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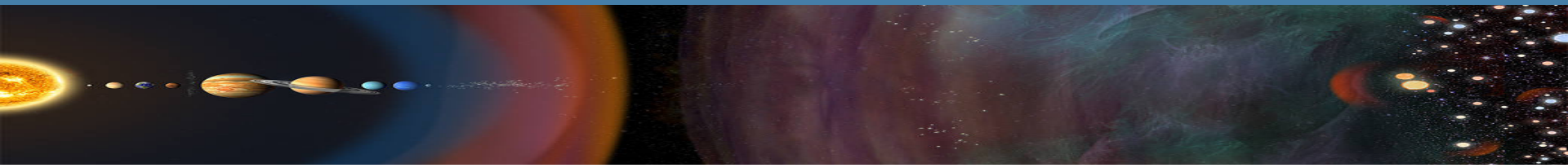
Long term evolution of planetary systems with a terrestrial planet and a giant planet

In collaboration with:

Ian Dobbs-Dixon & Michael Way

The 13th Hellenic Astronomical Conference, Heraklion, 05/06/2017



AIM

- To describe the motion of a system which consists of a star, a giant planet and an Earth-super Earth planet. Furthermore, the two planets are relatively close (e.g. a2/a1<3,4…?).
We are interested in the motion of the smaller planet.

Solar System

5.2 au

1 au

Sun Earth Jupiter

Kepler-87

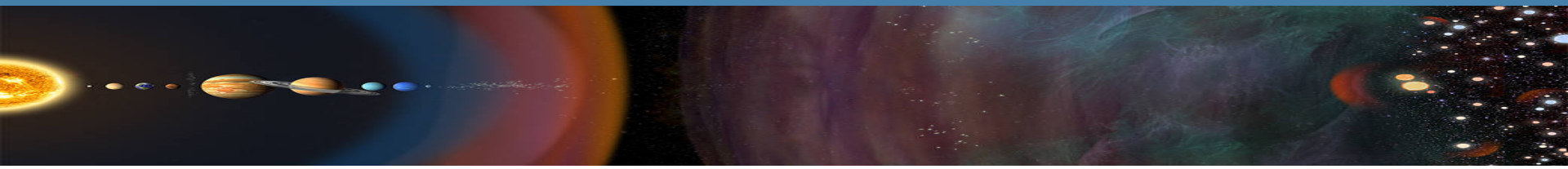
0.676 au

0.481 au

Kepler-87 Kepler-87b Kepler-87c

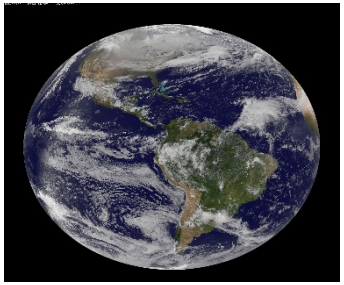
1.020 M_J 6.4 $M_\oplus$

(Ofir et al. 2014)



- This work has been motivated by two studies for habitable planets:

One that involves a 1D globally averaged atmospheric model
(Georgakarakos, Eggl & Dobbs-Dixon 2017, submitted)



One that involves a 3D general circulation model
(Way & Georgakarakos 2017).

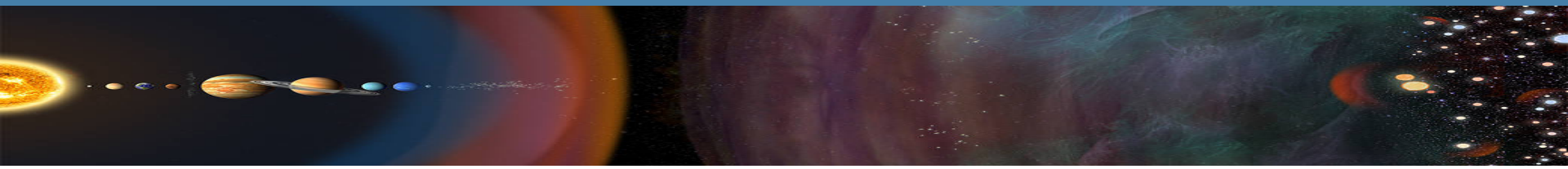
both of which
involve orbital evolution of the potential habitable world

For that purpose we thought to make use of:

Georgakarakos (2003)
(short period + P_3 secular – 2^{nd} order in the masses)

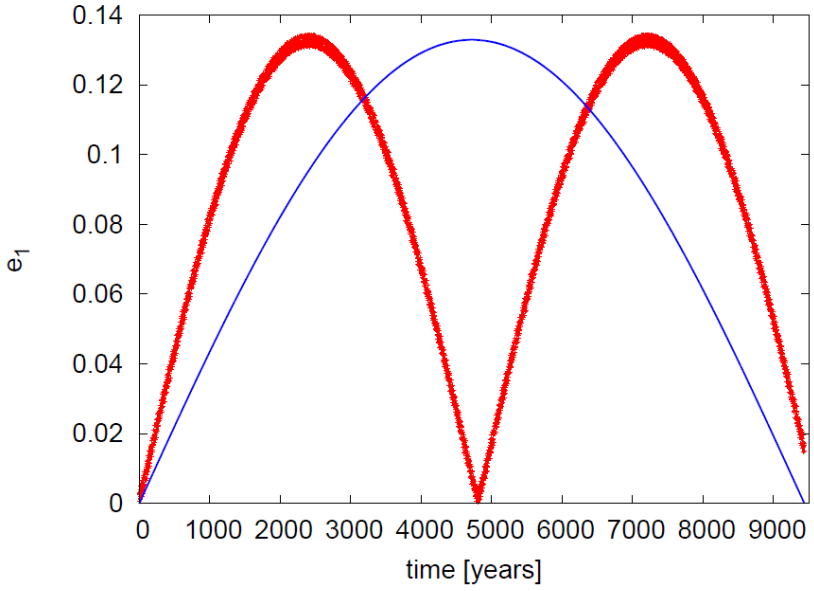
&

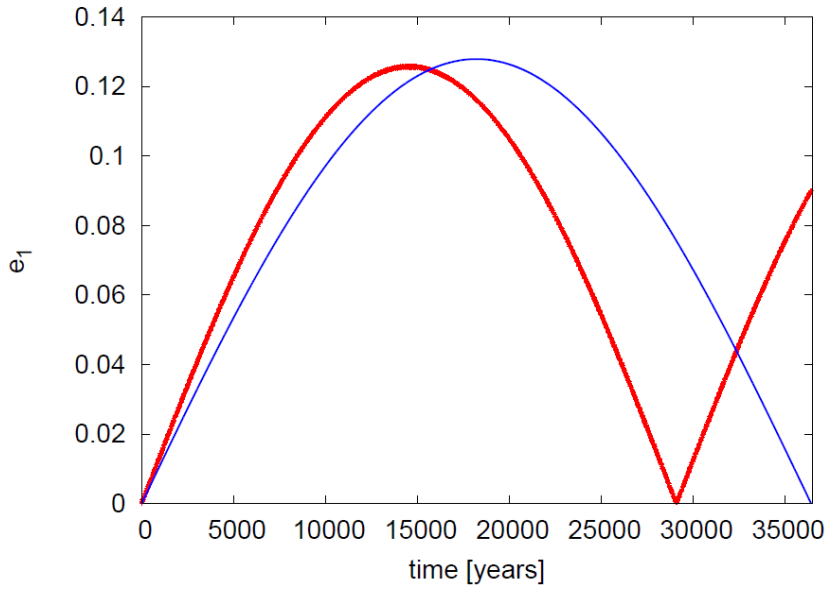
Georgakarakos & Eggl (2015)
(circumbinary planets: short period + P_3 secular + GR correction)



Full equations of motion

Georgakarakos 2003

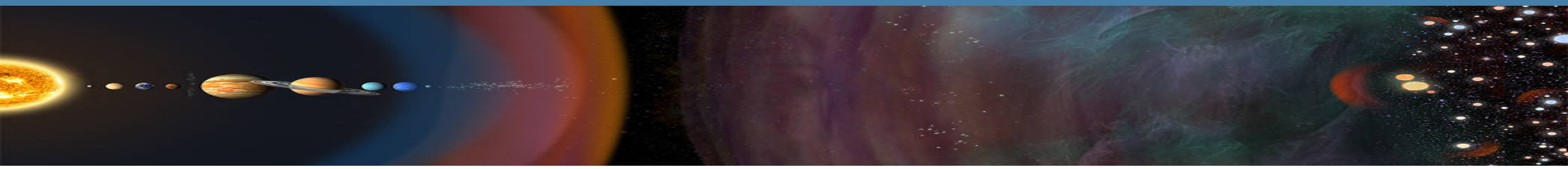




$1\ m_{Sun} - 1\ m_{Earth} - 1\ m_{Jupiter}$

a_1=1 au, a_2=1.9 au, e_2=0.1 a_1=1 au, a_2=3 au, e_2=0.15

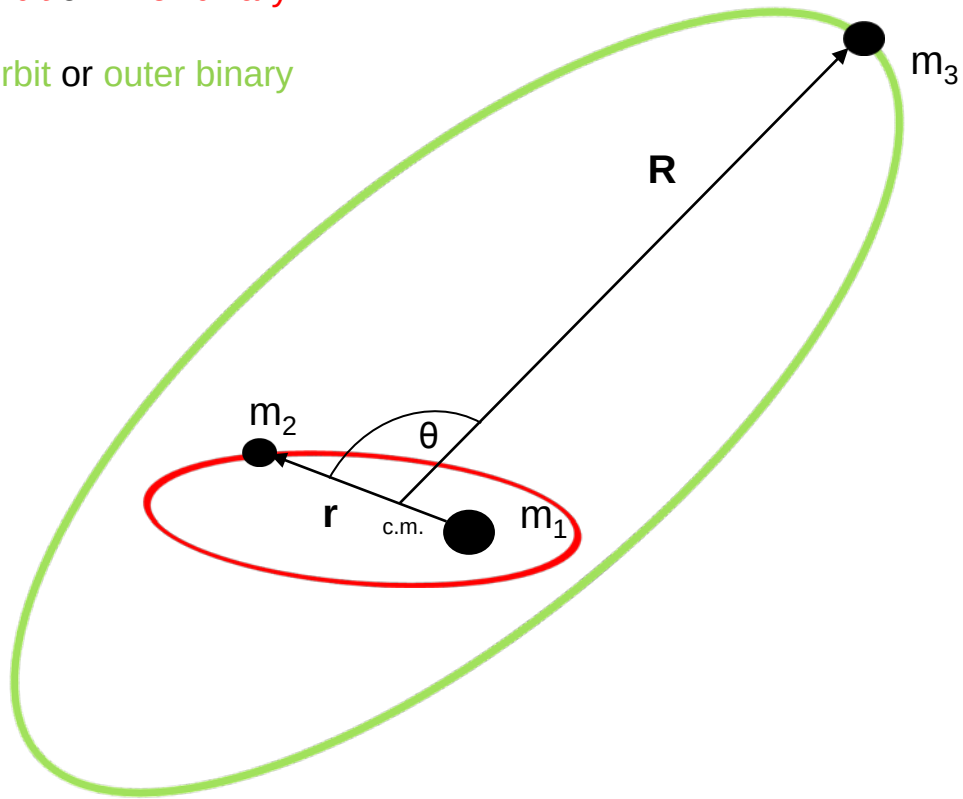
index 1 – inner planet, index 2 – outer planet



r, **R** Jacobi vectors

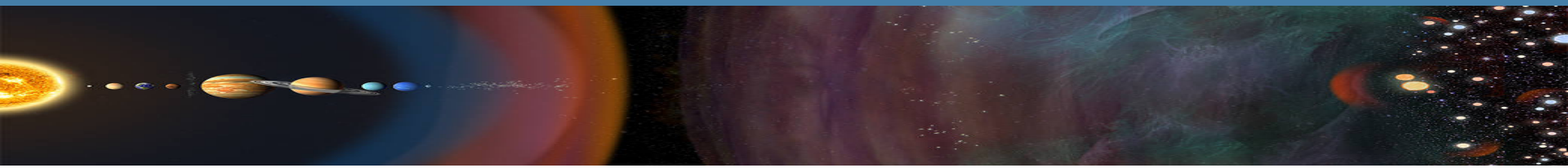
inner orbit or inner binary

outer orbit or outer binary



The setup:

- $m_1 >> m_2, m_3$
- the orbit of the large planet is assumed unchanged
- the small planet either inside (S-type) or outside the orbit of the larger planet (P-type)
- coplanar orbits
- we are interested in secular motion
- no mean motion resonances
- we assume that the eccentricity of the small planet is not large
- $r < R$



Hamiltonian Formulation:

$$H = -\frac{G^2 m_1^3 m_2^3}{2(m_1+m_2)L_1^2} - \frac{G^2 (m_1+m_2)^3 m_3^3}{2ML_2^2} + Gm_3\left(\frac{m_1+m_2}{R} - \frac{m_1}{r_{13}} - \frac{m_2}{r_{23}}\right)$$

inner planet — outer planet — perturbing Hamiltonian H_p

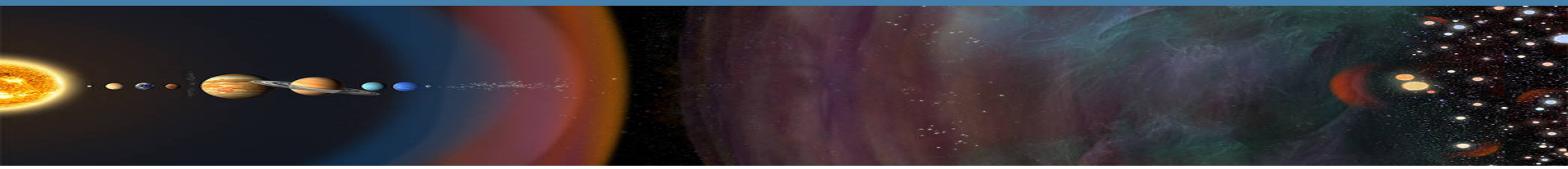
where r_{13} and r_{23} are the distances between m_1 and m_3 and m_2 and m_3 respectively and $L_{1,2} \sim a^{1/2}_{1,2}$ Delaunay variables

The perturbing Hamiltonian can be expressed in terms of Legendre polynomials P_n:

$$H_p = -\frac{Gm_1m_2m_3}{R}\sum_{n=2}^{\infty} M_n \left(\frac{r}{R}\right)^n P_n(\cos\theta)$$ where

$$M_n = \frac{m_1^{n-1} - (-m_2)^{n-1}}{(m_1+m_2)^n}$$

$$P_n(\cos\theta) = 2^n \sum_{k=0}^{n} \binom{n}{k} \binom{\frac{n+k-1}{2}}{n} \cos^k\theta$$



Average H over fast motions ⟶ obtain secular H_s as a power series in semi-major axis ratio a_1/a_2 ⟶ neglect terms of $O(e^3)$ in H_s (e being the smaller planet eccentricity) ⟶ derive Hamilton's equations.

S-type system:

$$\dot{x}_1 = A\sin 2\varpi_2 x_1 + (B - 2A\cos^2\varpi_2)y_1 + C\sin\varpi_2$$

$$\dot{y}_1 = -(B - 2A\sin^2\varpi_2)x_1 - A\sin 2\varpi_2 y_1 - C\cos\varpi_2$$

+4th order expansion

$$\dot{x}_1 = By_1 + C\sin\varpi_2$$

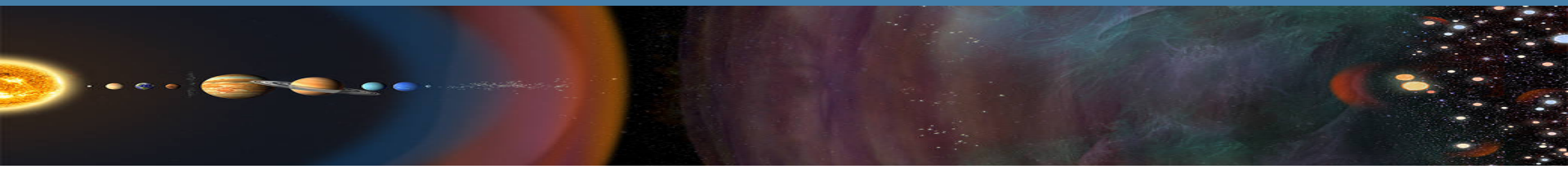
$$\dot{y}_1 = -Bx_1 - C\cos\varpi_2$$

3rd order expansion

where $x_1 = e_1\cos\varpi_1,\ y_1 = e_1\sin\varpi_1$

ϖ longitude of pericentre

semi-major axes remain constant (e.g. Harrington 1968)



where

$$A = A_{p4} + A_{p6} + A_{p8} + A_{p10}$$

$$A_{p4} = -\frac{315}{128}\frac{\sqrt{G}m_3(m_1^3+m_2^3)}{(m_1+m_2)^{7/2}(1-e_2^2)^{7/2}}\frac{a_1^{7/2}}{a_2^5}e_2^2$$

$$A_{p6} = -\frac{4725}{1024}\frac{\sqrt{G}m_3(m_1^5+m_2^5)}{(m_1+m_2)^{11/2}(1-e_2^2)^{11/2}}\frac{a_1^{11/2}}{a_2^7}e_2^2(2+e_2^2)$$

$$A_{p8} = -\frac{11\,025}{262\,144}\frac{\sqrt{G}m_3(m_1^7+m_2^7)}{(m_1+m_2)^{15/2}(1-e_2^2)^{15/2}}\frac{a_1^{15/2}}{a_2^9}e_2^2\left(528+880e_2^2+165e_2^4\right)$$

$$A_{p10} = -\frac{218\,295}{8388\,608}\frac{\sqrt{G}m_3(m_1^9+m_2^9)}{(m_1+m_2)^{19/2}(1-e_2^2)^{19/2}}\frac{a_1^{19/2}}{a_2^{11}}e_2^2\left(1664+5824e_2^2+3640e_2^4+364e_2^6\right)$$

$$B = B_{p2} + B_{p4} + B_{p6} + B_{p8} + B_{p10}$$

$$B_{p2} = -\frac{3}{4}\frac{\sqrt{G}m_3}{(m_1+m_2)^{1/2}(1-e_2^2)^{3/2}}\frac{a_1^{3/2}}{a_2^3}$$

$$B_{p4} = -\frac{45}{128}\frac{\sqrt{G}m_3(m_1^3+m_2^3)}{(m_1+m_2)^{7/2}(1-e_2^2)^{7/2}}\frac{a_1^{7/2}}{a_2^5}\left(4+13e_2^2\right)$$

$$B_{p6} = -\frac{525}{2048}\frac{\sqrt{G}m_3(m_1^5+m_2^5)}{(m_1+m_2)^{11/2}(1-e_2^2)^{11/2}}\frac{a_1^{11/2}}{a_2^7}\left(8+76e_2^2+33e_2^4\right)$$

$$B_{p8} = -\frac{11\,025}{262\,144}\frac{\sqrt{G}m_3(m_1^7+m_2^7)}{(m_1+m_2)^{15/2}(1-e_2^2)^{15/2}}\frac{a_1^{15/2}}{a_2^9}\left(64+1200e_2^2+1720e_2^4+305e_2^6\right)$$

$$B_{p10} = -\frac{218\,295}{8388\,608}\frac{\sqrt{G}m_3(m_1^9+m_2^9)}{(m_1+m_2)^{19/2}(1-e_2^2)^{19/2}}\frac{a_1^{19/2}}{a_2^{11}}\left(128+3968e_2^2+11\,872e_2^4+7000e_2^6+679e_2^8\right)$$

$$C = C_{p3} + C_{p5} + C_{p7} + C_{p9} + C_{p11}$$

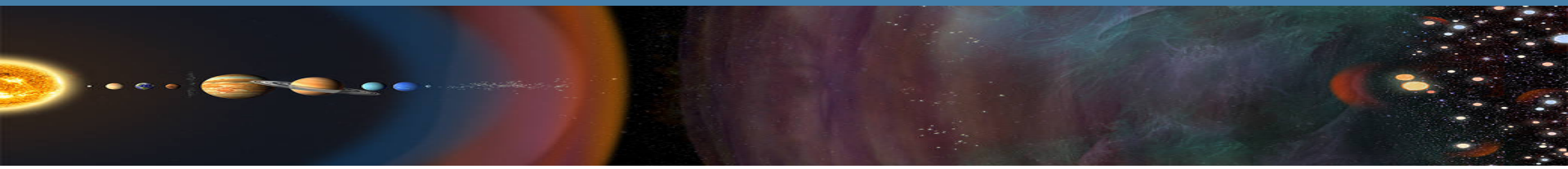
$$C_{p3} = \frac{15}{16}\frac{\sqrt{G}m_3(m_1-m_2)}{(m_1+m_2)^{3/2}(1-e_2^2)^{5/2}}\frac{a_1^{5/2}}{a_2^4}e_2$$

$$C_{p5} = \frac{105}{256}\frac{\sqrt{G}m_3(m_1^4-m_2^4)}{(m_1+m_2)^{9/2}(1-e_2^2)^{9/2}}\frac{a_1^{9/2}}{a_2^6}(4+3e_2^2)e_2$$

$$C_{p7} = \frac{4725}{16\,384}\frac{\sqrt{G}m_3(m_1^6-m_2^6)}{(m_1+m_2)^{13/2}(1-e_2^2)^{13/2}}\frac{a_1^{13/2}}{a_2^8}\left(8+20e_2^2+5e_2^4\right)e_2$$

$$C_{p9} = \frac{24\,255}{524\,288}\frac{\sqrt{G}m_3(m_1^8-m_2^8)}{(m_1+m_2)^{17/2}(1-e_2^2)^{17/2}}\frac{a_1^{17/2}}{a_2^{10}}\left(64+336e_2^2+280e_2^4+35e_2^6\right)e_2$$

$$C_{p11} = \frac{945\,945}{33\,554\,432}\frac{\sqrt{G}m_3(m_1^{10}-m_2^{10})}{(m_1+m_2)^{21/2}(1-e_2^2)^{21/2}}\frac{a_1^{21/2}}{a_2^{12}}\left(128+1152e_2^2+2016e_2^4+840e_2^6+63e_2^8\right)e_2.$$



The solution to our system is

$$x_1 = C_1 \cos\sqrt{B^2 - 2AB}t + C_2 \sin\sqrt{B^2 - 2AB}t - \frac{C}{B}\cos\varpi_2$$

$$y_1 = \frac{1}{B - 2A\cos^2\varpi_2}\Big[(C_2\sqrt{B^2 - 2AB} - AC_1 \sin 2\varpi_2)\cos\sqrt{B^2 - 2AB}t - (C_1\sqrt{B^2 - 2AB} + AC_2 \sin 2\varpi_2)\sin\sqrt{B^2 - 2AB}t\Big] - \frac{C}{B}\sin\varpi_2,$$

C_1, C_2 constants of integration,

and consequently $e_1 = \sqrt{x_1^2 + y_1^2}$

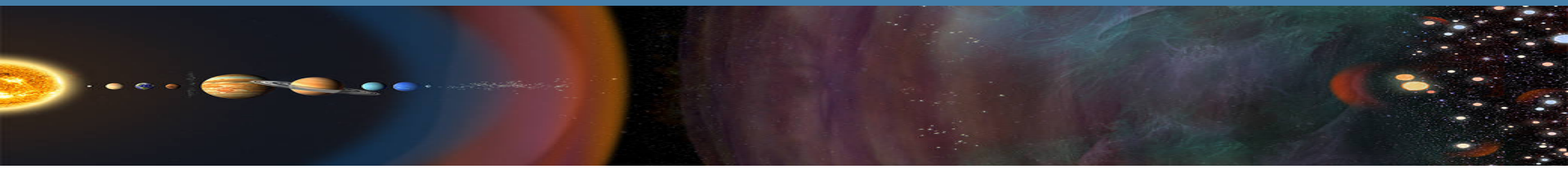
$$\varpi_1 = \arctan\left(\frac{y_1}{x_1}\right).$$

Period of the oscillation: $T = \dfrac{2\pi}{\sqrt{B^2 - 2AB}}$

Also: $e_1^{\max} = -2\dfrac{C}{B}$ and $< e_1^2 >= \left(\dfrac{C}{B}\right)^2 \dfrac{2B - 3A}{B - A}$. (e_{10}=0)

.

For the P-type systems we obtain a solution of the same form as for S-type systems. The secular solution of Georgakarakos (2003) can be obtained from the new solution by setting A=0.

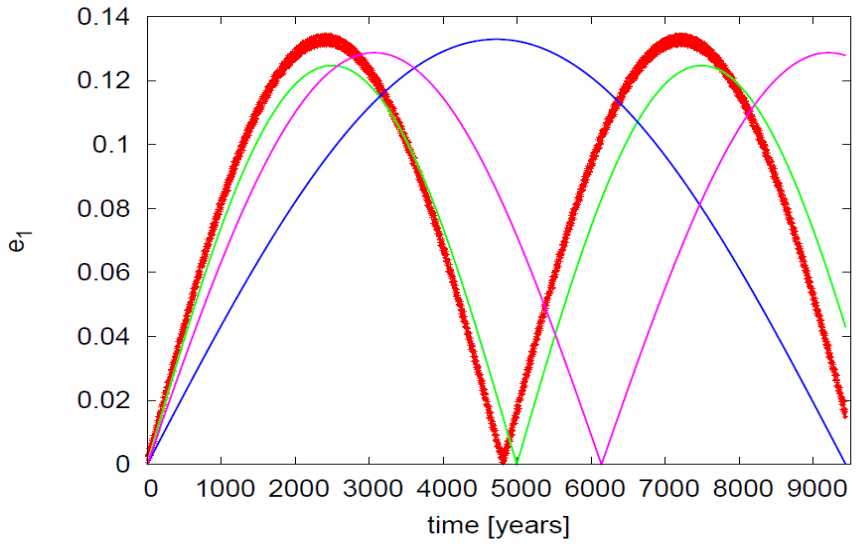


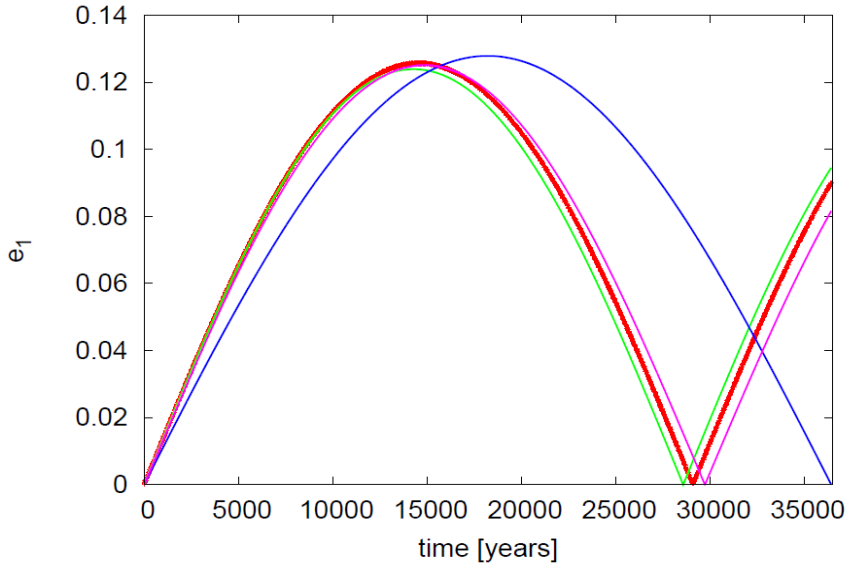
Full equations of motion

Georgakarakos 2003

Georgakarakos et al. 2016 (P_5)

Georgakarakos et al. 2016 (P_{11})

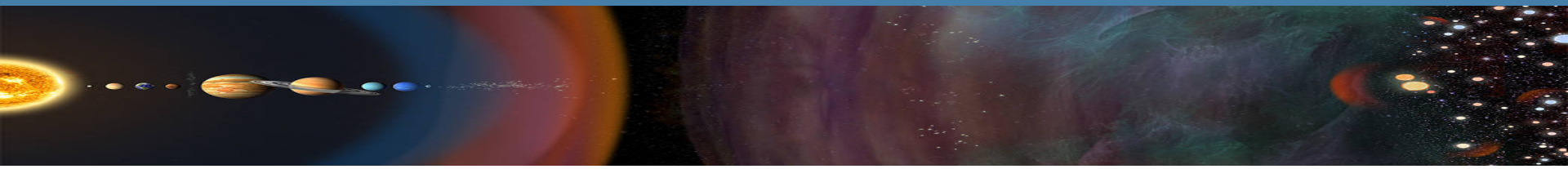




$1\ m_{Sun} - 1\ m_{Earth} - 1\ m_{Jupiter}$

a_1=1 au, a_2=1.9 au, e_2=0.1

a_1=1 au, a_2=3 au, e_2=0.15



Comparison of different order models – S-type systems

(taken from Georgakarakos et al. 2016)

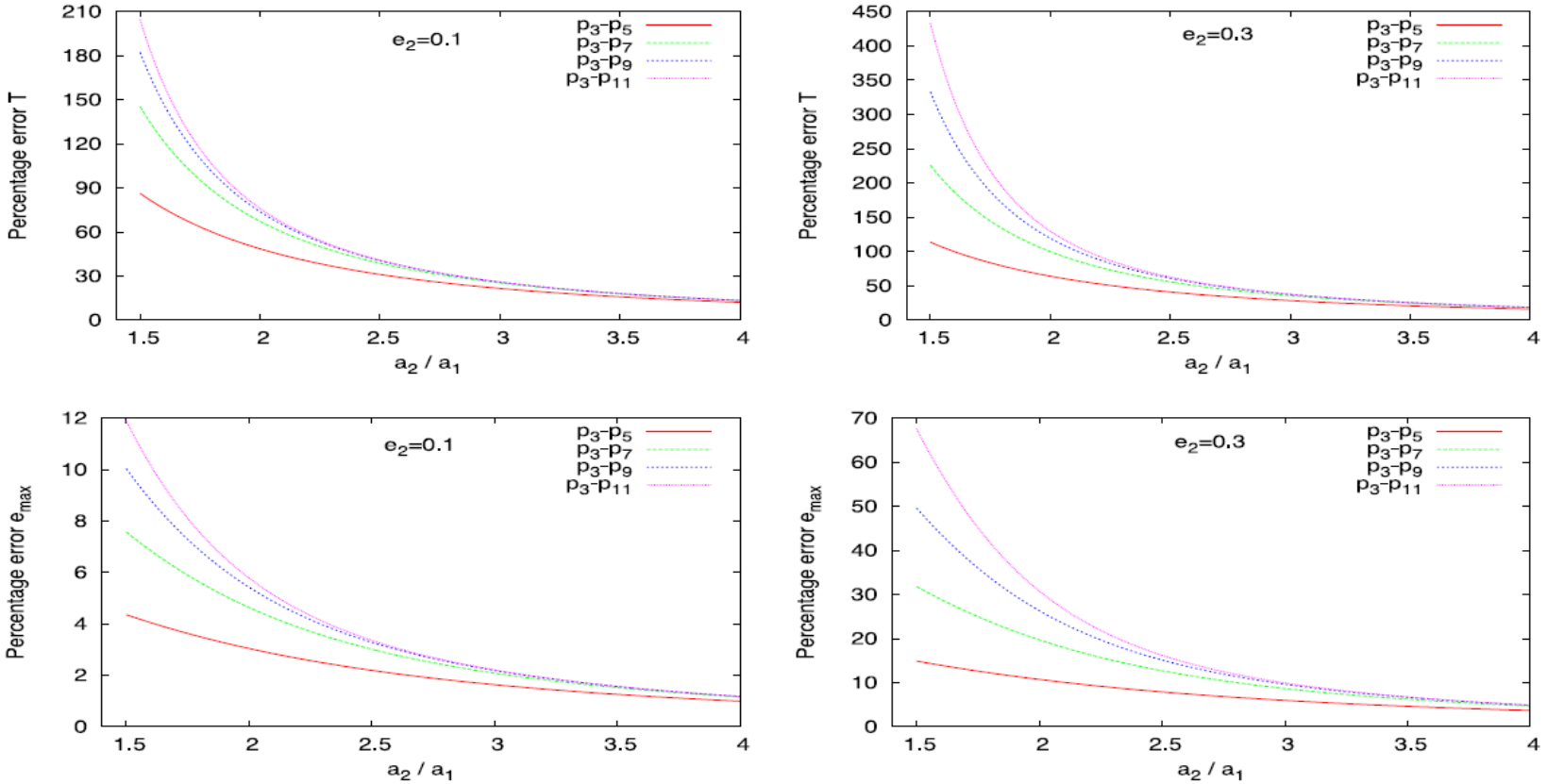
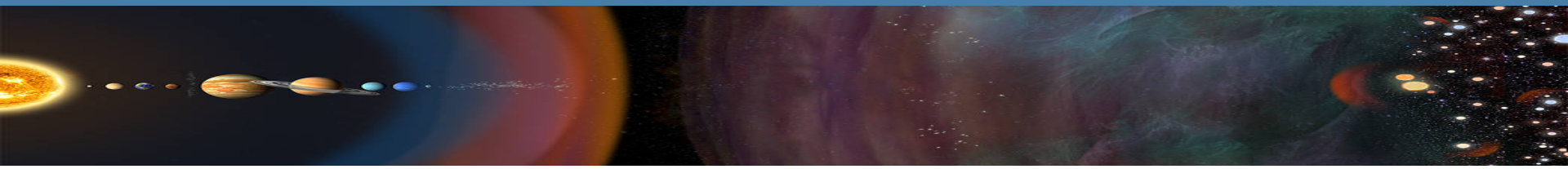
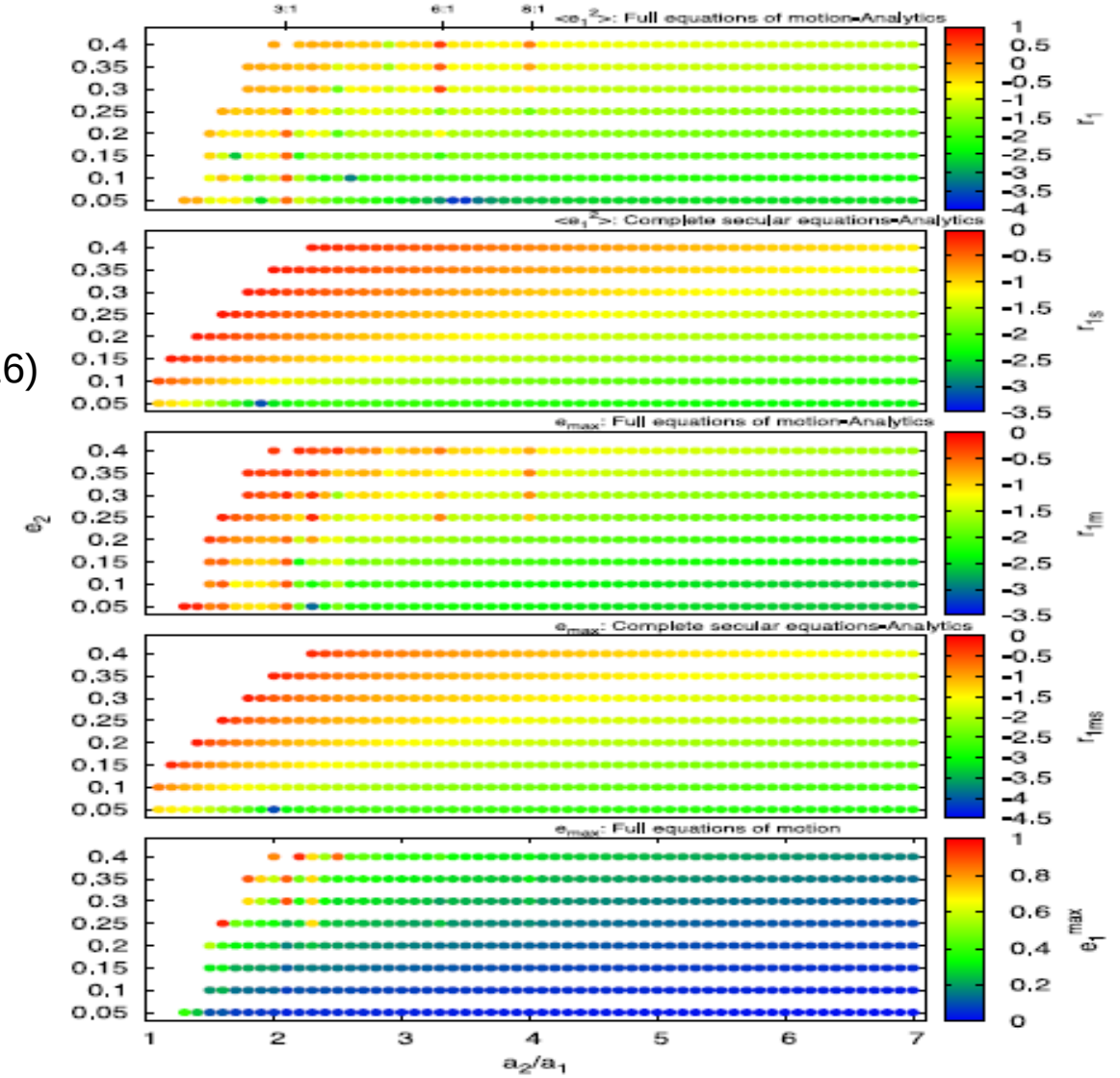


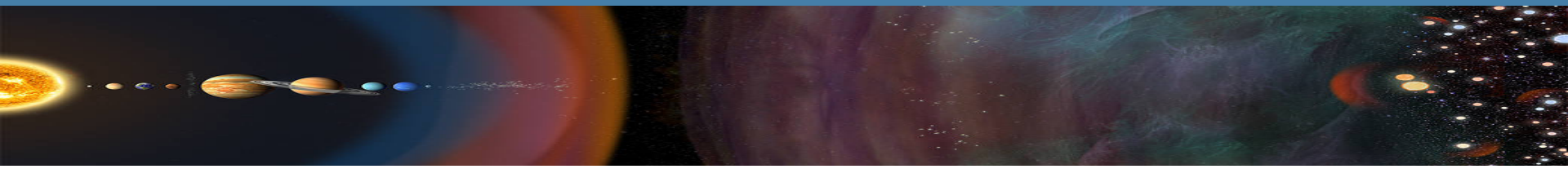
Figure 3. Percentage error for the period T and maximum eccentricity $e_{\max}$ between analytical solutions based on different expansion orders for a system with $m_0 = 1\,M_\odot$, $m_1 = 1\,M_\oplus$, $m_2 = 1 M_J$. For the left column plots, we have $e_2 = 0.1$, while for the right ones we have $e_2 = 0.3$. The error curves shown here are the result of the comparison between the following analytical models: $P_3 - P_5$, $P_3 - P_7$, $P_3 - P_9$, $P_3 - P_{11}$, where P_n ($n = 3, 5, 7, 9, 11$) denotes an analytical solution that is based on a Legendre polynomial expansion to order n.



$1\ m_{Sun} - 1\ m_{Earth} - 1\ m_{Jupiter}$

(taken from Georgakarakos et al. 2016)



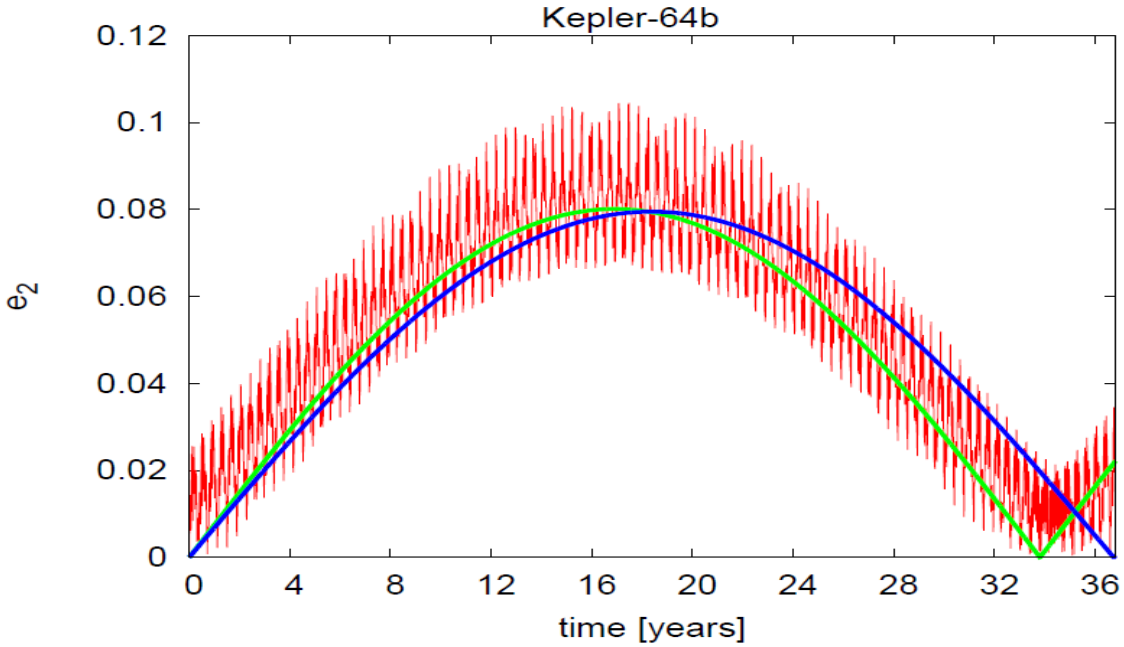


Full equations of motion

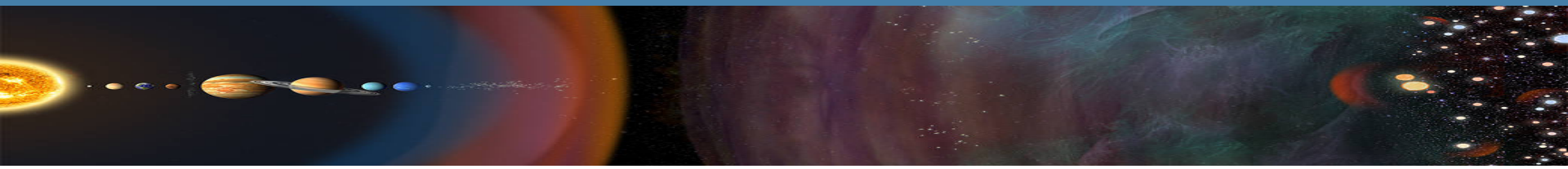
Georgakarakos et al. 2016

Georgakarakos & Eggl 2015

Kepler-64



m_1=1.384 m_{sun}
m_2=0.386 m_{sun}
m_3=0.532 m_{jup}
a_1=0.1744 au
e_1=0.2117
a_2=0.634 au



Some interesting issues and potential future work

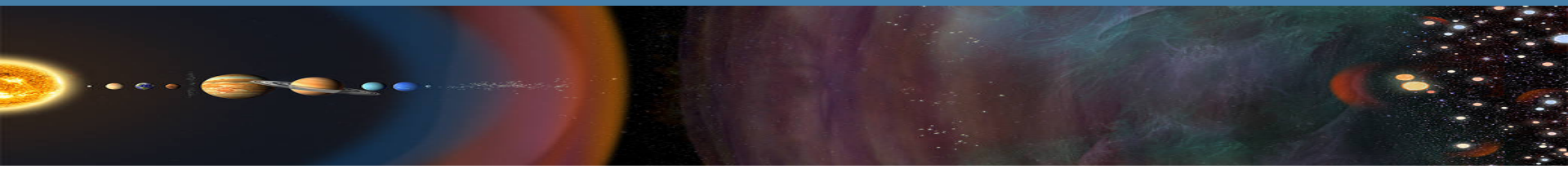
1) Slow convergence as a_2 / a_1 gets small in the inner case (similar situation for the outer case)

2) Inclusion of mean motion resonances

3) Orbital evolution of the second planet (variable pericentre?)

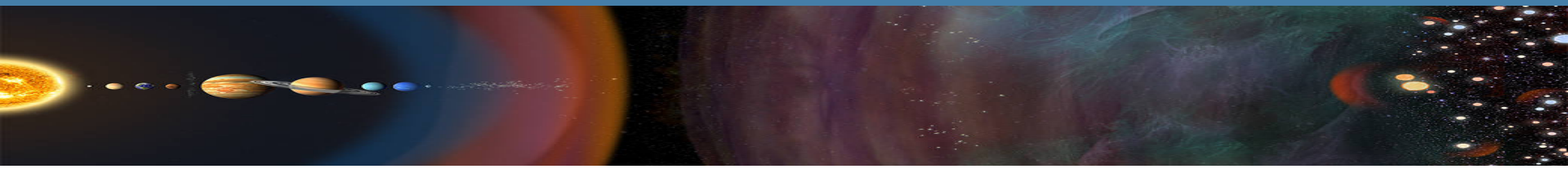
$$\dot{x}_1 = A \sin 2\varpi_2 x_1 + (B - 2A \cos^2 \varpi_2) y_1 + C \sin \varpi_2$$

$$\dot{y}_1 = -(B - 2A \sin^2 \varpi_2) x_1 - A \sin 2\varpi_2 y_1 - C \cos \varpi_2$$



Conclusions:

- We have extended some previous secular results
- Greater improvement in the period of the oscillation than in the amplitude of the eccentricity
- Easy to employ and choose the desired order of the expansion
- May have a wider range of applications than the cases discussed here



Thanks for your attention!

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