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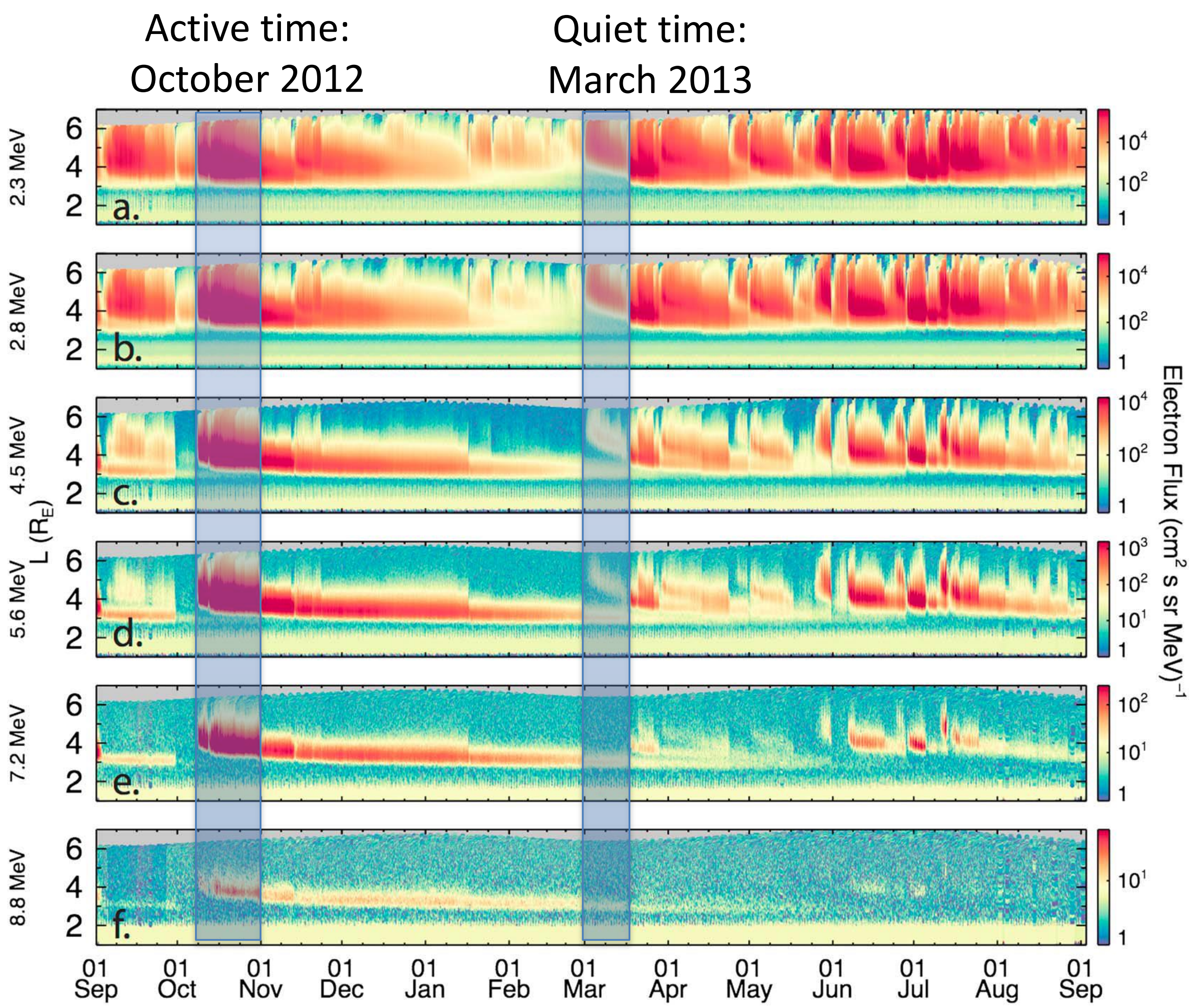
Modeling populations of energetic particles in the radiation belts using novel estimates of the radial diffusion coefficients

Theodoros E. Sarris

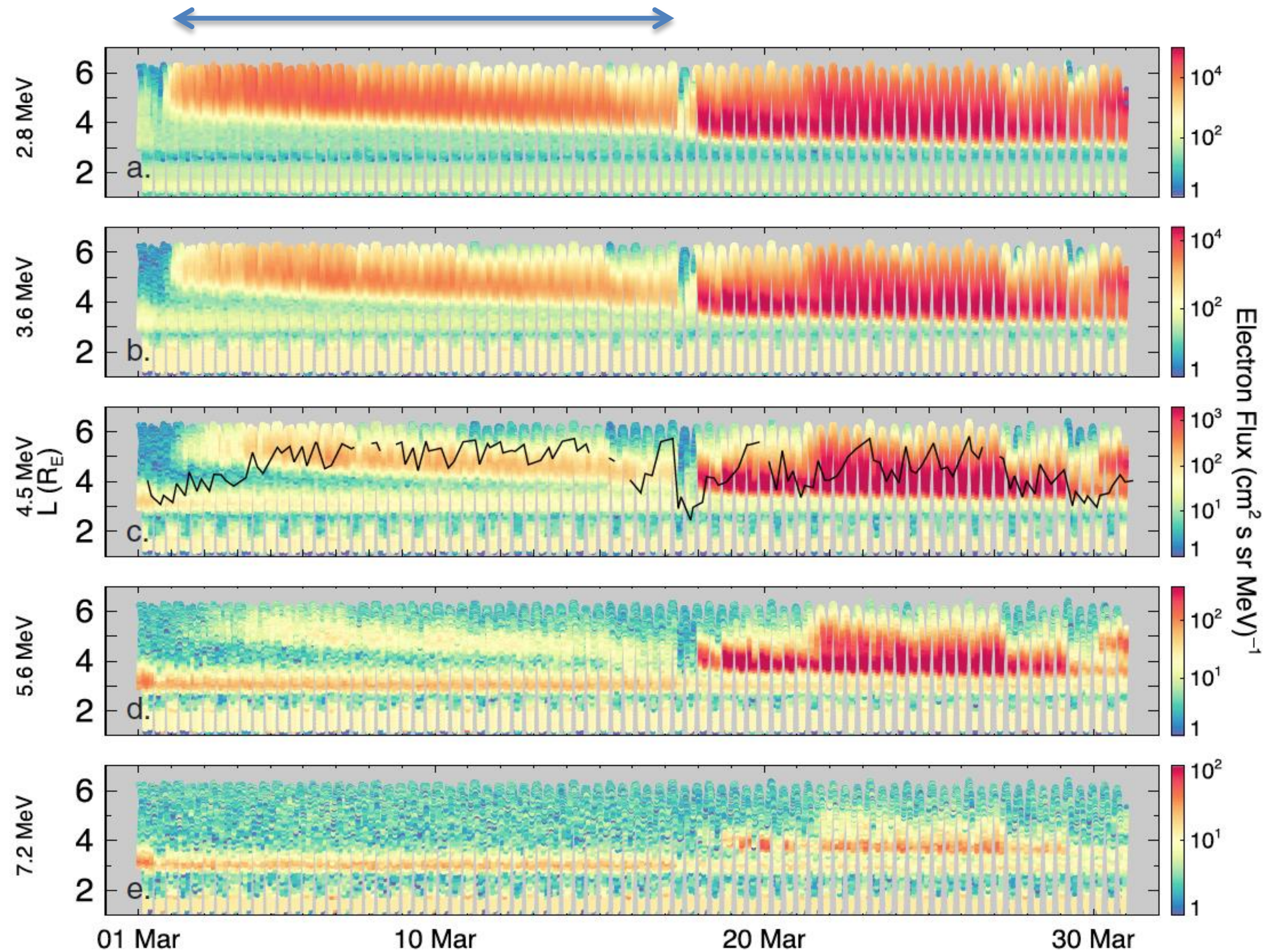
Background: The radiation belts as seen by the Van Allen Probes



***Baker at al.**, GRL, 2014*

Background: The radiation belts as seen by the Van Allen Probes

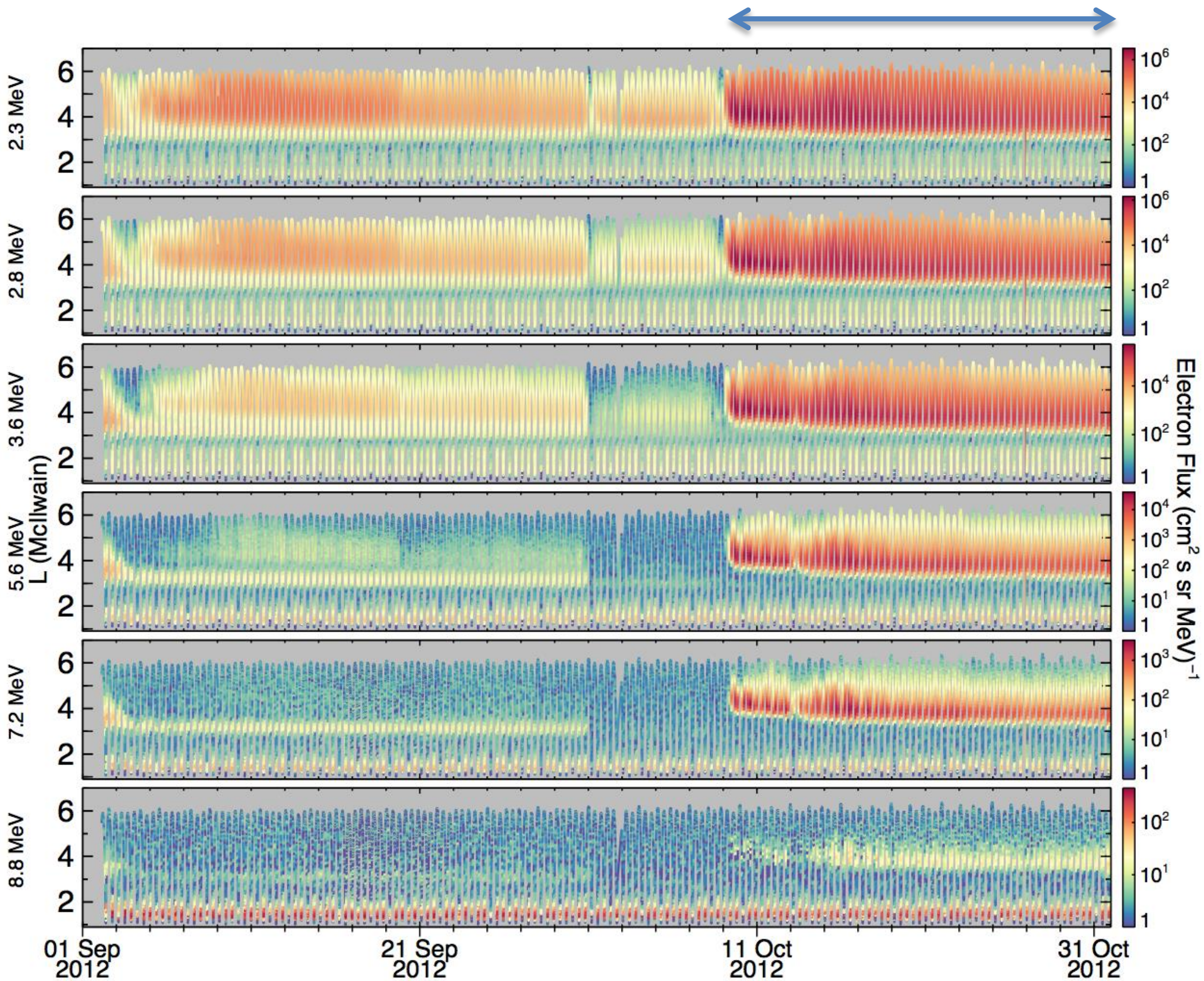
Quiet time diffusion:
March 2-17, 2013



***Baker at al.**, GRL, 2014*

Background: The radiation belts as seen by the Van Allen Probes

Active time diffusion:
October 11-31, 2012



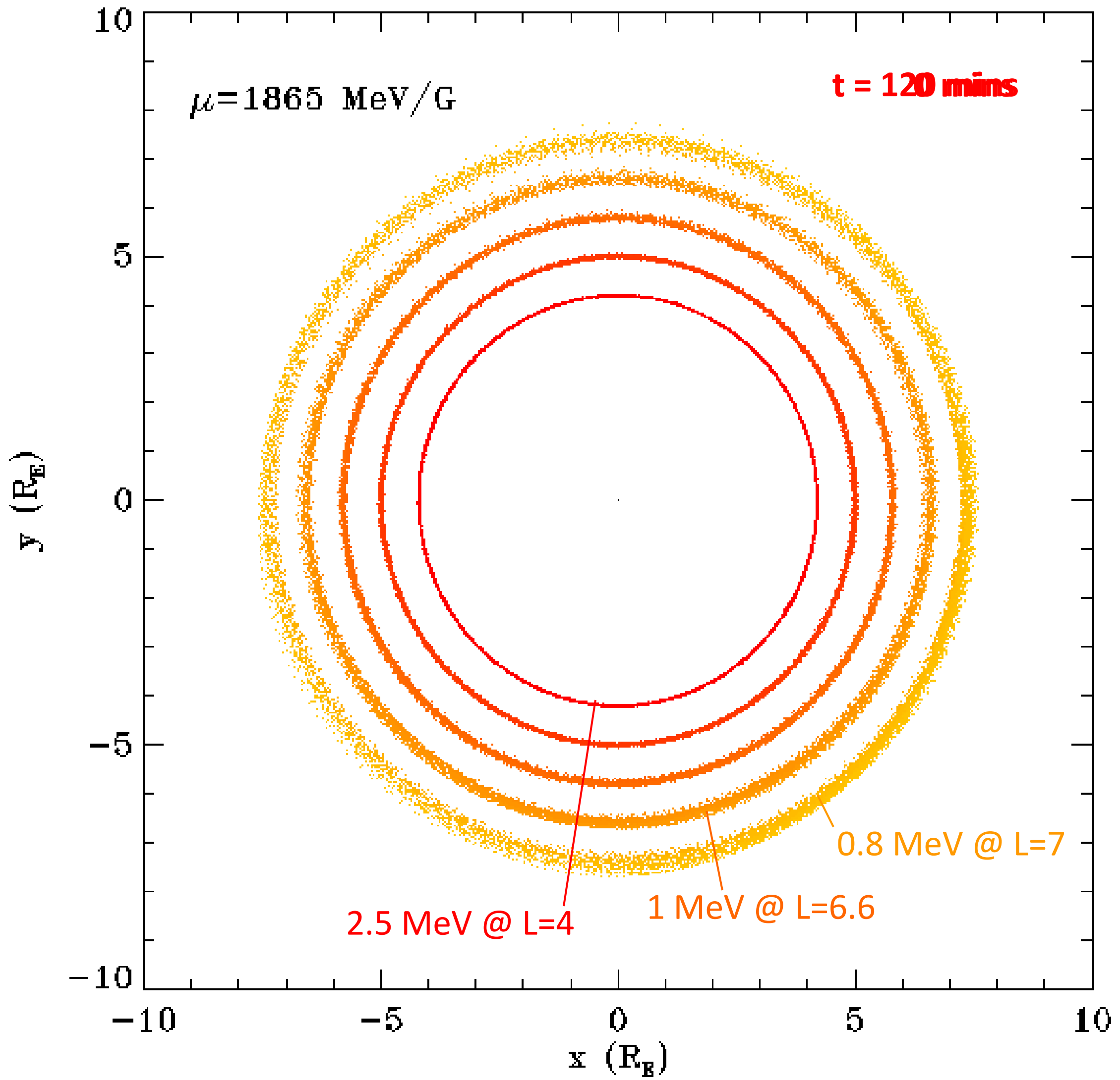
*(Figures by **Allison Jaynes**)*

Background: Particle Diffusion in a Dipole Field

Definition of the diffusion coefficient:
Rate of Average Squared Displacement

$$D_{LL}^{B} = \frac{\langle (\Delta L)^2 \rangle}{2t}$$

Tracing Particle Rings:



Background: The Diffusion Coefficient, D_{LL}^{B}

The Diffusion Coefficient:

$$D_{LL}^{B,Sym} = \frac{\mu^2}{8q^2 B_E^2 R_E^4} \frac{L^4}{\gamma^2} \sum_m m^2 P_m^B (m\omega_d)$$

Theoretical derivation:

- ***Falthammar*** *[1968]*
- ***Schultz & Lanzerotti*** *[1974]*
- ***Fei et al.*** *[2005]*

Empirical Approximations:

- ***Brautigam & Albert*** *[2000]*
- ***Elkington et al.*** *[2003]*
- ***Sarris et al.*** *[2005]*
- ***Huang et al.*** *[2010]*
- ***Ozeke et al.*** *[2013]*

- **Unknown Distribution of Power vs. Mode Number, *m***
  - Mode number calculations require multi-point measurements
  - Most often a single mode number is considered; e.g., all power is in m = 1
  - Existing models do not account for power at higher mode numbers
- **Unknown Distribution of Power vs. L**
  - Brautigam & Albert [2000]: based on only 18 days of gnd mag & 1 month of in situ sc data
  - Elkington et al. [2003]: based on MHD, which often is not a good approximation of ULF activity
  - Sarris et al. [2005]: based on GOES geosynchronous data only
  - Ozeke et al [2013]: based on THEMIS measurements in 2007-2011 (prolonged solar min)

Background: Mode number and Particle Interactions with ULF waves

The Diffusion Coefficient:

$$D_{LL}^{B,Sym} = \frac{\mu^2}{8q^2 B_E^2 R_E^4} \frac{L^4}{\gamma^2} \sum_m m^2 P_m^B(m\omega_d)$$

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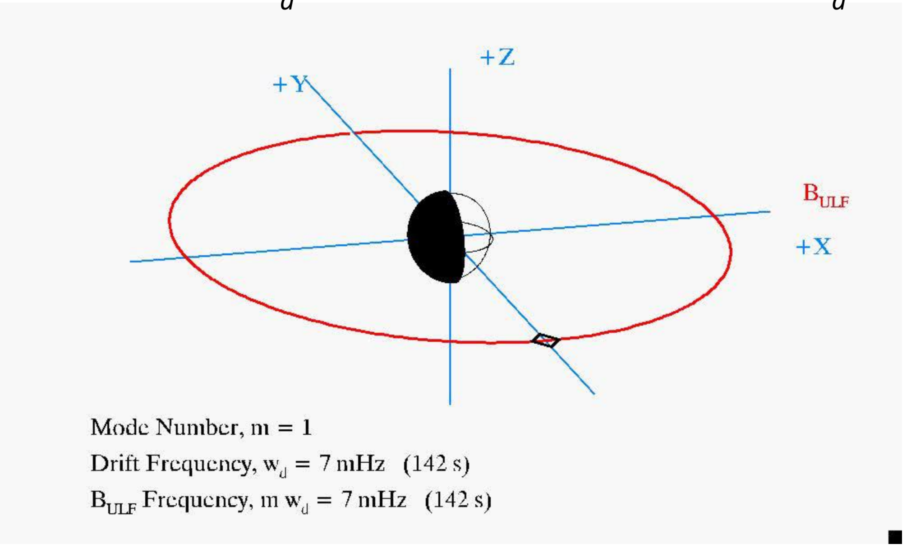
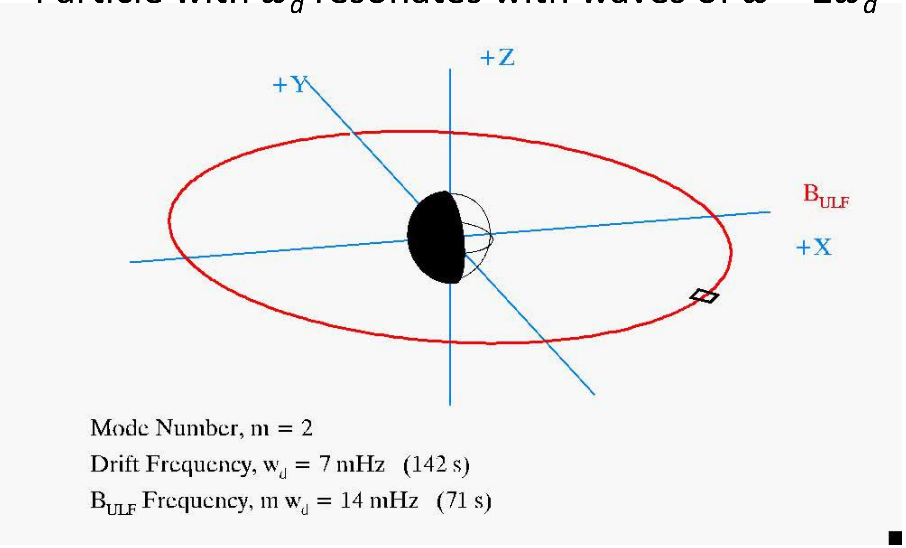
- ***Ozeke et al.*** *[2013]*

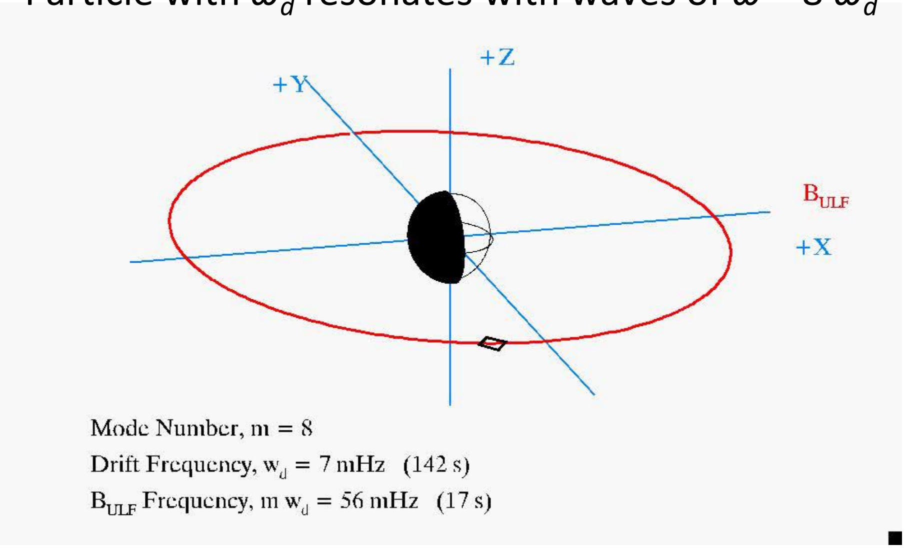
Particle with ω_d resonates with waves of $\omega = \omega_d$



Particle with ω_d resonates with waves of $\omega = 2\omega_d$



Particle with ω_d resonates with waves of $\omega = 8\ \omega_d$



Motivation: Limitations in existing D_{LL} estimates

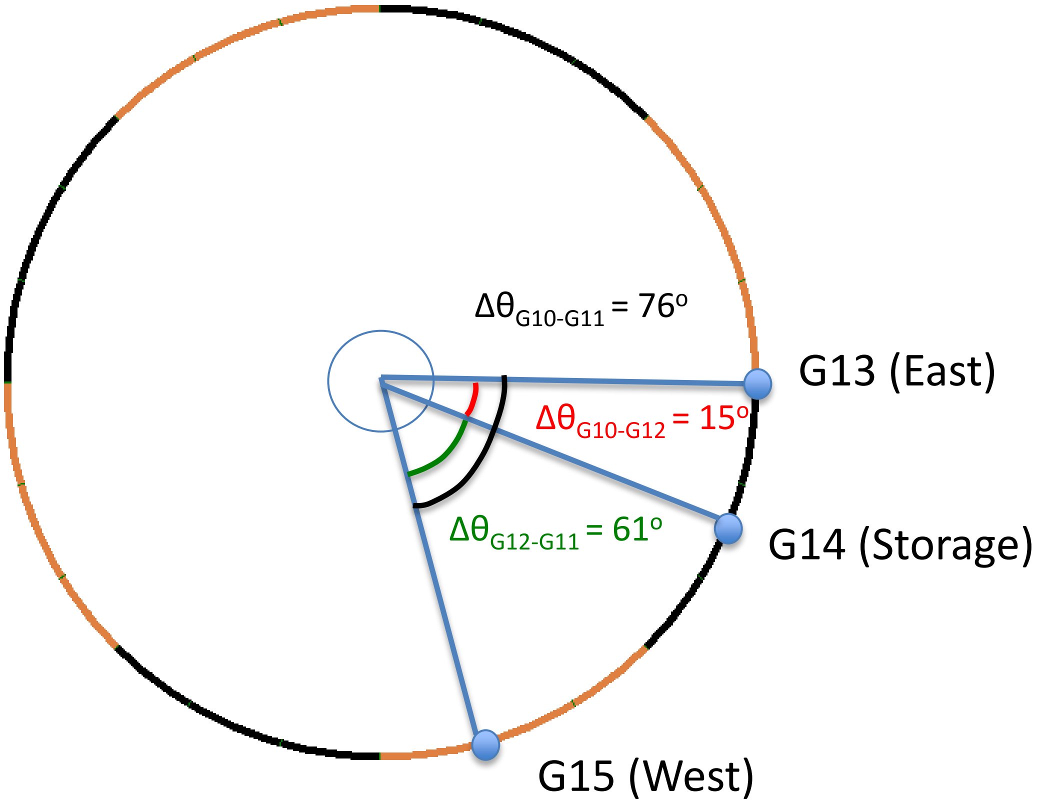
- **Most commonly used D_{LL}^{B} : Brautigam & Albert, 2000 (B & A)**
    - expressions based on only 18 days of ground magnetometer & 1 month of in situ sc measurements;
    - single mode number: all power in m = 1;
    - is often considered to provide unrealistic diffusion

- **B & A Radial Diffusion Coefficients are used in multiple radiation belt models:**
    - Versatile Electron Radiation Belt [Subbotin and Shprits, 2009]
    - SALAMMBO [e.g., Beutier and Boscher, 1995; Varotsou et al., 2008],
    - Dynamic Radiation Environment Assimilation Model (DREAM) [Koller et al., 2007],
    - Modified Kalman Filter model by Schiller et al., 2013.

- **Latest efforts to update D_{LL}^{B}: by Ozeke et al, 2013**
    - derived empirical expressions of power based on statistical THEMIS measurements in 2007-2011
    - measurements used in empirical expression are during prolonged solar minimum
    - does not account for power at higher mode numbers

Approximating the Distribution of Power in the various Mode Numbers

Background: Detection of mode number through Phase Differences



$\Delta\phi = m\ \Delta\theta = 360^o$

G13 – G14: $m_{max} = 360^o / 15^o = 24$

G14 – G15: $m_{max} = 360^o / 61^o = 6$

G13 – G15: $m_{max} = 360^o / 76^o = 5$

Methodology: Detection of mode number through Phase Differences

Overview of the technique:

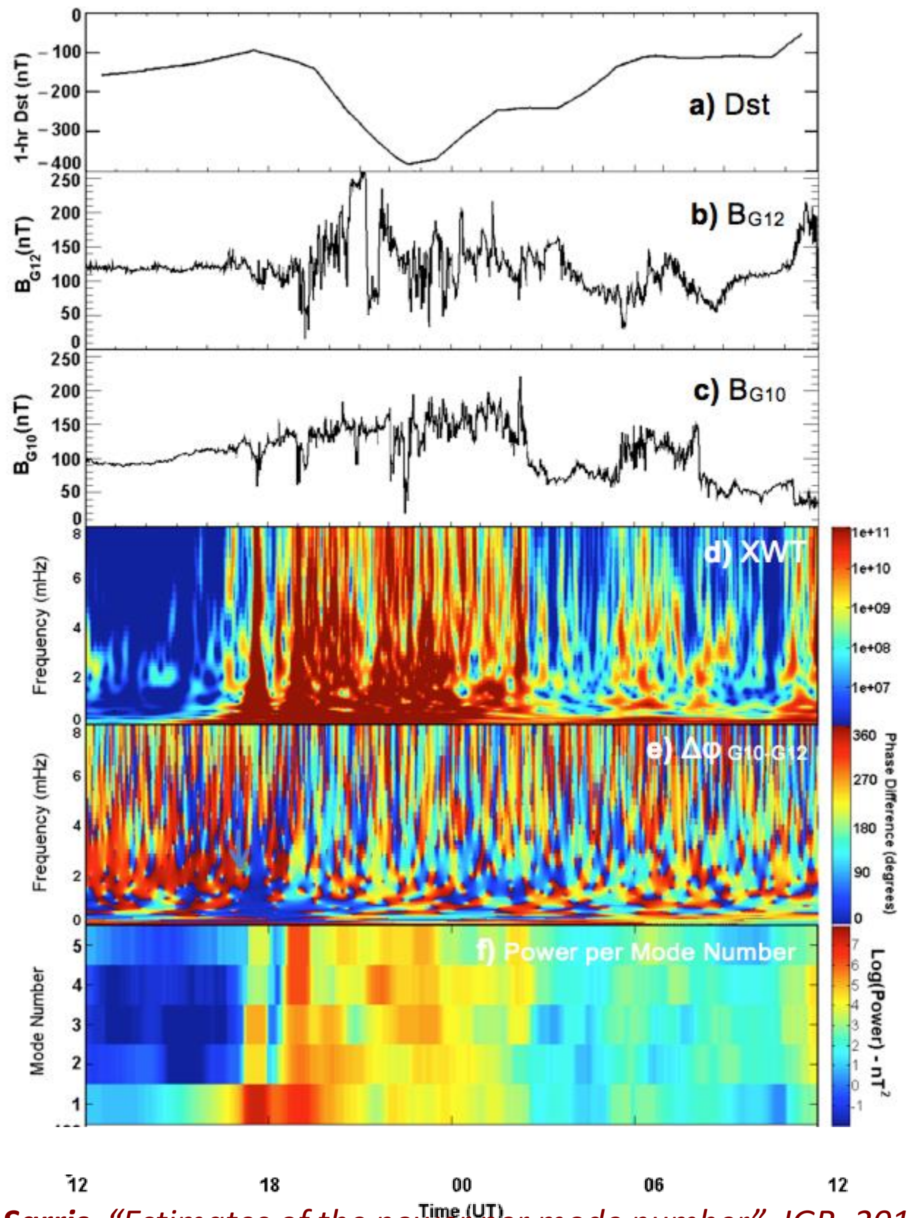
1st time series:

2nd time series:

Cross-Spectrogram Power:

Phase differences (Δϕ) vs. frequency:

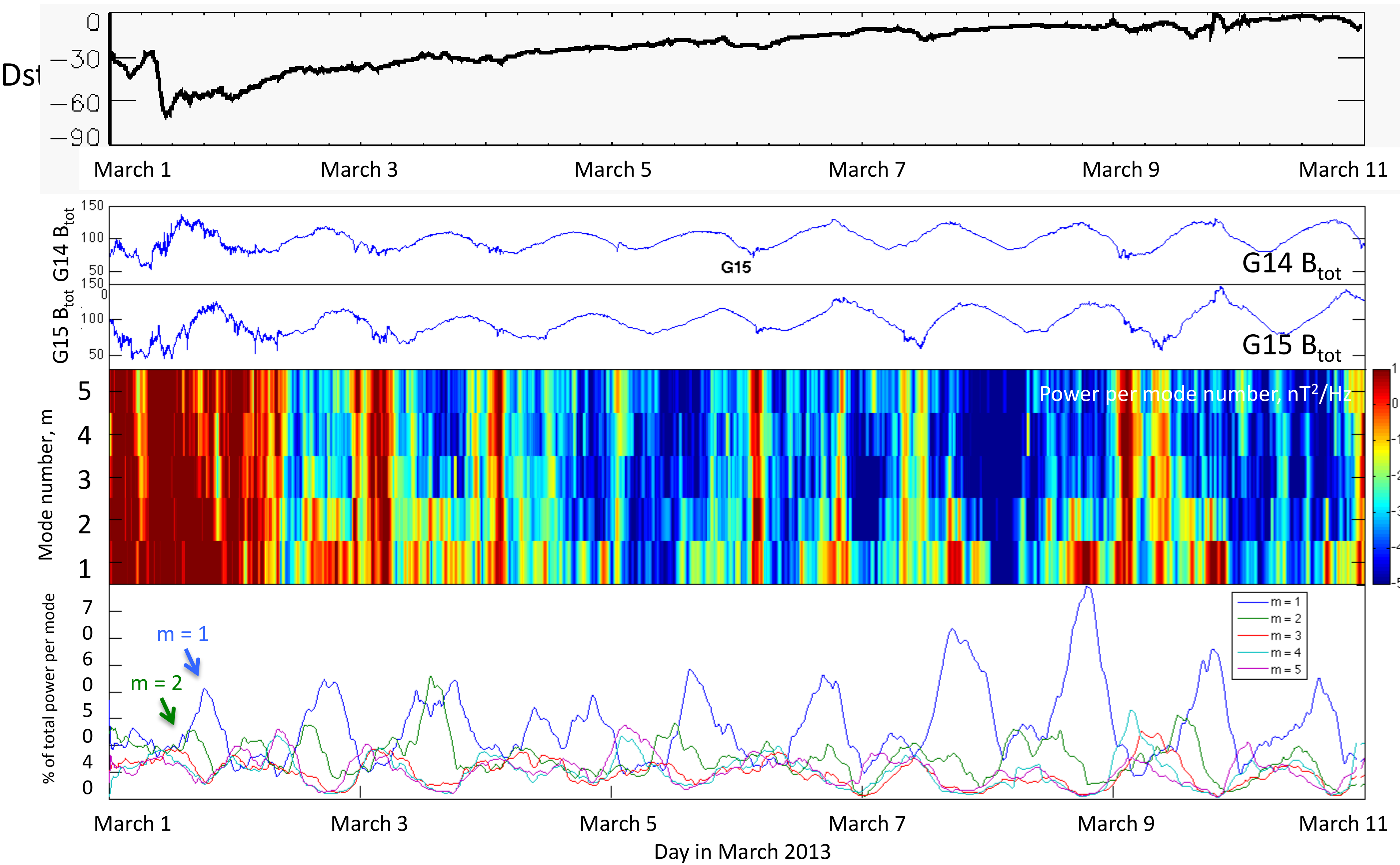
Sum of power at Δϕ's corresponding to each m:



***Sarris**, "Estimates of the power per mode number", JGR, 2014*

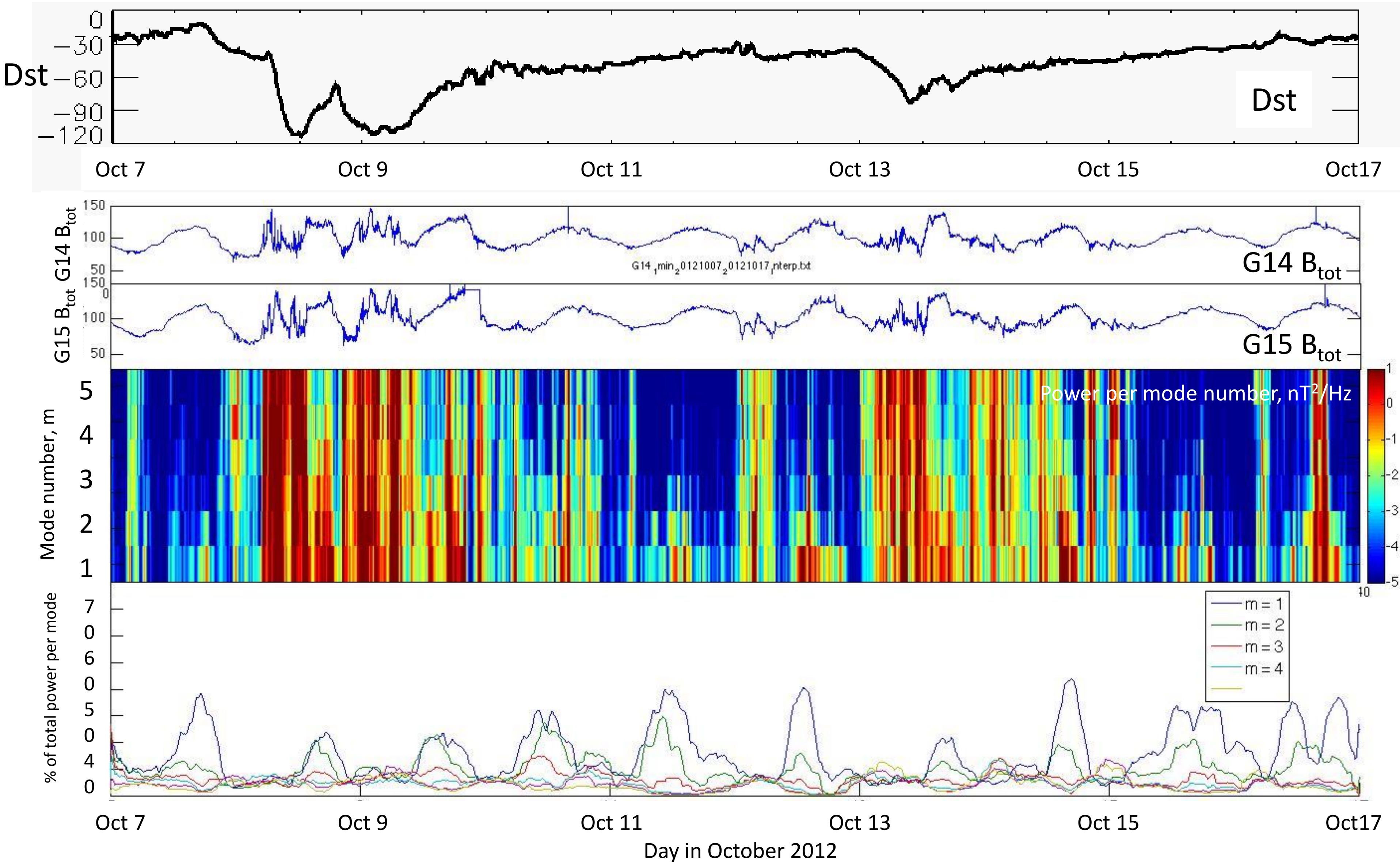
Result 1: Local Time Dependence of the Power per Mode number

Quiet time: March 1-11, 2013

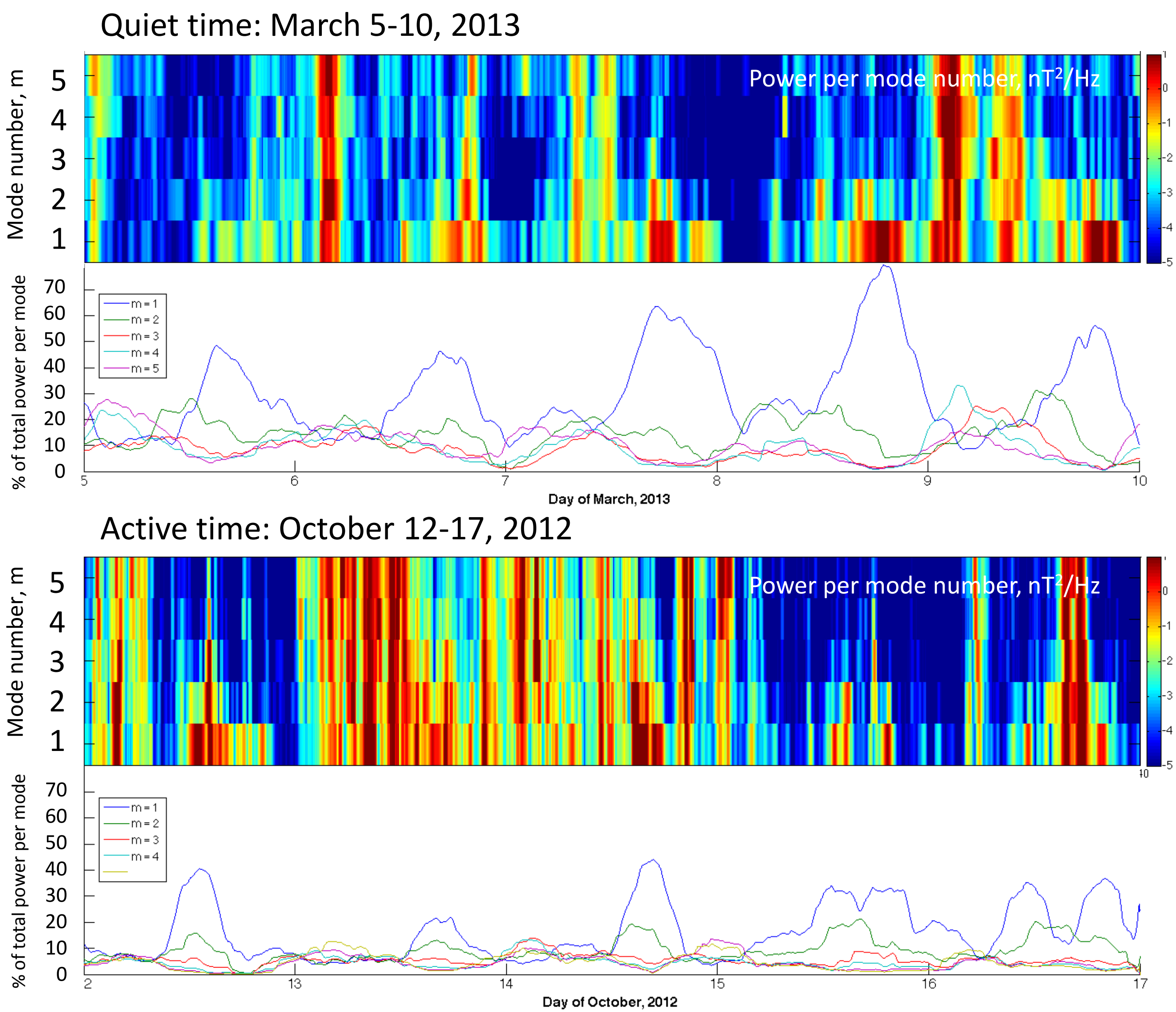


Result 1: Local Time Dependence of the Power per Mode number

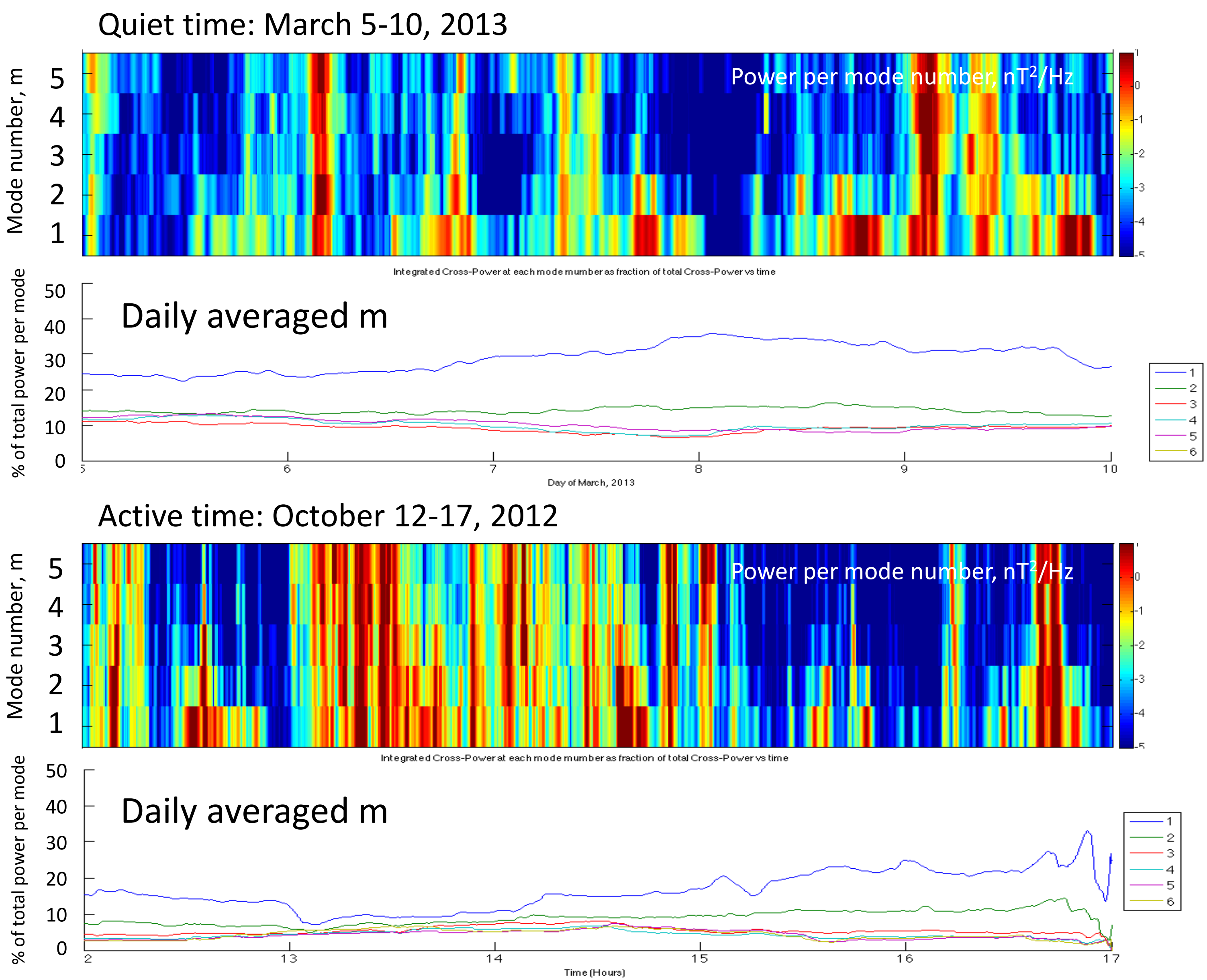
Active time: October 7-17, 2012



Result 2: Solar Wind - Dst Dependence of the Power per Mode number



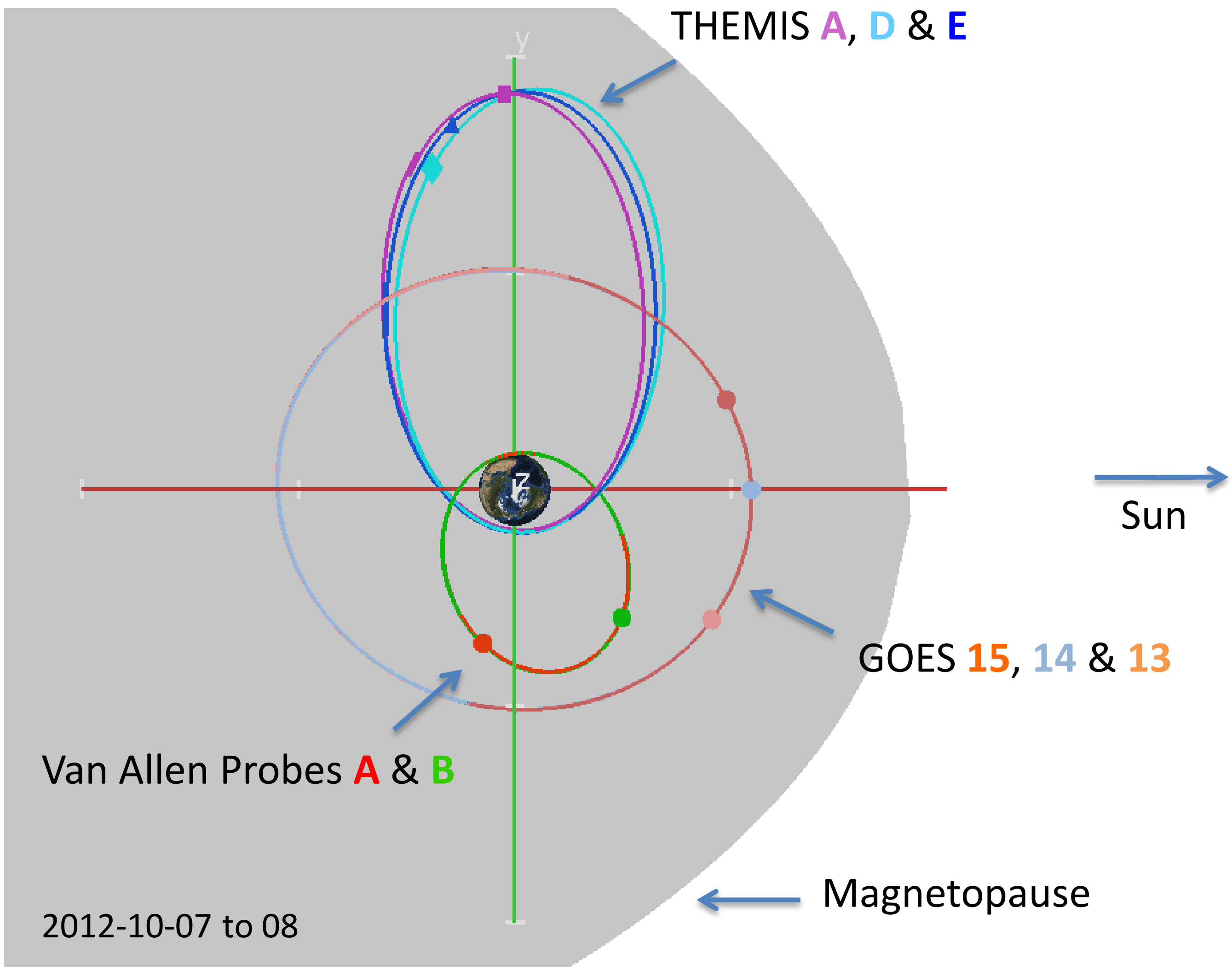
Result 2: Solar Wind - Dst Dependence of the Power per Mode number



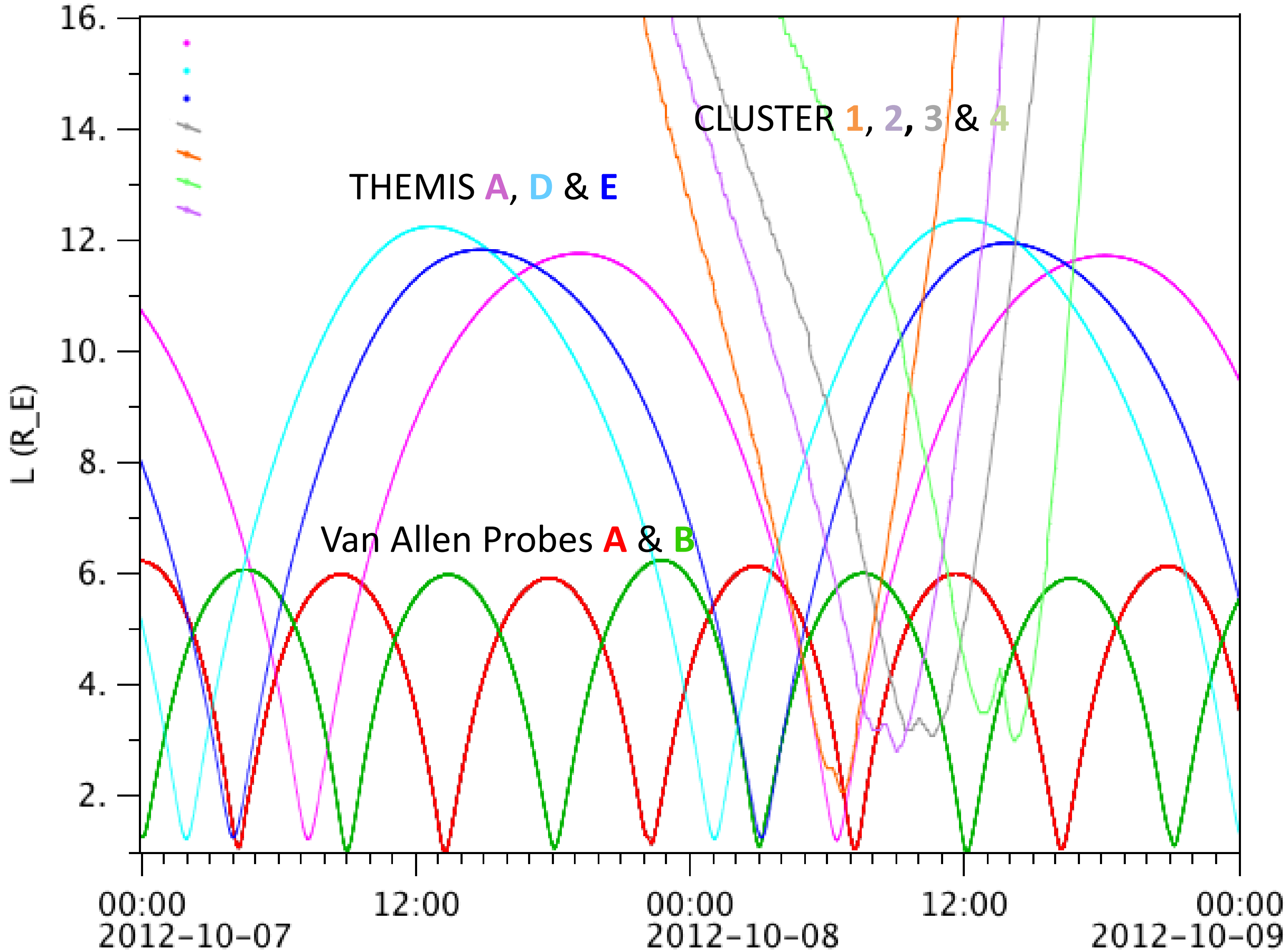
Estimates of ULF wave Power vs. L

using the Van Allen Probes, THEMIS and GOES

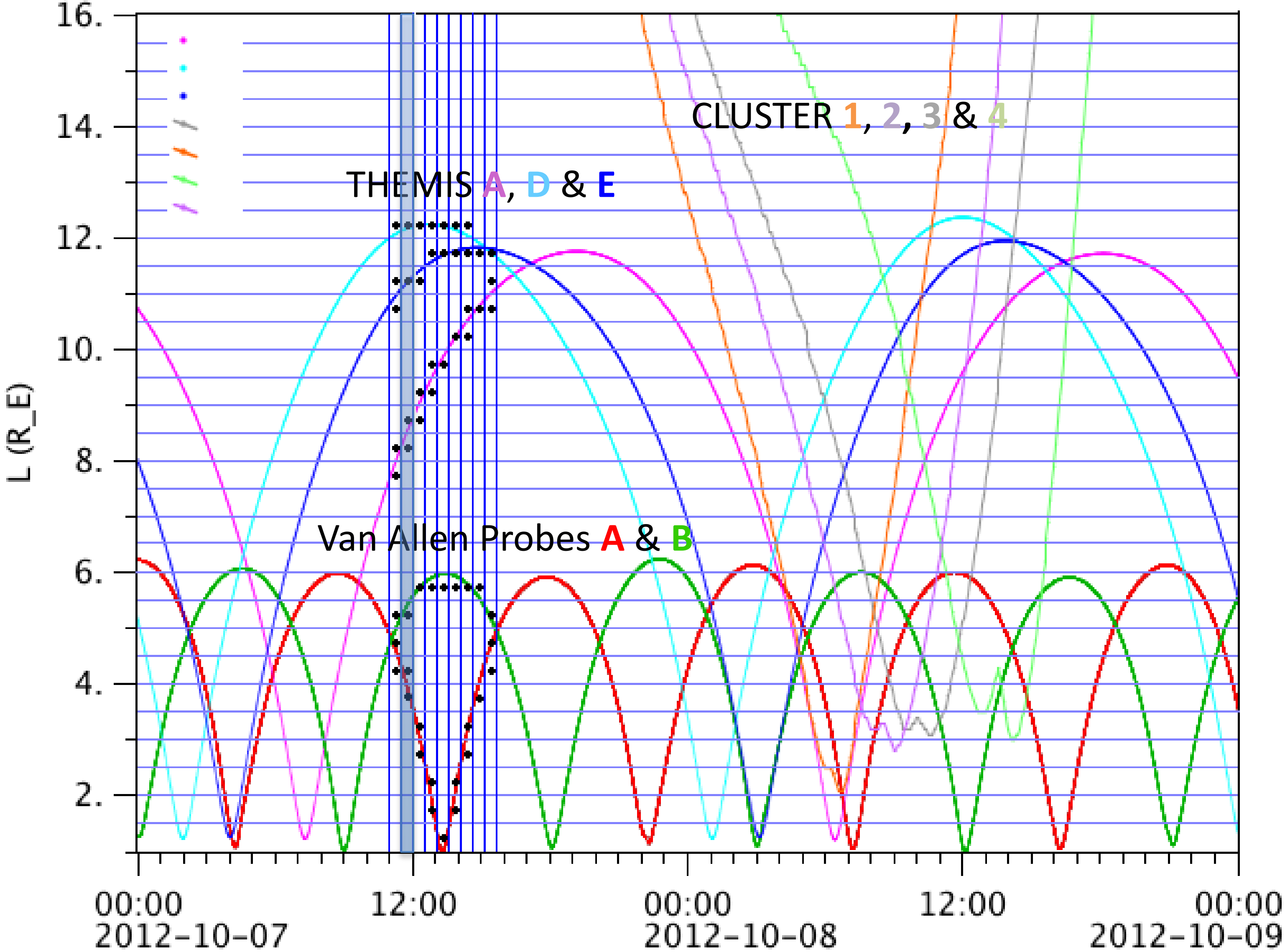
Measurements: ULF wave power using THEMIS, GOES, Van Allen Probes



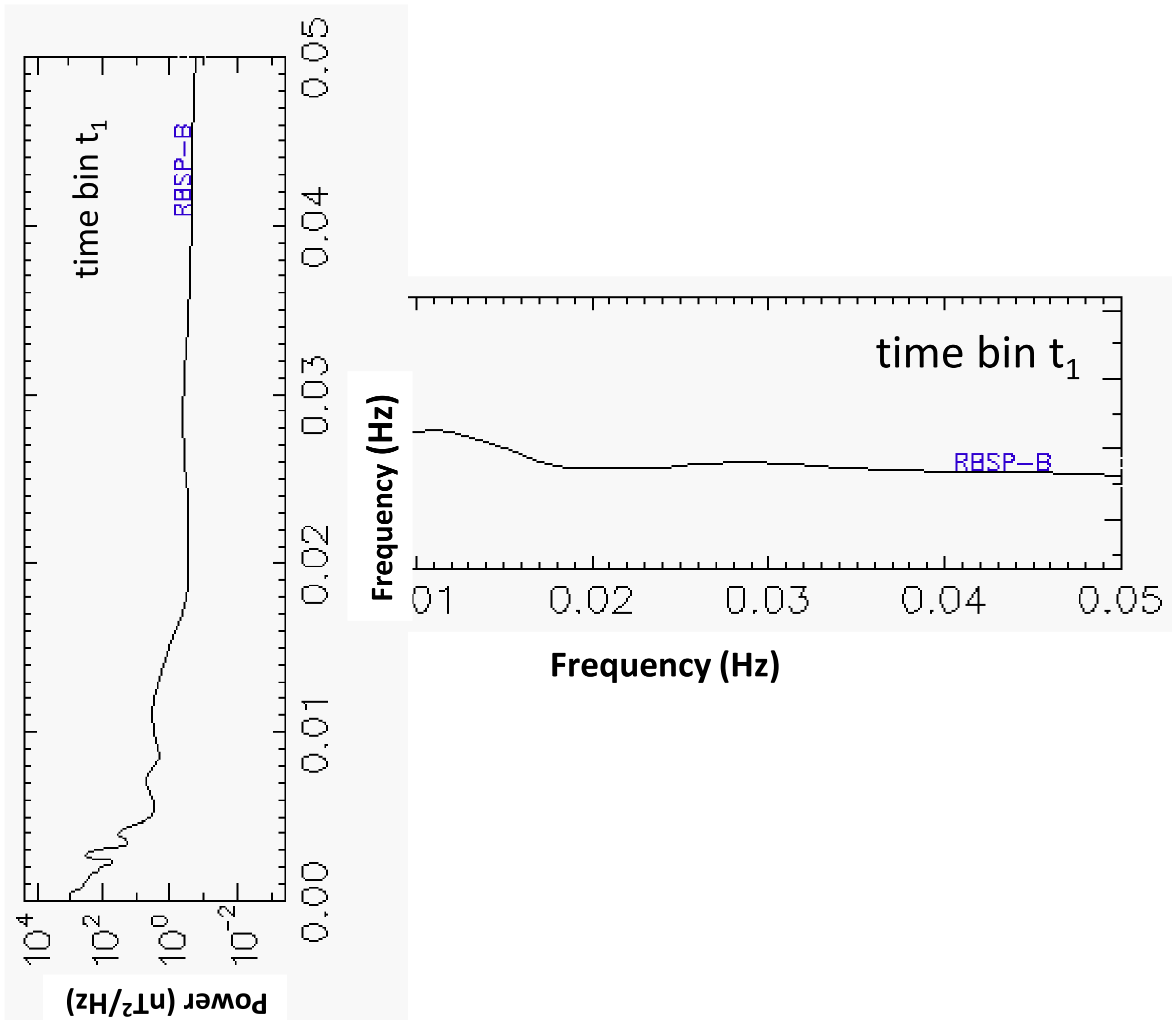
Measurements: Spacecraft L vs. time



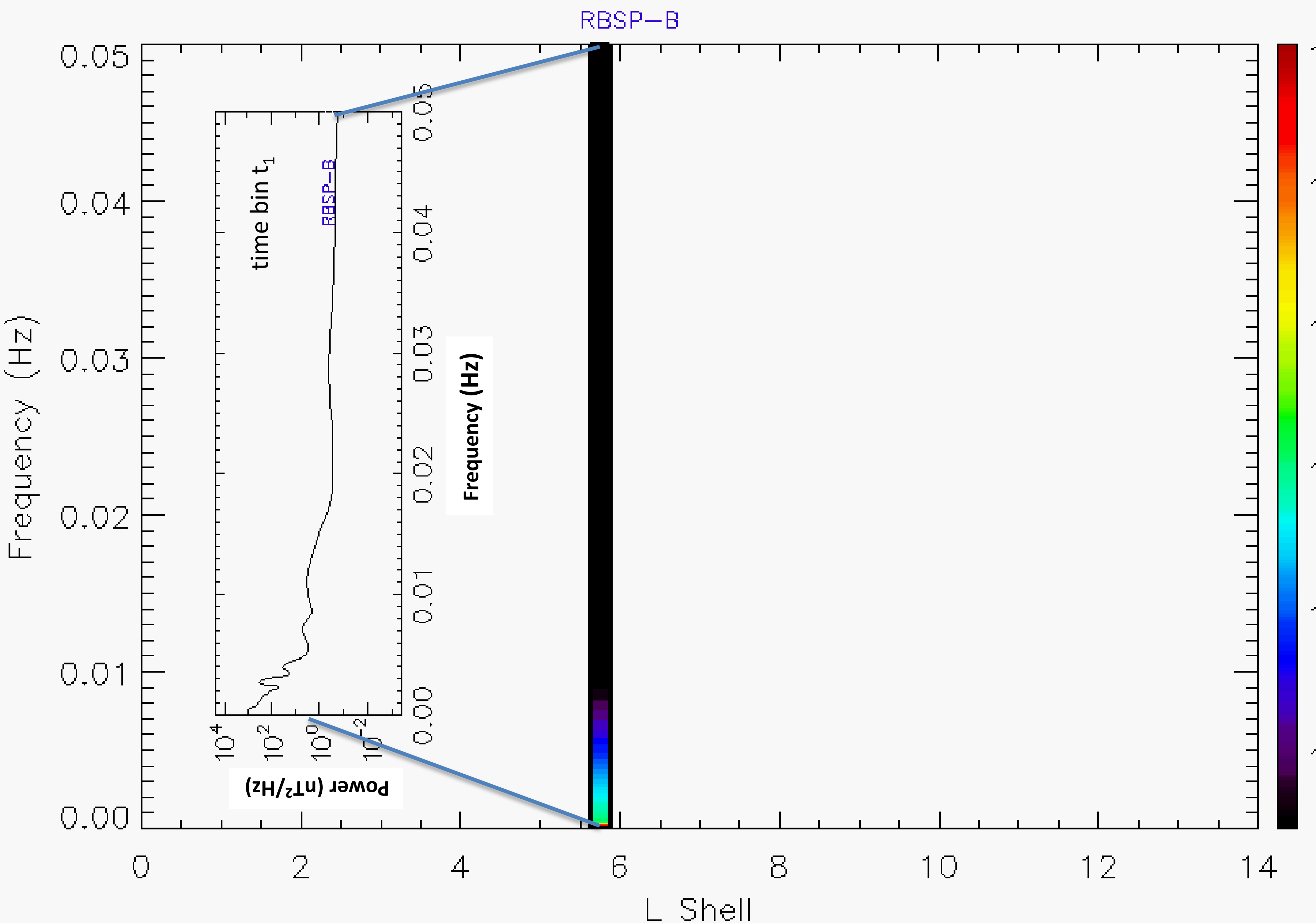
Measurements: Data in L bins and time bins



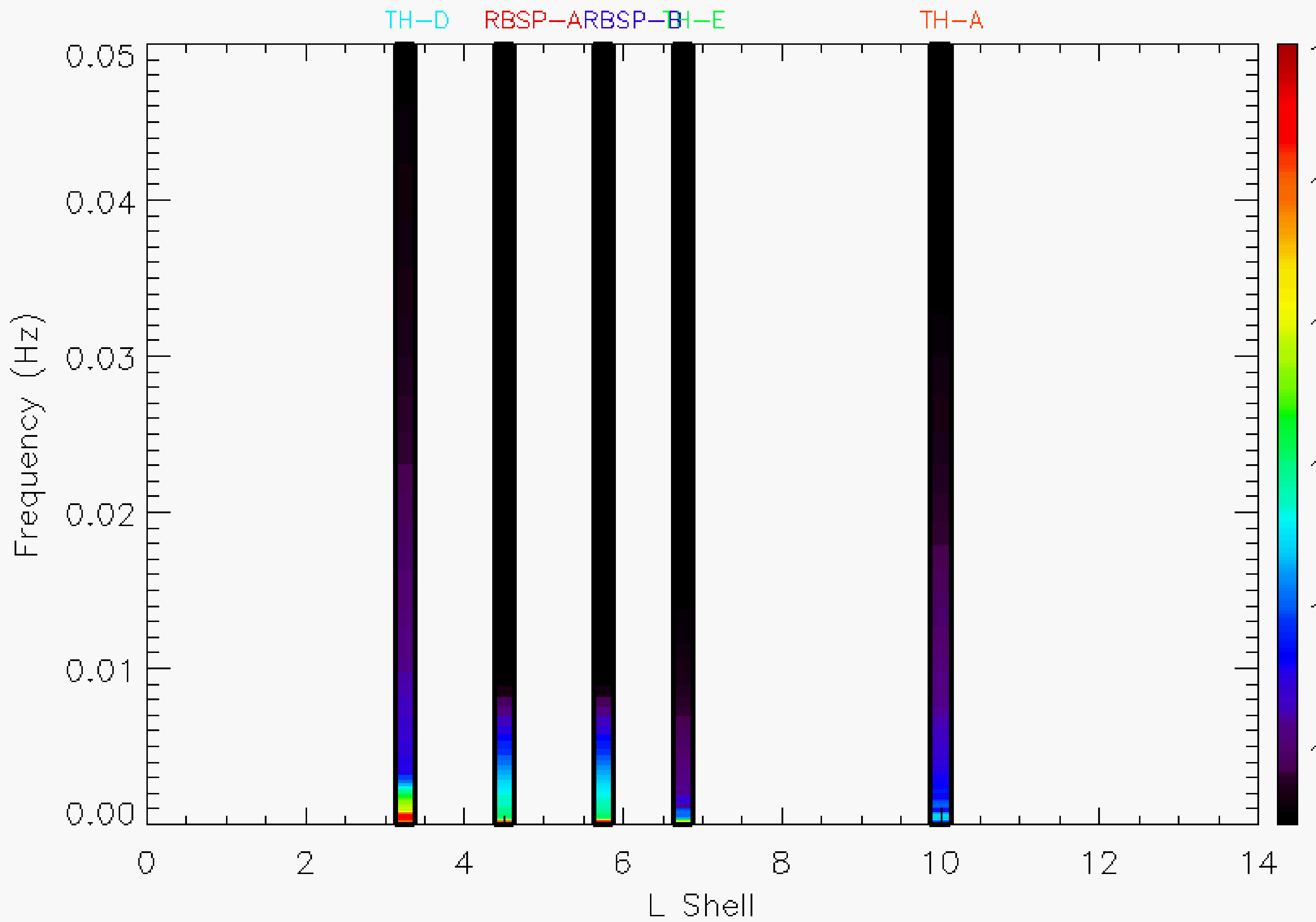
Measurements: Power Spectral Density in one time bin



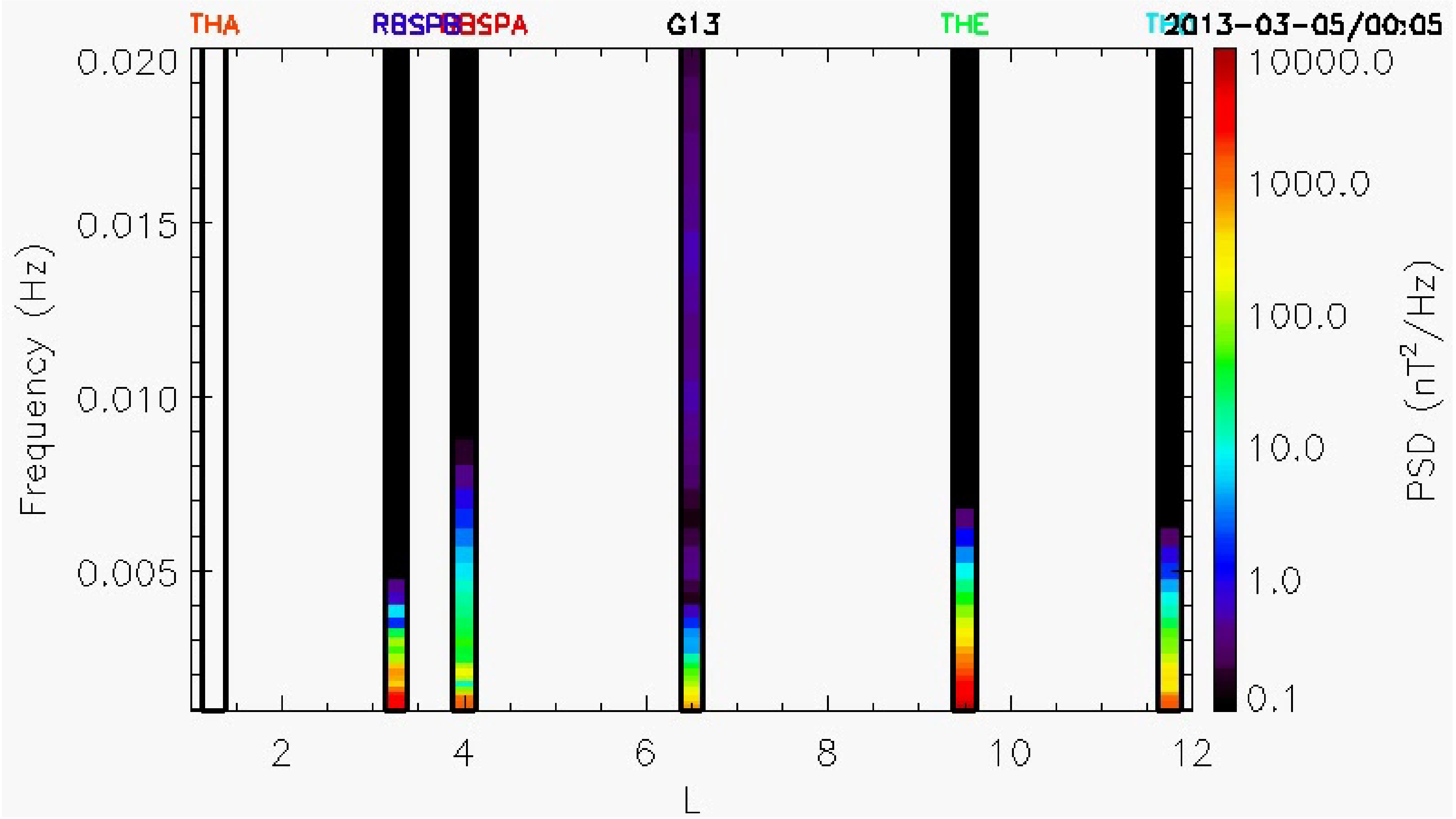
Measurements: Power Spectral Density vs. L and time



Measurements: Power Spectral Density vs. L and time



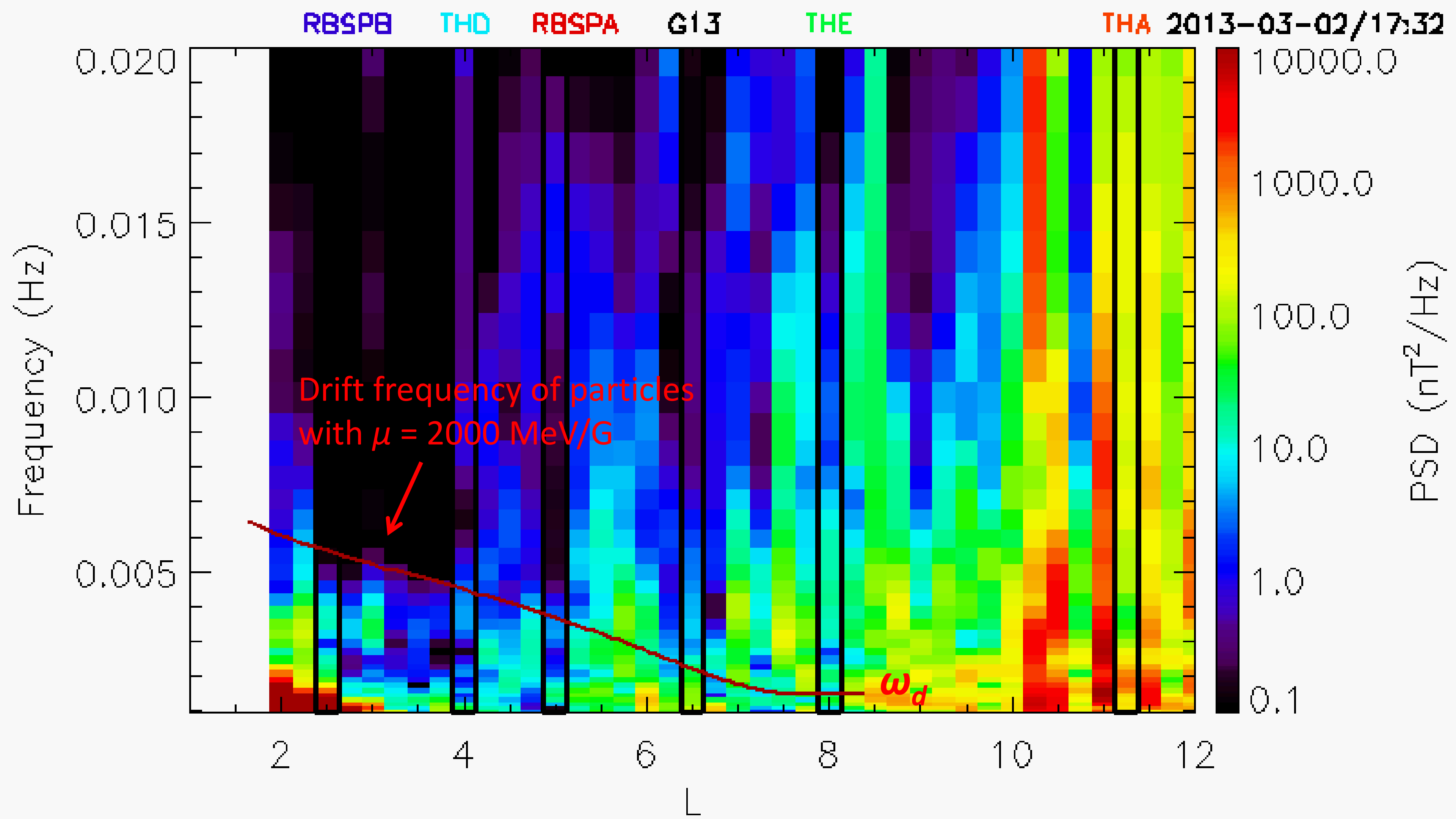
Measurements: Power Spectral Density vs. L and time



Calculations of D_{LL}^{B} :

Power Spectral Density at $m\omega_d$ is needed

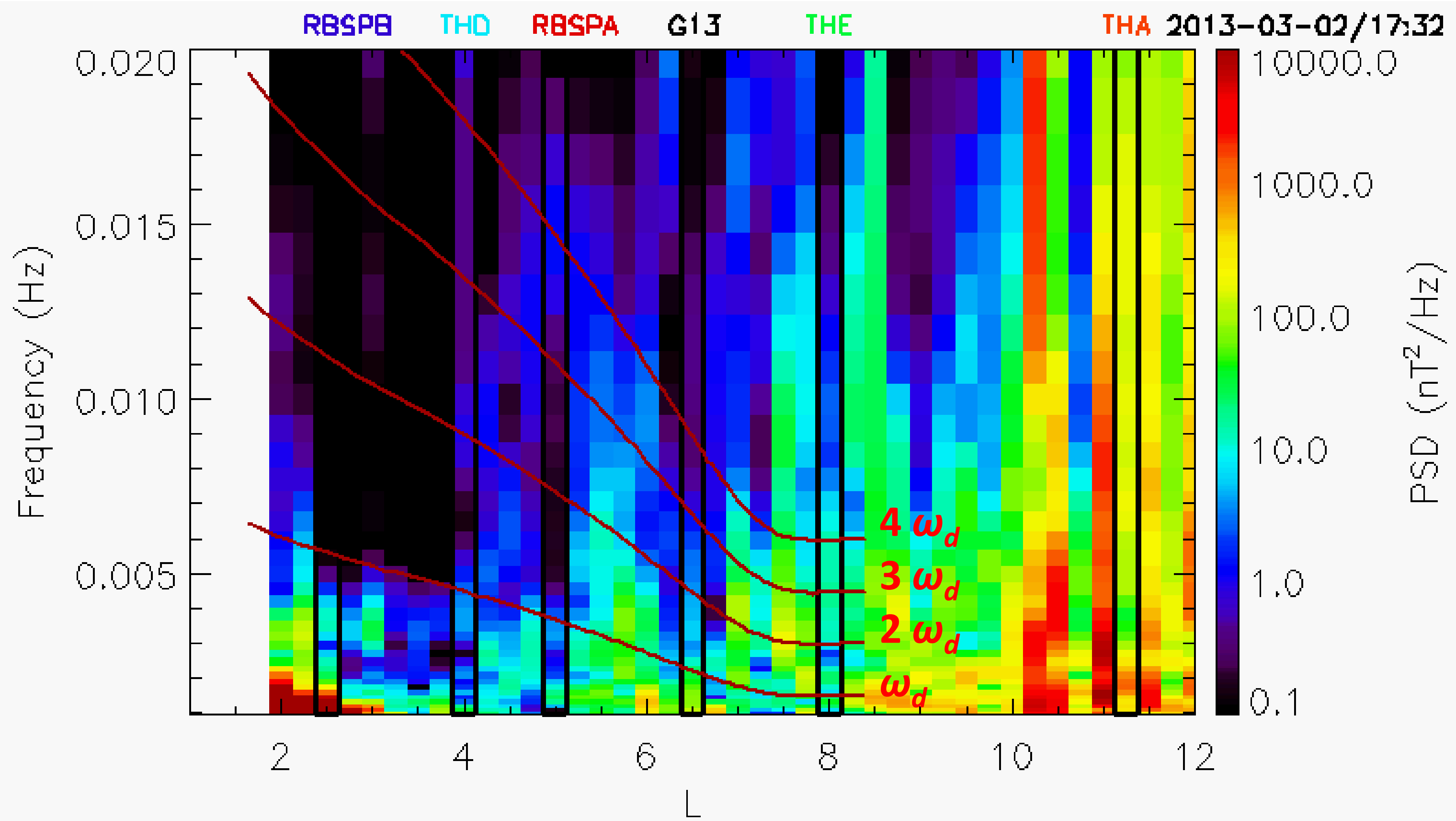
$$D_{LL}^{B,Sym} = \frac{\mu^2}{8q^2 B_E^2 R_E^4} \frac{L^4}{\gamma^2} \sum_m m^2 P_m^B (m\omega_d)$$



Calculations of D_{LL}^{B} :

Correct approach: Sum power in frequencies $m\omega_d$

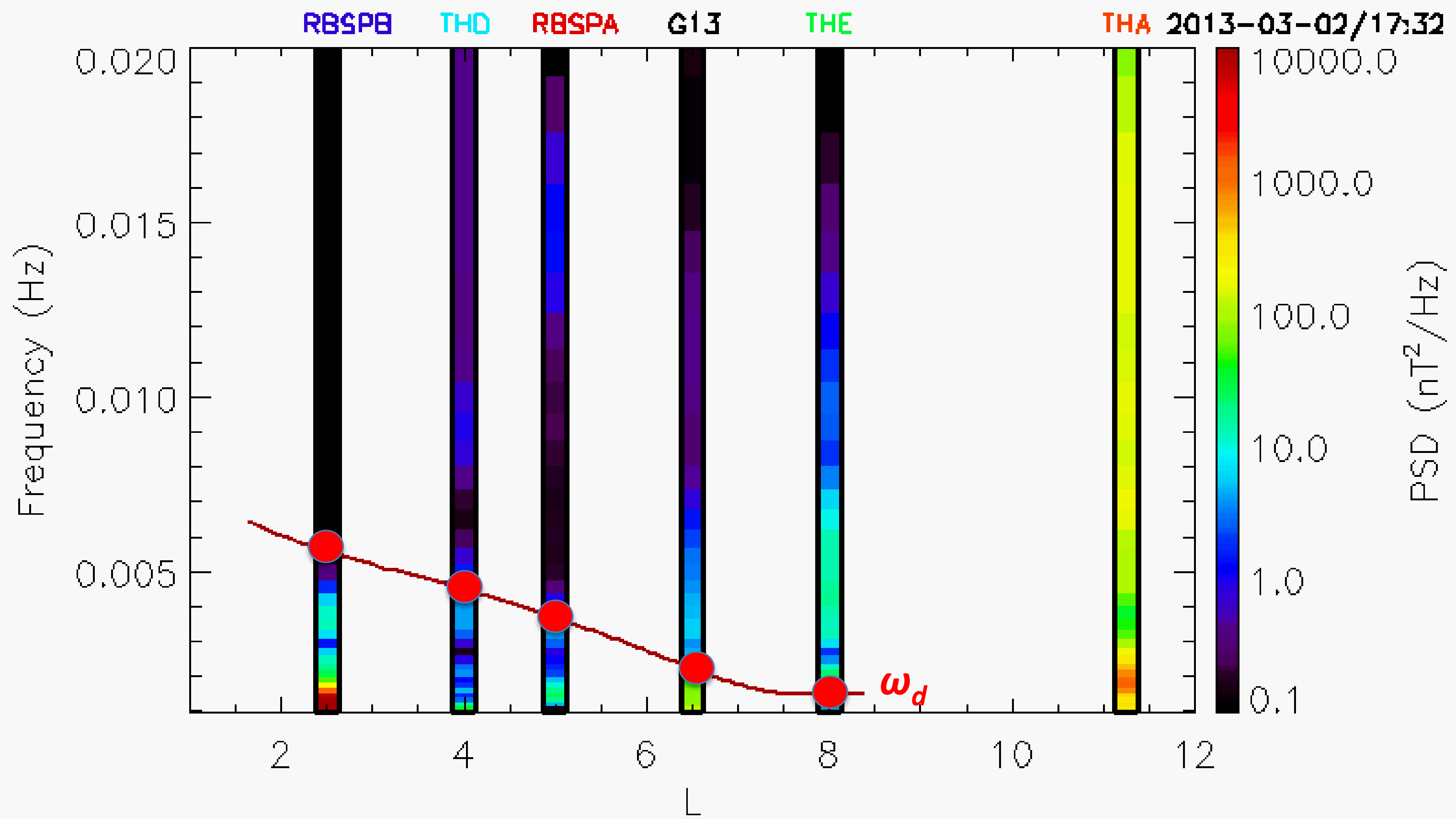
$$D_{LL}^{B,Sym} = \frac{\mu^2}{8q^2 B_E^2 R_E^4} \frac{L^4}{\gamma^2} \sum_m m^2 P_m^B(m\omega_d)$$



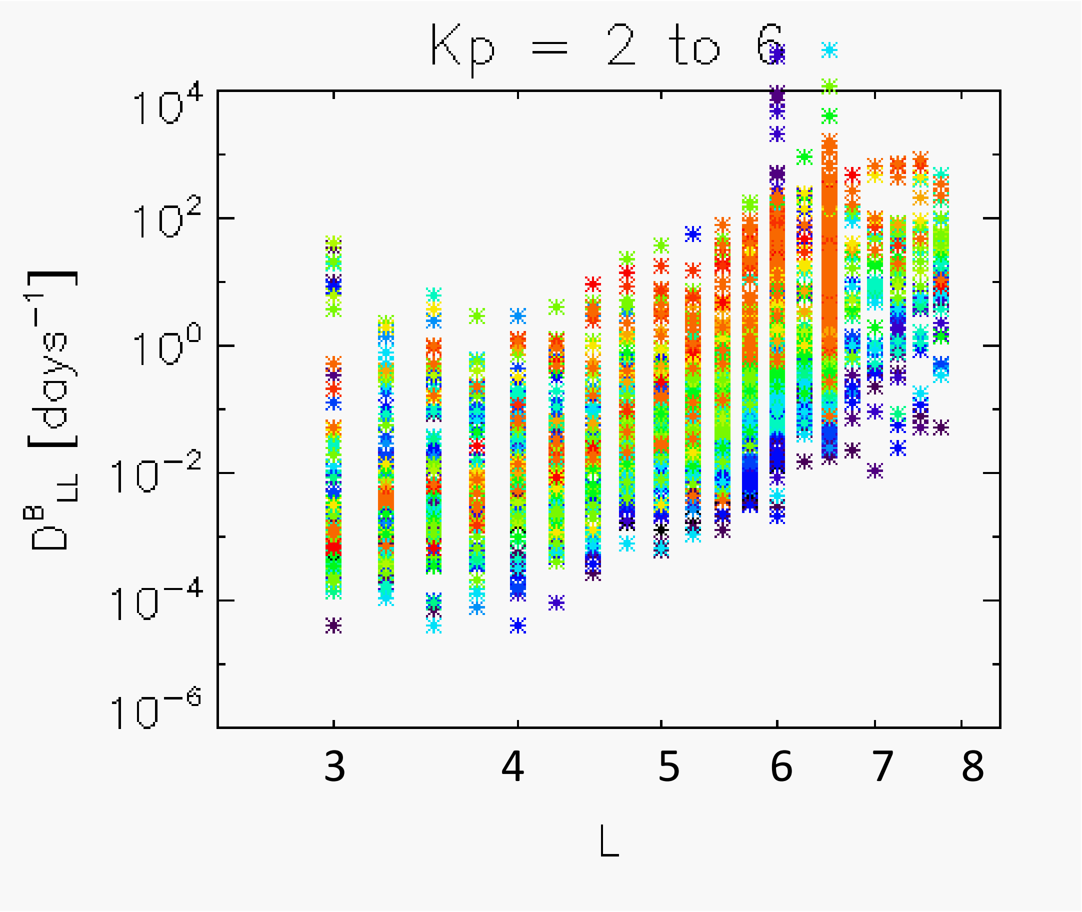
Calculations of D_{LL}^{B} :

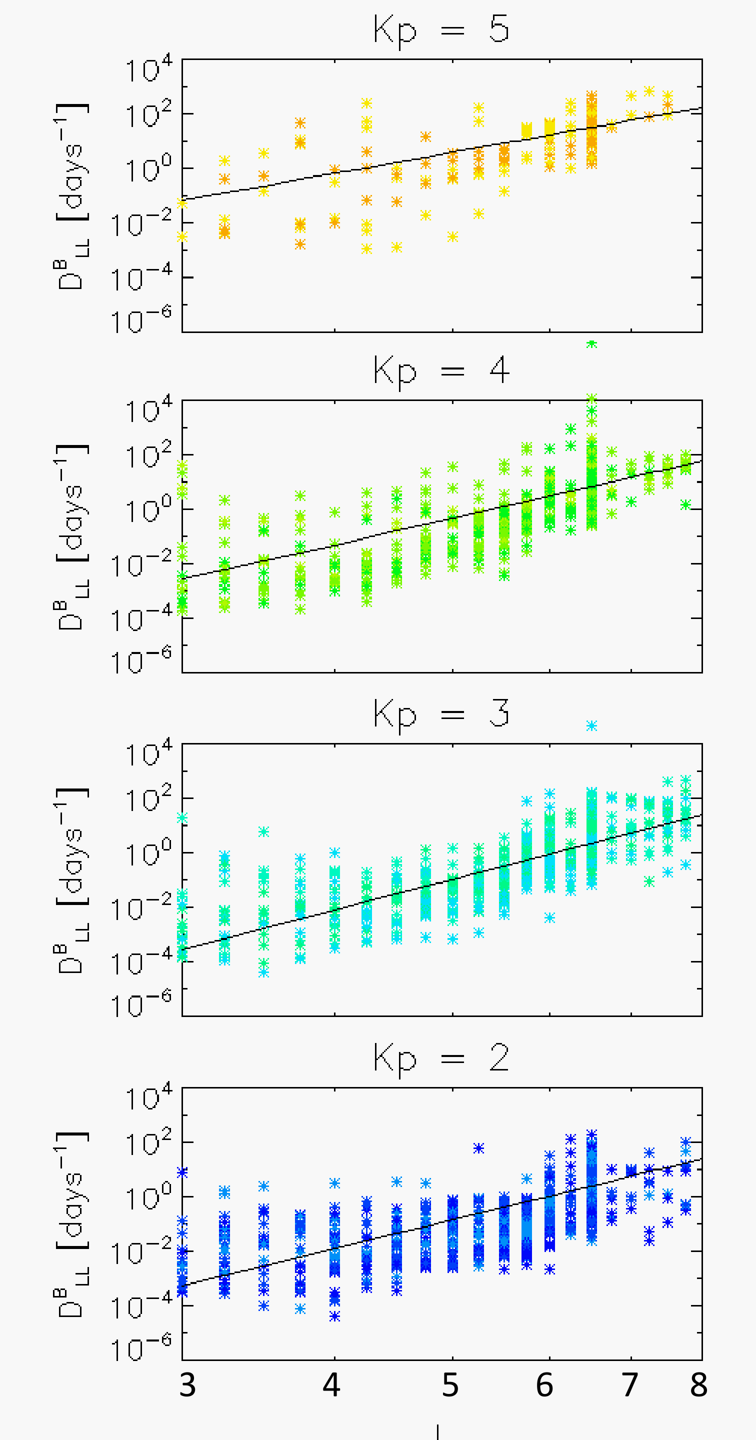
Common approach: assume all power is in m = 1

$$D_{LL}^{B,Sym} = \frac{m^2}{8q^2 B_E^2 R_E^4} \frac{L^4}{g^2} P_{m=1}^{B}(w_d)$$



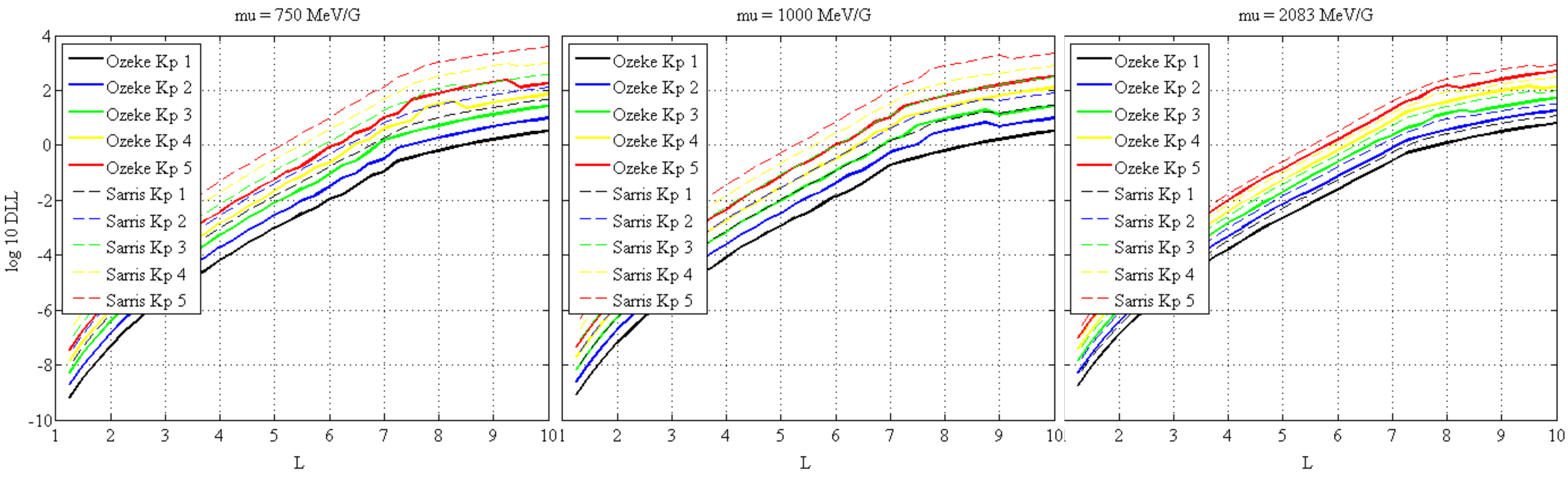
Calculations of D_{LL}^B :





New Radial Diffusion Coefficients:

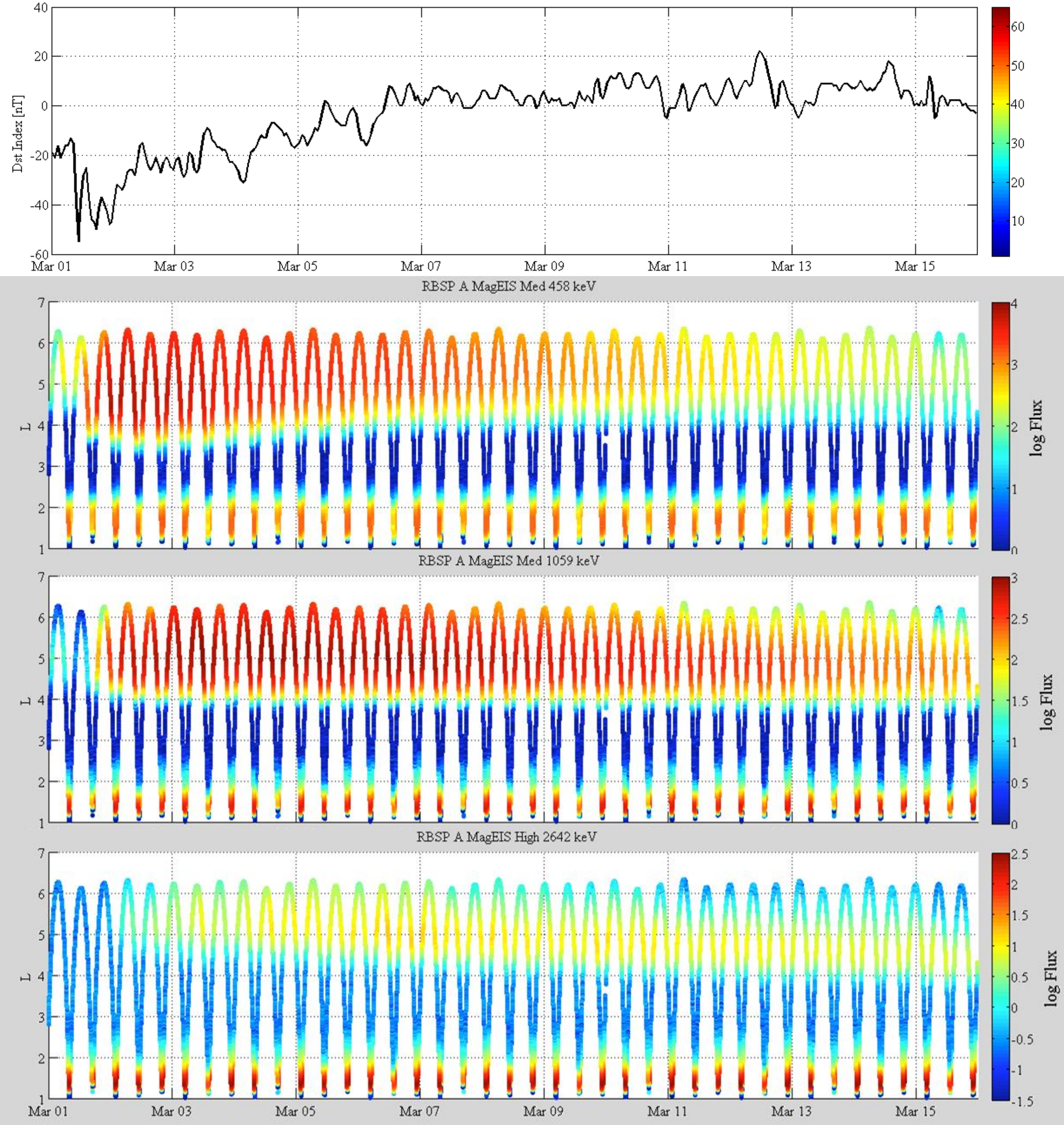
DLL comparison for Ozeke (2012) and Sarris (2015) for mu=750, 1000, 2083 MeV/G



Case Study: Simulations of Radial Diffusion by ULF Waves

Data Assimilation of Phase Space Densities using various D_{LL}

Energetic Electron Flux Measurements:



New Radial Diffusion Coefficients:

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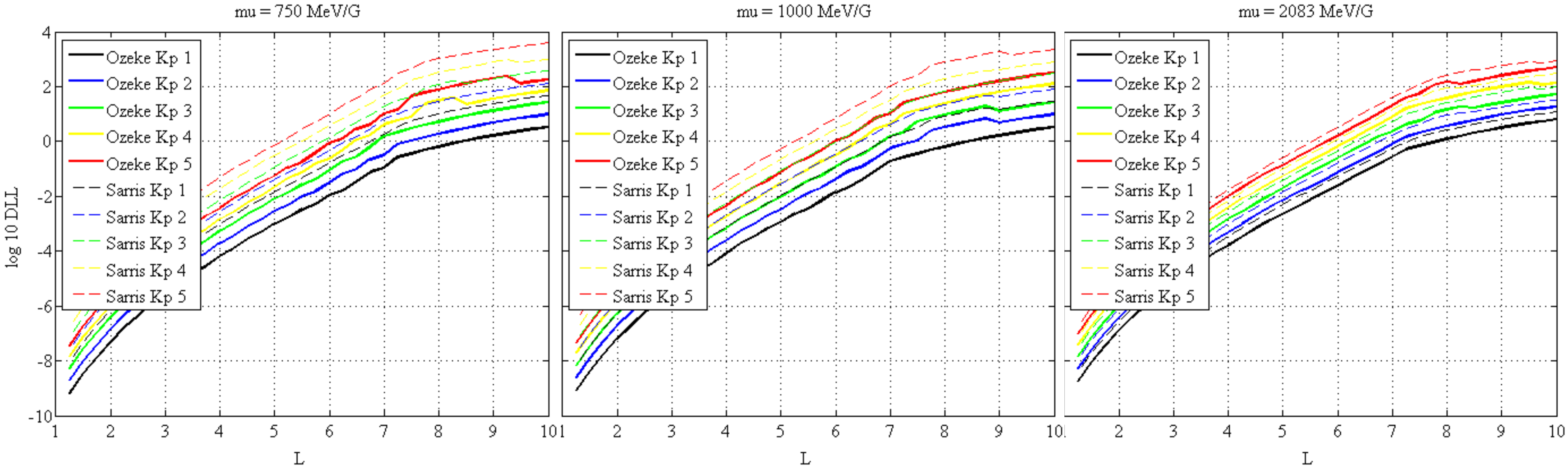
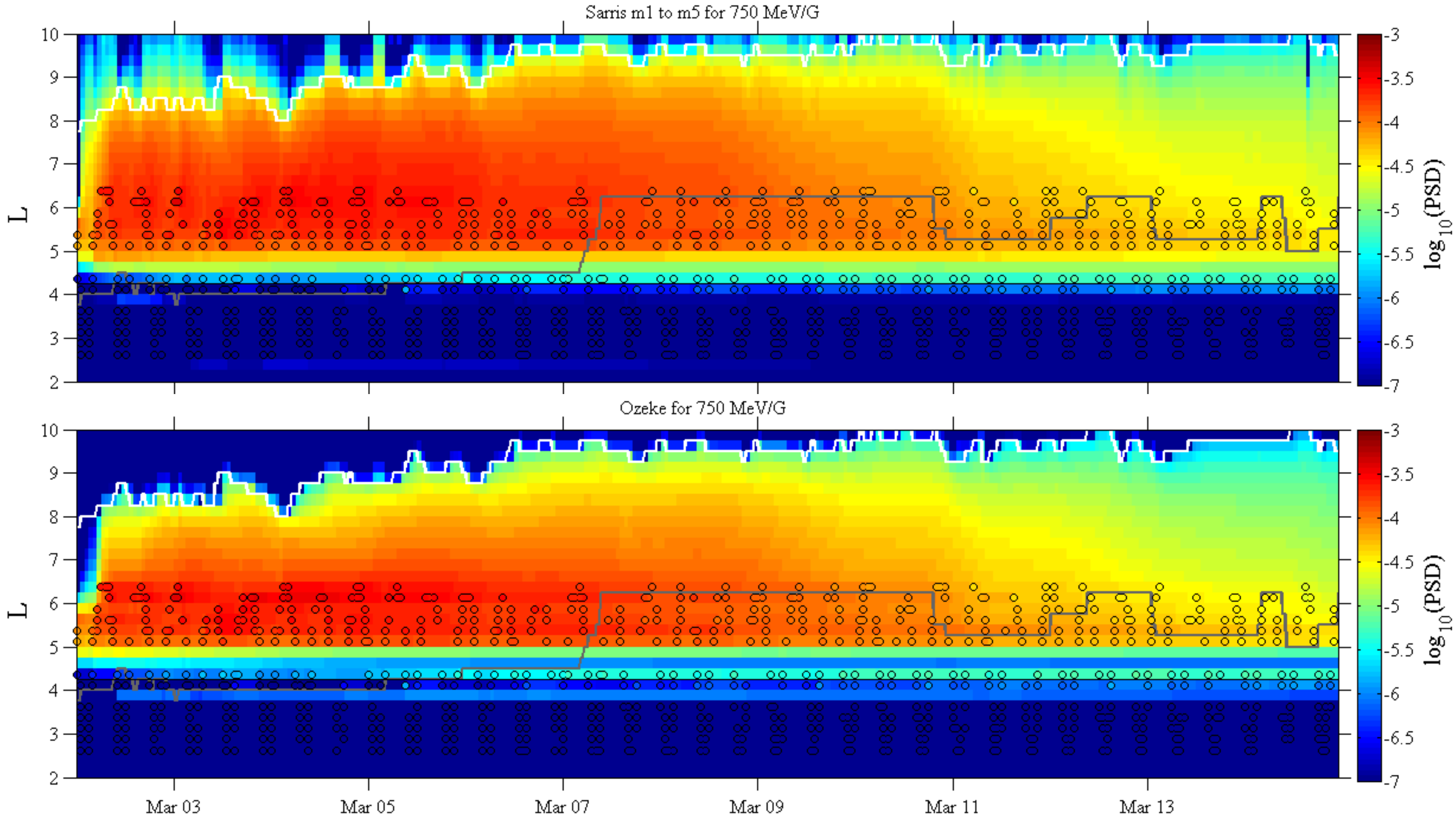
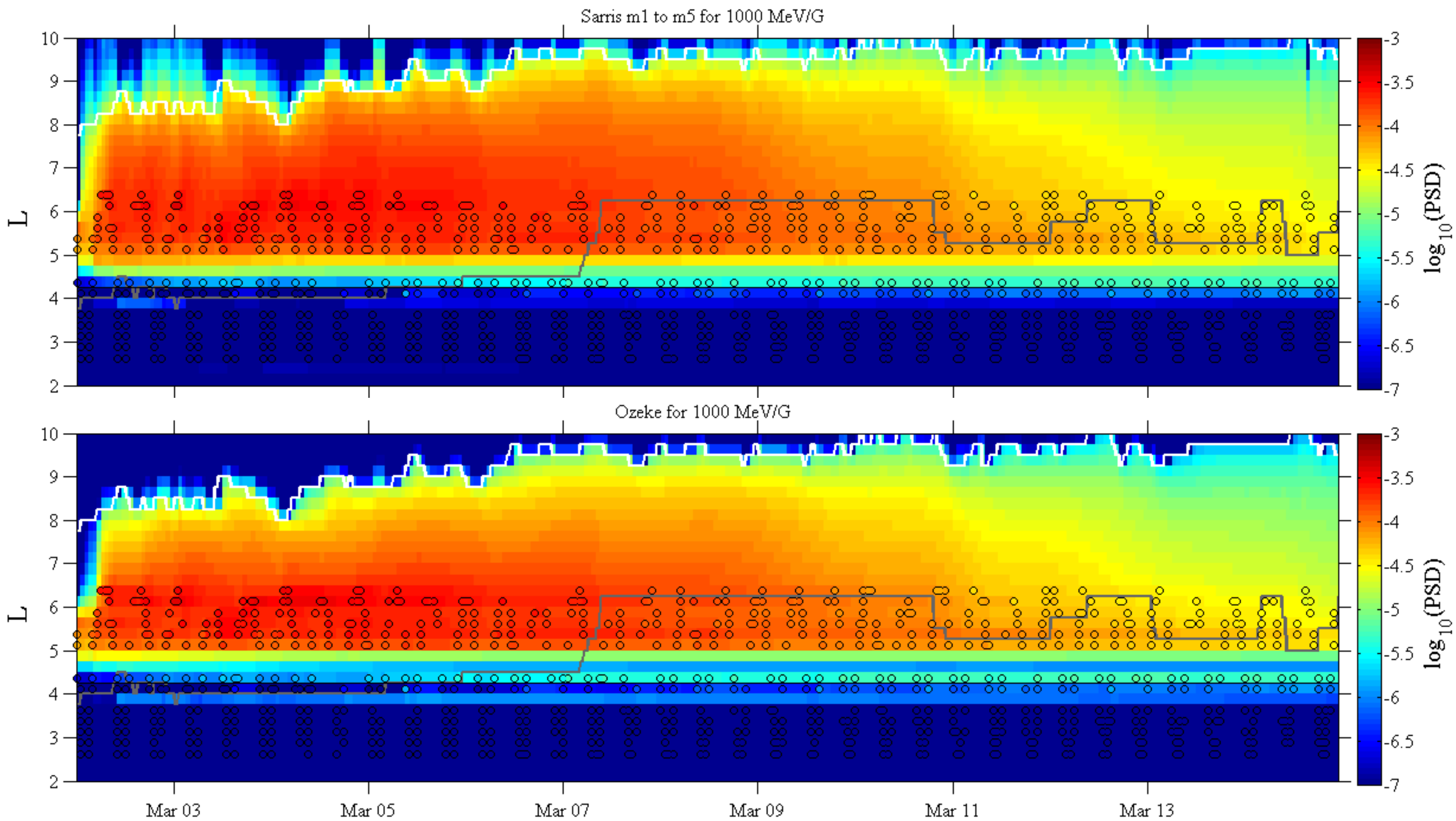


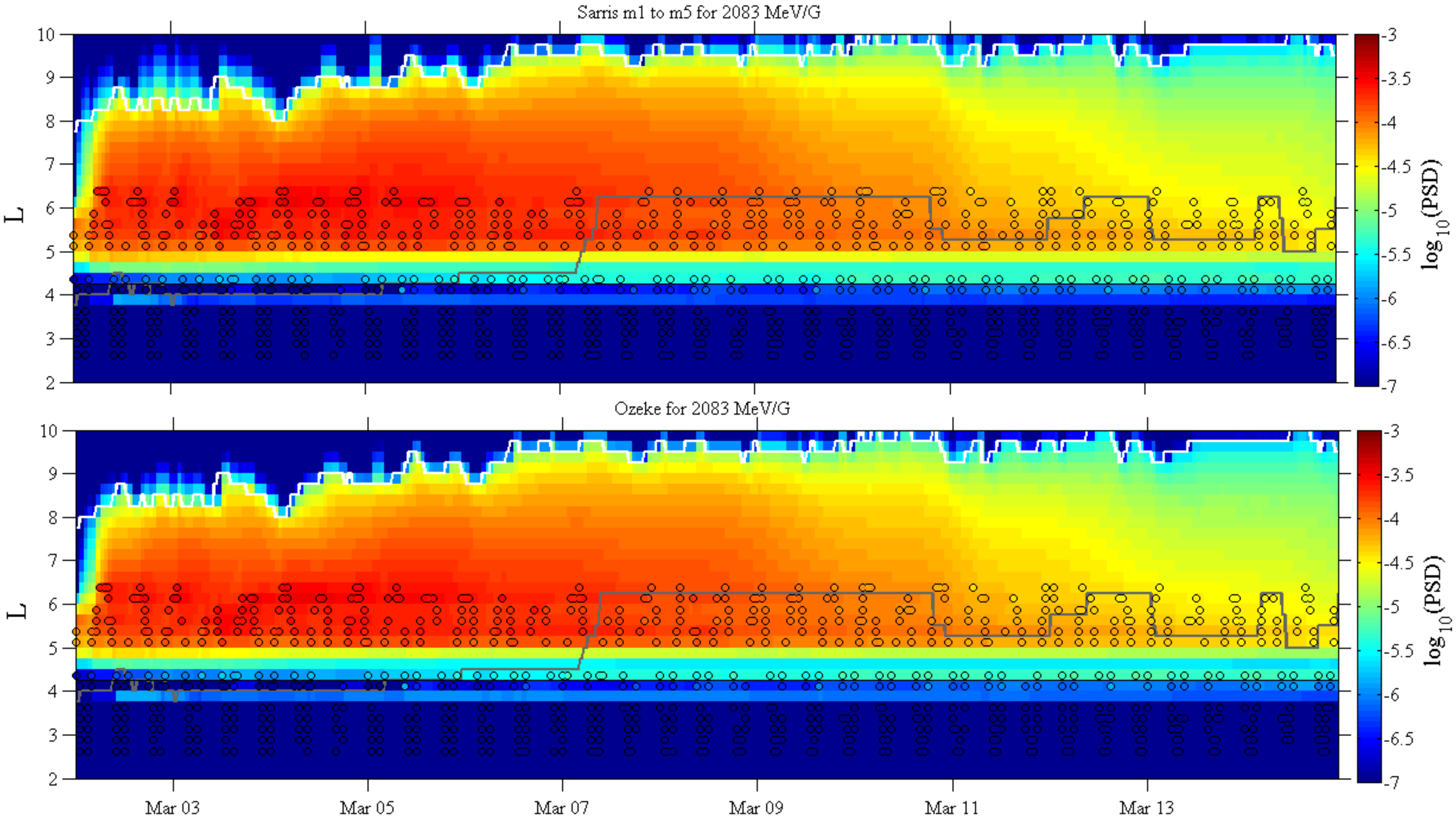
Data Assimilation of Phase Space Density:



Data Assimilation of Phase Space Density:



Data Assimilation of Phase Space Density:



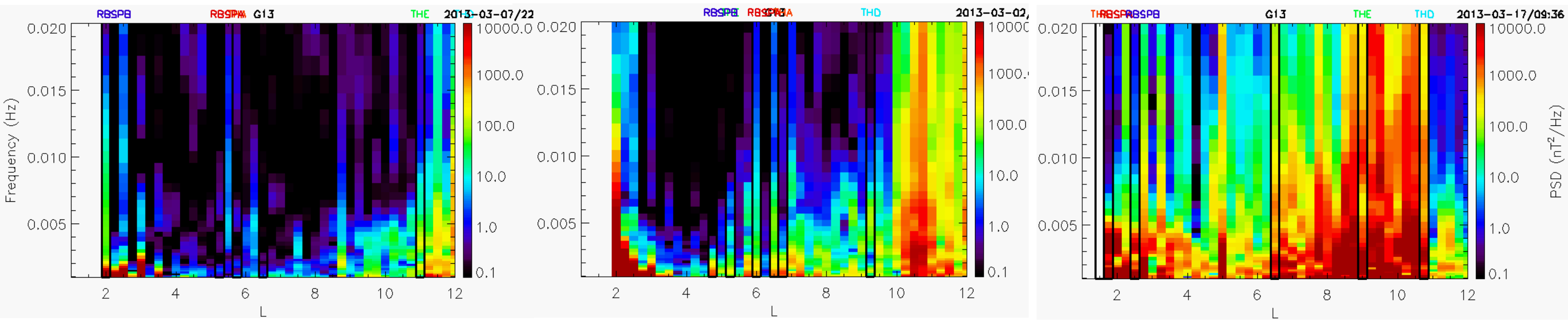
Summary, Conclusions and Next Steps:

- A diurnal variation is observed in the mode structure, with lower mode numbers (larger wavelengths) having more power in the dayside than in the nightside
- A magnetospheric activity dependence is also observed in the mode structure, with lower fractions of ULF wave power per mode number during active times
- Current diffusion coefficients underestimate radial diffusion, particularly during active times
- Maps and empirical relationships of the average ULF wave Power vs. L & Kp need to be constructed from Van Allen probes, THEMIS, GOES
- More accurate Diffusion Coefficients can lead to better estimates of Sources and Losses in the Radiation Belts

Measurements: Power Spectral Density vs. L and time

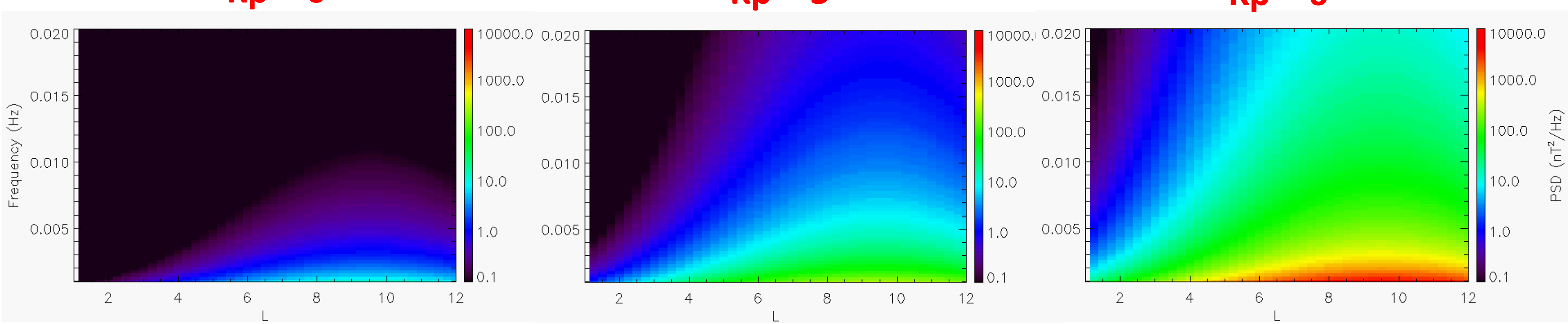
Kp dependence:

Kp = 0 **Kp = 3** **Kp = 6**



- Next step: produce maps and empirical functions of average power vs. L, frequency and Kp

Kp = 0 **Kp = 3** **Kp = 6**



- Existing attempt, by Ozeke et al 2012: based on THEMIS, 2007-2011; greatly underestimates power at high Kp

Background: Radial Diffusion in Phase Space Density