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Gravitational quantization of exoplanet orbits: The system TRAPPIST-1

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Abstract: We apply the so-called "global polytropic model" to the numerical study of the exoplanetary system TRAPPIST-1. We compare the computed exoplanet distances from their host star with corresponding observations, and quote certain orbit predictions given by the model.

Keywords: exoplanets; global polytropic model; quantized orbits; stars: individual (TRAPPIST-1)

1 Introduction

We study numerically the exoplanetary system TRAPPIST-1. This work is continuation of three previous papers regarding exoplanet systems ([1], [2], [3]); we shall not repeat here issues developed in these papers. A detailed account of the so-called "global polytropic model" can be found in [1] (Secs. 2, 3, and references therein).

2 Numerical Results

Results for the 7-planet TRAPPIST-1 system ([4], [5], [6], [7]) are shown in Table 1. The first root ξ_1 of the Lane–Emden function θ, coinciding with the radius of the host star, is expressed in both "classical polytropic units" (cpu) — in such units, the length unit is equal to the polytropic parameter α ([8], Eq. (3b)) — and solar radii $R_\odot$. All other orbit radii are expressed in AU.

The minimum sum of absolute percent errors is found to be

$$\Delta_{\rm min}\Big(n_{\rm opt}(\text{TRAPPIST-1}) = 2.525;\, q_{\rm b} = 6,\, q_{\rm c} = 7,\, q_{\rm d} = 8,\\ q_{\rm e} = 9,\, q_{\rm f} = 10,\, q_{\rm g} = 11,\, q_{\rm h} = 13\Big) \simeq 44.2. \tag{1}$$

It is worth clarifying here that the term "error" is used with the meaning of the deviation (the difference) of a computed value with respect to its corresponding observed value, and does not reflect any errors owing to the numerical methods used; the latter ones are very small in all computations of the present study.

The optimum polytropic index $n_{\rm opt} = 2.525$ for the star TRAPPIST-1 is close to the values $n_{\rm opt}$(Kepler-32) = 2.608 ([3], Sec. 3, Eq. (2), Table 2) and $n_{\rm opt}$(Kepler-186) = 2.530 ([3], Sec. 6, Eq. (5), Table 5). Note that both stars Kepler-32 and Kepler-186 have spectral type 'M', i.e. same to that of the star TRAPPIST-1.

The distance having the smaller error relative to its observed value, $\simeq 0.4\%$, is that of the planet e, while the larger error appears in the distance of the innermost planet b, $\simeq 14.7\%$. The average error for the computed orbit radii of the 7 planets relative to their corresponding observed orbits is $\simeq 6.3\%$.

Regarding the large error appearing in the distance of the innermost planet b, it may be due to its close proximity to the star TRAPPIST-1. Similar large errors have been also found for the innermost planet b of the exoplanetary system HD 10180, $\sim 44\%$ ([3], Sec. 2, Table 1), and for the innermost planet e of the exoplanetary system 55Cnc, $\sim 32\%$ ([1], Sec. 4, Table 1). It is worth emphasizing here that b is the closest planet to its host star, $\alpha_{\rm b} = 1.111 \times 10^{-2}$ AU, among all exoplanetary systems studied in [1], [2], and [3]. We conjecture that, forced by the host star, the planet b has been eventually expelled to an orbit close to the "right average-density orbit" (abbreviated "Right ADO"; [2], Sec. 2; also [3], Sec. 2) within the polytropic shell #6, $\alpha_{\rm R6} = 1.072 \times 10^{-2}$ AU. If so, then the deviation of $\alpha_{\rm R6}$ from the observed b's distance drops to $\simeq 3.5\%$.

3 Some Predictions

Regardind the large error involved in the orbit radius of the planet d, an interesting conjecture — made firstly for the planet f of the exoplanetary system HD 40307 ([2], Eq. (2) and Sec. 3.1; in Sec. 2 of [3] we have used the abbreviations "LADC" and "RADC" for the "left average-density orbit conjecture" and the "right average-density orbit conjecture", respectively) — is to associate this distance with the "left average-density orbit" (abbreviated "Left ADO") $\alpha_{\rm L8} \simeq 0.022$ AU, provided that the maximum-density orbit α_8 of the polytropic shell #8 is already occupied by another planet not yet observed. Then the error for the d's distance would drop to $\simeq 4.5\%$, and the closest distance of the two planets on their orbits would be $\sim 3 \times 10^5$ km ($\sim$ Earth–Moon distance).

Our numerical results for the exoplanetary system TRAPPIST-1 show that the polytropic shell #12 either remains unoccupied, or it points to prediction of one further planet not yet observed. For this shell, we have found $\alpha_{\rm L12} = 0.0576$ AU, $\alpha_{12} = 0.0605$ AU, and $\alpha_{\rm R12} = 0.0618$ AU. Likewise, in accordance with the up-to-now observations, the shells next to the polytropic shell #13 seem to be unoccupied. For the next seven shells we have found $\alpha_{14} = 0.0834$ AU, $\alpha_{15} = 0.0913$ AU, $\alpha_{16} = 0.1034$ AU, $\alpha_{17} = 0.1242$ AU, $\alpha_{18} = 0.1298$ AU, $\alpha_{19} = 0.1537$ AU, and $\alpha_{20} = 0.1777$ AU.

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Table 1: The system TRAPPIST-1: central body S_1, i.e. the host star TRAPPIST-1, and polytropic spherical shells of the planets b, c, d, e, f, g, h. For successive shells S_j and S_{j+1}, inner radius of S_{j+1} is the outer radius of S_j. All radii are expressed in AU, except for the host's radius ξ_1. Percent errors $\%E_j$ in the computed orbit radii α_j are given with respect to the corresponding observed radii A_j, $\%E_j = 100 \times |(A_j - \alpha_j)|/A_j$. Parenthesized signed integers following numerical values denote powers of 10.

Host star TRAPPIST-1 – Shell No	1		
$n_{\rm opt}$	2.525 (+00)		
ξ_1 (cpu)	5.4168(+00)		
ξ_1 ($R_\odot$)	1.17 (−01)		
		A	$\%E$
b – Shell No	6		
Inner radius, ξ_5	9.3829(−03)		
Outer radius, ξ_6	1.3307(−02)		
Left ADO, $\alpha_{\rm L6}$	9.4130(−03)		
Orbit radius, $\alpha_{\rm b} = \alpha_6$	9.4810(−03)	1.111(−02)	1.47(+01)
Right ADO, $\alpha_{\rm R6}$	1.0720(−02)		
c – Shell No	7		
Outer radius, ξ_7	1.9631(−02)		
Left ADO, $\alpha_{\rm L7}$	1.4536(−02)		
Orbit radius, $\alpha_{\rm c} = \alpha_7$	1.6063(−02)	1.522(−02)	5.54(+00)
Right ADO, $\alpha_{\rm R7}$	1.7811(−02)		
d – Shell No	8		
Outer radius, ξ_8	2.6154(−02)		
Left ADO, $\alpha_{\rm L8}$	2.2117(−02)		
Orbit radius, $\alpha_{\rm d} = \alpha_8$	2.4001(−02)	2.1(−02)	1.43(+01)
Right ADO, $\alpha_{\rm R8}$	2.5177(−02)		
e – Shell No	9		
Outer radius, ξ_9	3.1860(−02)		
Left ADO, $\alpha_{\rm L9}$	2.6800(−02)		
Orbit radius, $\alpha_{\rm e} = \alpha_9$	2.8111(−02)	2.8(−02)	3.98(−01)
Right ADO, $\alpha_{\rm R9}$	3.0247(−02)		
f – Shell No	10		
Outer radius, ξ_{10}	4.0928(−02)		
Left ADO, $\alpha_{\rm L10}$	3.3312(−02)		
Orbit radius, $\alpha_{\rm f} = \alpha_{10}$	3.5306(−02)	3.7(−02)	4.58(+00)
Right ADO, $\alpha_{\rm R10}$	3.8130(−02)		
g – Shell No	11		
Outer radius, ξ_{11}	5.2284(−02)		
Left ADO, $\alpha_{\rm L11}$	4.3295(−02)		
Orbit radius, $\alpha_{\rm g} = \alpha_{11}$	4.6360(−02)	4.5(−02)	3.02(+00)
Right ADO, $\alpha_{\rm R11}$	4.9450(−02)		
h – Shell No	13		
Inner radius, ξ_{12}	6.2345(−02)		
Outer radius, ξ_{13}	7.2790(−02)		
Left ADO, $\alpha_{\rm L13}$	6.2744(−02)		
Orbit radius, $\alpha_{\rm h} = \alpha_{13}$	6.4061(−02)	6.3(−02)	1.68(+00)
Right ADO, $\alpha_{\rm R13}$	6.8020(−02)		