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Families of multi multiplicity periodic orbits in the Sun-Jupiter-Asteroid-Spacecraft system

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Abstract: The circular restricted equilateral four body problem consists of three primary bodies m_1, m_2 and m_3 lying in a Lagrangian configuration i.e. the three bodies remain fixed at the apices of an equilateral triangle and moving in circular periodic orbits around their center of mass fixed at the origin of a rotating coordinate system. The fourth massless body m_4 is moving under the Newtonian gravitational attractions of the primaries and does not affect the motion of the three bodies. In this paper we consider as m_1 the Sun, as m_2 the Jupiter, as m_3 an Asteroid (say the 2797 Teucer of the Trojan asteroids) and as m_4 a spacecraft. We study numerically the network of the families of non-symmetric periodic orbits which have more than two intersections with the $x-$axis per period. We found a large number of families that were computed in detail covering their natural termination, the morphology, and stability of their member solutions. The vast majority of the families have stable periodic solutions. Characteristic curves of the families as well as non-symmetric periodic orbits of the problem are also presented.

1 Introduction

The choice of the Sun as the dominant primary body of the problem and the Jupiter as the second one, ensures that the necessary condition for the stability of the Lagrange central configuration is fulfilled. The equations of motion of the massless fourth body (Spacecraft) referred to a synodic rotating coordinate system with the same origin as the primaries are [?],

$$\ddot{x} - 2\dot{y} = x - \sum_{i=1}^{3} \frac{m_i(x - x_i)}{r_i^3}, \qquad \ddot{y} + 2\dot{x} = y - \sum_{i=1}^{3} \frac{m_i(y - y_i)}{r_i^3}$$

when $r_i^2 = (x - x_i)^2 + (y - y_i)^2,\ i = 1, 2, 3$ and $x_i,\ y_i$ are the coordinates of the primaries (for details see [?]). The gravitational potential in synodic coordinates is given by the equation $\Omega = \frac{1}{2}(x^2 + y^2) + \frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}$ and a Jacobian type of integral of the problem is $\dot{x}^2 + \dot{y}^2 = 2\Omega - C$ where C is the Jacobian constant.

2 Families of periodic orbits

In Fig. 1 (left) we present the positions of the three primary bodies m_1, m_2, m_3 and the eight equilibrium points of the problem as well, for $m_1 = m_{Sun}/M$, $m_2 = m_{Jupiter}/M$ and $m_3 = m_{Asteroid}/M$ when $M = m_{Sun} + m_{Jupiter} + m_{Asteroid}$, $m_{Sun} = 1.98892 \times 10^{30}$, $m_{Jupiter} = 1.8986 \times 10^{27}$ and $m_{Asteroid} = 1.4 \times 10^{18}$, which means that $m_1 \simeq 0.999046$, $m_2 \simeq 0.000953$ and $m_3 \simeq 0.703228 \times 10^{-12}$. We have considered as $m_{Asteroid}$ the mass of an actual asteroid of the Greek group of Trojan asteroids, 2797 Teucer. Because the very small mass of the Asteroid, with respect to the other primaries, all calculations reported in this paper were performed in quadruple precision. Using the grid method we found initial conditions for the numerical determination symmetric periodic orbits of the problem for $m_1 = m_{Sun}$ and $m_2 = m_3 = (1 - m_1)/2 \simeq 0.0004768$. In Fig. 1 (right) the results of this grid process

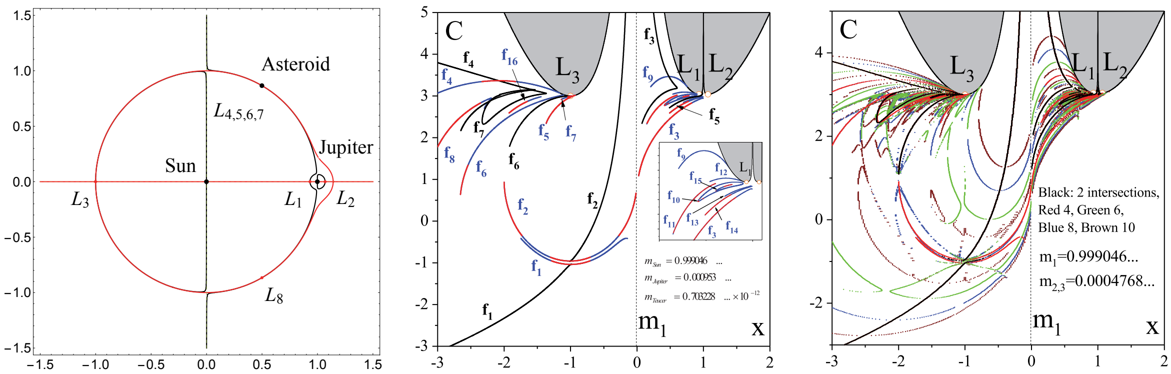

Figure 1: Left: The positions of the primary bodies and equilibrium points of the problem. Middle: The network of the families of the simple (black) and the double (blue) non-symmetric periodic orbits in the Sun - Jupiter - Teucer - Spacecraft system. The red arcs indicate horizontal stability. Right: The network of the families of symmetric periodic solutions re-entering after 2 to 10 oscillations per period for $m_2 = m_3$

are presented. We found the initial conditions of symmetric periodic orbits for 2 (black), 4 (red), 6 (green), 8 (blue) and 10 (brown) intersections with the x-axis. Using these initial conditions we found a large number of symmetric periodic orbits of the problem for 2 to 10 intersections. From these orbits we calculated series of asymmetric periodic orbits for various values of the primaries m_2 and m_3 and we stopped our calculations when $m_2 = m_{Jupiter}$ and $m_3 = m_{Teucer}$.

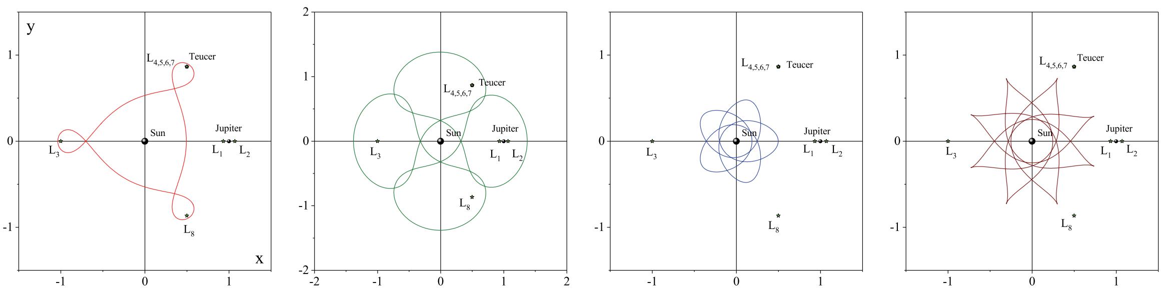

Figure 2: From left to right: Periodic orbits of the problem with 4, 6, 8 and 10 intersections with x-axis

Using these asymmetric periodic orbits we found 16 families of periodic orbits with four intersections with x-axis. In Fig. 1 (middle) we present the characteristic curves of these 16 families (blue lines). Their horizontal stability arcs are also shown (red lines). In the same figure we illustrate seven families of simple (two intersections) asymmetric periodic orbits (black lines) which we have found them in a previous work. The results obtained (Fig. 1, middle and right) indicate that these families of simple orbits are the basic families, i.e. families from which other families branch-off, correspond to families of periodic orbits with higher multiplicity and in addition it is them that dictate the shape of the solutions of the problem. The majority of 16 families, as they are evolved, change their multiplicity while some of them are led to collision with the Sun or the Jupiter. All the families have stable arcs (some of them are entirely stable), except the families f_9, f_{12} and f_{13} which consist of unstable periodic orbits. The morphology of the solutions of the families members corresponding to 4 to 10 intersections, is presented in Fig. 2, where one solution from four families is plotted in the physical plane (x, y). All these periodic orbits are stable except the third one (8 intersections).

References

[1] Baltagiannis, A. N. and Papadakis, K. E., 2013, "Periodic solutions in the Sun-Jupiter-Trojan Asteroid-Spacecraft system", *Planetary and Space Science*, **75**, pp. 148–157.