Stability of the Trojan asteroids

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Abstract

The problem of stability of the Trojan asteroids is investigated in the light of the Nekhoroshev theory of stability over large time intervals. We consider the two–dimensional (2D) planar, and the three–dimensional (3D) spatial restricted three body problem (Sun–Jupiter–asteroid) as simple models for describing the motion of an asteroid. Using these models we find regions of effective stability around the Lagrangian point L_4 such that if the initial conditions of an orbit are inside these regions the orbit is confined in a slightly larger neighborhood of the equilibrium for a very long time. We prove that stability over the age of the universe is guaranteed in a realistic region, big enough to include some real asteroids. This significantly improves previous works on the subject

1 Introduction

Almost one century after the discovery of the first Trojan asteroid (named 588 Achilles) by Max Wolf in 1906, the problem of the stability of these asteroids remains open. In recent years this problem has been investigated by a number of researchers, both numerically and analytically. The numerical investigations deal mainly, with the evolution in time of a sample of orbits, in sophisticated realistic models of the solar system, and the statistical study of these orbits (Milani 1993, 1994, Levison et al. 1997, Tsiganis et al. 2000, Dvorak & Tsiganis 2000).

The usual approach in analytical studies of the stability of the Trojan asteroids is to consider simple models for the system such as the two dimensional (2D) planar, and the three dimensional (3D) spatial restricted three body problem (RTBP) (Giorgilli et al. 1989, Simó 1989, Celletti & Giorgilli 1991, Celletti & Ferrara 1996). As an example of a more complicated model for the problem we refer to the model studied by Gabern & Jorba (2001) where the effect of Saturn on the motion of the asteroid has been taken into account. The techniques used in these papers are based in normal forms or first integrals calculations. Roughly speaking one shows that the system admits a number of approximate integrals, whose time variation can be controlled to be small for an extremely long time. In this case we have effective stability, i.e. even when an orbit is not stable, the time needed for it to leave the neighborhood of the equilibrium is larger than the expected lifetime of the physical system studied. This is the basis to derive the classical Nekhoroshev's estimates (Nekhoroshev 1977). The size of the regions of effective stability found in the above mentioned papers was not negligible but no real asteroids were actually found to be inside these regions.

In the present communication we pay attention to some somehow recent results obtained by Giorgilli & Skokos (1997–hereafter paper I) and Skokos & Dokoumetzidis (2001–hereafter paper II), where some real asteroids were found to be effectively stable.

2 Realistic estimations of the effective stability region

The first result that guaranties the effective stability of real asteroids was provided in paper I for the 2D RTBP. In the RTBP the Trojan asteroids are located in

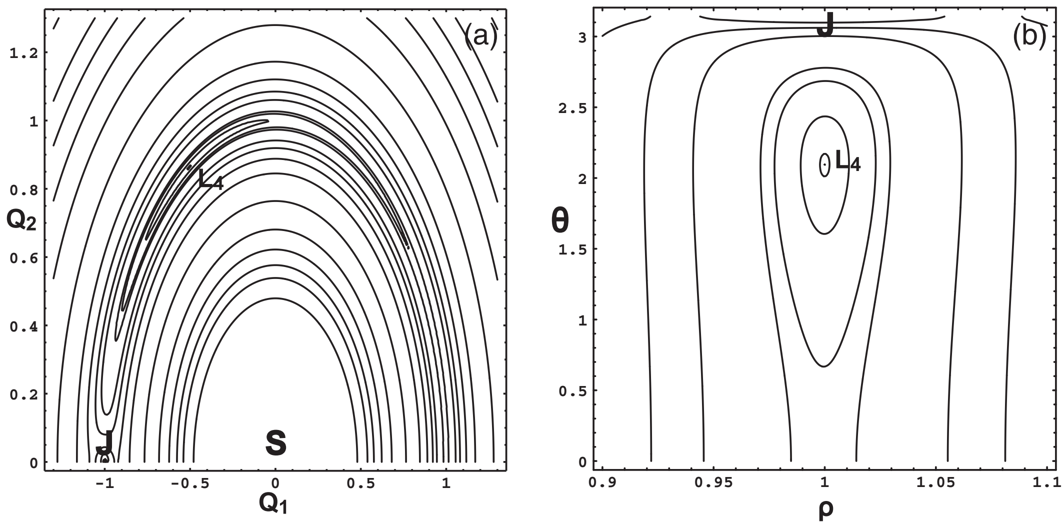

Figure 1: Curves of zero velocity for the RTBP on the plane of motion of Jupiter (J) in (a) heliocentric coordinates Q_1, Q_2 in a uniformly rotating frame with its origin on the Sun (S) and (b) polar coordinates ρ, θ, with $Q_1 = \rho \cos\theta$, $Q_2 = \rho \sin\theta$. In both frames the position of the elliptic Lagrangian point L_4 is also marked.

the neighborhood of the elliptic equilibrium Lagrangian points L_4 and L_5. In 2D systems the KAM theory guarantees (under generic conditions of non-resonance and non-degeneracy) the existence of 2D tori that act as confiners for the motion, since, they separate the 3D energy surface. So trajectories initially located near the elliptic equilibrium point and inside a 2D torus will always remain in the neighborhood of the equilibrium point, which ensures true stability. So, the result obtained in paper I underline the fact that Nekhoroshev theory can give meaningful estimates, but does not take into account the possibility of the so–called Arnold diffusion which appears only in the 3D spatial problem. In the spatial problem the tori around the elliptic equilibrium points L_4 and L_5 are 3D while the energy manifold is 5D. Thus the tori cannot act as barriers for the motion. So, some orbits with initial conditions near L_4 (or L_5) can be driven to regions of the phase space far away from it. The effective stability around L_4 for the 3D RTBP was studied in paper II and one real asteroid (out of the 98 tested) was found to be inside the estimated stability region.

Both papers were based on the construction of a normal form for the system in appropriate coordinates and in estimations of its time variation. The not negligible improvement of the size of the stability region achieved in these papers, compared to older attempts, is mainly due to the use of better coordinates. The curves of zero velocity on the plane of Jupiter's orbit form the banana–shaped region around L_4 shown in figure 1(a). All previous works were based on expansions in cartesian coordinates around L_4. It is evident that cartesian coordinates are not suitable to describe regions with circular shape. On the other hand the use of polar (for the 2D problem) or cylindrical coordinates (for the 3D problem) are better candidates as can be seen from figure 1(b).

Also the fact that the normal form was computed to higher orders than in previous studies, both in the 3D and the 2D case, helped in improving the estimations of the stability region's size. In particular in the 3D case the normal form was computed up to order 29, while in the 2D case up to order 49.

3 Summary

The basic results of papers I and II can be summarized as follows:

- For the first time the estimated size of the effective stability region is big enough to include real asteroids: 1 asteroid in the spatial case and 4 in the planar case. In the 3D case the region where the most remote asteroid is located (out of the 98 real asteroids checked), is larger by a factor 34 compared to the estimated stability region. This result improves significantly older estimates (Giorgilli et al. 1989, Celletti & Giorgilli 1991) where no real asteroid was inside the stability region and a factor 3,000 was needed for the most remote asteroid to be inside the stability region.

- The radii of the effective stability region in the spatial and planer cases are close to each other for the same order of expansion of the normal form, with the radius computed for the spatial case being always slightly smaller. Thus, Arnold diffusion does not affect the size of the effective stability region significantly.
- The theoretical framework used in papers I and II reached the limits of its effectiveness by providing the best possible results in the 2D case, and comparable results in the 3D case. In particular the optimal order of the expansion of the normal form was found to be equal to 38 in the 2D case, although the expansion of the normal form was performed up to order 49. So the computation of the normal form to orders higher than 38 does not improve the estimations in the 2D case. In order to have a non negligible improvement of the estimated size of the effective stability region, one has to choose better coordinates than the cylindrical ones, in the sense that these coordinates should be more adapted to describe the banana–shaped region of the actual stability region around L_4.

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