Neutron stars: Are they bald like black holes?

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12th Hellenic Astronomical Conference: June 28-July 2, 2015

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The (mathematical) perfection of black holes

- Black holes are the simplest classical objects that Nature itself can create. They have no internal structure.
- They depend only on two quantities: their mass and their angular momentum $a = S/M$. (2 black holes with the same M and a are completely identical.)
- The spacetime itself around an isolated object is completely described by its multipole moments. For a black hole these moments are simply

$$M_{2k} = M(-a^2)^k \quad , \quad S_{2k+1} = Ma(-a^2)^k$$

$$M_0 = M, \quad S_1 = Ma, \quad M_2 = -Ma^2, \quad S_3 = -Ma^3, \quad M_4 = Ma^4, \ldots \quad \ldots$$

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- Actually exotic matter, the exact behavior of which is not known.
- Two neutron stars with the same mass and rotation could have completely different radius and shape, depending on the actual equation of state of its matter.
- Its multipole moments (therefore the whole spacetime around it) are expected to depend strongly on the EoS (either stiff, or soft, or ...)

Neutron stars are **not** that simple. Are they? [B]

- Practically only the first few (4) moments are essential for the description of the outer spacetime of a neutron star.

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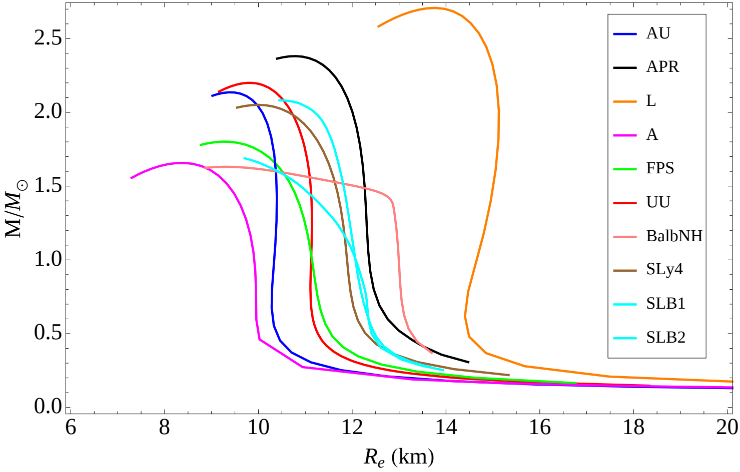
- An exact 4-parameter solution of the vacuum Einstein equation (based on these 4 moments) could be used to describe all kind of rotating neutron stars with quite high accuracy.
- How these parameters depend on the EoS-parameters?
- Could someone read from observations these parameters and use them to learn its EoS?

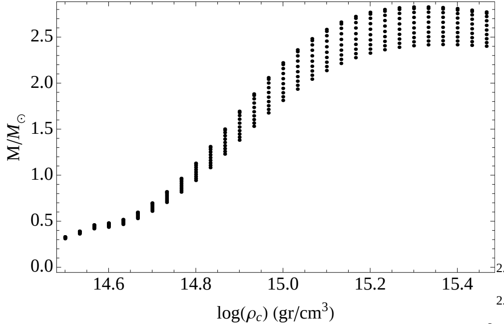
We constructed a wide range of neutron star models.

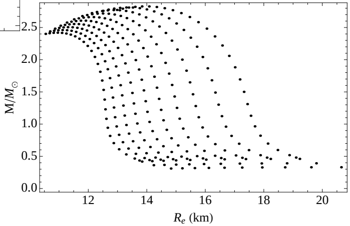


The quest for the EoS: An unanticipated result [A]

For each static model (based on different EoS) we constructed a sequence of rotating model up to the highest possible rate.

e.g. for APR EoS:





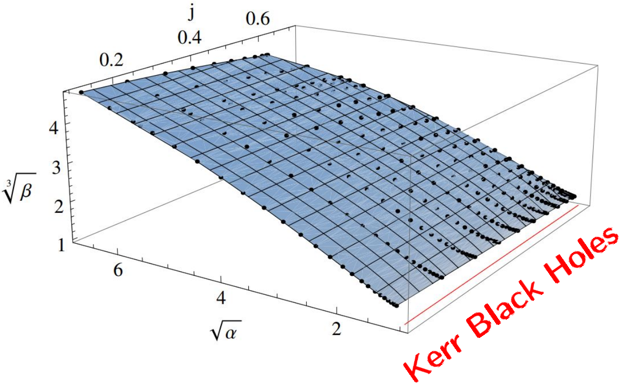
The quest for the EoS: An unanticipated result [B]

- For each model we measured the first 4 moments and (inspired from the simplicity of BH) we constructed the following 3 reduced moments:

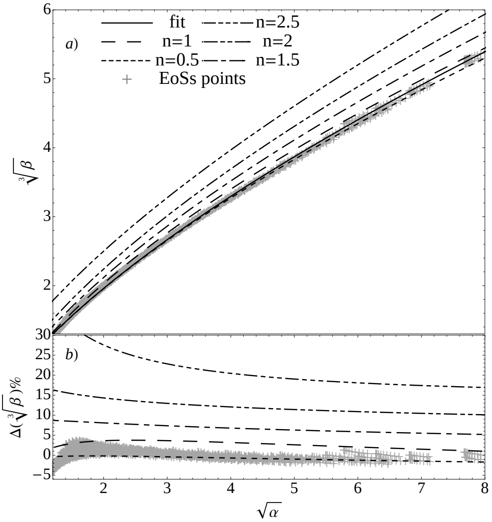
$$j = \frac{S_1}{M^2} \qquad = (a/M)_{BH}$$

$$\sqrt{\alpha} = \sqrt{\frac{-M_2}{M^3 j^2}} \qquad = 1_{BH}$$

$$\sqrt[3]{\beta} = \sqrt{\frac{-S_3}{M^4 j^3}} \qquad = 1_{BH}$$

- 
j
0.2
0.4
0.6
4
3
2
1
$\sqrt[3]{\beta}$
6
4
2
$\sqrt{\alpha}$
Kerr Black Holes

and from a different point of view...



FIT: $\sqrt[3]{\beta} = -0.36 + 1.48\sqrt{\alpha}^{0.65}$

(better than 5%)

What about Newtonian polytropes?

Dimensional analysis for slowly rotating Newtonian stars with polytropic index n:

$$S_1 = J \propto \rho_c^{-1/(2n)} \left(\frac{\Omega}{\Omega_K}\right)$$

$$M_2 = Q \propto \rho_c^{-2/n} \left(\frac{\Omega}{\Omega_K}\right)^2$$

$$S_3 \propto \rho_c^{-5/(2n)} \left(\frac{\Omega}{\Omega_K}\right)^3$$

$$\Rightarrow \sqrt[3]{\beta} \propto (\sqrt{\alpha})^{2/3}$$

But only for slowly rotating!

Questions arising

- Why all EoSs behave the same way (with respect to $\beta - \alpha - j$ relation)?

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- Why all EoSs behave the same way (with respect to $\beta - \alpha - j$ relation)?
- What about the higher moments?
- Do other universalities (I-Love-Q) are other aspects of the same physical cause?
- Are neutron stars the only 'universal' objects?

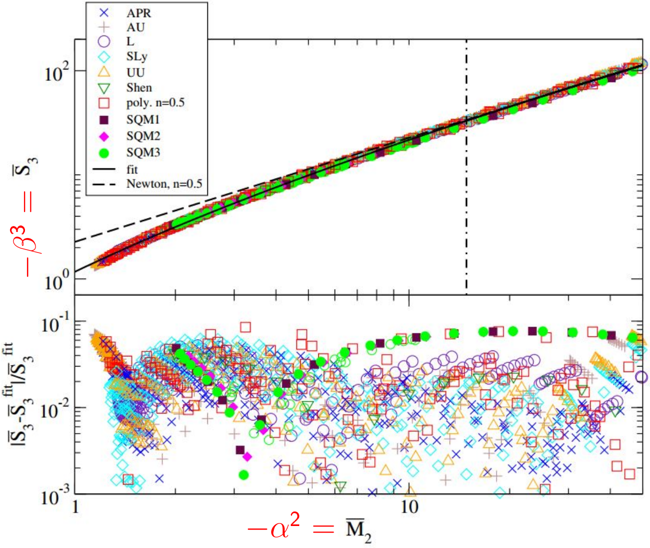
- Higher moments:

$$\gamma = \sqrt[4]{\frac{M_4}{M^5 j^4}} \quad = 1_{BH}$$

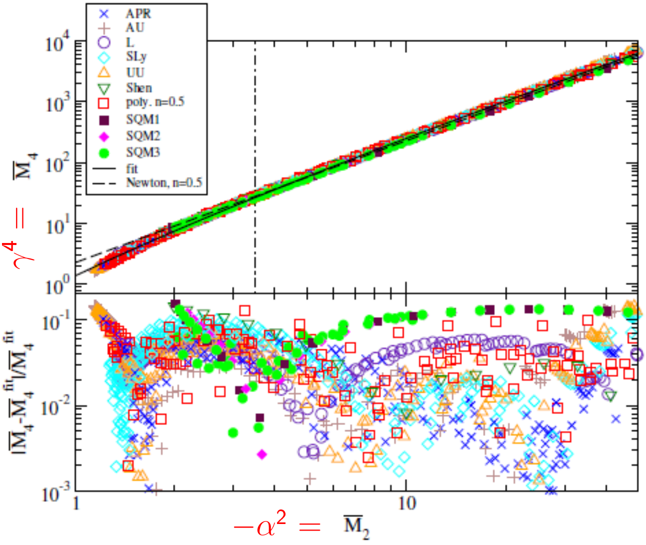
Is it universally connected with α, like β?

Answers... [A]

- ...remember



- hexadecapole-quadrupole is almost as good



Answers... [A]

- Universality of spin-octupole vs quadrupole relation better than 5%.

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- Universality of spin-octupole vs quadrupole relation better than 5%.
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- Technically difficult (and rather useless) to infer universalities for higher moments.
- Even quark stars (much different EoSs) still obey universal relations.
- Practically, ultra-compact objects are bald like BHs.

by-products

- Analytic expressions for the metric at quartic order with respect to $j = S_1/M^2$ were constructed, to test numerical codes, and estimate the systematic errors for future X-ray observations (X-ray pulse profiles as tracers of star deformations by *D. Psaltis et al ApJ* **787**,*10 (2014)*.)

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- Very accurate fits were found for

$$\begin{aligned} M_2 &\simeq -\alpha^2(EoS, M) M^3 j^2 \\ S_3 &\simeq -\beta^3(EoS, M) M^4 j^3 \\ M_4 &\simeq \gamma^4(EoS, M) M^5 j^4 \end{aligned}$$

Answers ...[B]

- There was a universal relation found, relating (moment of inertia) I, (deformability due to tidal effects) λ=Love number, (quadrupole moment) Q.

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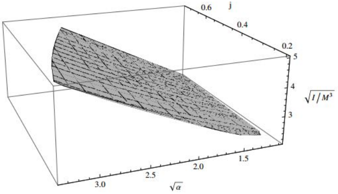
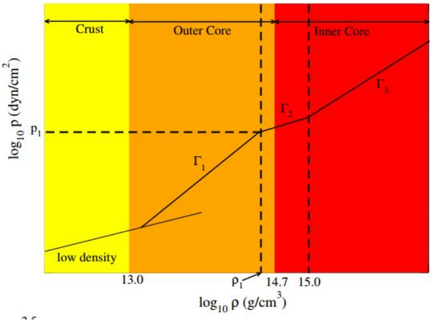
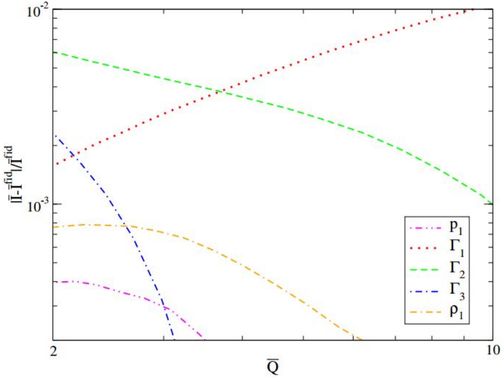
  Doneva D, Yazadjiev S, Stergioulas N, Kokkotas K, ApJL **781** *L6 (2014).*
- The universality is restored when the spin parameter j was used as the 'rotation rate' dimension. *Pappas G, Apostolatos T, PRL* **112** *121101 (2014).*



- The sensitivity of universality of I-Love-Q as a function of EoS parameters was tested.



- All parameters $\Gamma_1, \Gamma_2, \Gamma_3, p_1, \rho_1$ were modified by 30%.



No substantial deviations
from universallity was induced!

Answers ...[D]

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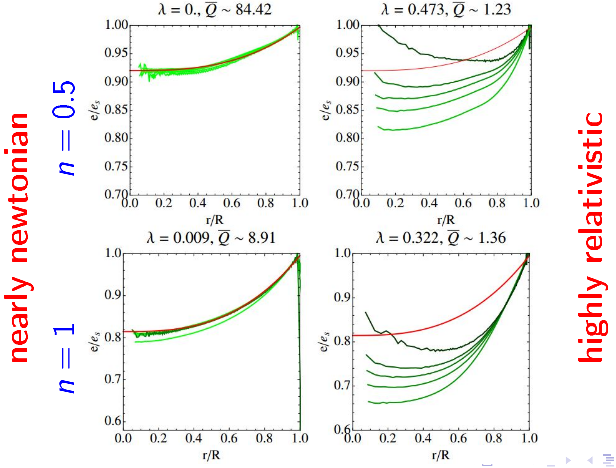
- For relativistic bodies:



Answers ...[E]

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- The variation of eceentricity is actually lower within the radii range that matters to the universallity.

- From Newtonian computations such eccentricity variations are sufficiently small to support universal relations between moments.

Answers ...[F]

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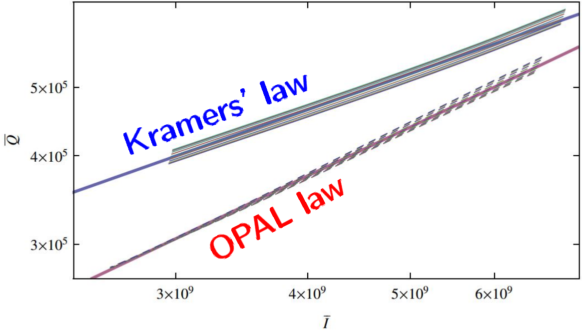

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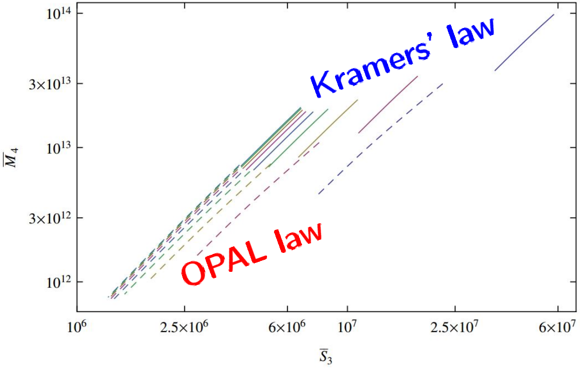

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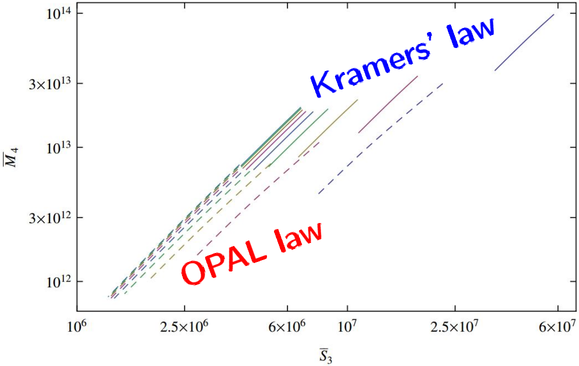

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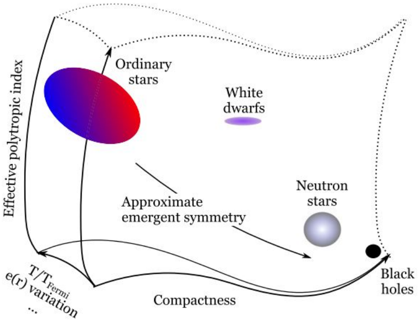
- No universality for non-compact stars.

The verdict



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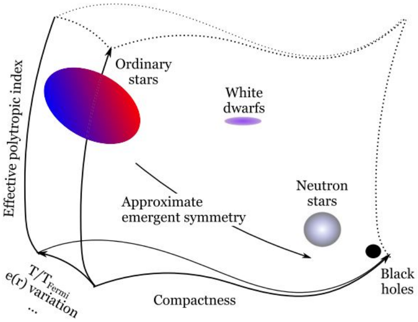

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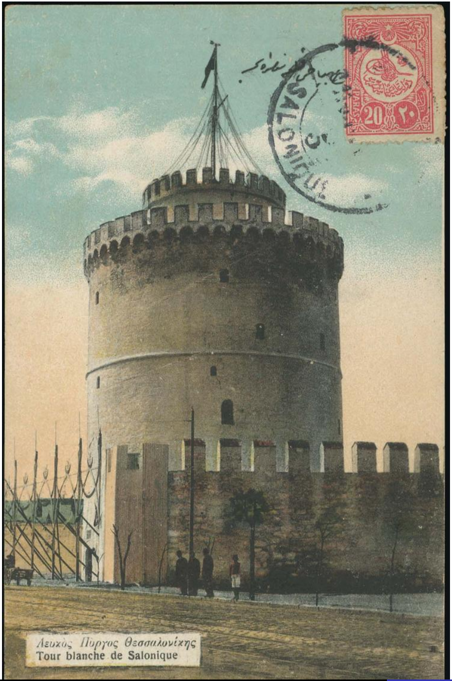
- Nuclear Physics at ultra-compact densities conspire so as to bring effective n at such values $(0.5 - 1)$ that eccentricity remains almost constant. This is essential for universality.



The End