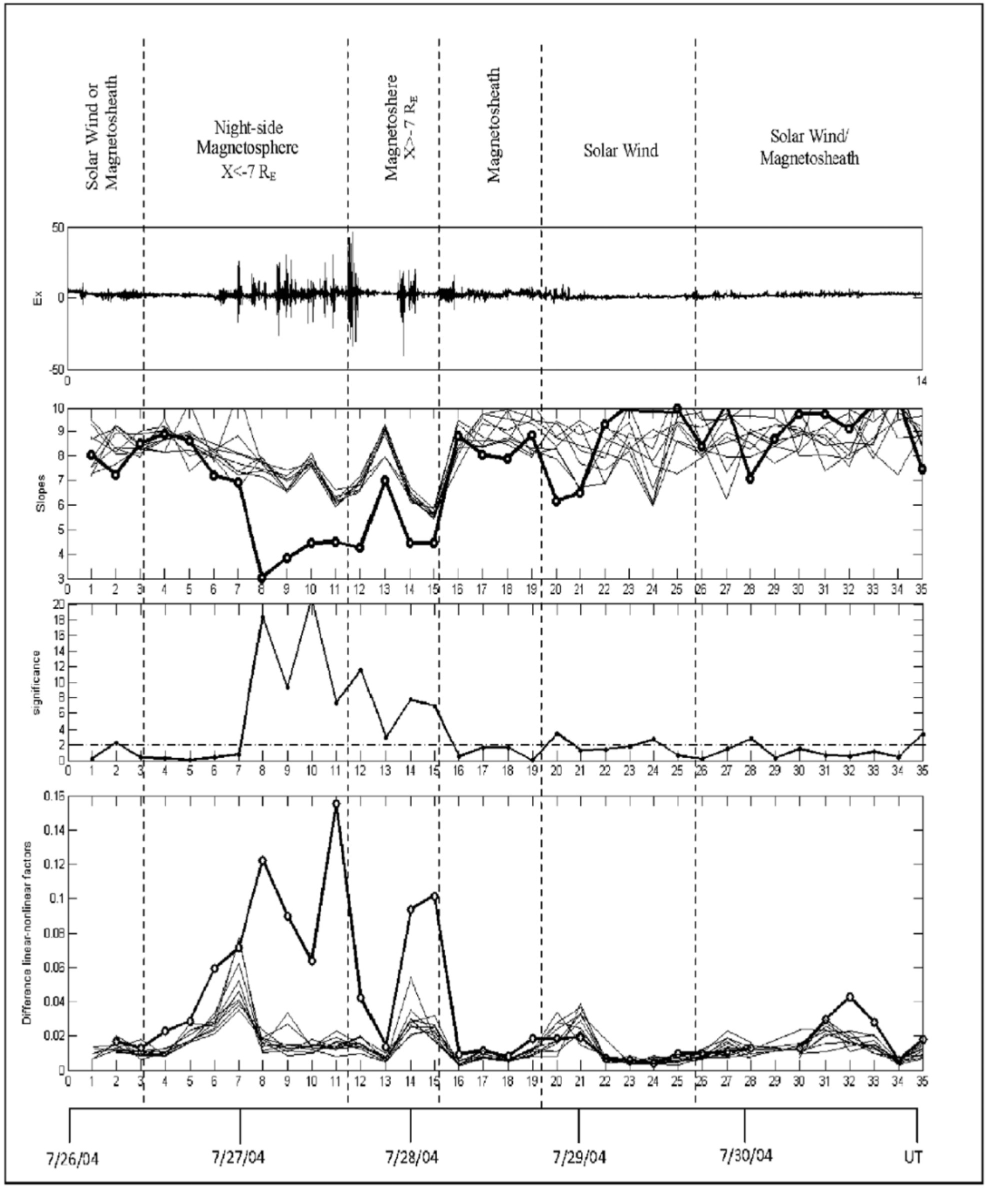


Magnetosphere 26-30 July 2004

Bz Component



Ex Component



Summarize parameter values of non-extensive statistics in Solar Plasmas The q-triplet (q_sen, q_stat, q_rel) of Tsallis.

System	q_sen	q_stat	q_rel (C(τ))
Solar Wind (Bz cloud)	0.484±0.009	2.02±0.04	32.25±2.31
Solar (Sunspot Index)	0.368±0.005	1.53 ± 0.04	5.67±0.13
Solar (Flares Index)	0.308±0.005	1.870±0.005	5.33±0.22
Solar (Protons)	0.817 ± 0.024	2.31±0.13	2.48±0.23
Solar (Electrons)	0.860±0.009	2.13±0.06	2.58±0.22
Cosmic Stars (Brigthness)	-0.568±0.042	1.64±0.03	7.71±0.58
Cosmic Ray (C)	-0.441±0.013	1.44±0.05	193.30±18.43

Conclusions

Everywhere in Solar plasma we have confirmed the following:

- Tsallis Sq entropy and non extensivity
- Nonequillibrium stationary states (NESS)
- Nonequillibrium topological phase transition processes
- Multifractal intermittent turbulence
- Anomalous diffusion
- Multiplicative processes
- Low Dimensional Chaos and/or SOC.

These experimental results are concluded theoretically by:

- Strange Dynamics
- Fractal extension of MHD theory
- Fractional extension of magnetized plasmas Langevin and Fokker – Planck equations
- Nonequillibrium Percolation Theory

THANK YOU

for your attention