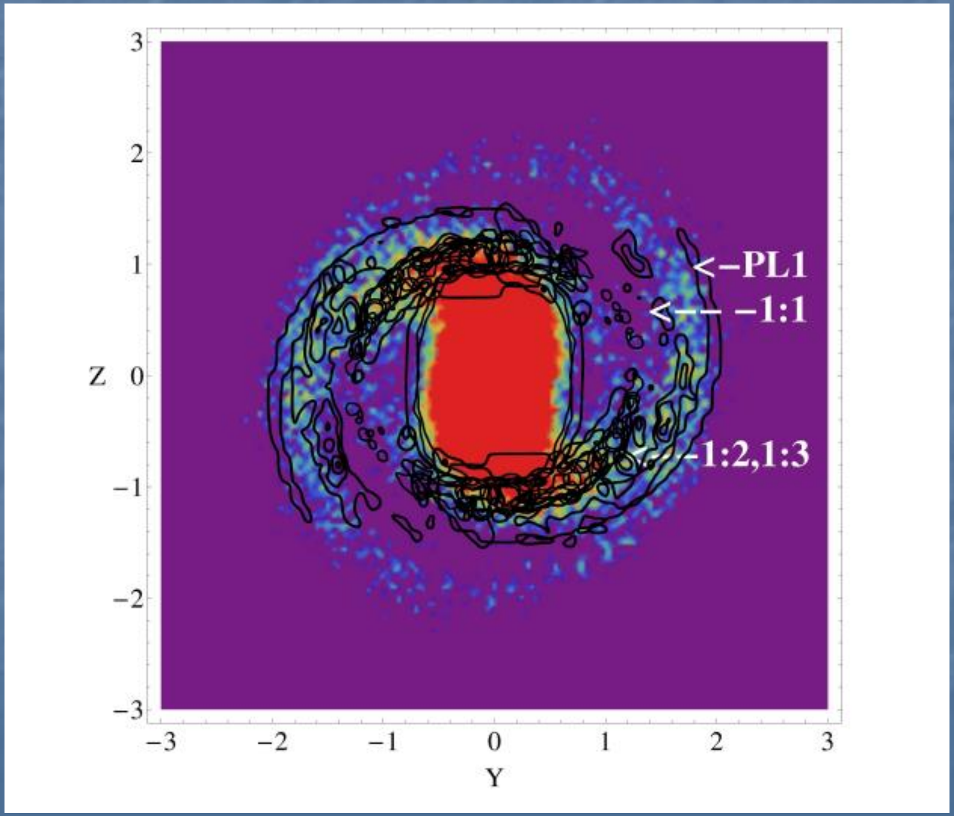
Convergence regions of the Moser normal forms and the structure of chaos



G. Contopoulos and M. Harsoula (MNRAS 2013)

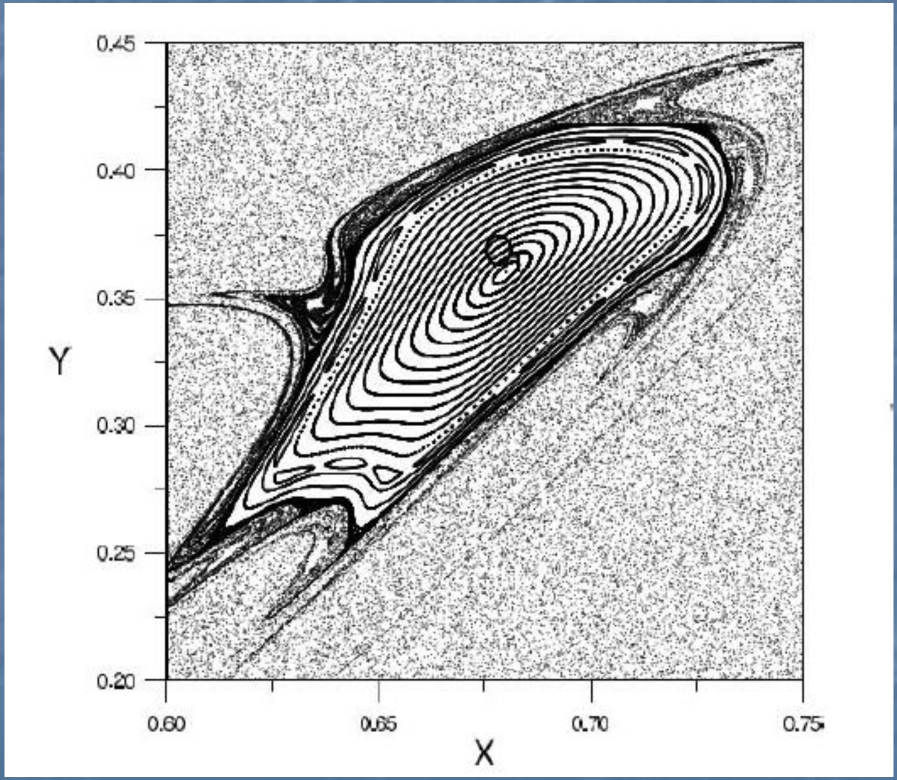
M. Harsoula

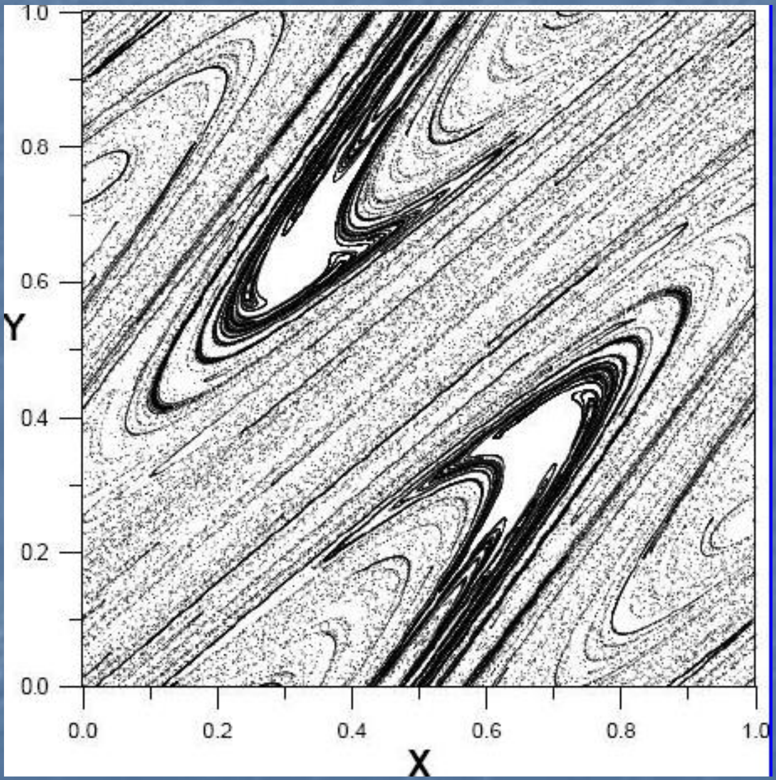
C. Efthymiopoulos

G. Contopoulos

RCAAM
Academy of Athens

Order and Chaos in Dynamical Systems





Standard map

Analytical formulas for the
“ch … ves

He … *ation*

$x' = ($ …

$y' = s$ …

… $)$

… $T]$

Taylor Series
expansion around
unstable periodic p…
(x0,y0)

…ical form

… $_3(u,v) + \ldots$

… $_3(u,v) + \ldots$

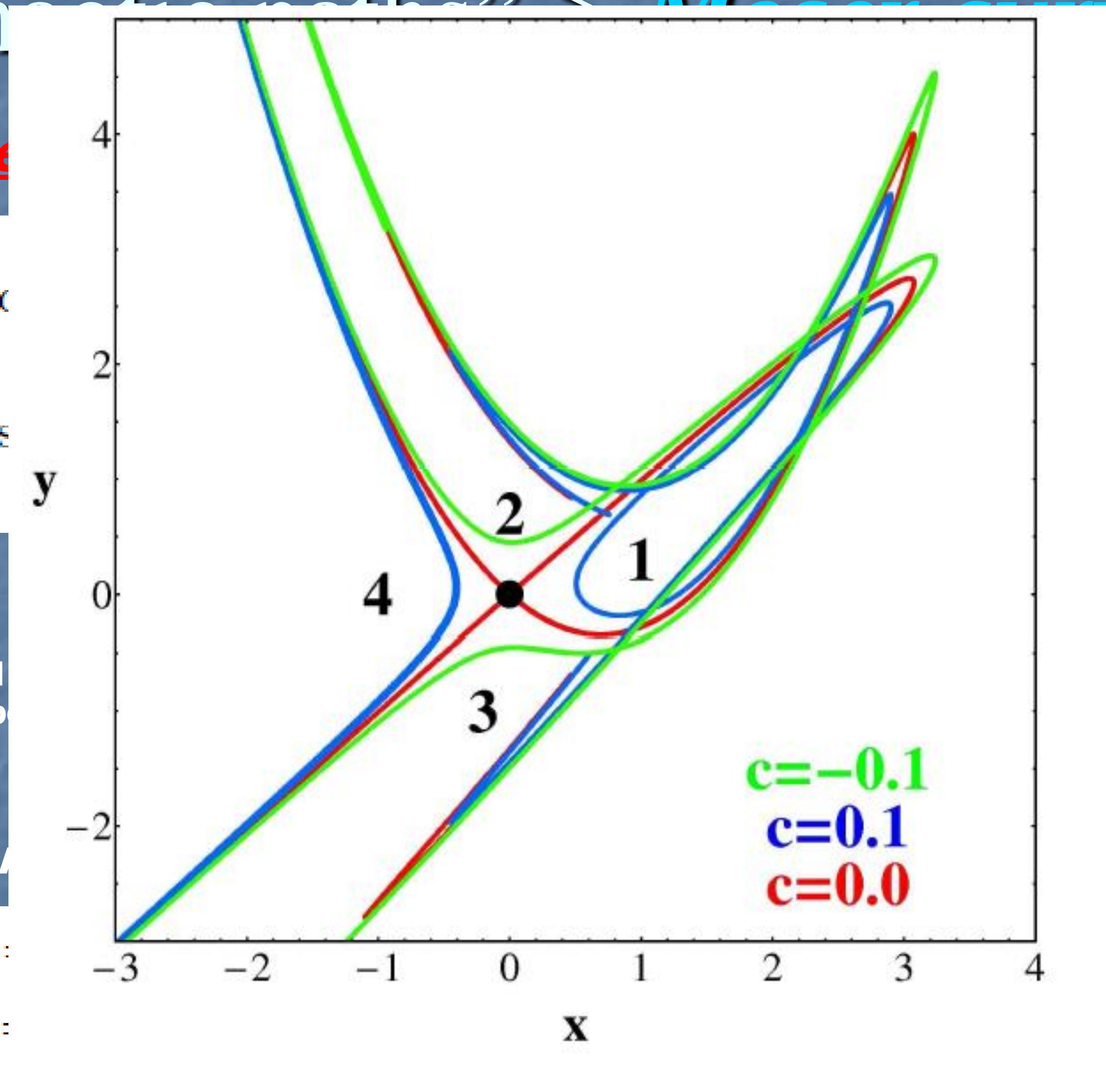
… $= v = 0).$

near-identity ca…

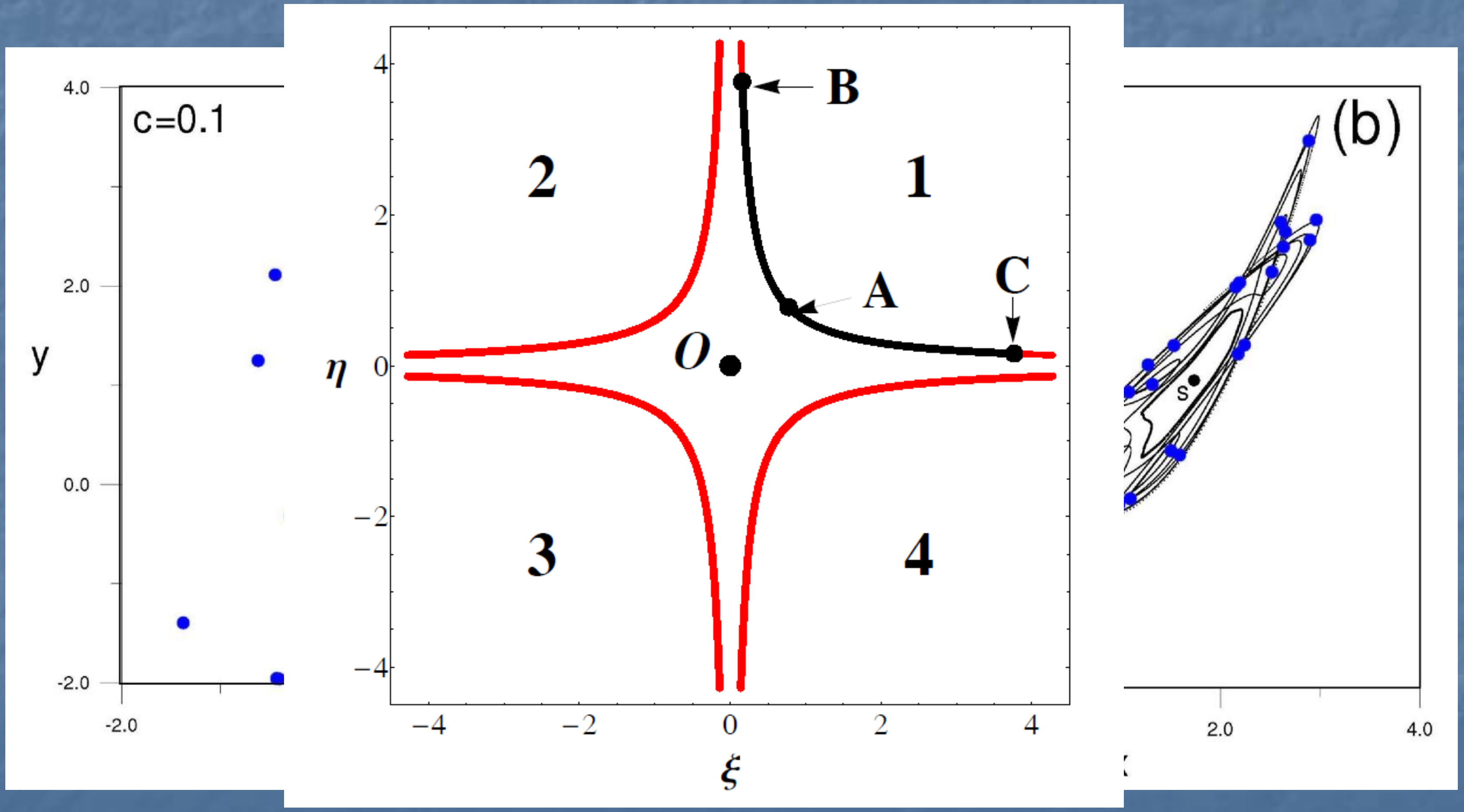
$u = \Phi_1(\xi,\eta) =$ … $w_2c + w_3c^2 + \ldots)\xi$

$v = \Phi_2(\xi,\eta) =$ … $+ v_2c + v_3c^2 \ldots)\eta$

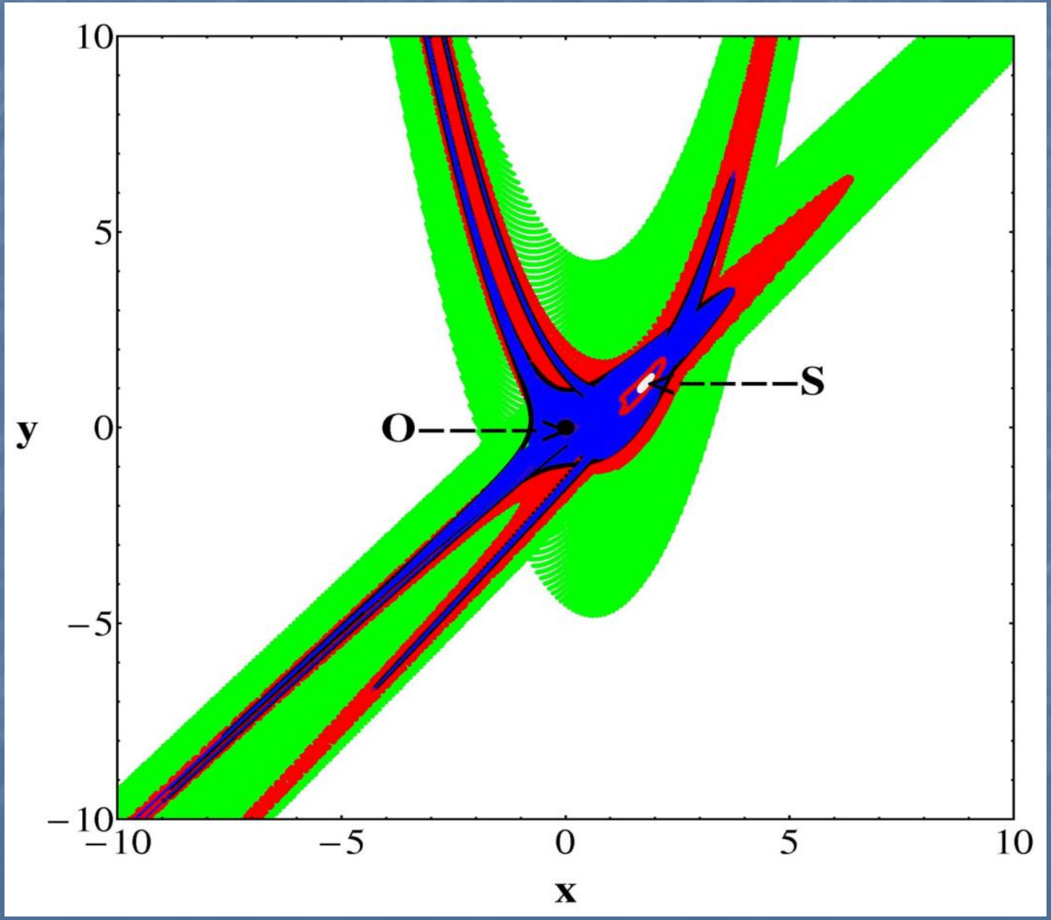
… $\eta;$



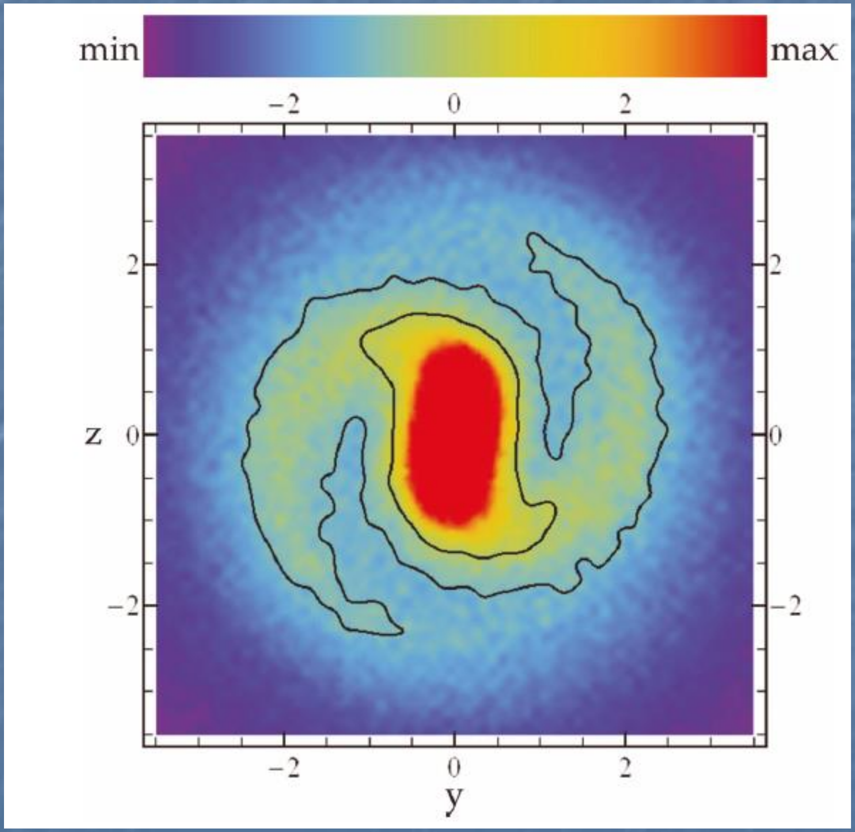
“The Road of Chaos”



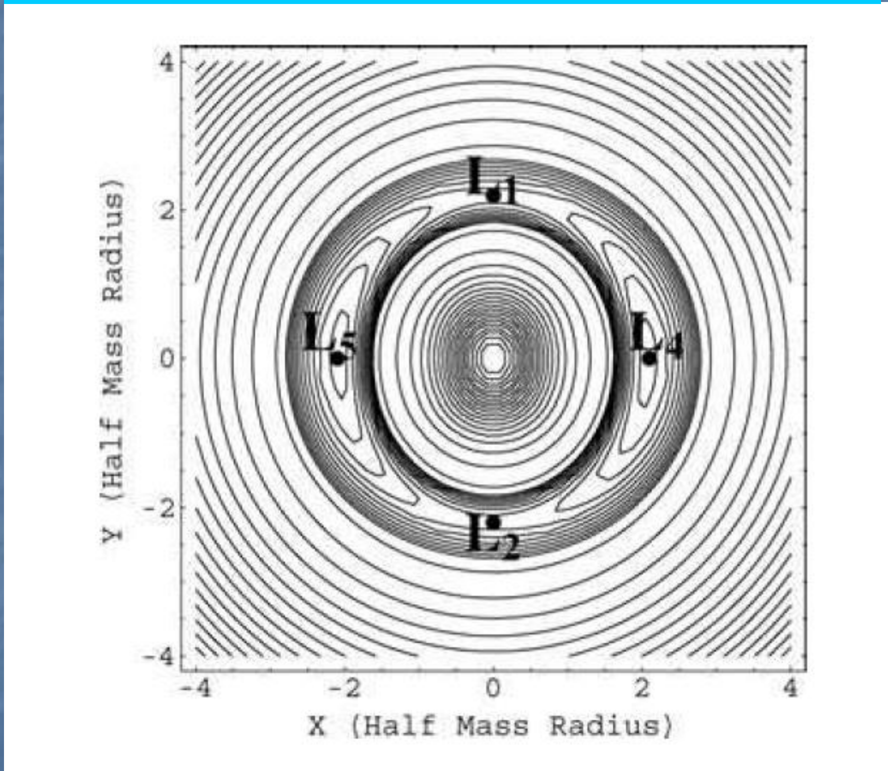
Moser Region of Convergence



Application in barred-spiral galaxies



equipotentials



Averaged Hamiltonian - Mapping

Hadjidemetriou Method

$$H=\frac{P_r^2}{2}+\frac{P_\varphi^2}{2r^2}-\Omega_P P_\varphi+\Phi(r,\varphi) \quad (1)$$

(φ, P_φ) action-angle variables

$$\Phi(r,\phi)=\Phi_0(r)+\Phi_1(r)\cos(2\phi)+\Phi_2(r)\sin(2\phi)$$

Epicyclic analysis around corotation and Taylor expansion

Normal form construction up to 2nd order in actions

$$H=Z(P_\phi, J_r, \phi)+\mathrm{O}(J^3,\ldots)$$

Epicyclic action angle variables

$$dr=r-r_c=\sqrt{\frac{2J_r}{\kappa_r}}\sin\phi_r$$

$$P_r=\sqrt{2k_r J_r}\cos\phi_r$$

(φ_r, J_r) action-angle variables

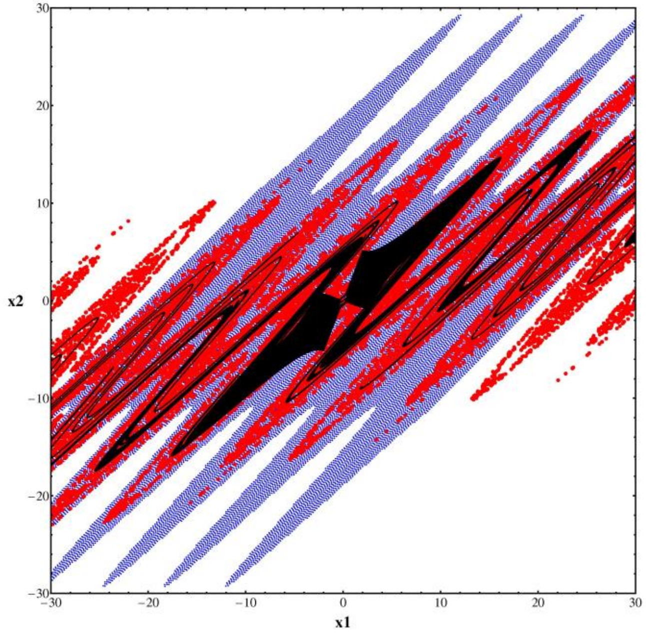
Canonical transformation via Lie series-> eliminating fast angle φ_r!

Convergence regions in barred-spiral galaxies

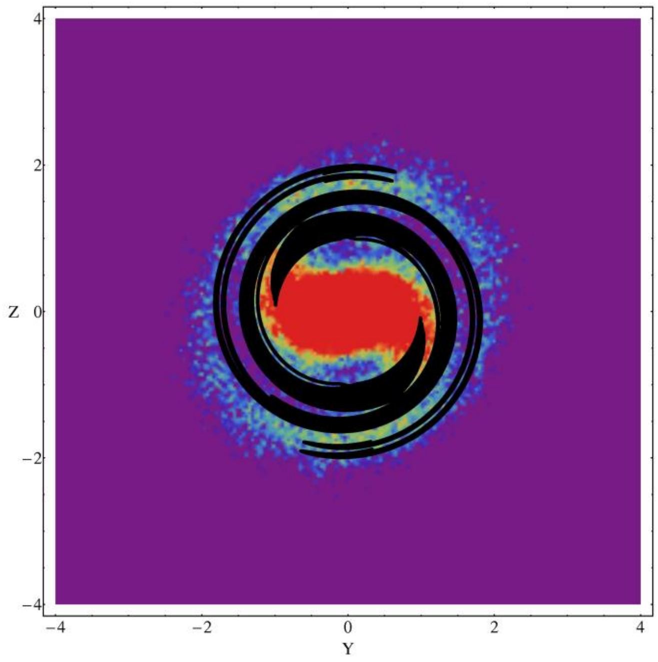
Averaged mapping :
Standard map

$$x' = x + y + K \times \mathrm{Sin}[x]$$

$$y' = y + K \times \mathrm{Sin}[x]$$



Convergence region
in configuration space:
Spirals

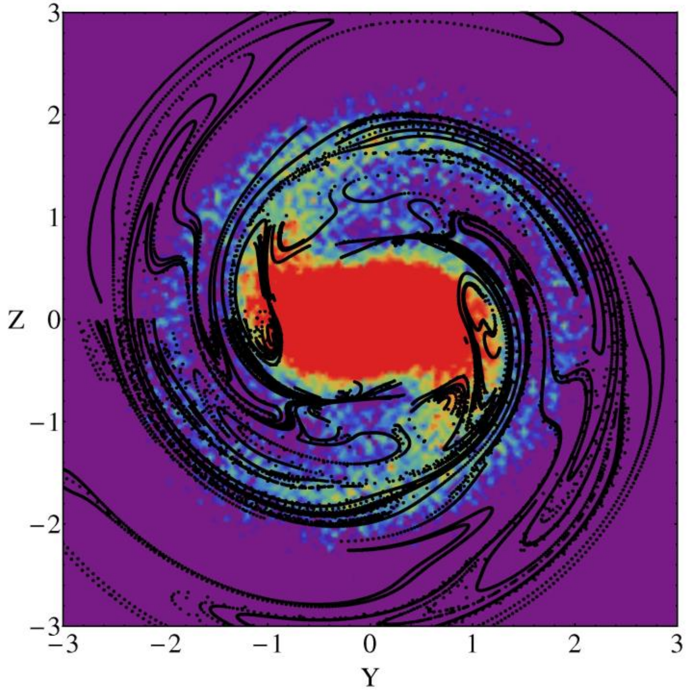


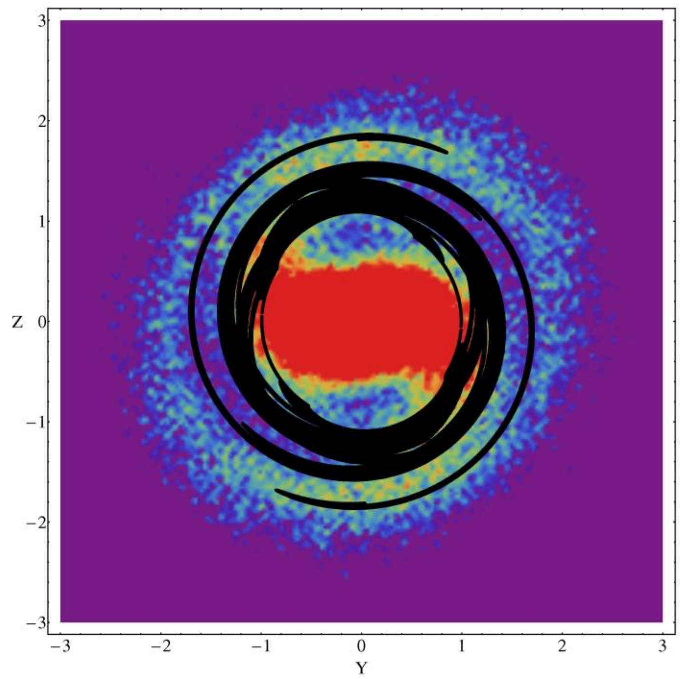
Numerical versus analytical results

Manifolds of the apocenters of the periodic orbit PL1

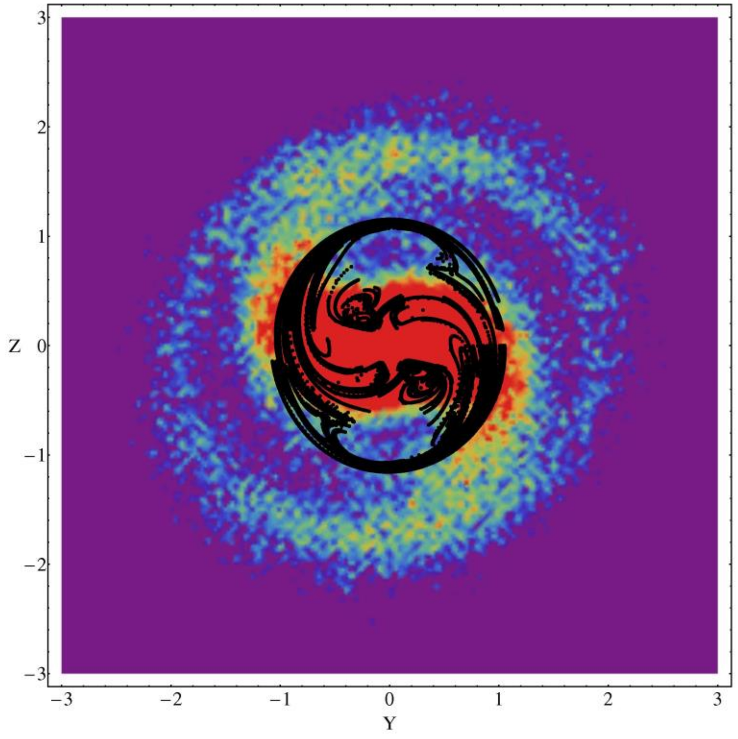
Voglis, N. Tsoutsis, P. Efthymiopoulos, C.

2006-2008

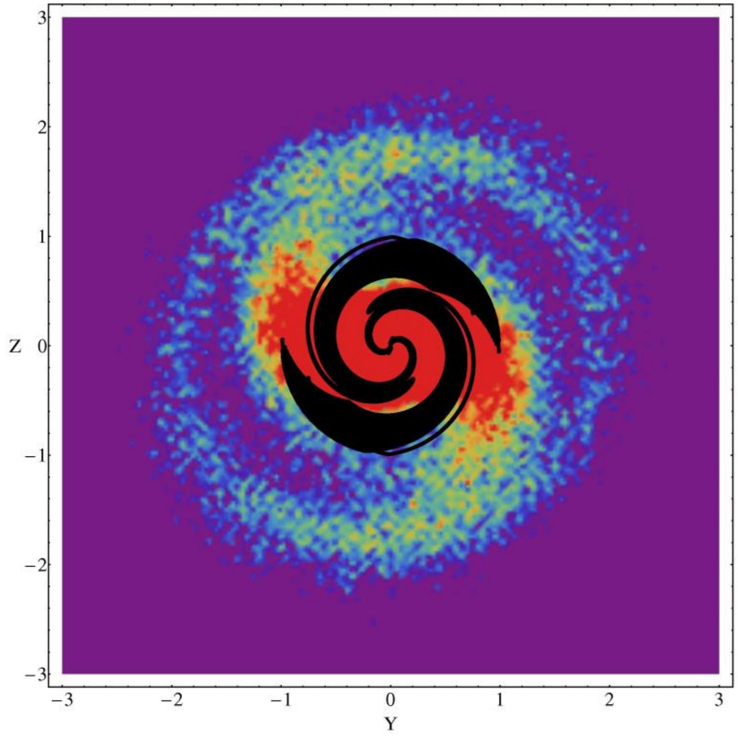




Manifolds of the pericenters of the periodic orbit PL1



Analytical convergence regions of the pericenters



Conclusions

- *Moser invariant curves* **determine the chaotic paths and the structure of chaos in dynamical systems around a hyperbolic fixed point via** *analytical convergent series*

- *The Moser region of convergence* **is a kind of attractor of all chaotic orbits and it does not communicate with the outer region of the phase space**

- **The Moser region of convergence has very interesting applications in barred-spiral galaxies explaining the stickiness and the very slow diffusion of the** *chaotic orbits***. It also explains the spiral shape of the arms.**

- **An interesting talk of the use of hyperbolic invariant manifolds in astrodynamics will be given by Dr. C. Efthymiopoulos on Wednesday "***Theory and applications of hyperbolic invariant manifolds in astrodynamics***"**