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Analytic Orbit Propagation for Transiting Circumbinary Planets

12th HELLENIC ASTRONOMICAL CONFERENCE
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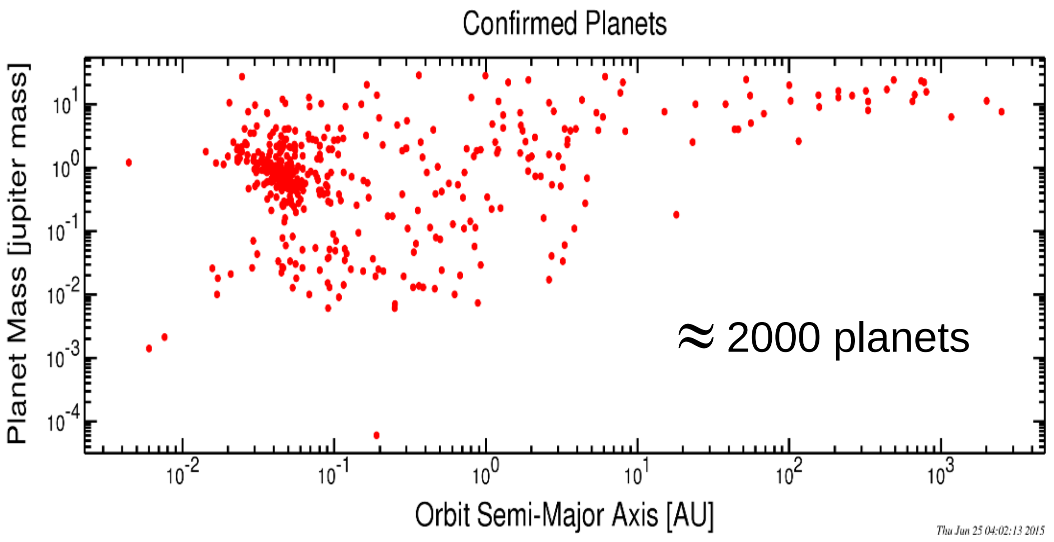
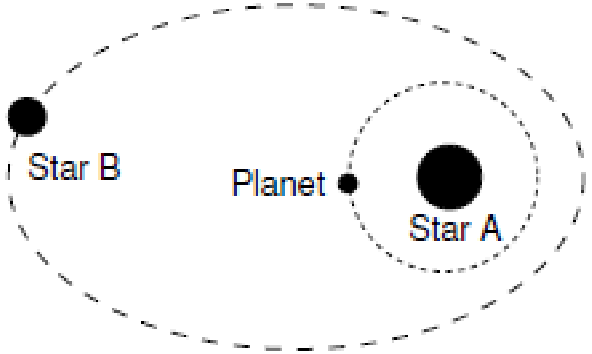
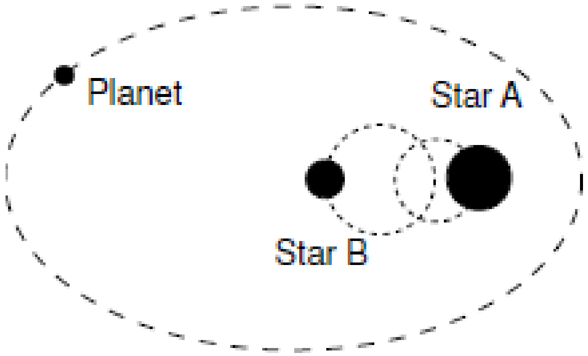


140 planets in binaries

114 S-type (circumstellar)



26 P-type (circumbinary)

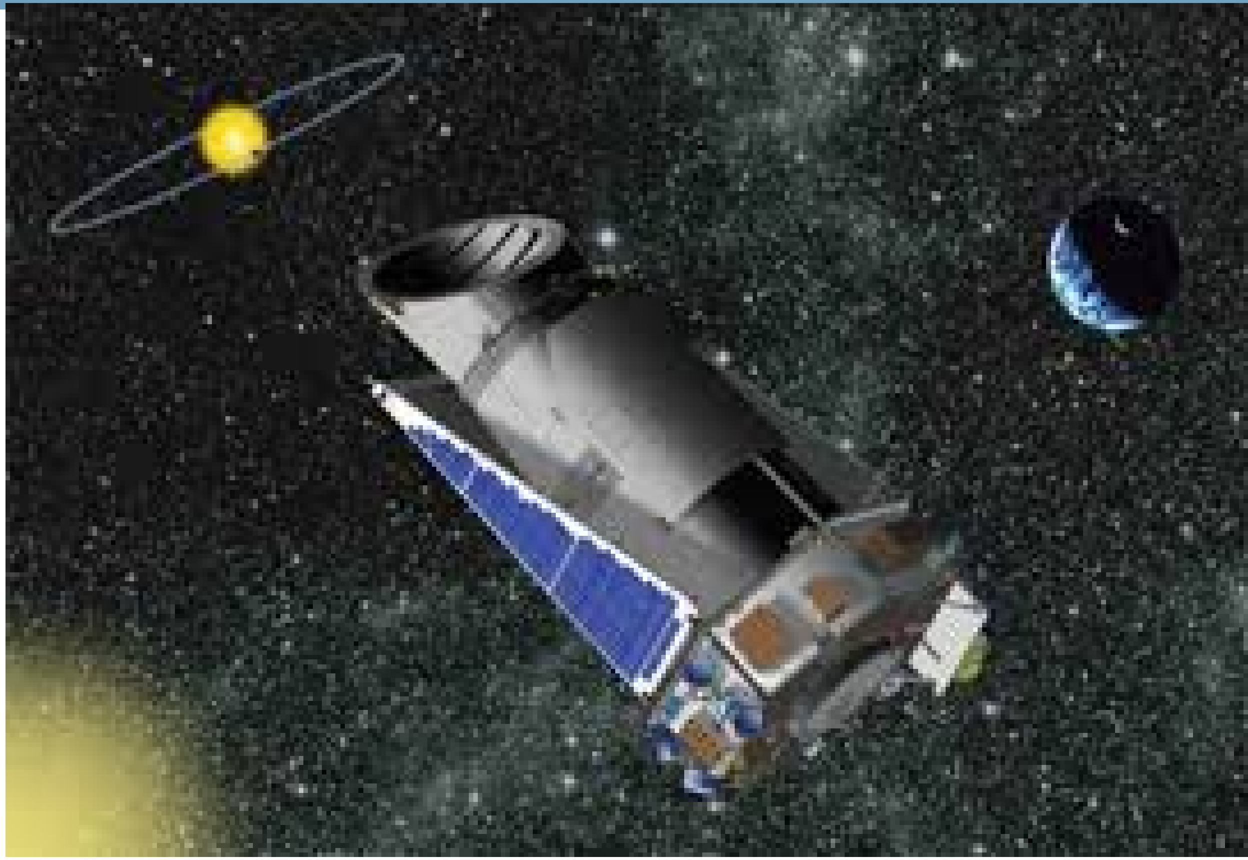


Confirmed planets: 1028

Planet candidates: 4664

Eclipsing binaries: 2165

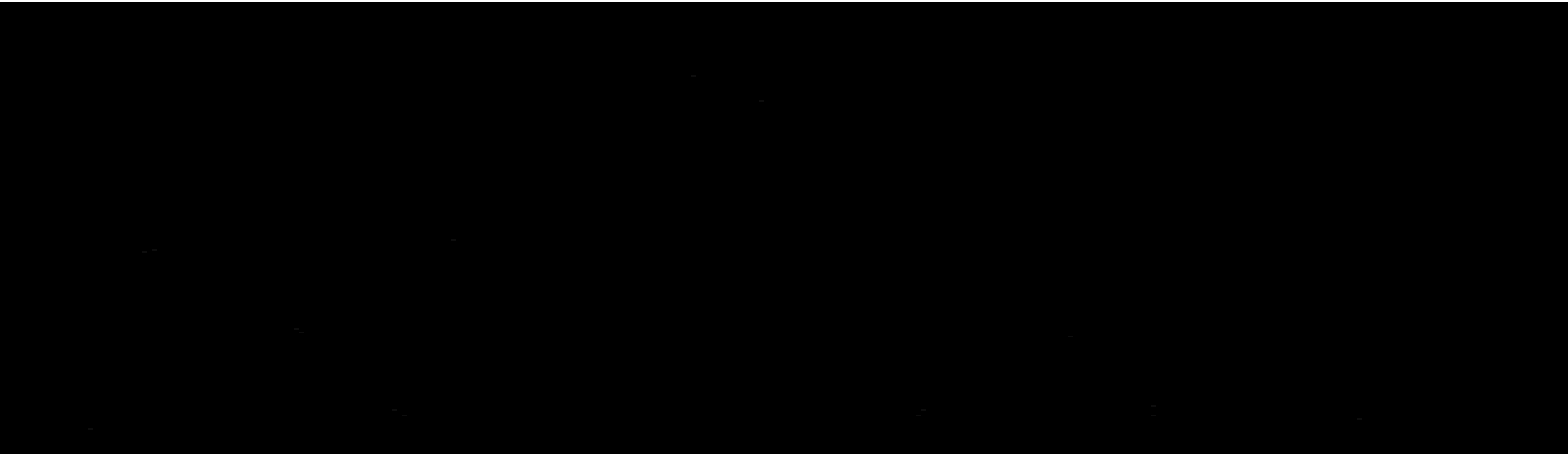
<u>Transit method</u>



<u>P-type</u>:
Kepler-16b, Kepler-34b, Kepler-35b, Kepler-38b, Kepler-47b, Kepler-47c, Kepler-47d, Kepler-64b, Kepler-413b, KIC 9632895

AIM: To provide an analytical description of a circumbinary planetary orbit

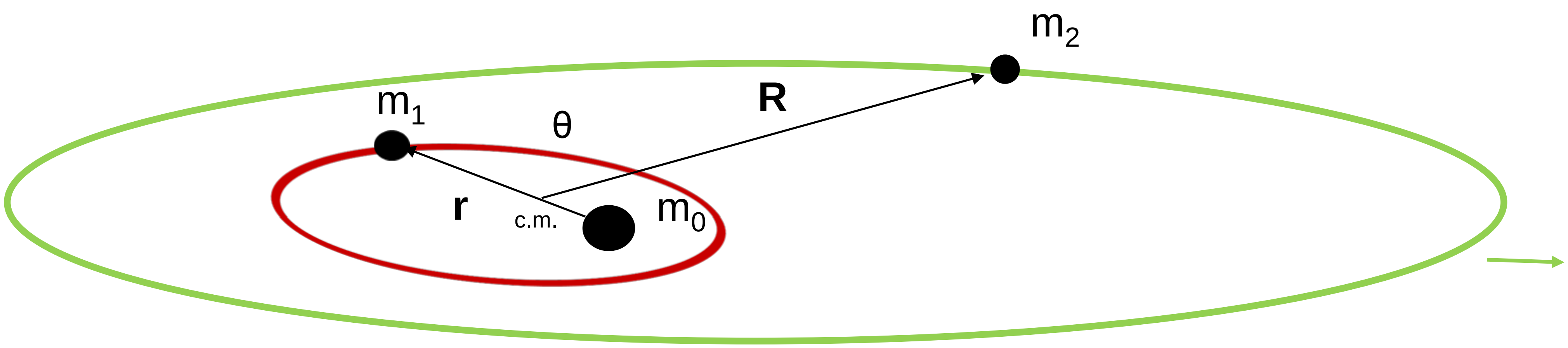
- modelling TTVs
- habitable zones in P-type systems (Eggl, Georgakarakos & Pilat-Lohinger 2015; in prep)
- low mass binaries
- planet formation in binaries



Hierarchical Triple System (HTS)

r, **R** Jacobi vectors

$m_2 \ll m_0, m_1$



coplanar system

Initially circular

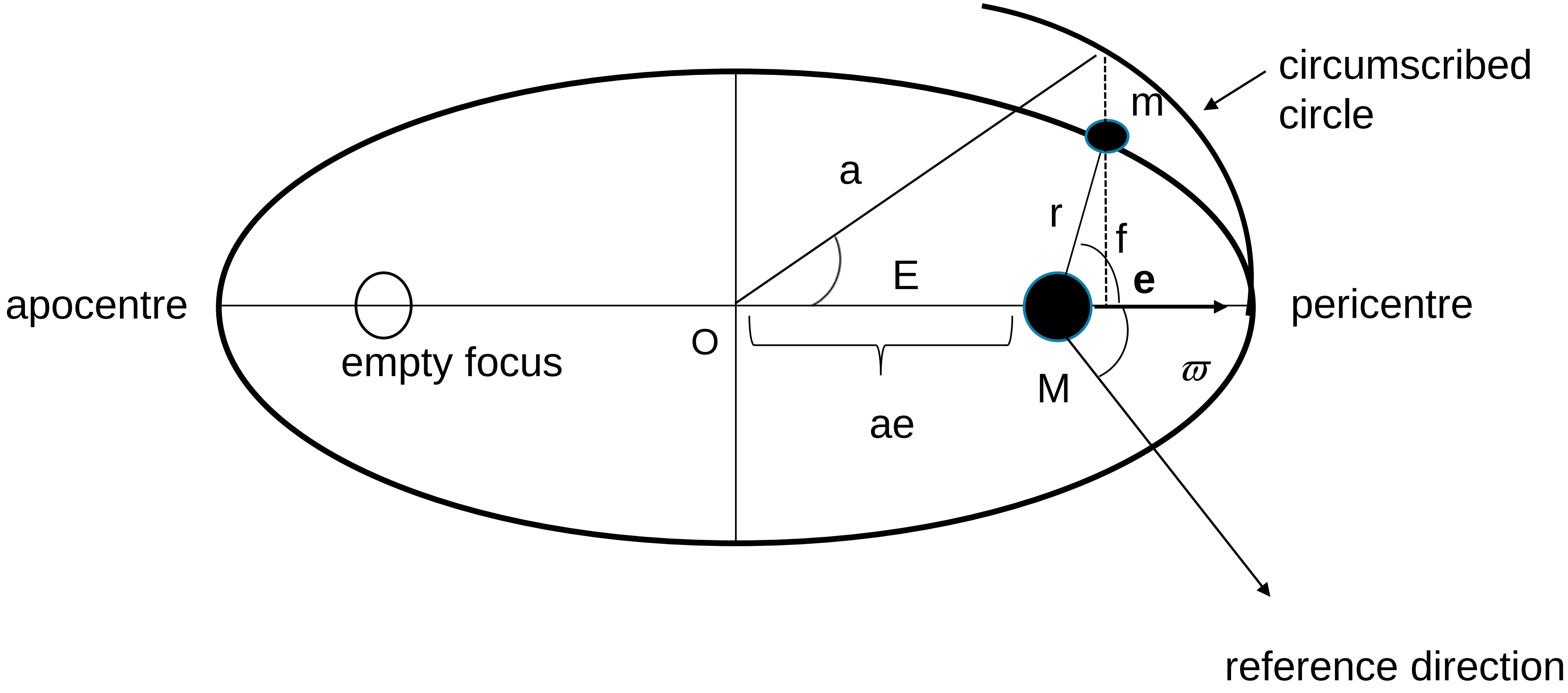
Hierarchical Triple System (HTS)

● No long term changes in the semi-major axes of the orbits (e.g. Harrington 1968).

+ coplanarity

the dynamical evolution of the hierarchical triple is dominated by the time dependent changes in the eccentric (Runge-Lenz) vectors of the inner and outer orbit

Two body problem



a = semi-major axis,
e = eccentricity,
e = Runge-Lenz (eccentric) vector,
ϖ = longitude of pericentre,
E=eccentric anomaly,
f=true anomaly
T = period of the orbit,
n = mean motion ($T=2\pi/n$),
$\ell=nt+c$ mean anomaly (c is a constant).

METHOD:

- long term (secular) evolution: Hamiltonian formulation
- mid term ($\sim T_2$) and short term ($\sim T_1$) evolution: Runge-Lenz vector
- no mean motion resonances ($n_1 \neq kn_2, \; k \in \mathbb{Z}$)
- semi-major axes and binary eccentricity constant

LONG TERM MOTION

$$H = H_0 + H_1 + H_p$$

r/R small

where

$$H_0 = -\frac{\mathcal{G}^2 m_0^3 m_1^3}{2(m_0 + m_1)L_1^2}$$

is the Keplerian energy of the inner orbit,

$$H_1 = -\frac{\mathcal{G}^2 (m_0 + m_1)^3 m_2^3}{2ML_2^2}$$

is the Keplerian energy of the outer orbit, and

$$H_p = \mathcal{G}m_2\left(\frac{m_0 + m_1}{R} - \frac{m_0}{r_{02}} - \frac{m_1}{r_{12}}\right)$$

$$H_p = -\frac{\mathcal{G}m_0 m_1 m_2}{R}\sum_{j=2}^{\infty} M_j \left(\frac{r}{R}\right)^j \mathcal{P}_j(\cos\theta)$$

$$P_n(x) = \frac{(-1)^n}{2^n n!}\frac{d^n}{dx^n}(1 - x^2)^n$$

is the perturbing Hamiltonian, with r_{02} and r_{12} being the distances between m_0 and m_2 and m_1 and m_2 respectively

MID AND SHORT TERM EVOLUTION

Planetary equation of motion:

$$\ddot{\boldsymbol{R}} = -\mathcal{G}M\left(\mu_0\frac{\boldsymbol{R}+\mu_1\boldsymbol{r}}{|\boldsymbol{R}+\mu_1\boldsymbol{r}|^3} + \mu_1\frac{\boldsymbol{R}-\mu_0\boldsymbol{r}}{|\boldsymbol{R}-\mu_0\boldsymbol{r}|^3}\right) = \mathcal{G}M\frac{\partial}{\partial \boldsymbol{R}}\left(\frac{\mu_0}{|\boldsymbol{R}+\mu_1\boldsymbol{r}|} + \frac{\mu_1}{|\boldsymbol{R}-\mu_0\boldsymbol{r}|}\right)$$

with

$$\mu_i = \frac{m_i}{m_0+m_1}, \qquad i = 0, 1$$

As previously, HTS - r/R small - use Legendre polynomials

$$\frac{1}{|\boldsymbol{R}+\mu_1\boldsymbol{r}|} = \frac{1}{R}\sum_{n=0}^{\infty}\left(-\frac{\mu_1 r}{R}\right)^n \mathcal{P}_{\rm n}(\cos\theta),$$

and

$$\frac{1}{|\boldsymbol{R}-\mu_0\boldsymbol{r}|} = \frac{1}{R}\sum_{n=0}^{\infty}\left(\frac{\mu_0 r}{R}\right)^n \mathcal{P}_{\rm n}(\cos\theta).$$

With $\cos\theta = \dfrac{\boldsymbol{r}\cdot\boldsymbol{R}}{rR}$

Runge-Lenz vector:

$$\begin{aligned}\boldsymbol{e}_2 &= -\frac{\boldsymbol{R}}{R} + \frac{1}{\mathcal{G}M}(\dot{\boldsymbol{R}}\times\boldsymbol{h}) \Rightarrow \\ \dot{\boldsymbol{e}}_2 &= -\frac{\dot{\boldsymbol{R}}}{R} + \frac{\dot{R}}{R^2}\boldsymbol{R} + \frac{1}{\mathcal{G}M}[2(\dot{\boldsymbol{R}}\cdot\ddot{\boldsymbol{R}})\boldsymbol{R} - (\boldsymbol{R}\cdot\ddot{\boldsymbol{R}})\dot{\boldsymbol{R}} - (\boldsymbol{R}\cdot\dot{\boldsymbol{R}})\ddot{\boldsymbol{R}}],\end{aligned}$$

where $\boldsymbol{h} = \boldsymbol{R}\times\dot{\boldsymbol{R}}$.

$$e_{21}(t) = \frac{m_0 m_1}{(m_0+m_1)^{\frac{4}{3}} M^{\frac{2}{3}}} \frac{1}{X^{\frac{4}{3}}} \left[\frac{3}{4}\cos l_2 + e_1^2\left(\frac{33}{16}\cos l_2 + \frac{35}{16}\cos 3l_2\right) + \frac{P_1(t)}{X}\right] +$$
$$C_1 \cos K_1 t + C_2 \sin K_1 t + \frac{K_2}{K_1 - K_3}\cos K_3 t$$
$$e_{22}(t) = \frac{m_0 m_1}{(m_0+m_1)^{\frac{4}{3}} M^{\frac{2}{3}}} \frac{1}{X^{\frac{4}{3}}} \left[\frac{3}{4}\sin l_2 + e_1^2\left(\frac{3}{16}\sin l_2 + \frac{35}{16}\sin 3l_2\right) + \frac{P_2(t)}{X}\right] +$$
$$C_1 \sin K_1 t - C_2 \cos K_1 t + \frac{K_2}{K_1 - K_3}\sin K_3 t,$$

where

$$C_1 = -\frac{m_0 m_1}{(m_0+m_1)^{\frac{4}{3}} M^{\frac{2}{3}}} \frac{1}{X^{\frac{4}{3}}} \left[\frac{3}{4}\cos l_{20} + e_1^2\left(\frac{33}{16}\cos l_{20} + \frac{35}{16}\cos 3l_{20}\right) + \frac{P_1(t_0)}{X}\right] - \frac{K_2}{K_1 - K_3}$$

$$C_2 = \frac{m_0 m_1}{(m_0+m_1)^{\frac{4}{3}} M^{\frac{2}{3}}} \frac{1}{X^{\frac{4}{3}}} \left[\frac{3}{4}\sin l_{20} + e_1^2\left(\frac{3}{16}\sin l_{20} + \frac{35}{16}\sin 3l_{20}\right) + \frac{P_2(t_0)}{X}\right].$$

X – period ratio

l_2 planetary mean anomaly

$$K_1 = \frac{3}{8}\frac{\sqrt{\mathcal{G}M}\, m_0 m_1 a_{1l}^2}{(m_0+m_1)^2 a_{2l}^{\frac{7}{2}}}\left(2 + 3e_{1l}^2\right)$$

$$K_2 = \frac{15}{64}\frac{\sqrt{\mathcal{G}M}\, m_0 m_1 (m_0 - m_1) a_{1l}^3}{(m_0+m_1)^3 a_{2l}^{\frac{9}{2}}} e_{1l}\left(4 + 3e_{1l}^2\right)$$

$$K_3 = \frac{3}{4}\frac{\sqrt{\mathcal{G}}\, m_2 a_{1l}^{\frac{3}{2}}\sqrt{1 - e_{1l}^2}}{(m_0+m_1)^{\frac{1}{2}} a_{2l}^3}$$

$$
\begin{aligned}
P_1(t) \;=\;& \frac{21}{32}(1-\sqrt{1-e_1^2})\cos(2E_1+3l_2)+\frac{3}{32}(1-\sqrt{1-e_1^2})\cos(2E_1+l_2)-\frac{3}{32}(1+\\
&+\sqrt{1-e_1^2})\cos(2E_1-l_2)-\frac{21}{32}(1+\sqrt{1-e_1^2})\cos(2E_1-3l_2)+e_1\Big[-\frac{21}{96}(1-\\
&-\sqrt{1-e_1^2})\cos(3E_1+3l_2)-\frac{3}{96}(1-\sqrt{1-e_1^2})\cos(3E_1+l_2)+\frac{3}{32}(13+\\
&+5\sqrt{1-e_1^2})\cos(E_1-l_2)+\frac{3}{96}(1+\sqrt{1-e_1^2})\cos(3E_1-l_2)-\frac{105}{32}(1-\\
&-\sqrt{1-e_1^2})\cos(E_1+3l_2)+\frac{21}{96}(1+\sqrt{1-e_1^2})\cos(3E_1-3l_2)-\frac{3}{32}(13-\\
&-5\sqrt{1-e_1^2})\cos(E_1+l_2)+\frac{105}{32}(1+\sqrt{1-e_1^2})\cos(E_1-3l_2)\Big]+e_1^2\Big[\frac{3}{32}(\frac{7}{2}-\\
&-\sqrt{1-e_1^2})\cos(2E_1+l_2)+\frac{21}{32}(\frac{1}{2}-\sqrt{1-e_1^2})\cos(2E_1+3l_2)-\frac{3}{32}(\frac{7}{2}+\\
&+\sqrt{1-e_1^2})\cos(2E_1-l_2)-\frac{21}{32}(\frac{1}{2}+\sqrt{1-e_1^2})\cos(2E_1-3l_2)\Big]+e_1^3\Big[-\frac{1}{64}\times\\
&\times\cos(3E_1+l_2)-\frac{7}{64}\cos(3E_1-3l_2)+\frac{3}{64}\cos(E_1-l_2)+\frac{7}{64}\cos(3E_1+3l_2)+\\
&+\frac{21}{64}\cos(E_1+3l_2)-\frac{3}{64}\cos(E_1+l_2)+\frac{1}{64}\cos(3E_1-l_2)-\\
&-\frac{21}{64}\cos(E_1-3l_2)\Big]
\end{aligned}
$$

$$
\begin{aligned}
P_2(t) \;=\;& \frac{21}{32}(1-\sqrt{1-e_1^2})\sin(2E_1+3l_2)-\frac{3}{32}(1-\sqrt{1-e_1^2})\sin(2E_1+l_2)-\frac{3}{32}(1+\\
&+\sqrt{1-e_1^2})\sin(2E_1-l_2)+\frac{21}{32}(1+\sqrt{1-e_1^2})\sin(2E_1-3l_2)+e_1\Big[-\frac{7}{32}(1-\\
&-\sqrt{1-e_1^2})\sin(3E_1+3l_2)+\frac{1}{32}(1-\sqrt{1-e_1^2})\sin(3E_1+l_2)-\frac{3}{32}(3-\\
&-5\sqrt{1-e_1^2})\sin(E_1-l_2)+\frac{1}{32}(1+\sqrt{1-e_1^2})\sin(3E_1-l_2)-\frac{105}{32}(1-\\
&-\sqrt{1-e_1^2})\sin(E_1+3l_2)-\frac{7}{32}(1+\sqrt{1-e_1^2})\sin(3E_1-3l_2)-\frac{3}{32}(3+\\
&+5\sqrt{1-e_1^2})\sin(E_1+l_2)-\frac{105}{32}(1+\sqrt{1-e_1^2})\sin(E_1-3l_2)\Big]+e_1^2\Big[\frac{3}{32}(\frac{5}{2}+\\
&+\sqrt{1-e_1^2})\sin(2E_1+l_2)+\frac{21}{32}(\frac{1}{2}-\sqrt{1-e_1^2})\sin(2E_1+3l_2)+\frac{3}{32}(\frac{5}{2}-\\
&-\sqrt{1-e_1^2})\sin(2E_1-l_2)+\frac{21}{32}(\frac{1}{2}+\sqrt{1-e_1^2})\sin(2E_1-3l_2)\Big]+e_1^3\Big[-\frac{3}{64}\times\\
&\times\sin(3E_1+l_2)+\frac{7}{64}\sin(3E_1-3l_2)-\frac{9}{64}\sin(E_1-l_2)+\frac{7}{64}\sin(3E_1+3l_2)+\\
&+\frac{21}{64}\sin(E_1+3l_2)-\frac{9}{64}\sin(E_1+l_2)-\frac{3}{64}\sin(3E_1-l_2)+\\
&+\frac{21}{64}\sin(E_1-3l_2)\Big]
\end{aligned}
$$

Maximum and averaged squared eccentricity

$$\begin{aligned} e_2^{max} = &\; e_{2sm}^{max} + e_{2l}^{max} \\ = &\; \frac{m_0 m_1}{(m_0+m_1)^{\frac{4}{3}} M^{\frac{2}{3}}} \frac{1}{X^{\frac{4}{3}}} \left[\frac{3}{2} + \frac{17}{2} e_1^2 + \frac{1}{X}\left(3 + 19 e_1 + \frac{21}{8} e_1^2 - \frac{3}{2} e_1^3\right)\right] \\ &+ \frac{2K_2}{K_1 - K_3}. \end{aligned}$$

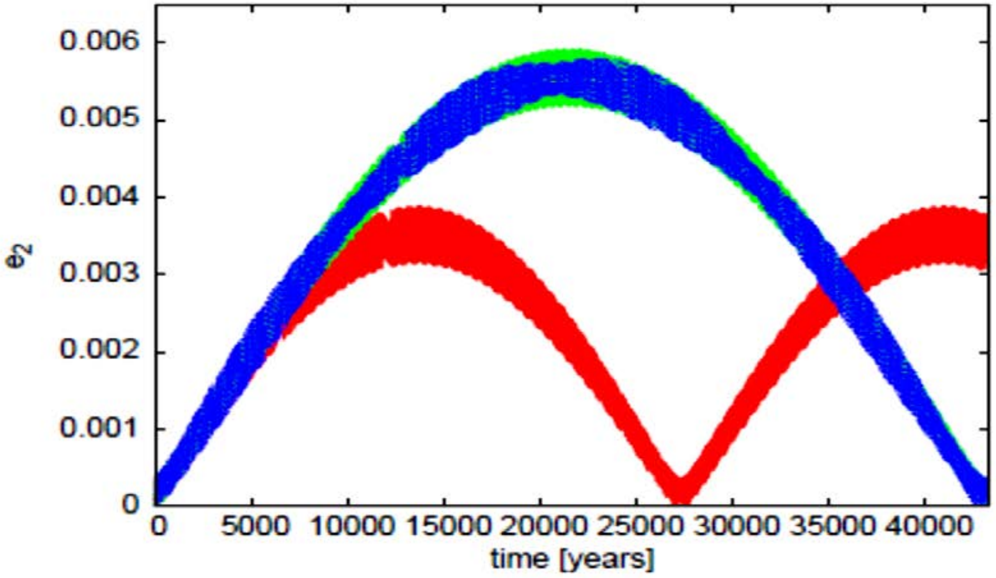
$$\begin{aligned} \langle e_2^2 \rangle = &\; \langle e_{21}^2(t) + e_{22}^2(t) \rangle = \frac{m_0^2 m_1^2}{(m_0+m_1)^{\frac{8}{3}} M^{\frac{4}{3}}} \frac{1}{X^{\frac{8}{3}}} \left[\frac{9}{8} + \frac{27}{8} e_1^2 + \frac{887}{64} e_1^4 - \right. \\ & \left. -\frac{975}{64} \frac{1}{X} e_1^4 \sqrt{1-e_1^2} + \frac{1}{X^2}\left(\frac{225}{64} + \frac{6619}{64} e_1^2 - \frac{26309}{512} e_1^4 - \frac{393}{64} e_1^6\right)\right] + \\ & +2\left(\frac{K_2}{K_1 - K_3}\right)^2. \end{aligned}$$

Post-Newtonian correction

$$\frac{dg_{1PN}}{dt} = \frac{3\mathcal{G}^{\frac{3}{2}}(m_0+m_1)^{\frac{3}{2}}}{c^2 a_{1l}^{\frac{5}{2}}(1-e_{1l}^2)}$$

$$K_{3PN} = \frac{3}{4}\frac{\mathcal{G}^{\frac{1}{2}} m_2 a_{1l}^{\frac{3}{2}}(1-e_{1l}^2)^{\frac{1}{2}}}{(m_0+m_1)^{\frac{1}{2}} a_{2l}^3} + \frac{3\mathcal{G}^{\frac{3}{2}}(m_0+m_1)^{\frac{3}{2}}}{c^2 a_{1l}^{\frac{5}{2}}(1-e_{1l}^2)}$$

Red – non-relativistic full equations of motion
Green – relativistic full equations of motion
Blue – analytical solution



$m_0=5\ M_\odot$, $m_1=4\ M_\odot$, $m_2=1\ M_J$

$a_1=0.2$ au, $a_2=6.84$ au, $e_1=0.5$

$E_{10}=0^\circ$, $\ell_{20}=90^\circ$

Analytic description of the system's evolution

Binary orbit

$$\varpi_1 \approx g_1 = K_{3PN} t,$$

$$l_1 = n_1 t + l_{10},$$

$$r \cos f_1 = a_1(\cos E_1 - e_1),$$

$$r \sin f_1 = a_1\left(1 - e_1^2\right)^{1/2} \sin E_1,$$

$$n_1 = \mathcal{G}^{1/2}(m_0 + m_1)^{1/2} a_1^{-3/2}$$

$$\mathbf{r} = W_1\left(r \cos f_1, r \sin f_1\right)^T$$

$$W_i = \begin{pmatrix} \cos \varpi_i & -\sin \varpi_i \\ \sin \varpi_i & \cos \varpi_i \end{pmatrix}, \qquad i = 1, 2$$

$$\dot{a}_1 = 0, \dot{a}_2 = 0, \dot{e}_1 = 0$$

Planetary orbit

$$\varpi_2 = \arctan(e_{22}/e_{21}),$$

$$l_2 = n_2 t + \varpi_2 + l_{20},$$

$$e_2(t) = |\mathbf{e}_2|(t),$$

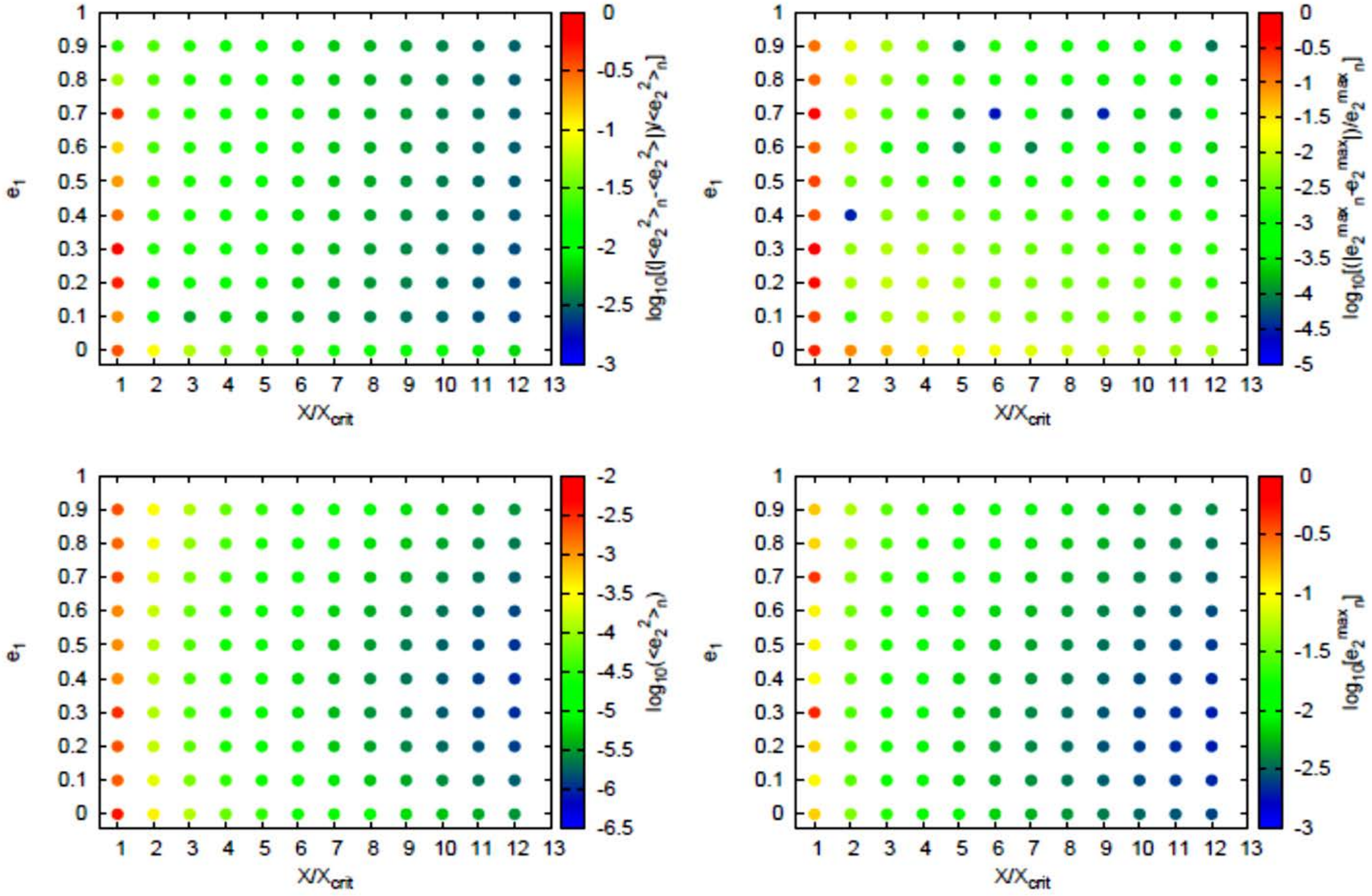
$$R \cos f_2 = a_2(\cos E_2 - e_2),$$

$$R \sin f_2 = a_2\left(1 - e_2^2\right)^{1/2} \sin E_2$$

$$n_2 = (\mathcal{G}M)^{1/2} a_2^{-3/2}$$

$$\mathbf{R} = W_2\left(R \cos f_2, R \sin f_2\right)^T$$

Numerical Testing



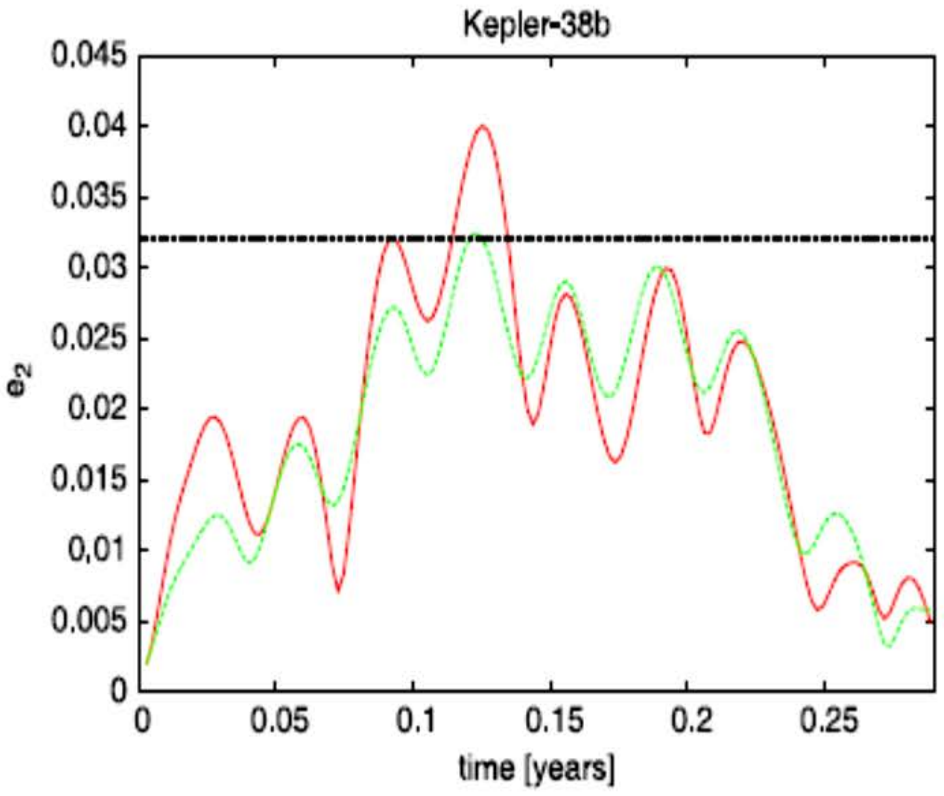
$m_1/(m_0+m_1)=0.3$
$m_2/(m_0+m_1)=0.001$

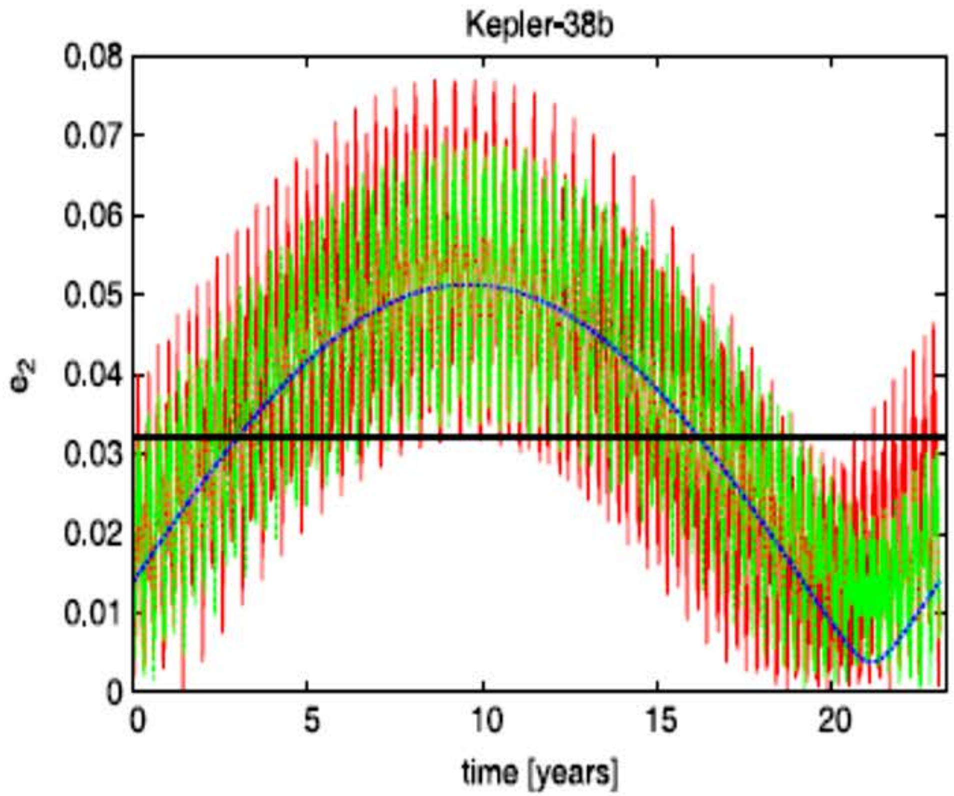
X_{crit} is the critical period ratio based on Holman & Wiegert (1999).

Integration time= 1 analytical secular period

APPLICATION TO REAL SYSTEMS

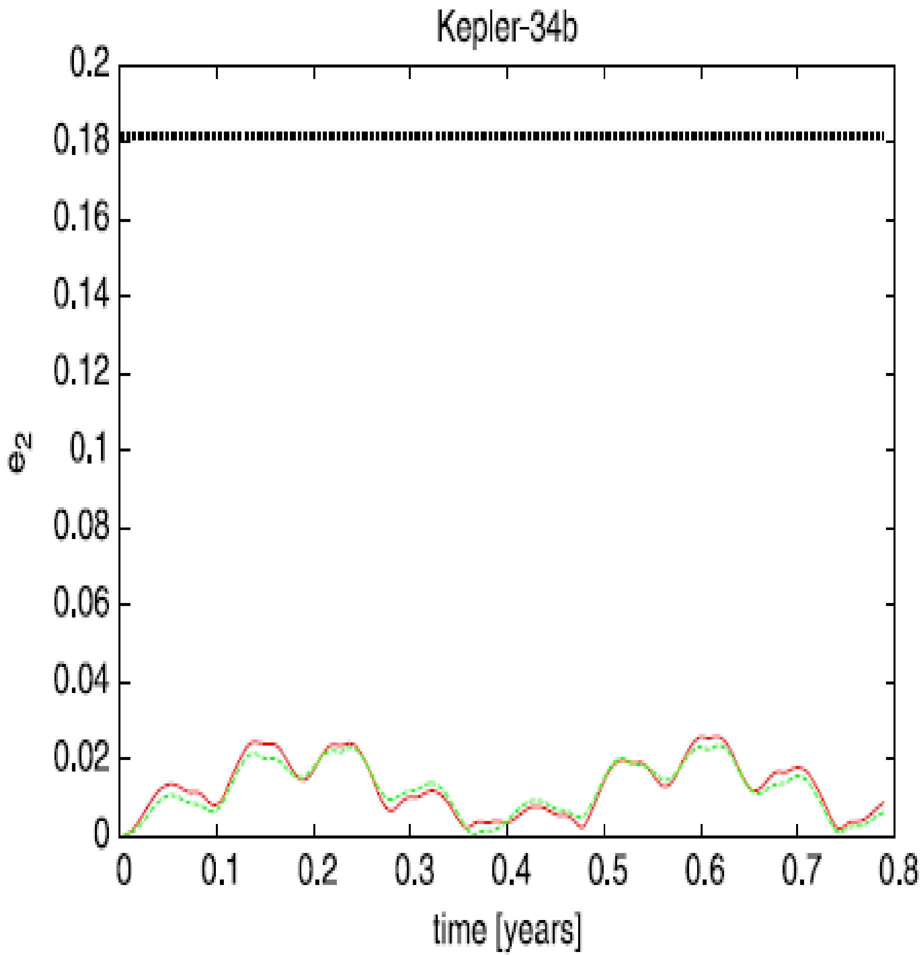
Kepler-16, Kepler-34, Kepler-35, Kepler-38, Kepler-64, Kepler-413

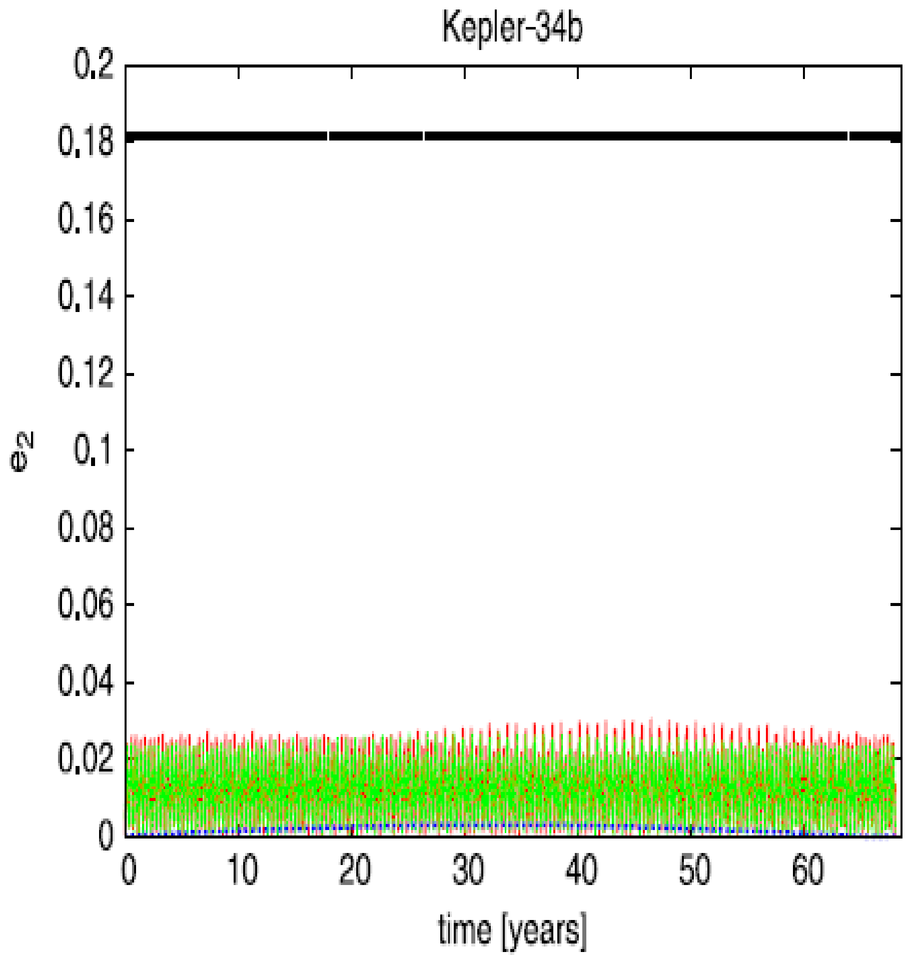




Red – Numerical solution
Green – Analytical solution
Blue – Analytical secular solution
Black – Observed value

m_0=0.949 $M_\odot$
m_1=0.249 $M_\odot$
m_2=0.384 M_J
a_1=0.1469 au
a_2=0.4644 au
e_1=0.1032
E_{10}=0°
$\varpi_1 = 0^\circ$
ℓ_{20}=90°



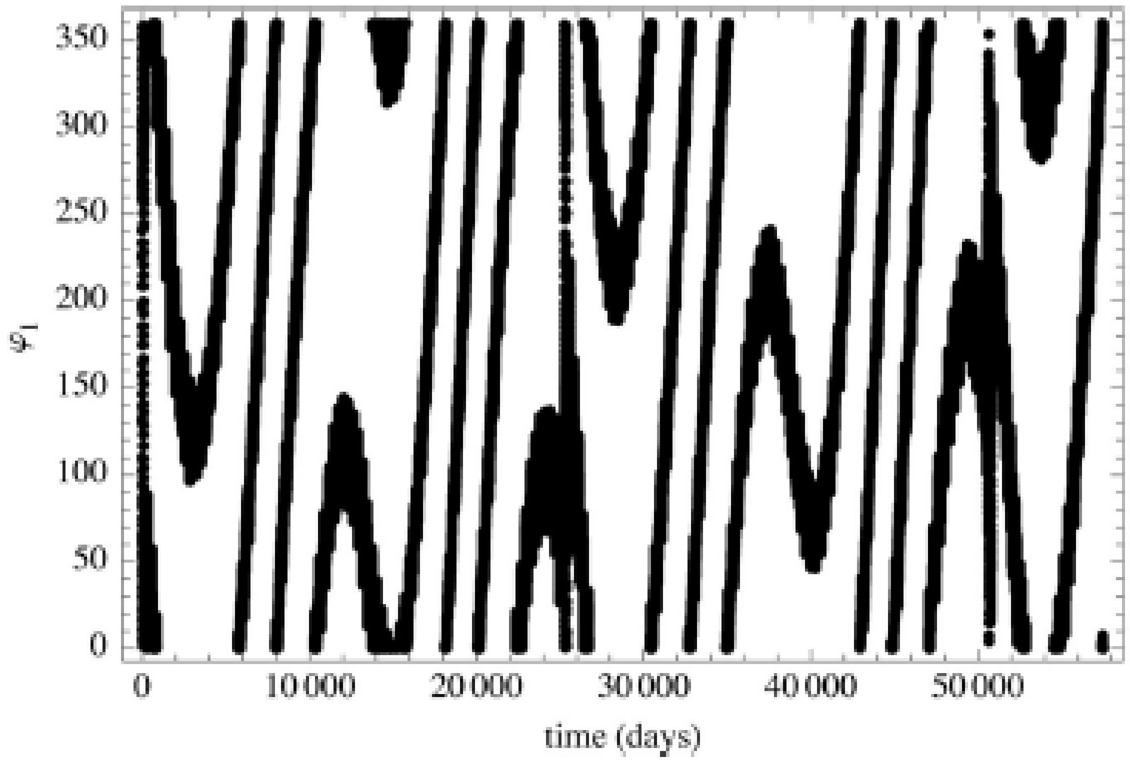


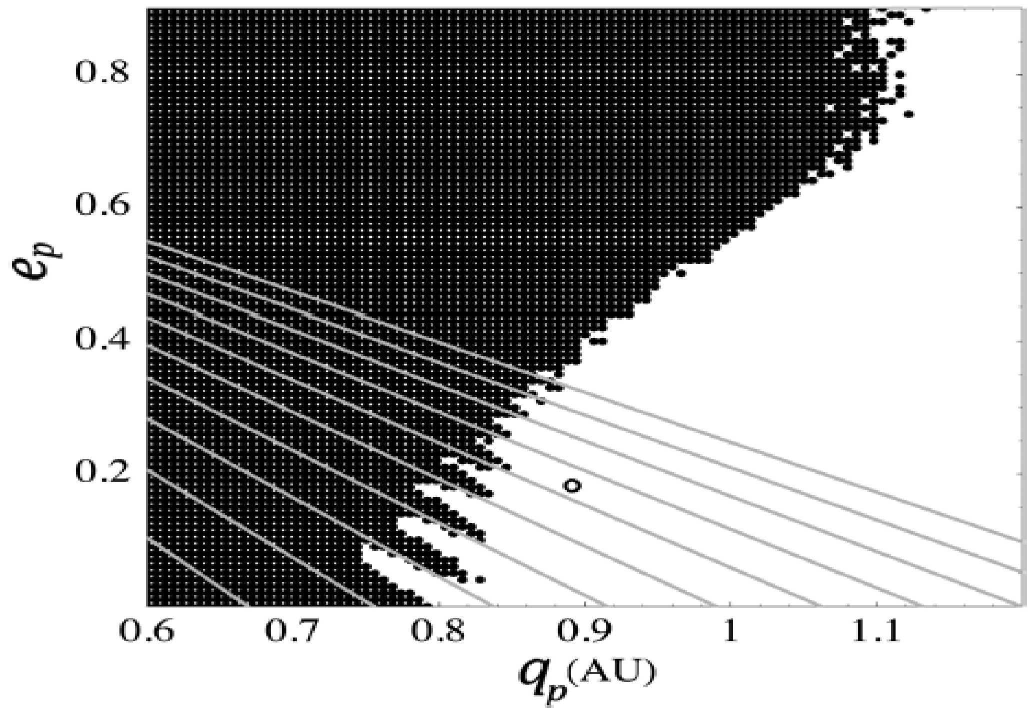
Red – Numerical solution
Green – Analytical solution
Blue – Analytical secular solution
Black – Observed value

$m_0=1.0479\ M_\odot$
$m_1=1.0208\ M_\odot$
$m_2=0.220\ M_J$
$a_1=0.22882$ au
$a_2=1.0896$ au
$e_1=0.52087$
$E_{10}=0^\circ$
$\varpi_1=0^\circ$
$l_{20}=90^\circ$

Chavez et al. (2015)

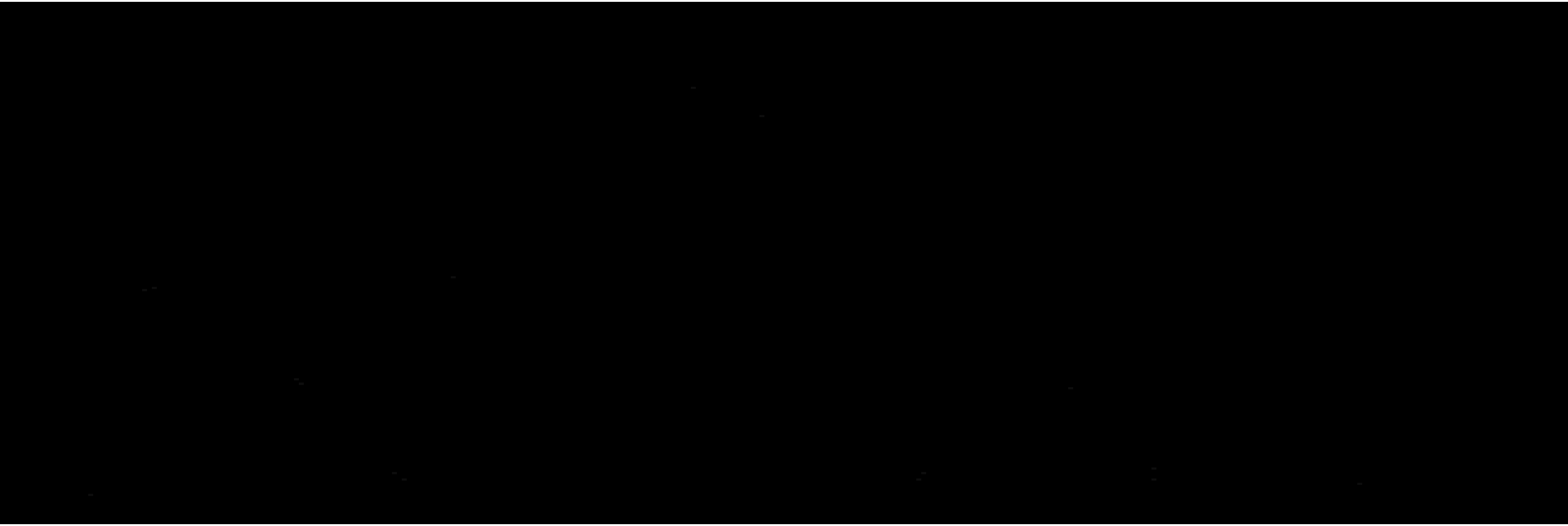
$$\varphi_1 = \lambda_b - 10\lambda_p + 9\varpi_p$$



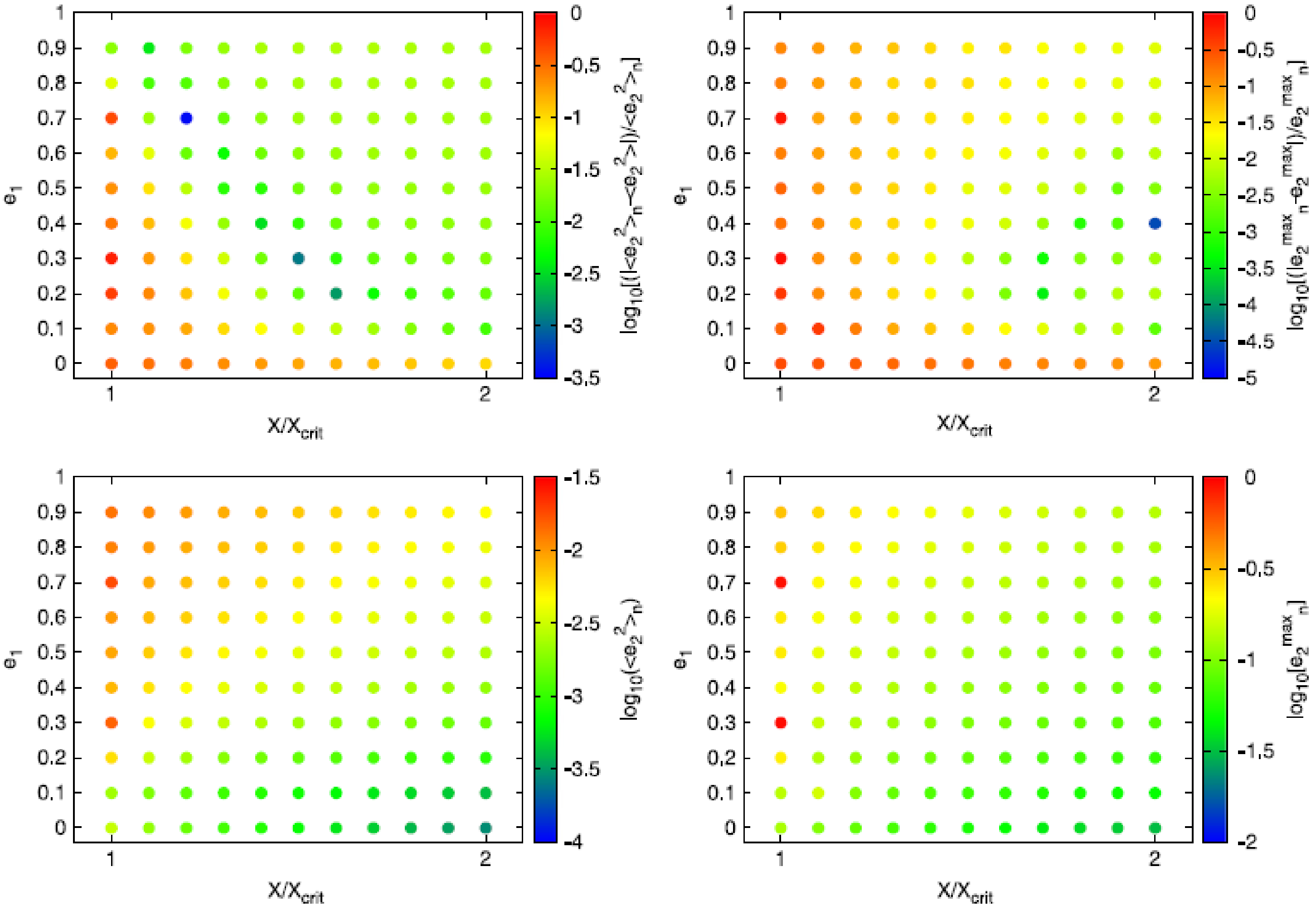


resonance trapping? (Kley & Haghighipour 2014) + 2nd planet? (Kley & Haghighipour 2015)

More details can be found in: Georgakarakos, N., Eggl, S. (2015), *ApJ*, 802, 94



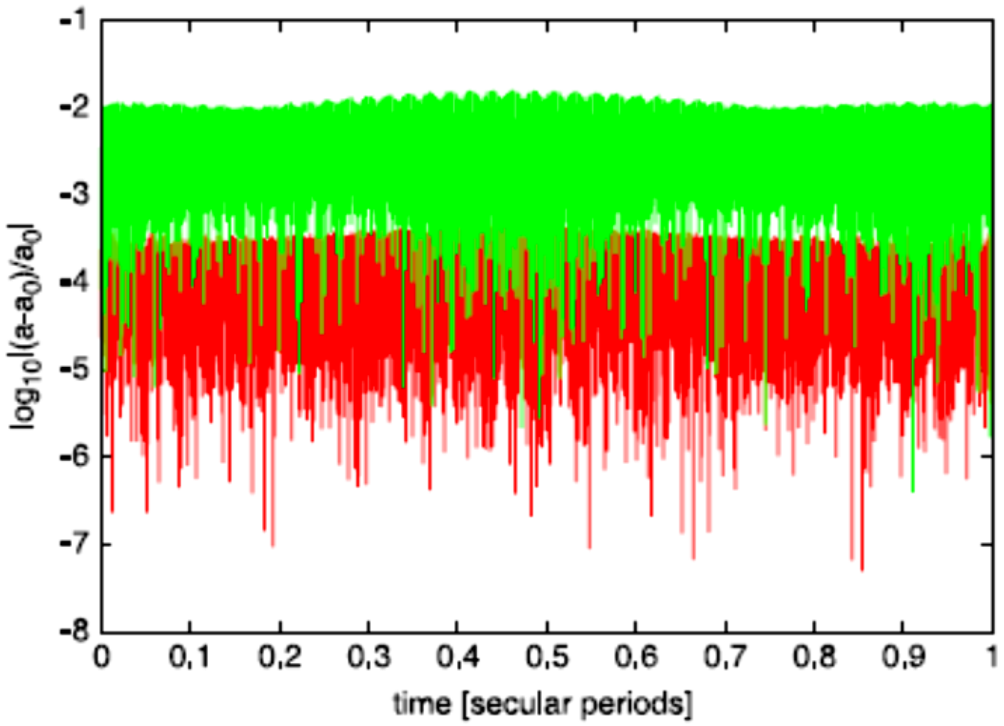
THANK YOU FOR YOUR ATTENTION



$m_1/(m_0+m_1)=0.3$
$m_2/(m_0+m_1)=0.001$

X_{crit} is the critical period ratio based on Holman & Wiegert (1999).

Integration time= 1 analytical secular period


-1
-2
-3
-4
-5
-6
-7
-8
log10|(a-a0)/a0|
0
0,1
0,2
0,3
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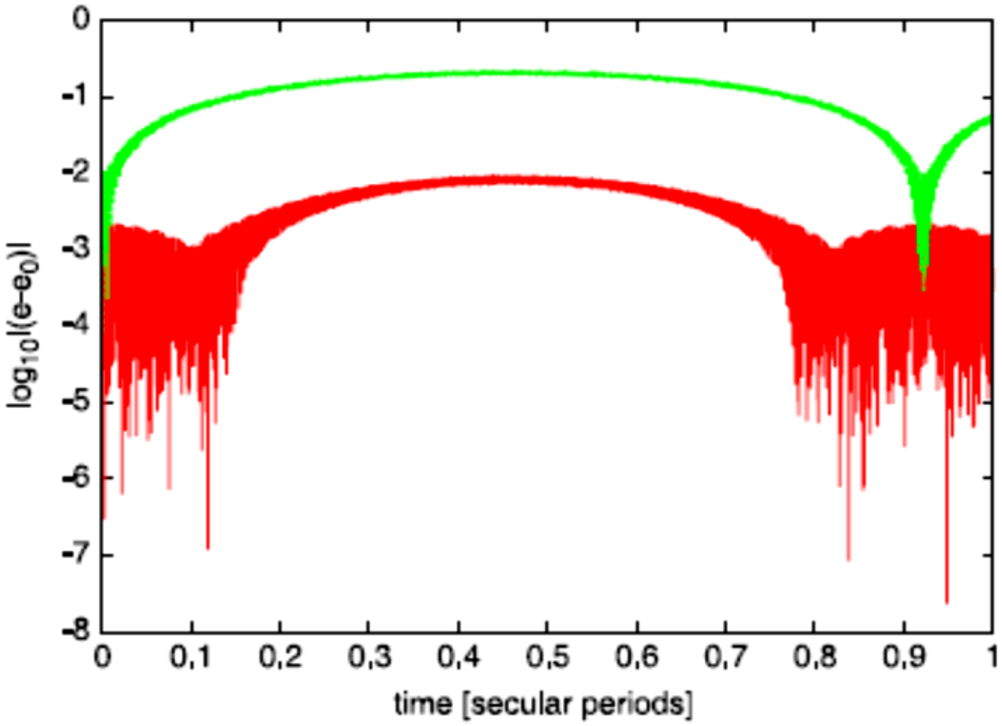
0,9
1
time [secular periods]


0
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log10|(e-e0)|
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0,1
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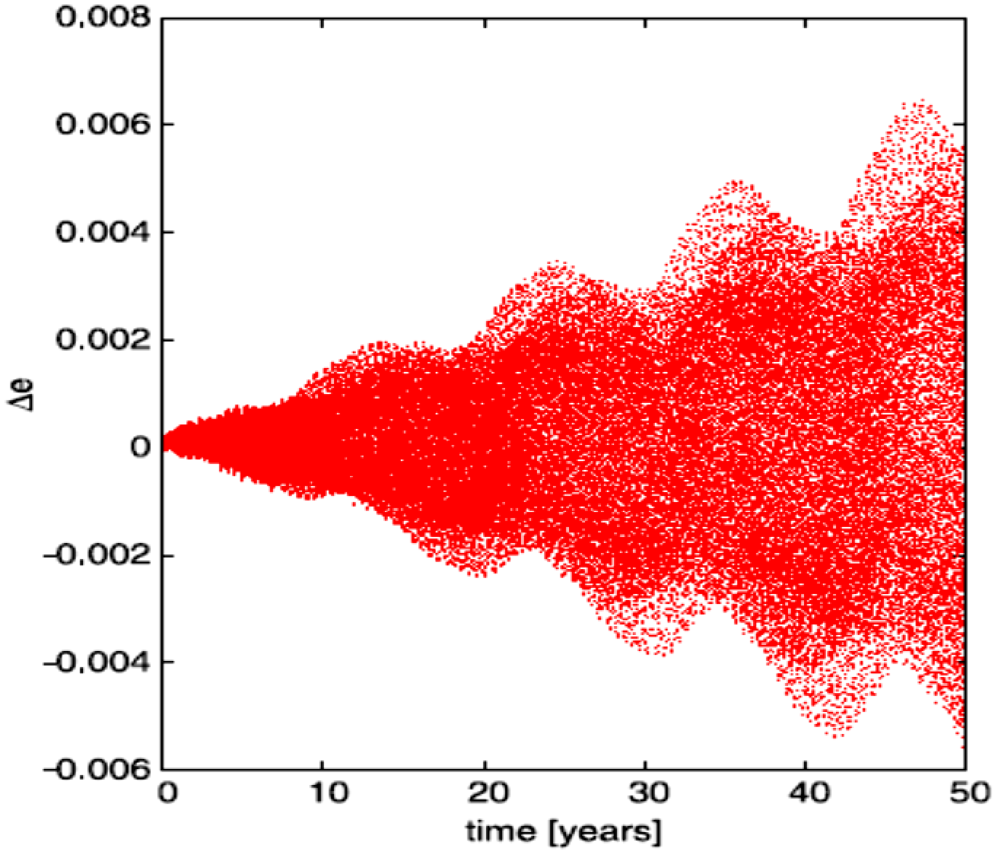
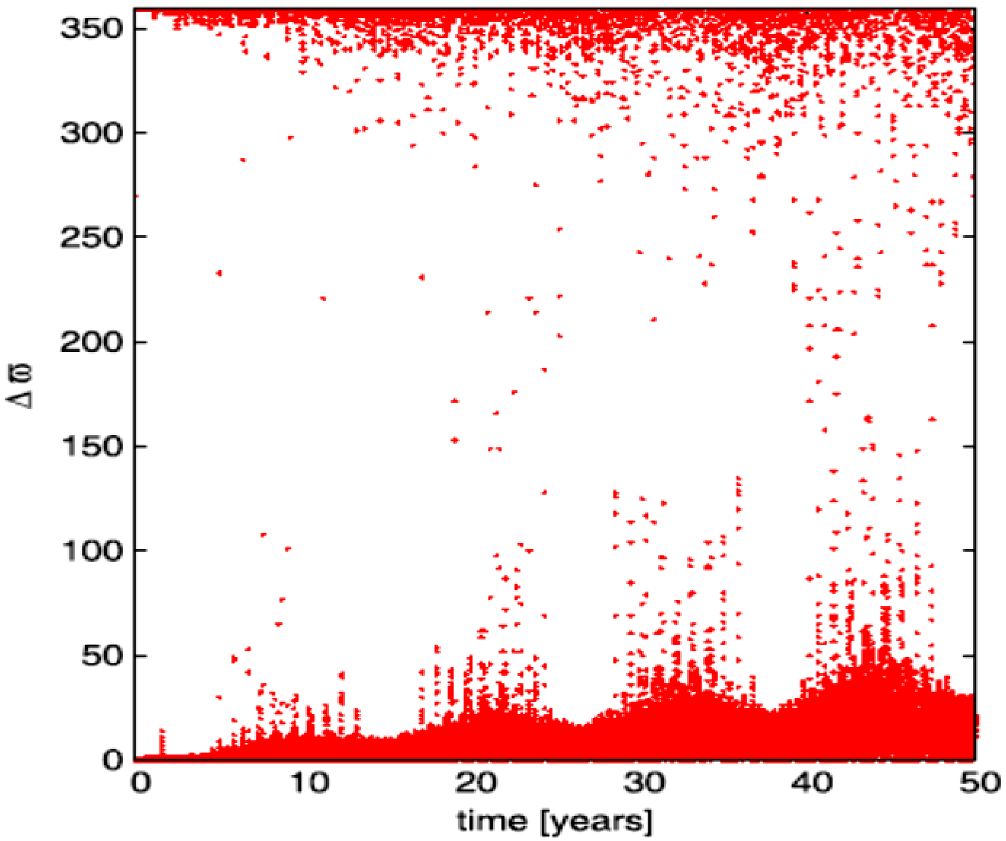
0,4
0,5
0,6
0,7
0,8
0,9
1
time [secular periods]

Kepler-413


0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
Δe
0
10
20
30
40
50
time [years]


350
300
250
200
150
100
50
0
Δϖ
0
10
20
30
40
50
time [years]