Research Center for Astronomy
and Applied Mathematics
of the Academy of Athens

Cosmological Geometrodynamics:
from Planck to modified gravity

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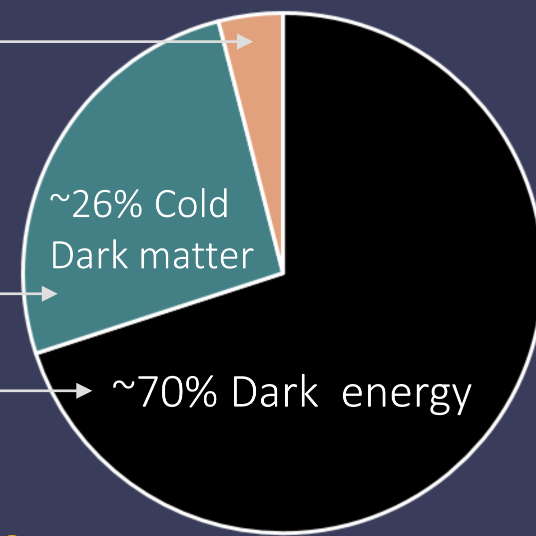
OUTLINE OF THIS LECTURE

- Introduction
- Observational evidence of the cosmic acceleration
- Recent Planck results
- Dark energy as a modification of gravity
- The modified gravity models
- Cosmological Implications
- Summary-Conclusions

THE CURRENT COSMOLOGICAL STATUS

$$\Omega_{DE} + \Omega_{DM} + \Omega_{BAR} = 1$$



We live in a very exciting period for the advancement of our knowledge for the Cosmos.

- Current observational data strongly support a flat and accelerating Universe with Ho ~67-73Km/sec/Mpc and To ~14 Gyr.

- The mystery of dark energy poses a challenge of such magnitude that, as stated by the Dark Energy Task Force (DETF – advising DOE, NASA and NSF), ''Nothing short of a revolution in our understanding of fundamental physics will be required to achieve a full understanding of the cosmic acceleration'' (Albrecht et al. 2006).

COSMOLOGICAL OBSERVATIONS & SPACETIME

S. Perlmutter, A. Reiss & B. Schmidt: Nobel Prize 2011

Hubble diagram (SNIa) → $\Omega_m + \Omega_\Lambda$

Temperature fluctuations of the CMB → spatial geometry Ω_κ

Large-Scale Structure → Ω_m (independent from dark energy)

Extragalactic sources (galaxies, AGNs, GRBs, HII, LBGs) at large redshifts → $\Omega_m + \Omega_\Lambda$

Growth data + gravitational lensing to check gravity on cosmological scales

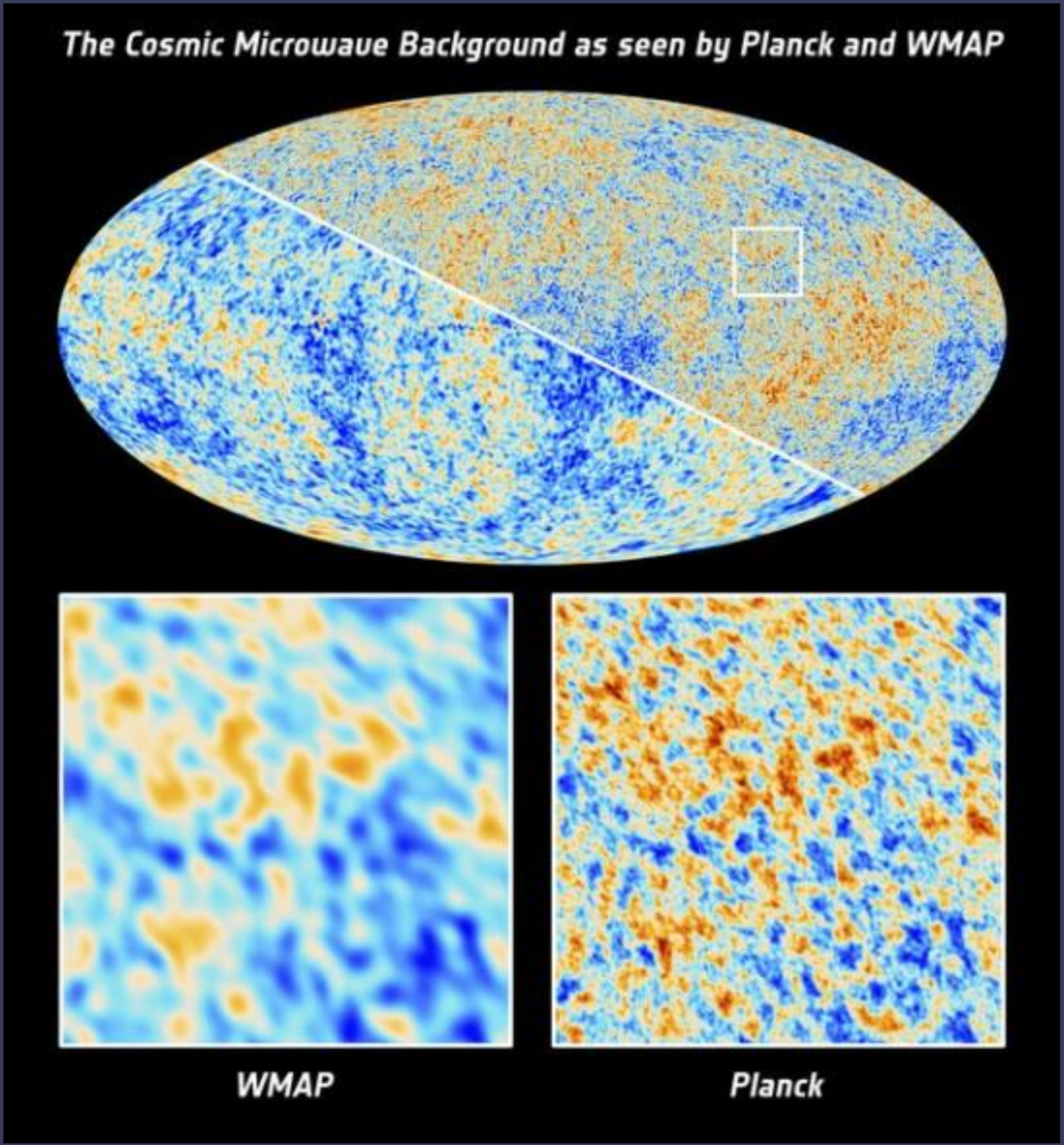
The Friedmann-Lemaitre-Robertson-Walker (FRLW) spacetime

$$ds^2 = \underbrace{-c^2 dt^2}_{\text{Time Part}} + \underbrace{\alpha^2(t)}_{\text{The scale factor}} \underbrace{\left[\frac{dr^2}{1-Kr^2} + r^2\left(d\vartheta^2 + \sin^2\theta d\varphi^2\right)\right]}_{\text{Spatial part of the metric}}$$

The expansion rate of the Universe, Hubble function

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

The analysis of the CMB data (Planck: Ade et al. 2015; Spergel et al. 2015) points a spatially flat (K=0) the FRW metric.


The Cosmic Microwave Background as seen by Planck and WMAP
WMAP
Planck

CMB temperature anisotropies provide a standard ruler.

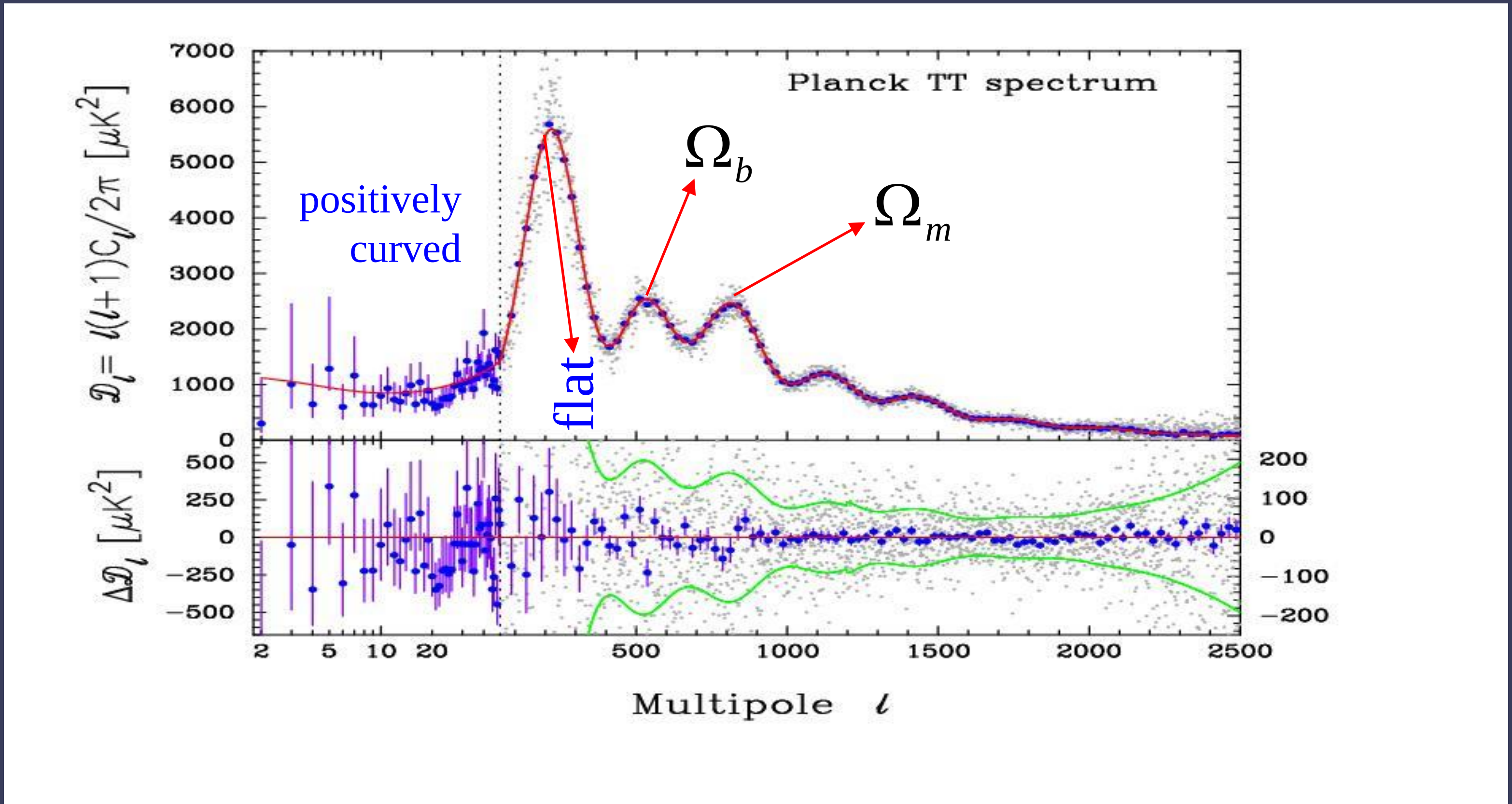
They were produced about 400,000 years after the Big Bang,and should be most prominent at a physical size of 400,000 light years across.

$[\theta_{peak}(\mathrm{deg})]^{-1/2}=1+K/H_0 R_0^2$

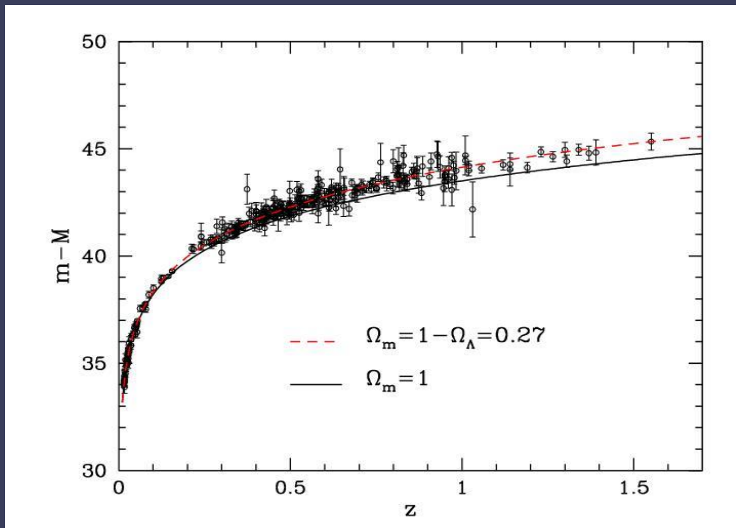
Observation: $\theta_{peak} = 1^o$

The universe is spatially flat (K=0) to an accuracy of better than a percent:

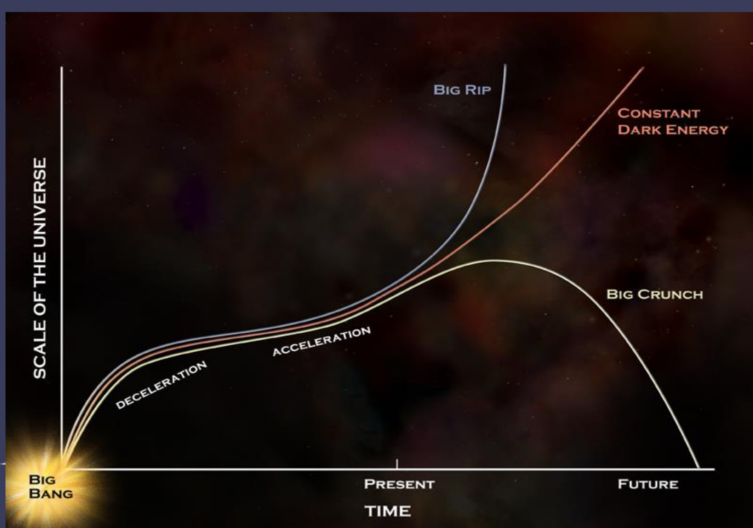
$$l_{peak} \approx 220$$



Evidence of cosmic acceleration



S. Perlmutter, A. Reiss & B. Schmidt: Nobel Prize 2011



Observationally assuming a matter dominated Universe we have:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 > \frac{8\pi G}{3}\rho_m$$

OR

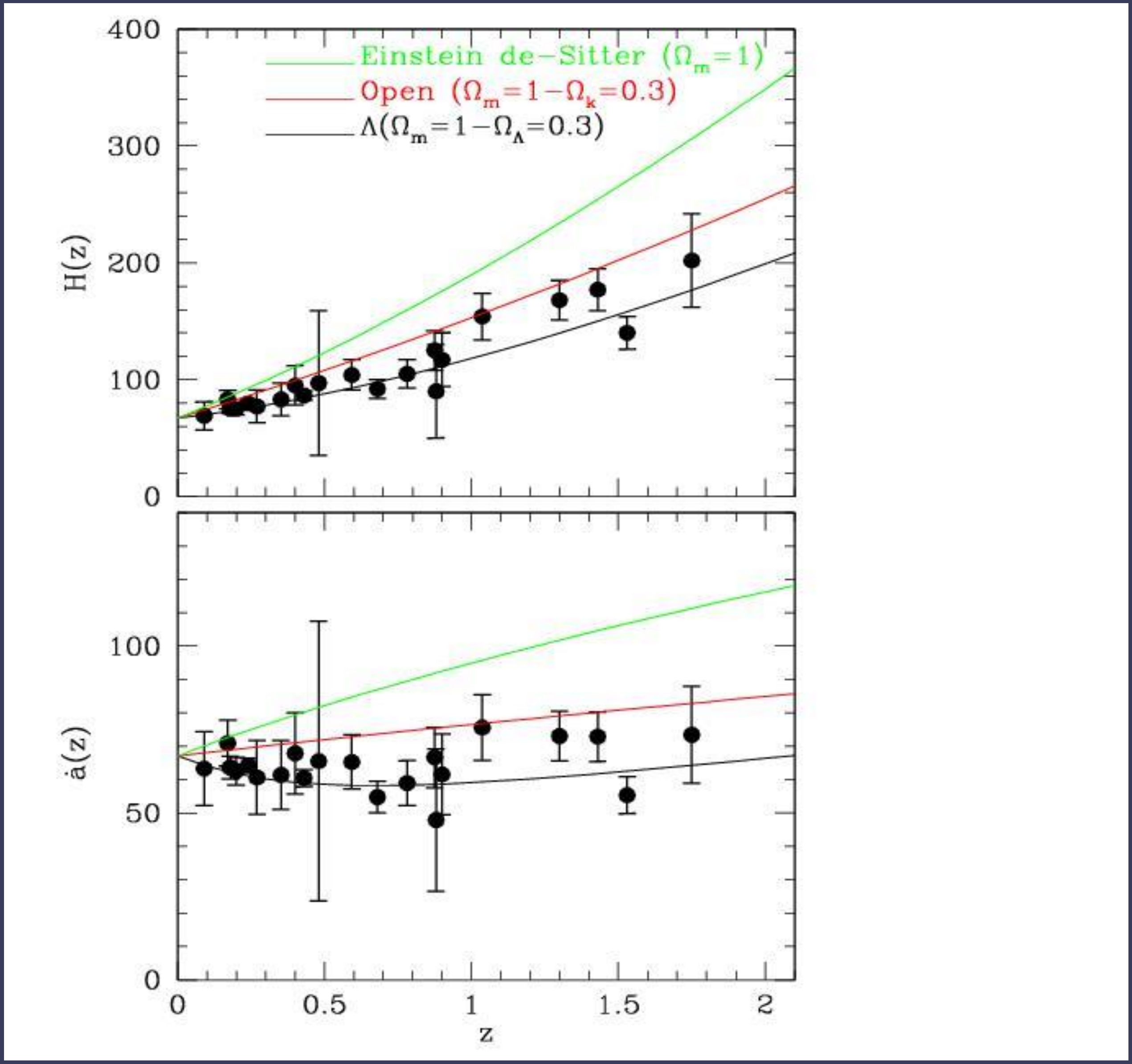
Change Gravity. GR is not valid at Cosmological scales. Modify the "law of gravity"

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{eff}}{3}\rho_m$$

Change the cosmic fluid. We add a "dark energy"

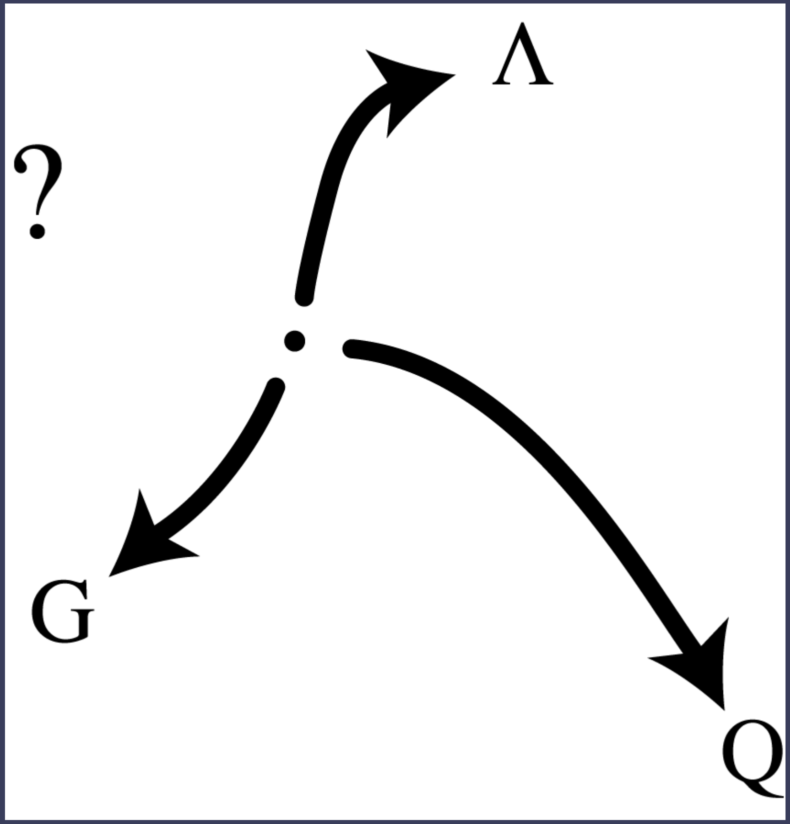
$$H^2 \equiv \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_Q)$$

HUBBLE DATA – EXPANSION RATE



"Dark Energy" introduces a New Physics

Artistic view of a universe filled by a turbulent sea of dark energy R. Caldwell (2005)



THE DYNAMICS OF THE UNIVERSE

Friedman Equations:

$$\left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{\alpha^2}$$

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}(\rho + 3P) \ .$$

- For Matter: $\rho_m(\alpha) = \rho_{m0}\alpha^{-3}$ and $P_m = 0$.
- For Radiation: $\rho_r(\alpha) = \rho_{r0}\alpha^{-4}$ and $P_r = \frac{1}{3}\rho_r$
- For Dark Energy: $\rho_Q(\alpha) = \rho_{Q0}X(a)$

$$X(a) = \exp\left(3\int_a^1 [1 + w(a)]d\ln a\right)$$

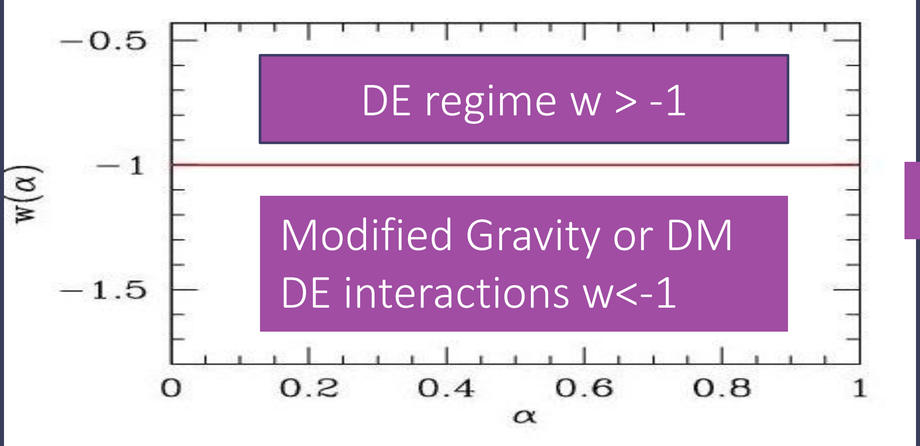
$$P_Q(a) = w(a)\rho_Q(a) \qquad w(a) < -1/3$$

Equation of State parameter

In the matter dominated era and for spatially flat models we get:

$$H^2 \equiv \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_Q)$$

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}[\rho_m + (3w+1)\rho_Q]$$



Λ-vacuum w=-1

The cosmological parameters

Scale factor – Density Parameters :

$$\left(\Omega_m + \Omega_Q + \Omega_r\right) = 1$$

$$X(\alpha) = \exp\left(-3\int_1^{\alpha} \frac{w(x)}{x} dx\right)$$

$$P_Q = w\rho_Q \quad w(\alpha) < -1/3$$

The Friedmann-Lemaitre equation devided by $H = \dot{\alpha}/\alpha$ becomes:

$$1 = \frac{8\pi G\rho_m}{3H^2} + \frac{8\pi G\rho_Q}{3H^2} + \frac{8\pi G\rho_r}{3H^2}$$

$$\Omega_m(\alpha) \equiv \frac{\rho_m}{\rho_m + \rho_Q + \rho_r} \equiv \frac{\Omega_m \alpha^{-3}}{E^2(\alpha)}$$

$$\Omega_Q(\alpha) \equiv \frac{\rho_Q}{\rho_m + \rho_Q + \rho_r} \equiv \frac{\Omega_Q X(\alpha)}{E^2(\alpha)}$$

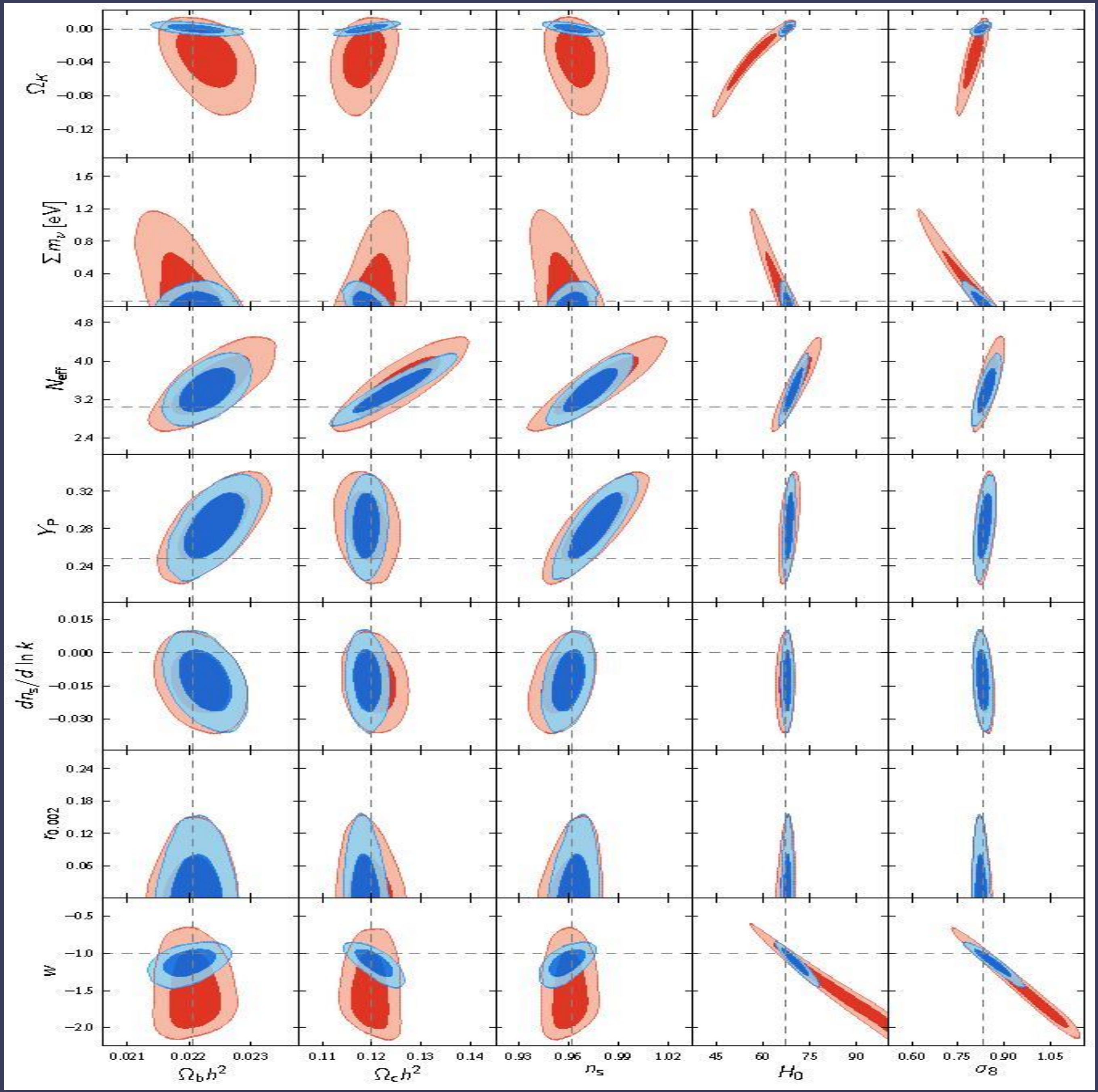
$$\Omega_r(\alpha) \equiv \frac{\rho_r}{\rho_m + \rho_Q + \rho_r} \equiv \frac{\Omega_r \alpha^{-4}}{E^2(\alpha)}$$

with

$$E(\alpha) = \left(\Omega_m \alpha^{-3} + \Omega_Q X(\alpha) + \Omega_r \alpha^{-4}\right)^{1/2}$$

Note that for w=-1 or X(α)=1 we get the usual Λ cosmology. Performing a Cosmo-Statistics we can but constraints on the cosmological parameters

Planck results Ade et al. 2014



Planck Results: Ade al al. (2015-arXiv:1502.01589)

$$\Omega_m = 0.308 \pm 0.012$$

$$H_0 = 67.8 \pm 0.9 Km/s/Mpc$$

$$\sigma_8 = 0.815 \pm 0.009$$

$$\Omega_b h^2 = 0.02226 \pm 0.00023$$

$$Y_p = 0.251 \pm 0.04$$

$$z_{dec} = 1089.94 \pm 0.42$$

$$z_{eq} = 3365 \pm 44$$

$$N_{eff} = 3.15 \pm 0.23$$

$$N = 50 - 60$$

$$n_s = 0.968 \pm 0.006$$

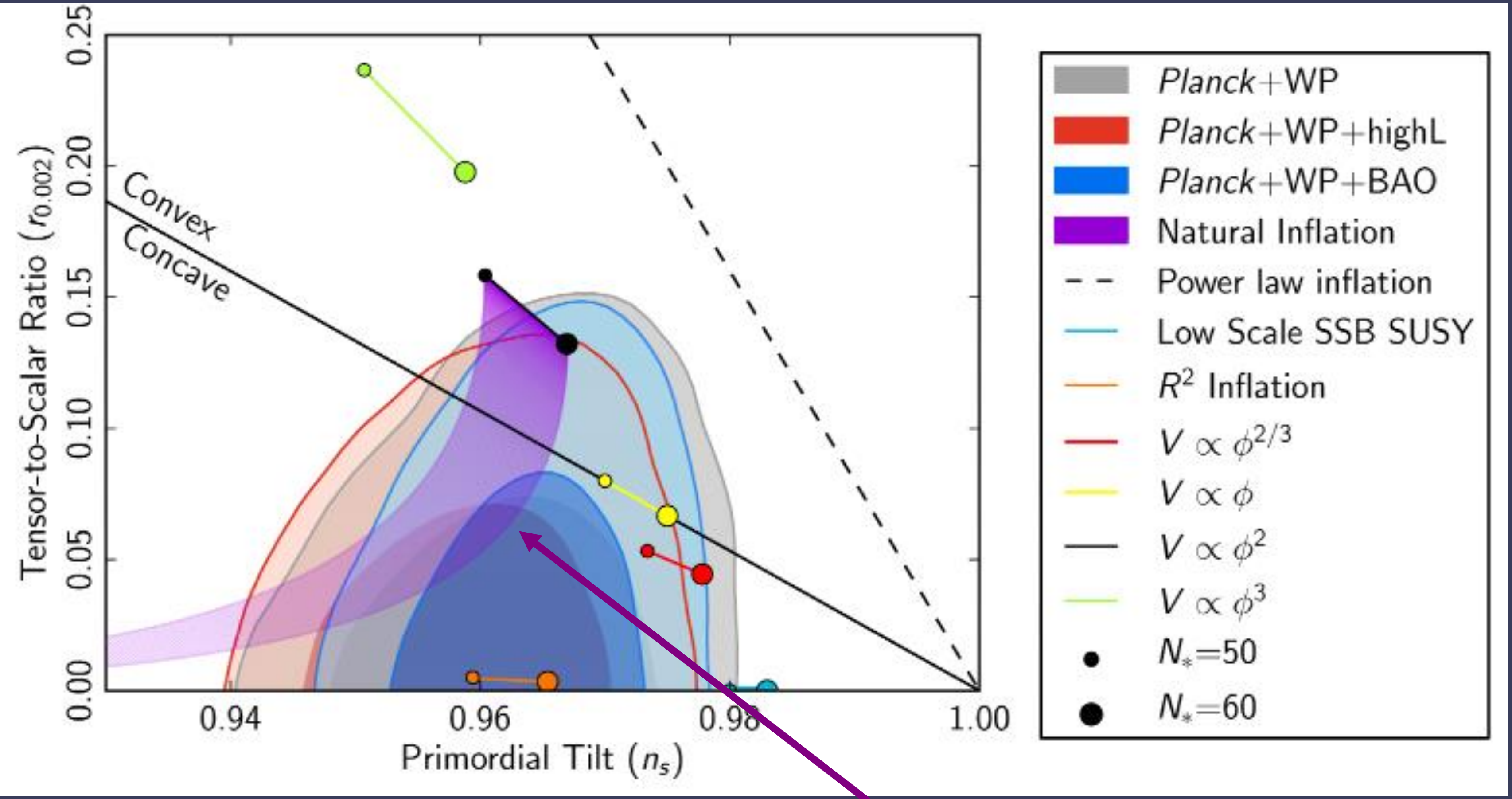
$$r = \frac{tensor}{scalar} < 0.11$$

If we include dark energy then
Planck + WMAP + SNIa

$$w = \frac{P_{de}}{\rho_{de}} = -1.006 \pm 0.045$$

Planck versus Inflation

$$n_s = 0.968 \pm 0.006 \qquad r < 0.11$$



Hyperbolic inflation Basilakos & Barrow 2015 Phys. Rev. D. In press

$$V(\phi) \propto \sinh^b(\phi / f)$$

Dark Energy as a modification of gravity?

An alternative approach to cosmic acceleration is to change gravity ‘’geometrical dark energy’’

Here we alter general relativity by modifying the usual Einstein’s field equations

The alternative-gravity theories have to satisfy the following criteria:

- For systems such as the solar system or galaxies, the alternative gravity must be very close to general relativity
- At cosmological scales we must have $g_{MR} < g_{GR}$ which implies that the cosmic acceleration is due to the weak gravity nature. The dark energy reflects on the physics of gravity (geometrical dark energy)

Tests of Gravity by R. Caldwell 2005

Locally Einstein's General Relativity is the standard model of gravitation

$$\eta = 2\frac{a_1 - a_2}{a_1 + a_2} < 4 \times 10^{-13}$$

local acceleration of bodies of different composition
Eot-Wash: Baessler et al, PRL 83 (1999) 3585

$$\dot{G}/G = (4 \pm 9) \times 10^{-13}\ /\mathrm{yr}$$

Lunar laser ranging: Williams et al, PRL 93 (2004) 261101

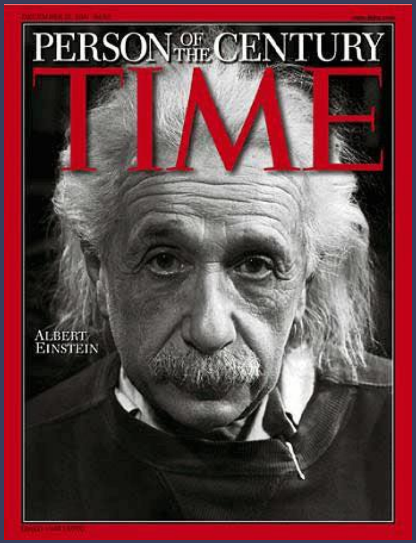
$$\frac{M_G}{M_I}|_{earth} - \frac{M_G}{M_I}|_{moon} = (-1 \pm 1.4) \times 10^{-13}$$

Nordtvedt effect: observations of the acceleration of bodies with different gravitational binding energies tests the Strong Equivalence Principle

Mass definitions:

$$\vec{F} = M_I \vec{a}$$

$$\Phi = -G M_G / r$$



The main steps are:

1. Define the modified Einstein-Hilbert action (S) in which we include all the ingredients (modified gravity, scalar fields, matter etc).

2. Varying the action ($\delta S=0$) in order to obtain the modified Einstein's field equations as well as the Klein Gordon equation (if a scalar field is present).

3. Using the FRW metric we derive the so called Friedmann equations (equations of motion) which describe the cosmic dynamics of the Universe.

SCALAR TENSOR THEORIES

These are probably the simplest example of modified gravity models and as such one of the most intensely studied alternatives to General Relativity. The general Einstein-Hilbert action is

$$S = \frac{1}{8\pi G}\int d^4x\sqrt{-g}\left[\frac{1}{2}f(\phi,R) - \frac{1}{2}\varsigma(\phi)(\nabla\phi)^2\right] + S_m + S_r$$

Is a general function of the scalar field and the Ricci scalar

Is a function of the scalar field (inflaton, dilaton string theory etc)

Are the matter-radiation actions that depend on the metric and matter fields

For more details see Fujii & Maeda 2003; Nojiri, Odintsov & Tretyakov 2007; Amendola & Tsujikawa 2010; Capozziello & de Laurentis 2011 (and references therein)

The above general action includes a wide variety of theories. Indeed some of these are:

GENERAL RELATIVITY

GR is a particular case of the scalar tensor theories.
Varying the action we have the Einstein's equations

$$f(R) = R - 2\Lambda$$
$$\varsigma(\phi) = 0$$

Λ is the Einstein's cosmological constant

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

The energy momentum tensor $\longrightarrow$ $T^{\mu}_{\nu} = diag\,(-\rho, P, P, P)$

$$\rho = \rho_m + \rho_r \qquad\qquad P = P_m + P_r$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G_N} = \rho_{\Lambda 0}$$

The Bianchi identities insure the covariance of the theory $\longrightarrow$ $\nabla_\mu G^{\mu}_{\nu} = \nabla_\mu T^{\mu}_{\nu} = 0$

For the spatially flat FRW metric we get the Friedmann equations of motion

From (00)-components

$$H^2 = \frac{8\pi G_N}{3}(\rho_m + \rho_r + \rho_\Lambda)$$

From (ii)-components

$$3H^2 + 2\dot{H} = -8\pi G_N(P_m + P_r - \rho_\Lambda)$$

Bianchi identities assuming no interactions

$$\dot{\rho}_m + 3H(\rho_m + P_m) = 0$$

For dust

$$\xrightarrow{P_m = 0} \rho_m = \rho_{m0} a^{-3}$$

For radiation

$$\xrightarrow{P_r = \rho_r / 3} \rho_r = \rho_{r0} a^{-4}$$

$$\dot{\rho}_r + 3H(\rho_r + P_r) = 0$$

$$\Omega_m = 8\pi G_N \rho_m / 3H_0^2$$

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda$$

$$\Omega_\Lambda = 8\pi G_N \rho_\Lambda / 3H_0^2$$

$$\Omega_r = 8\pi G_N \rho_r / 3H_0^2$$

This is the Λ Cosmology

The scalar field dark energy models

These dark energy models adhere to General Relativity. The corresponding Einstein-Hilbert action includes:

$$S = \frac{1}{8\pi G}\int d^4x\sqrt{-g}\left[\frac{1}{2}f(\phi,R) - \frac{1}{2}\varsigma(\phi)(\nabla\phi)^2\right] + S_m + S_r$$

$$f(\phi,R) = R - 2V(\phi)$$

For a homogeneous scalar field

$$(\nabla\phi)^2 = \dot{\phi}^2(t)$$

The dark energy equation of state parameter

$$w = \frac{\frac{1}{2}\zeta\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\zeta\dot{\phi}^2 + V(\phi)}$$

The scalar field dark includes a wide variety of theories depending on

$$V(\phi)$$

$$\zeta(\phi)$$

I) f(R) gravity models

$$f(\phi, R) = f(R) \qquad \varsigma(\phi) = 0$$

Varying the action we derive the modified Einstein field equations.

In the context of FRW geometry (spatially flat) the Friedmann equations become:

$$3 f_R H^2 - \frac{f_R R - f}{2} + 3 H f_{RR} \dot{R} = 16\pi G(\rho_m + \rho_r)$$

$$-2 f_R \dot{H} = 16\pi G(\rho_m + 4\rho_r / 3) + \ddot{f}_R - H\dot{f}_R$$

$$w(a) = -1 - \frac{2a}{3H}\frac{dH}{da}$$

$$R = g^{\mu\nu} R_{\mu\nu} = 6(2H^2 + \dot{H})$$

Notice that the concordance Λ-cosmology can be found for

$$f(R) = R - 2\Lambda$$

The effective Newton's parameter depends also from scale.

$$\frac{G_{eff}(a,k)}{G_N} = \frac{1}{f_R}\frac{1+4(k^2 f_{RR}/a^2 f_R)}{1+3(k^2 f_{RR}/a^2 f_R)}$$

$$k = 1/\lambda \approx 0.1 hMpc^{-1}$$

Obviously the concordance Λ cosmology admits General Relativity

$$f(R) = R - 2\Lambda$$

$$G_{eff}(a,k) = G_N$$

For more details see: Sotiriou & Faraoni 2008; Amendola & Tsujikawa 2010; Capozziello & de Laurentis 2011; Elizalde et al. 2012 (and references therein)

In the literature we usually use either the Hu & Sawicki model:

$$f(R) = R - m^2 \frac{c_1 \left(R / m^2\right)^n}{1 + c_2 \left(R / m^2\right)^n}$$

or the Starobinsky model:

$$f(R) = R - c_1 m^2 \left[1 - \frac{1}{(1 + R^2 / m^4)^n}\right]$$

In Basilakos, Nesseris & Perivolaropoulos (Phys. Rev. D. 2013) we prove that the above f(R) models can be written as perturbations around the Λ cosmology (see also Nojiri, Odintsov, Saez-Gomez 2009).

II) Generalized Brans-Dicke Theories

$$f(\phi, R) = F(\phi)R - 2U(\phi) \qquad \varsigma(\phi) = (1 - 6Q^2)F(\phi)$$

$$Q \equiv -\frac{F_{,\phi}}{F}$$

Varying the action (based on a spatially flat FRW) we have

$$\frac{3FH^2}{8\pi G} = \frac{1}{2}(1 - 6Q^2)F\dot{\phi}^2 + U - 3H\dot{F} + \rho_m + \rho_r$$

$$(1 - 6Q^2)F\left[\ddot{\phi} + 3H\dot{\phi} + (\dot{F}/2F)\dot{\phi}\right] + U_{,\phi} + QFR = 0$$

The effective Newton's parameter becomes

$$\frac{G_{eff}(a)}{G_N} = \frac{1}{F_0} \frac{4 + 2\omega_{BD}}{3 + 2\omega_{BD}} \qquad 3 + 2\omega_{BD} = \frac{1}{2Q^2}$$

Using the Solar System constrain namely at very small scales the modified gravity tends to General Relativity we have

$$\omega_{BD} > 4.3 \times 10^4 \qquad |Q| < 2.4 \times 10^{-3}$$

For more details see: Nojiri et al. 2005; Damour & Nordvedt 1993; Fujii & Maeda 2003; Nesseris & Perivolaropoulos 2006; Amendola & Tsujikawa 2010 (and references therein)

III) Gauss-Bonnet gravity

$$f(\phi, R) = R - 2V(\phi) - 2g(\phi)R_{GB}^2$$

Gauss-Bonnet term

$$R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

Varying the action (based on a spatially flat FRW) we have

$$\frac{3H^2}{8\pi G} = \frac{1}{2}\dot{\phi}^2 + V(\phi) + 24g_{,\phi}H^3\dot{\phi} + \rho_m + \rho_r$$

$$\ddot{\phi} + 3H\phi + V_{,\phi} + 24g_{,\phi}H^2(H^2 + \dot{H}) = 0$$

A particular form is used $g(\phi) = (g_0/\lambda)e^{\lambda\phi}$

For more details see: Nojiri et al. 2005; Carter & Neupane 2006; Amendola et al. 2006; Amendola & Tsujikawa 2010; Basilakos, Bauer & Sola 2012; Capozziello et al. 2013 (and references therein)

IV) The braneworld - Dvali, Gabadadze & Porati (DGP) gravity

$$S = \frac{1}{16\pi G_{eff}} \int d^5X \sqrt{-\tilde{g}}\,\tilde{R} + \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\,R - \int d^5X \sqrt{-\tilde{g}}\,(L_m + L_r)$$

$\tilde{g}_{AB}$

Is the metric in the 5D brane

$g_{\mu\nu}$

Is the usual 4D metric which is embedded into the 5D brane. This space is maximally symmetric with a constant curvature K(=0, flat).

$$ds^2 = -n^2(\tau, y)\,d\tau^2 + \alpha^2(\tau, y)\,\gamma_{ij}\,dx^i dx^j + dy^2$$

In the context of a DGP cosmological model the ''accelerated'' expansion of the universe can be explained by a modification of the gravitational interaction in which gravity becomes weak at cosmological scales owing to the fact that our 4D brane survives into an extra dimensional manifold.

The 5D Einstein equations are given by

$$G_{AB} \equiv \widetilde{R}_{AB} - \frac{1}{2}\widetilde{R}\widetilde{g}_{AB} = 8\pi G_{eff}(T_{AB} + \widetilde{U}_{AB})$$

$$T_{B}^{A(brane)} = \delta(y)\, diag(-\rho_m, P_m, P_m, P_m, 0)$$

Also the components from the scalar curvature of the brane are

$$\widetilde{U}_{00} = -\frac{3}{8\pi G_N}\left(\frac{\dot{a}^2}{a^2} + K\frac{n^2}{a^2}\right)\delta(y)$$

$$\widetilde{U}_{ij} = -\frac{1}{8\pi G_N}\left[\frac{a^2}{n^2}\left(-\frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{n}}{n} - 2\frac{\ddot{a}}{a}\right) - K\right]\gamma_{ij}\delta(y)$$

The Friedmann equation (for flat K=0) becomes

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_m a^{-3} + 2\Omega_{bw} + 2\sqrt{\Omega_{bw}}\sqrt{\Omega_m a^{-3} + \Omega_{bw}}$$

$$\Omega_{bw} = \frac{(1-\Omega_m)^2}{4}$$

$$\frac{G_{eff}(a)}{G_N} = \frac{2 + 4\Omega_m^2(a)}{3 + 3\Omega_m^2(a)}$$

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{E^2(a)}$$

V) Finsler – Randers Geometry

The Finslerian geomtery extends the usual Riemannian geometry. Note that the Riemannian geometry is also a Finslerian. Generally a Finsler space is derived from a generating function F(x,y) on a tangent bundle on a manifold M (Randers 1941, Goenner & Bogoslovsky 1999; Stavrinos & Diakogiannis 2004; Kouretsis et al. 2009; Skakala & Visser 2011; Vacaru 2012)

$$F : TM \rightarrow \Re$$

Particular attention has been paid on the so called Finsler-Randers cosmological model.

$$F(x,y) = \sigma(x,y) + u_\mu(x) y^\mu \qquad \sigma(x,y) = \sqrt{g_{\mu\nu} y^\mu y^\nu}$$

$$u_\mu = (u_0, 0, 0, 0) \qquad u_0 = 2C_{000} \qquad \tilde{g}_{\mu\nu} = F g_{\mu\nu} / \sigma$$

Is a weak primordial vector field

Cartan tensor

$$\widetilde{R}_{\mu\nu} - \frac{1}{2}\widetilde{R}\widetilde{g}_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

The Friedmann equation (for flat K=0) becomes

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_m a^{-3} + 2\Omega_Z + 2\sqrt{\Omega_z}\sqrt{\Omega_m a^{-3} + \Omega_z}$$

$$\Omega_z = \frac{\dot{u}_0^2}{4H_0^2} \qquad \Omega_z = \frac{(1-\Omega_m)^2}{4} = \Omega_{bw}$$

In Basilakos & Stavrinos (2013) we prove that the Finsler-Randers model is cosmologically equivalent with that of the DGP gravity, despite the fact that the two models have a completely different geometrical origin.

Conclusions - Future work

- Testing gravity on cosmological scales is one of the main problems in cosmology
- We compare the above models with expansion data in order to put constrains on the free parameters of the models.
- Then we have to compare the models against growth data in order to check possible departures from GR
- How many of the above geometrical models can provide exactly the same Hubble function?
- Of course there are also other geometrical models which explain cosmic acceleration such as [f(T), massive gravity etc]

Planck versus Inflation