Relativistic magnetised cosmological perturbations in the post-recombination era

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Heraklion, July 2017

Large Scale Magnetic Fields

Magnetic Fields are Everywhere

From the Earth and the Sun, to distant galaxies and clusters ($B \sim 10^{-6}\ G$)

Even in the intergalactic voids ? ($B \sim 10^{-15}\ G$)

Origin of Large Scale Magnetic Fields (*open question*)

- Astrophysical (*after recombination*)
- Cosmological (*before recombination*) { During inflation / After inflation }

Implications for Large Scale Structure Formation

- How magnetic fields affect the evolution of density perturbations
- Relativistic studies in the *dust era* are *incomplete*

Magnetic Fields in GR and Ideal MHD

Fluid Description

Magnetic energy-momentum tensor

$$T_{ab}^{(B)} = \frac{1}{2}B^2 u_a u_b + \frac{1}{6}B^2 h_{ab} + \Pi_{ab}$$

with

$$\rho_B = \frac{1}{2}B^2 \,, \qquad p_B = \frac{1}{6}B^2$$

and

$$\Pi_{ab} = \frac{1}{3}B^2 h_{ab} - B_a B_b \Rightarrow \begin{cases} k^b\Pi_{ab} = +\frac{1}{3}B^2 k_a, & \text{magnetic pressure} \quad (k_a \perp B_a) \\ n^b\Pi_{ab} = -\frac{2}{3}B^2 n_a, & \text{magnetic tension} \quad (n_a // B_a) \end{cases}$$

Interaction with Geometry/Gravity

Einstein equations (*Standard interaction*)

$$G_{ab} = \kappa({T_{ab}}^{(m)} + {T_{ab}}^{(B)})$$

Ricci identities (*Geometric interaction*)

$$2\nabla_{[a}\nabla_{b]}B_c = R_{abcd}B^d \qquad (R_{abcd}\text{: 4-Riemann tensor})$$

Inhomogeneity Variables

Key Variable

Inhomogeneities in the matter density

$$\Delta_a = \frac{a}{\rho} D_a \rho \qquad (a\text{: } \textit{scale factor})$$

as measured by two neighbouring observers.

Auxiliary Variables

Inhomogeneities in the expansion

$$\mathcal{Z}_a = a D_a \Theta \qquad (\Theta\text{: } \textit{volume expansion})$$

Inhomogeneities in the magnetic energy density

$$\mathcal{B}_a = \frac{a}{B^2} D_a B^2 \qquad (B^2 = B^a B_a)$$

Basic Equations (Ideal MHD)

Conservation Laws and Kinematic Equations

Continuity equation

$$\dot{\rho} = -\Theta(\rho + p)$$

Equation of motion (Navier - Stokes)

$$\left(\rho + p + \frac{2}{3}B^2\right) A_a = -D_a p - \frac{1}{2} D_a B^2 + B^b D_b B_a - \Pi_{ab} A^b$$

Raychaudhuri's equation (basic kinematic formula)

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}\left(\rho + 3p + B^2\right) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a$$

Maxwell's Equations in the Ideal MHD

$$\dot{B}_a = -\frac{2}{3} B_a + (\sigma_{ab} + \omega_{ab}) B^b \qquad (\textit{Magnetic Induction Law})$$

and

$$D^a B_a = 0 \qquad (\textit{Gauss Law})$$

FRW Background ($K = 0 = \Lambda$)

Non-magnetised Background

Zero-perturbative order Variables

$$\rho \to \bar{\rho} = \bar{\rho}(t), \qquad p \to \bar{p} = \bar{p}(t) \quad \text{and} \quad \frac{\Theta}{3} \to H = H(t)$$

Zero-perturbative order Equations

$$H^2 = \frac{1}{3}\kappa\rho \qquad \text{and} \qquad \dot{H} = -H^2 - \frac{1}{6}\kappa(\rho + 3p)$$

Background Magnetic Field

Sufficiently random and weak B-field so that:

$$\langle \bar{B}_a \rangle = 0, \qquad \langle \bar{B}^2 \rangle \neq 0 \quad \text{and} \quad \frac{\langle \bar{B}^2 \rangle}{\bar{\rho}} \ll 1$$

Magnetic evolution

$$(\bar{B}^2)^{\cdot} = -4H\bar{B}^2 \;\Rightarrow\; \bar{B}^2 \propto a^{-4}$$

Linear Evolution Equations

Evolution of the Inhomogeneities

In matter density

$$\dot{\Delta}_a = 3wH\Delta - (1+w)\mathcal{Z}_a + \frac{3aH}{\rho}\left(\frac{1}{2}D_aB^2 - B^bD_bB_a\right) + 2{c_a}^2(1+w)aHA_a$$

In volume expansion

$$\dot{\mathcal{Z}}_a = -2H\mathcal{Z}_a - \frac{1}{2}\rho\Delta_a - \frac{1}{2}B^2\mathcal{B}_a + \frac{3}{2}a\left(\frac{1}{2}D_aB^2 - B^bD_bB_a\right) + aD_aA$$

In magnetic energy density

$$\dot{\mathcal{B}}_a = \frac{4}{3(1+w)}\dot{\Delta}_a - \frac{4wH}{1+w}\Delta_a - \frac{4aH}{\rho(1+w)}\left(\frac{1}{2}D_aB^2 - B^bD_bB_a\right) - 4aH\left(1+\frac{2}{3}{c_a}^2\right)A_a$$

Parameters

- Barotropic index: $w = \bar{p}/\bar{\rho} = \text{constant} \Rightarrow w = {c_s}^2 = d\bar{p}/d\bar{\rho}$ (*sound speed*)
- Alfvén speed: ${c_a}^2 = \frac{\bar{B}^2}{\bar{\rho}(1+w)} \Rightarrow {c_a}^2 \ll 1$

Density Perturbations

Three Types of Inhomogeneity for Matter Density

$$\Delta_{ab} = a\mathrm{D}_b\Delta_a = \frac{1}{3}\Delta h_{ab} + \Delta_{[ab]} + \Delta_{\langle ab\rangle}$$

- $\Delta = a\mathrm{D}^a\Delta_a$: describes density perturbations (*scalar*) $\left(\Delta \to \delta = \delta\rho/\bar{\rho} \gtreqless 0\right)$
- $\Delta_{[ab]} = a\mathrm{D}_{[b}\Delta_{a]}$: depicts density vortices (*vector*)
- $\Delta_{\langle ab\rangle} = a\mathrm{D}_{\langle b}\Delta_{a\rangle}$: monitors shape distortions (*trace-free tensor*)

Three Types of Inhomogeneity for $\mathcal{Z}_a$ and $\mathcal{B}_a$

$$\mathcal{Z}_{ab} = a\mathrm{D}_b\mathcal{Z}_a = \frac{1}{3}\mathcal{Z}h_{ab} + \mathcal{Z}_{[ab]} + \mathcal{Z}_{\langle ab\rangle}$$

$$\mathcal{B}_{ab} = a\mathrm{D}_b\mathcal{B}_a = \frac{1}{3}\mathcal{B}h_{ab} + \mathcal{B}_{[ab]} + \mathcal{B}_{\langle ab\rangle}$$

Linear Density Perturbations ($w = 0$, ${c_a}^2 \ll 1$)

Matter Density Perturbations

$$\dot{\Delta} = -\mathcal{Z} + \frac{3}{2} H {c_a}^2 \mathcal{B} - {c_a}^2 H a^2 \mathcal{R} - 6H\frac{a^2}{\rho}\left({\sigma_B}^2 - {\omega_B}^2\right)$$

where

- $\mathcal{R}$: perturbed 3-Ricci scalar ($\mathcal{R} \gtrless 0$)
- ${\sigma_B}^2 = D_{\langle b} B_{a\rangle} D^{\langle b} B^{a\rangle}/2$: **magnetic shear** (*deformation of the B-field lines*)
- ${\omega_B}^2 = D_{[b} B_{a]} D^{[B} B^{a]}/2$: **magnetic vorticity** (*twisting of the B-field lines*)

Expansion Perturbations

$$\begin{aligned}\dot{\mathcal{Z}} = & - 2H\mathcal{Z} - \frac{1}{2}\rho\Delta + \frac{1}{4}{c_a}^2\rho\mathcal{B} - \frac{1}{2}{c_a}^2 D^2\mathcal{B} \\ & - \frac{1}{2}{c_a}^2\rho a^2\mathcal{R} - 3a^2\left({\sigma_B}^2 - {\omega_B}^2\right) + \frac{2a^2}{\rho}D^2\left({\sigma_B}^2 - {\omega_B}^2\right)\end{aligned}$$

Magnetic Energy Density Perturbations

$$\dot{\mathcal{B}} = \frac{4}{3}\dot{\Delta} \Rightarrow \mathcal{B} = \frac{4}{3}\Delta + C$$

Wave-like Equation

Matter Density Perturbations

$$\ddot{\Delta} = -2H\dot{\Delta} + \frac{1}{2}\kappa\rho\Delta + \frac{2}{3}{c_a}^2 D^2\Delta + \frac{2}{3}{c_a}^2\rho a^2\mathcal{R} + 4a^2\left({\sigma_B}^2 - {\omega_B}^2\right) - 2\frac{a^2}{\rho}D^2\left({\sigma_B}^2 - {\omega_B}^2\right)$$

where c_a: Alfvén speed = wave speed

Harmonic Decomposition

$$\ddot{\Delta}_{(n)} = -2H\dot{\Delta}_{(n)} + \frac{1}{2}\kappa\rho\left[1 - \left(\frac{\lambda_J}{\lambda_n}\right)^2\right]\Delta_{(n)} + 4a^2\left[1 + \frac{1}{6}\left(\frac{\lambda_H}{\lambda_n}\right)^2\right]({\sigma_B}^2 - {\omega_B}^2)_{(n)} + \frac{2}{3}\rho {c_a}^2 a^2\mathcal{R}_{(n)}$$

where

- $\lambda_n = a/n$: physical scale of the perturbation
- $\lambda_H = 1/H$: Hubble horizon ($\lambda_H = 3 \times 10^3\ Mpc$)
- $\lambda_J = \frac{2}{3}c_a\lambda_H \ll \lambda_H$: **Magnetic Jeans length** $\begin{cases} \lambda_J \sim 1Mpc & \text{if } B \sim 10^{-6}\ G \\ \lambda_J \sim 10kpc & \text{if } B \sim 10^{-7}\ G \end{cases}$

Solutions I

Dust with magnetic pressure ($\mathcal{R} = 0 =$magnetic tension)

$$\Delta_{(n)} = C_1 t^{-\frac{1}{6}+\frac{1}{6}\sqrt{25-24\alpha}} + C_2 t^{-\frac{1}{6}-\frac{1}{6}\sqrt{25-24\alpha}} \qquad (\textit{power law})$$

where $\alpha = (\lambda_J/\lambda_n)^2 \;=\; \textit{constant}$

Cases

- $\lambda_n \gg \lambda_J$

$$\Delta_{(n)} = C_1 t^{\frac{2}{3}} + C_2 t^{-1} \qquad (\textit{standard non-magnetised})$$

- $\lambda_n \ll \lambda_J$

$$\Delta_{(n)} = t^{-\frac{1}{6}} \left(C_1 t^{+i\frac{\sqrt{24\alpha}}{6}} + C_2 t^{-i\frac{\sqrt{24\alpha}}{6}} \right)$$

- $\lambda_n = \lambda_J$

$$\Delta_{(n)} = C_1 + C_2 t^{\frac{-1}{3}}$$

Conclusion

Magnetic pressure *inhibits* the growth of matter density perturbations

Solutions II

Dust with magnetic pressure and tension ($\mathcal{R} = 0$)

Linear system of differential equations:

$$\ddot{\Delta}_{(n)} = -2H\dot{\Delta}_{(n)} + \frac{1}{2}\kappa\rho\left[1 - \left(\frac{\lambda_J}{\lambda_n}\right)^2\right]\Delta_{(n)} + 4a^2\left[1 + \frac{1}{6}\left(\frac{\lambda_H}{\lambda_n}\right)^2\right]\left({\sigma_B}^2 - {\omega_B}^2\right)_{(n)}$$

$$\left({\sigma_B}^2 - {\omega_B}^2\right)^{\cdot} = -6H\left({\sigma_B}^2 - {\omega_B}^2\right) \qquad \Rightarrow \qquad \left({\sigma_B}^2 - {\omega_B}^2\right) \propto a^{-6}$$

Cases

- $\lambda_J \ll \lambda_n \ll \lambda_H$

$$\Delta_{(n)} = C_1 t^{\frac{2}{3}} + C_2 t^{-1} + C_3$$

- $\lambda_n \ll \lambda_J$

Decreasing oscillations: $\Delta_{(n)} \propto t^{-\frac{1}{6}}$

- $\lambda_n = \lambda_J$

$$\Delta_{(n)} = C_1 + C_2 t^{-\frac{1}{3}} + C_3 \ln t$$

Conclusion

Magnetic tension has a *positive* but *small* contribution

Summary

Relativistic Magnetised Baryonic Perturbations Revisited

Dust era: magnetic pressure + magnetic tension

Results

Magnetic pressure *vs* magnetic tension

- $\lambda \gg \lambda_J$: no magnetic effect (confirmed)
- $\lambda \ll \lambda_J$: decaying oscillations, $\Delta \propto t^{-1/6}$ (demonstrated)
- $\lambda \sim \lambda_J$: $\Delta = const.$ *vs* $\Delta \propto \ln t$ (demonstrated)

Future Work

- Include the magneto-curvature effects ($\mathcal{R} \gtrless 0$)
- Allow for curved background spacetimes (?)


Thank you!