Periodic motions and stability in the regular polygon problem of (N+1) bodies with quasi-homogeneous potentials

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THE PROBLEM

The regular polygon problem of (N+1) bodies is a N-body model where the N-1 of the bodies have equal masses m and are located at the vertices of an imaginary regular polygon, while another body with mass m_0 is located at the center of mass of the system. In the resultant force field created by the N primaries, a very small body moves without affecting their motion. The initial statement of the problem was based on the assumption that all big bodies create Newtonian force fields. Here we assume an inverse square Manev-like corrective term in the Newtonian potentials of all the big bodies so as the created potentials are quasi-homogeneous.

EQUATIONS OF MOTION IN A SYNODIC SYSTEM

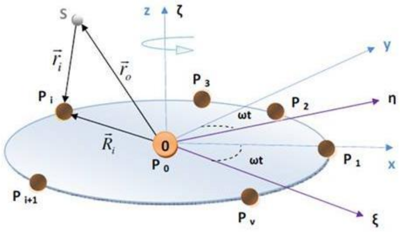
In a synodic coordinate system with ω=1, the dimensionless equations of motion are,

$$\ddot{x}-2\dot{y}=\frac{\partial U}{\partial x}=U_x\,,\ \ddot{y}+2\dot{x}=\frac{\partial U}{\partial y}=U_y\,,\ddot{z}=\frac{\partial U}{\partial z}=U_z$$

POTENTIAL FUNCTION

$$U(x,y,z)=\frac{1}{2}(x^2+y^2)+\frac{1}{\Delta}\left[\beta\left(\frac{1}{r_0}+\frac{e_o}{r_0^2}\right)+\sum_{i=1}^{\nu}\left(\frac{1}{r_i}+\frac{e}{r_i^2}\right)\right]$$

- $\beta=m_0/m$ is the ratio of the central mass to a peripheral one
- e_0 and e are the Manev's parameters of the central and the peripheral primaries respectively



$$\Delta=M(\Lambda+2e\Lambda'+\beta M^2+2\beta e_o M^3)$$

$$\Lambda=\sum_{i=2}^{\nu}\frac{\sin^2(\pi/\nu)}{\sin[(i-1)(\pi/\nu)]}\quad \Lambda'=\sum_{i=2}^{\nu}\frac{\sin^3(\pi/\nu)}{\sin^2[(i-1)(\pi/\nu)]}\quad M=2\sin(\pi/\nu)$$

Condition for existing solutions: $\Delta>0$

$$r_0=\left(x^2+y^2+z^2\right)^{\frac{1}{2}}\quad r_i=\left[(x-x_i)^2+(y-y_i)^2+z^2)\right]^{\frac{1}{2}}$$

distances of the particle from the central and the peripheral primaries respectively

JACOBIAN INTEGRAL: $\dot{x}^2+\dot{y}^2+\dot{z}^2=2U(x,y,z)-C$

SEARCHING THE SYMMETRIC PERIODIC ORBITS

Conditions of periodic planar motions which are symmetric with respect to the *x*-axis

$x_0\neq 0, y_0=0, \dot{x}_0=0, \dot{y}_0\neq 0$ **at the initial position (t=0)**

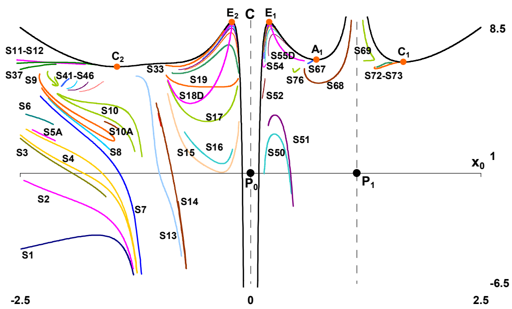
$x_{T/2}\neq 0, y_{T/2}=0, \dot{x}_{T/2}=0, \dot{y}_{T/2}\neq 0$ **at the half- period (t=T/2)**

Method of searching: We applied a grid search method or scanning procedure in the phase space of the initial conditions which is the plane x_0-C, where C is the Jacobian constant

Linear stability: we use the reduced form of the monodromy matrix, $J=\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}\partial x/\partial x_0 & \partial x/\partial\dot{x}_0\\ \partial\dot{x}/\partial x_0 & \partial\dot{x}/\partial\dot{x}_0\end{pmatrix}$ and its trace *k=a+d (stability index)*.

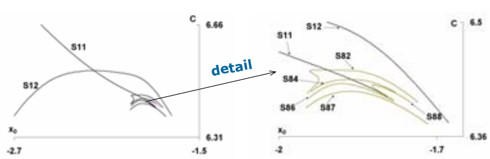
When symmetric orbits are considered, then *a=d*,and the criterion for stability becomes *-1≤a≤1.* Families of simple periodic orbits may emanate from points of the main family where *a=1.*

Case where only the central primary creates a Manev-like potential ($\nu=7, \beta=5, e_o=-0.1, e=0$)

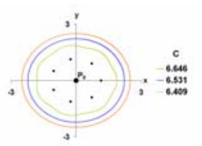


Simple symmetric periodic families for $-2.5<x_0<2.5$

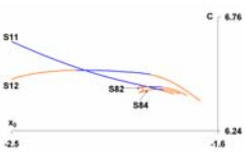
Study of simple periodic orbits of "fish-type" families: It is a formation of curves, two of which intersect (S11 and S12) thus forming the outline of a fish. This is the reason why we refer to them as "fish-type families". All the orbits of that group are planetary-type, retrograde and are described around and outside of the formation of the primaries.



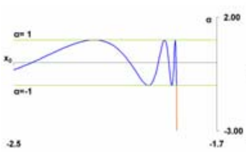
Characteristic curves of simple periodic orbits of "fish-type" families



Evolution of s.p.orbits of the S11



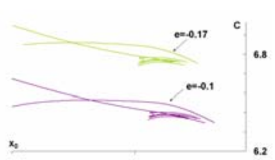
*** Stability of the parts of characteristic curves**



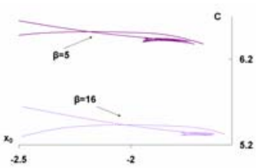
*** Stability diagram of S11**

*** Stability diagram of S12**

***The blue regions denote the parts where orbits are stable, while the red ones indicate the parts where orbits are unstable**

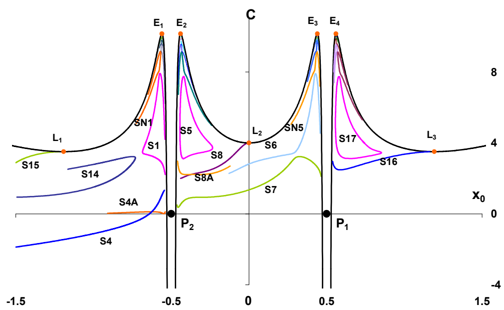


Parametric variation of characteristic curves of the "fish-type" families of s.p.o. with e, for β=5



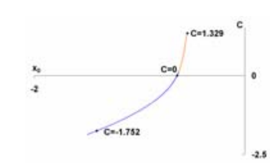
Parametric variation of characteristic curves of the "fish-type" families of s.p.o. with β, for e=-0.1

Case where only the peripheral primaries create a Manev-like potential Application to Copenhagen problem($\nu=2, \beta=0, e_o=0, e=-0.03$)

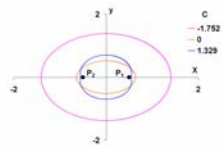


Simple symmetric periodic families for $-1.5<x_0<1.5$

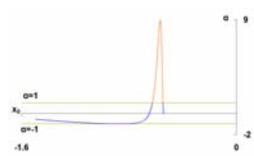
Study of various simple periodic orbits: The following orbits are retrograde planetary-type (S4 and S8) and orbits around an equilibrium point(S16). In the first and third column of the figures, the blue regions denote the parts where orbits are stable, while the red ones indicate the parts where orbits are unstable



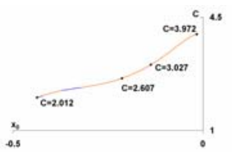
Characteristic curve of S4 family



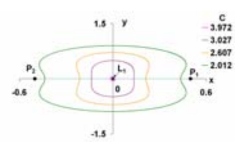
Evolution of s.p.orbits of the S4



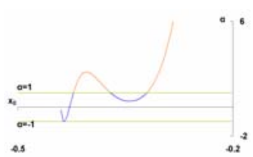
Stability diagram of S4



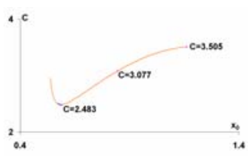
Characteristic curve of S8 family



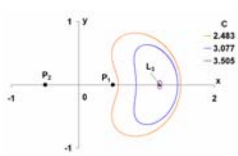
Evolution of s.p.orbits of the S8



Stability diagram of S8



Characteristic curve of S16 family



Evolution of s.p.orbits of the S16

Stability diagram of S16

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