3-D Numerical Simulations of Rotating and Pulsating Relativistic Stars

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Abstract

A summary of new 3-D numerical simulations of relativistic stars is presented, which were done within the NASA Neutron Star Grand Challenge project. A numerical code that evolves both the hydrodynamics and the spacetime is used to study pulsating stars, emission of gravitational radiation, rotation and hydrodynamical shocks in neutron star formation. We obtain the first normal mode frequencies of rapidly rotating relativistic stars. In a separate study, and evolving only the hydrodynamics, we find nonlinear saturation of r-modes in rotating stars and present an explanation for their maximum dynamical amplitude.

1 3-D Simulations of Single Relativistic Stars

Computational general relativistic astrophysics is an increasingly important field of research. Its development is being driven by a number of factors. Firstly, the large amount of observational data by high-energy X-ray and γ-ray satellites such as Chandra, XMM and others. Secondly, the new generation of gravitational wave detectors coming online in the next few years, and thirdly, the rapid increase in computing power through massively parallel supercomputers and the associated advance in software technologies, which make large-scale, multi-dimensional numerical simulations possible. Three-dimensional (3D) simulations of general relativistic astrophysical events such as stellar gravitational collapse or collisions of compact stars and black holes are needed to fully understand the incoming wealth of observations from high-energy astronomy and gravitational wave astronomy. It is thus not surprising that in recent years hydrodynamical simulations of compact objects in numerical relativity has become the focus of several research groups (see [1] and references therein).

Within the NASA Neutron Star Grand Challenge project, a 3-D relativistic code has been developed mainly by Washington University, St. Louis and the Albert Einstein Institute, Potsdam with additional contributions from Thessaloniki, Valencia, Trieste and other groups [2, 4, 1]. The code has the capability of solving the coupled set of the Einstein equations and the general relativistic hydrodynamic equations. The code has been validated in long-term, accurate simulations of the dynamics of isolated stars in strong gravitational fields. Single relativistic stars are indeed expected as the end-point of a number of astrophysical scenarios (such as gravitational collapse and binary neutron star merging) and should provide important information about strong field physics both through electromagnetic and gravitational wave emissions. A number of new numerical techniques have been incorporated in the present code leading to a much improved ability to simulate relativistic stars. These techniques concern both the evolution of the field equations, for which a new conformal-traceless formulation of the Einstein equations is used and the evolution of the hydrodynamical variables, for which the use of the “monotonized central differencing” (MC) limiter has provided us with the small error growth-rates necessary for simulations over several dynamical timescales.

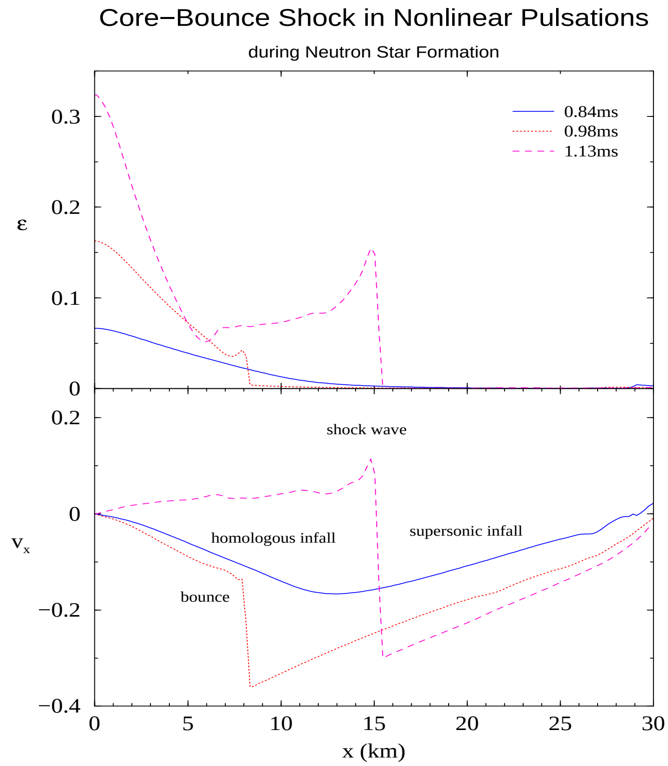

Figure 1: Shock formation in the outer core mantle, during the first bounce at equilibrium densities of an unstable star, evolved with an ideal fluid EOS. The top and bottom panels show the internal energy ε and radial velocity v_x, respectively, at three different times: the homologous infall phase, the inner core bounce and the outwards shock propagation. The oscillations of the inner core are damped by shock heating.

More precisely, we have focused on the accuracy of the code during long-term evolution of spherical and rapidly rotating stellar models. We also investigated the nonlinear dynamics of stellar models that are unstable to the fundamental radial mode of pulsation. Upon perturbation, the unstable models will either collapse to a black hole, or migrate to a configuration in the stable branch of equilibrium configurations. In the case of migration to the stable branch, we are able to accurately follow the nonlinear oscillations that accompany this process and that can give rise to strong shocks (Fig. 1). The ability to simulate large amplitude oscillations is important as we expect a neutron star formed in a supernova core-collapse or in the accretion-induced collapse of a white dwarf to oscillate violently in its early stages of life.

Particularly important for their astrophysical implications, we study the linear pulsations of spherical and rapidly rotating stars. The computed frequencies of radial, quasi-radial and quadrupolar oscillations have been compared with the corresponding frequencies obtained with lower-dimensional numerical codes or with alternative techniques such as the Cowling approximation (in which the spacetime is held fixed and only the hydrodynamical equations are evolved) or relativistic perturbative methods. The comparison shows an excellent agreement confirming the ability of the code to extract physically relevant information from tiny perturbations. The successful determination of the eigenfrequencies for rapidly rotating stars computed with our code (Fig. 2) is noteworthy. Such frequencies have not been obtained before, with the system of equations still being too complicated for perturbative techniques.

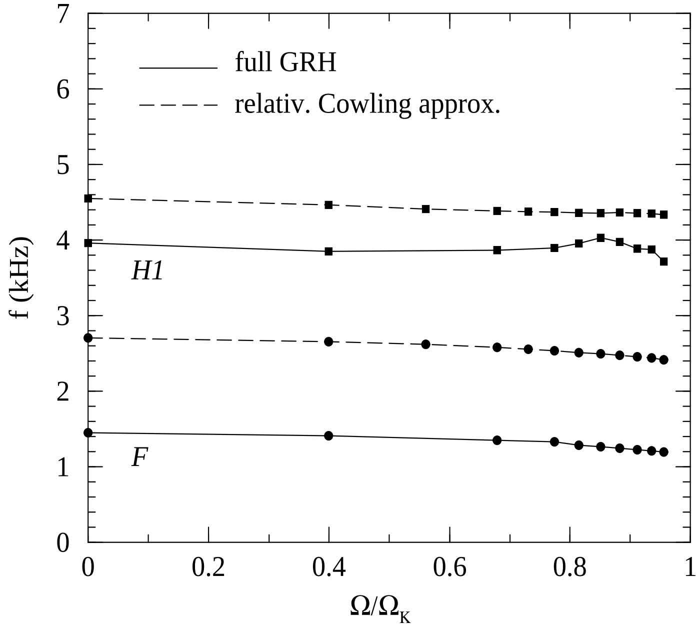

Figure 2: Quasi-radial pulsation frequencies for a sequence of rotating $N = 1$ polytropes and a number of different rotation rates. The frequencies of the fundamental mode F (filled squares) and of the first overtone $H1$ (filled circles) are computed from *coupled* hydrodynamical and spacetime evolutions (solid lines). The sequences are also compared with the corresponding results obtained from computations in the relativistic Cowling approximation.

2 Non-linear R-Modes in Relativistic Stars

Considerable effort has been spent in the last two years, in determining the properties of *r*-modes in rotating compact stars, since the discovery that these modes are unstable to the emission of gravitational radiation (see [5] for a recent review). This is motivated by the current understanding that the *r*-mode instability may have several important astrophysical consequences: it provides an explanation for the spin-down of rapidly rotating proto-neutron stars to Crab-like spin-periods and for the spin-distribution of millisecond pulsars and accreting neutron stars, while being a strong source of detectable continuous gravitational radiation [6, 7]. In addition, if *r*-modes induce differential rotation, then the interaction between them and the magnetic field in neutron stars has been used to explain one type of gamma-ray bursts [8] and as a mechanism for enhancing the star's toroidal magnetic field, which could, in turn, limit the *r*-mode amplitude [9].

Still, several important uncertainties remain, the resolution of which could significantly modify the above conclusions. We have addressed some of these uncertainties [10]. Specifically, we performed 3-D simulations using perturbed uniformly rotating relativistic stars and evolving the relativistic hydrodynamical equations. We find that nonlinear saturation sets in when the *r*-mode amplitude has exceeded a few times unity. Shock waves on the surface of the rapidly rotating star are seen during saturation (Fig. 3), as in [11]. The appearence of shock waves does not necessarily imply that they are the primary cause of saturation. Our simulations apply on a dynamical timescale and it cannot be excluded that weak hydrodynamical couplings saturate the *r*-mode amplitude on longer timescales (but shorter than the growth timescale due to gravitational radiation reaction). We have further confirmed the existence of discrete *r*-modes in isentropic relativistic stars and our results indicate that there exists indeed a kinematical differential rotation, which could have implications on the coupling of the magnetic field with the *r*-mode oscillations.

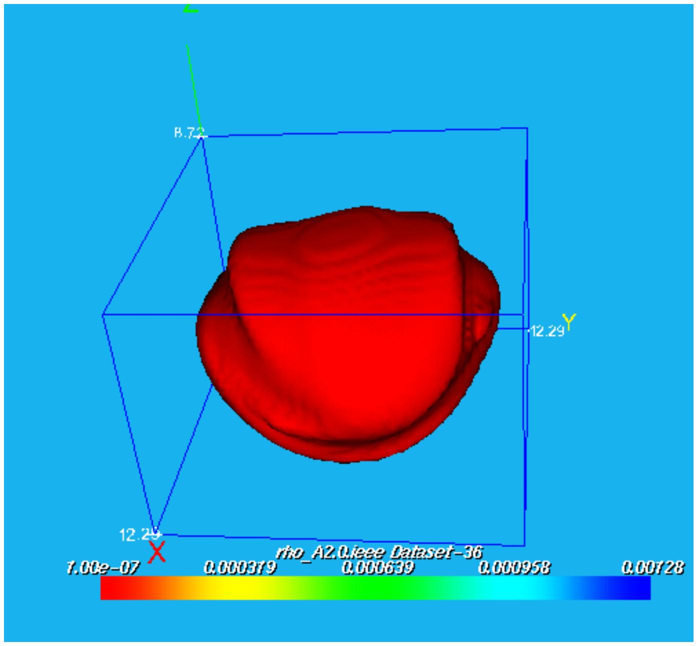

Figure 3: Shock waves developing on the surface of a rapily rotating relativistic star when the simulation is started with an initial r-mode amplitude of a few times unity.

These are the first simulations of nonaxisymmetric oscillation modes in rotating relativistic stars (see [12] for a recent news coverage of this work) and further detailed study will be required to arrive at final conclusions regarding the nonlinear development of the *r*-mode instability.

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