The central supermassive Black Hole of the Milky Way

George Pappas

4th Summer school of the Hel.A.S. - 27 July 2022

ARISTOTLE UNIVERSITY OF THESSALONIKI

The central supermassive Black Hole of the Milky Way

and its Shadow

George Pappas

4th Summer school of the Hel.A.S. - 27 July 2022

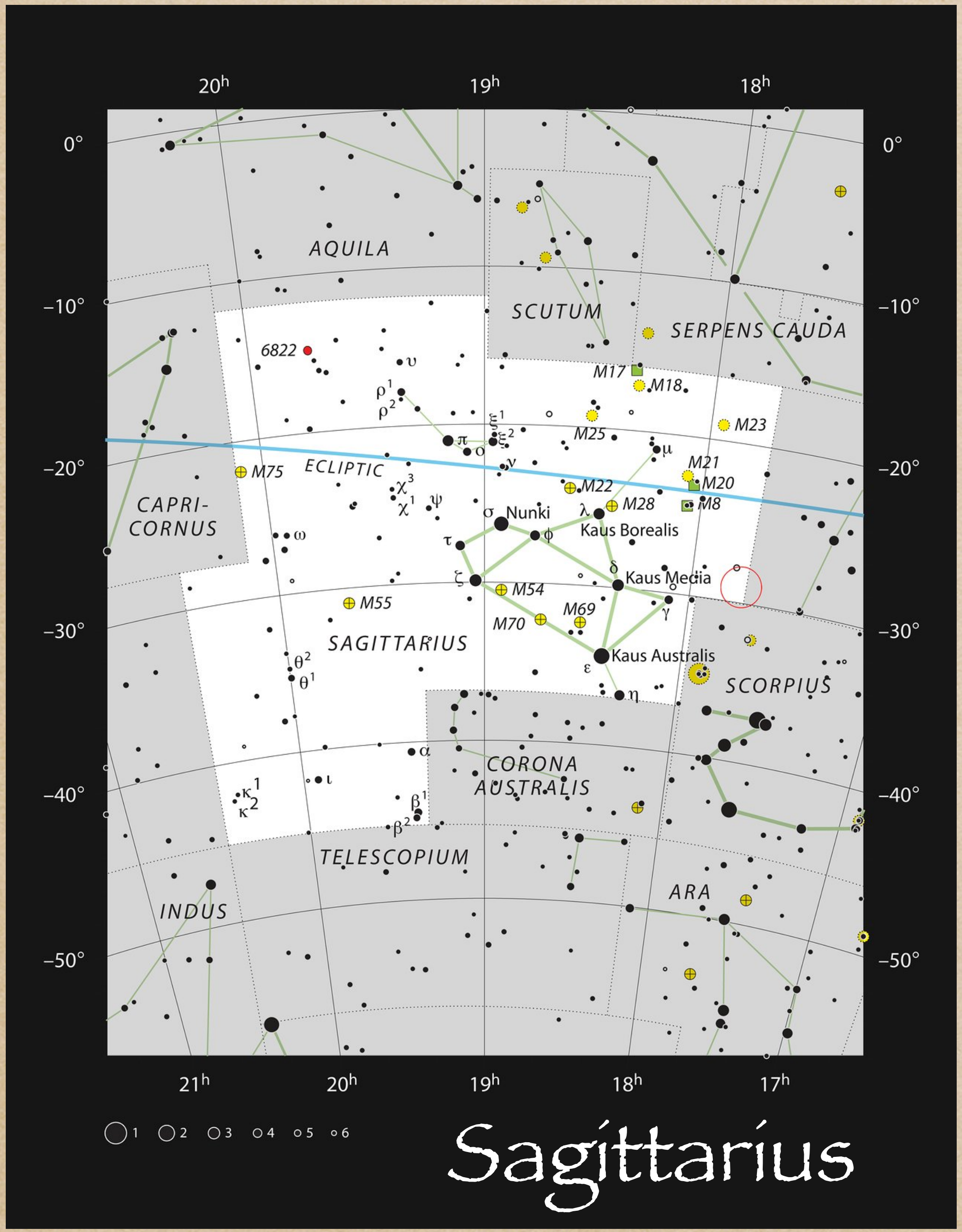
ARISTOTLE UNIVERSITY OF THESSALONIKI

Outline of the lecture

- A compact object at the Milky Way center
- What are Black Holes
- Photon orbits around Black Holes
- What is the Shadow of a Black Hole
- The Shadow of Sagittarius A*

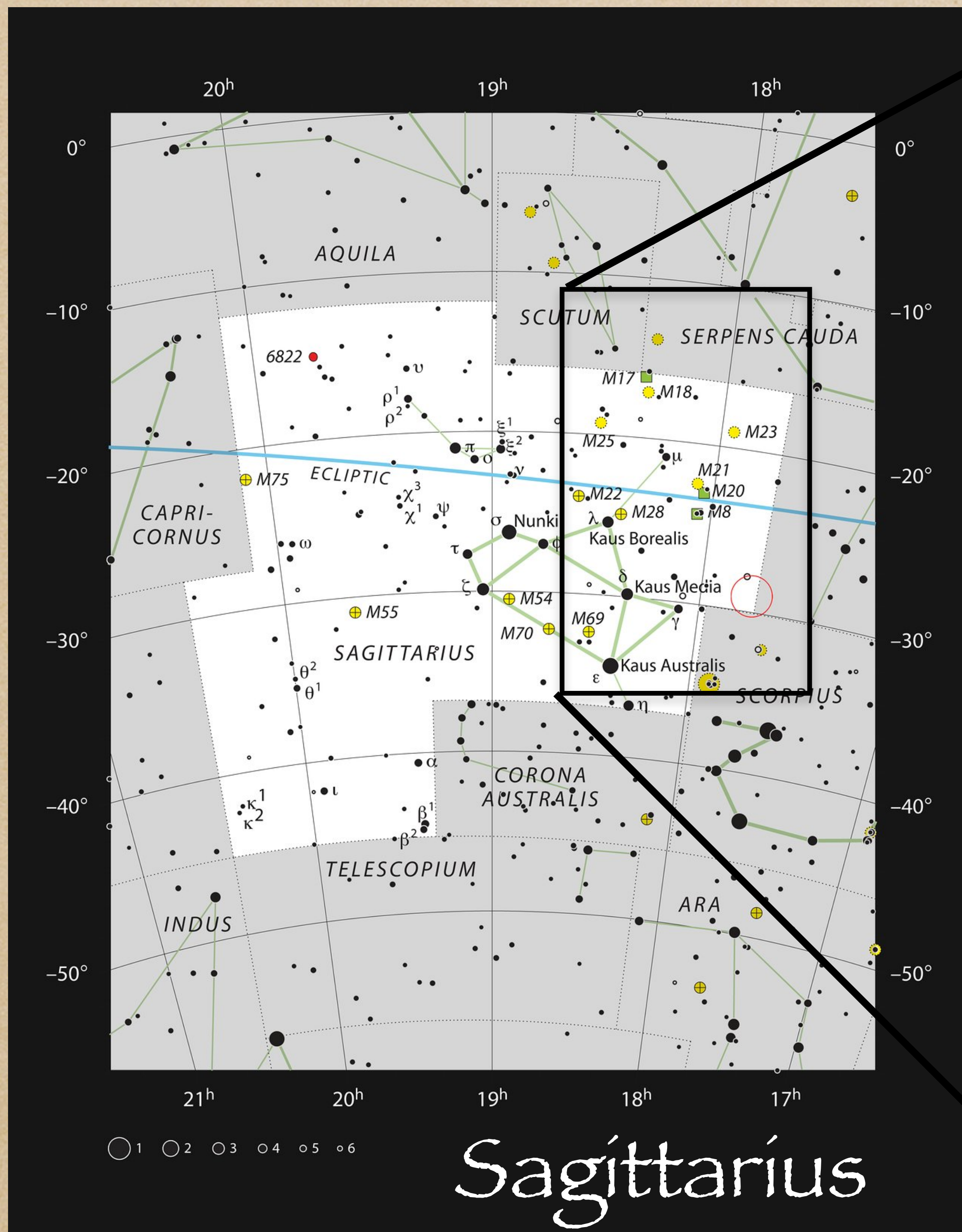
The Milky Way center

The Milky Way center

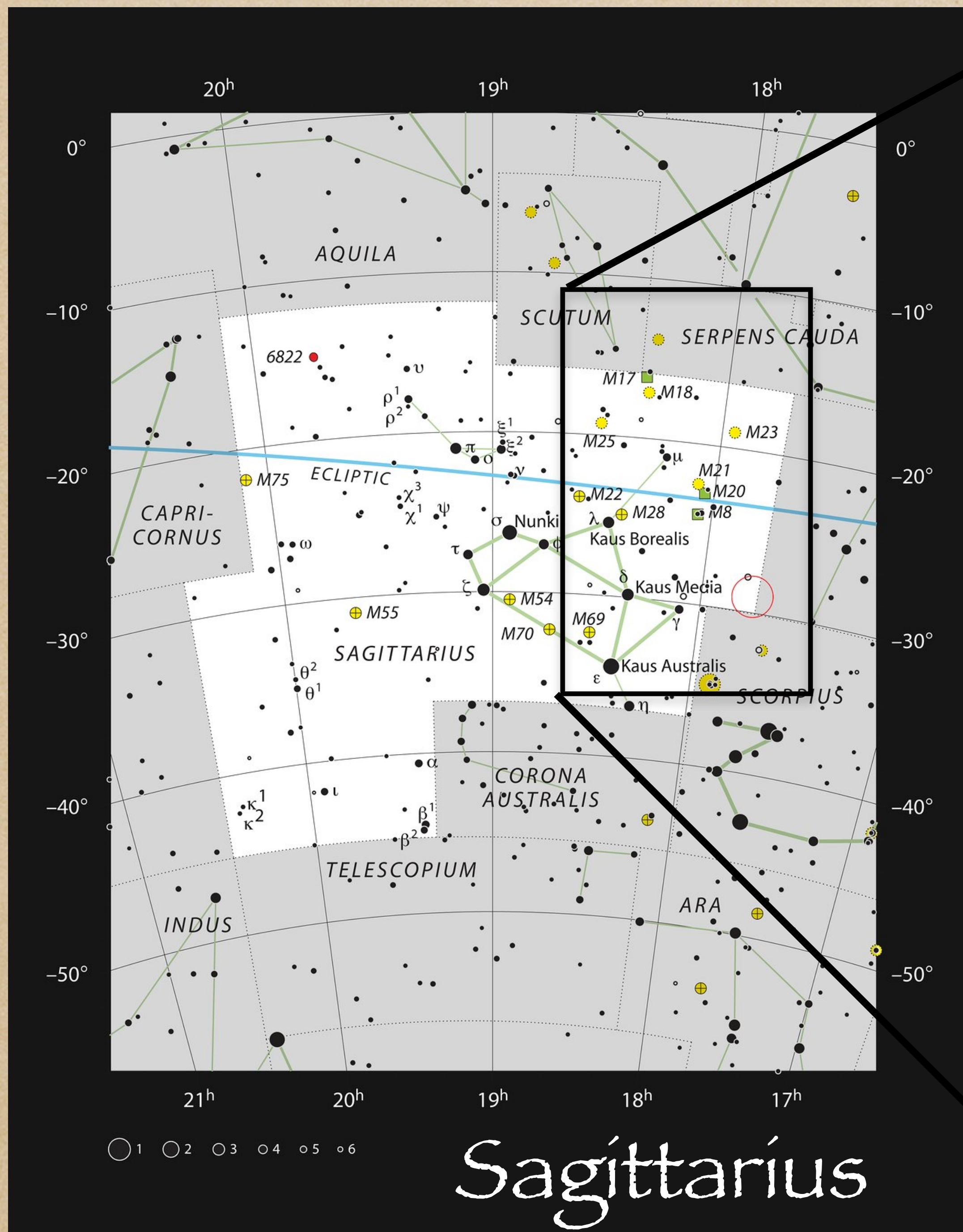


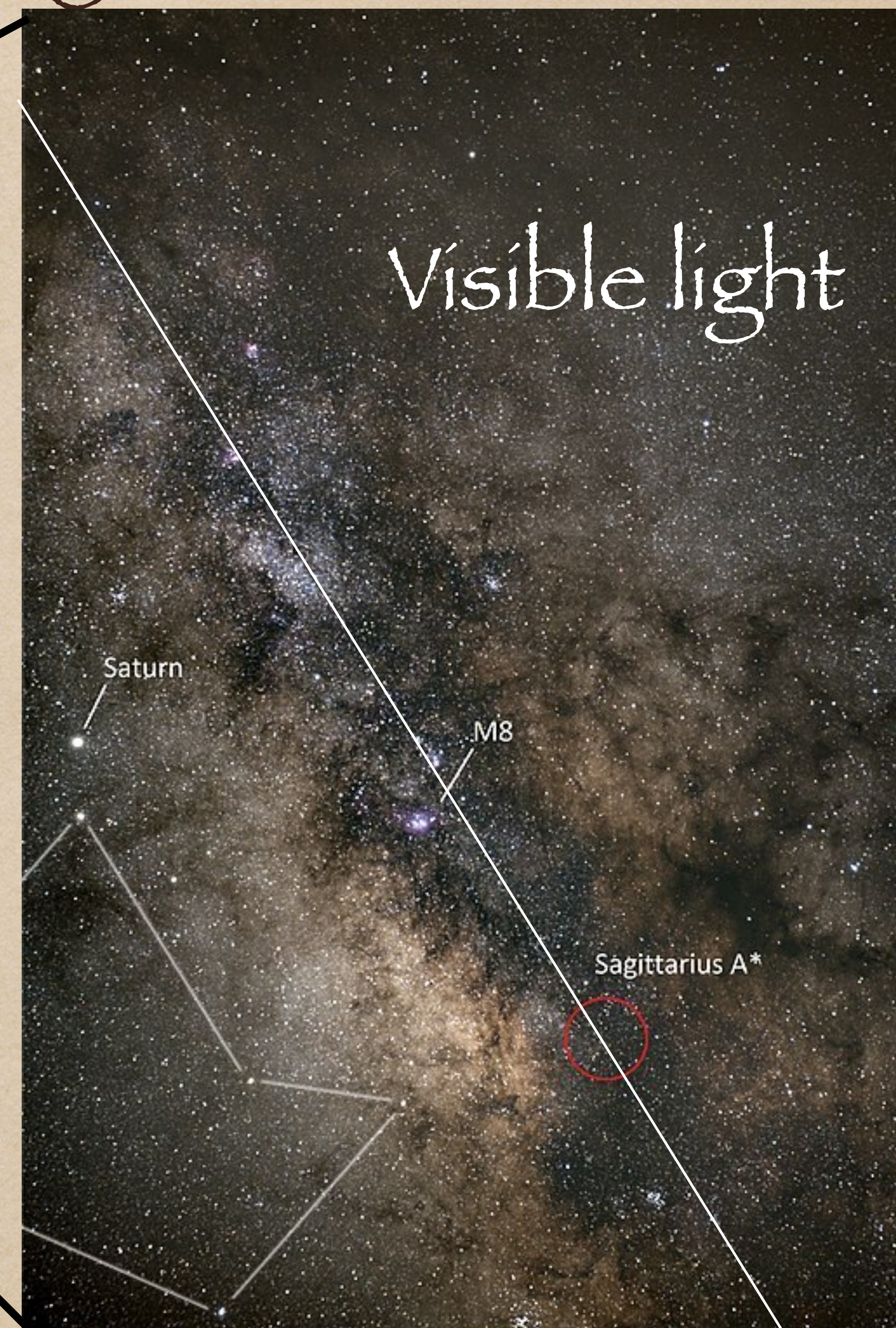
Sagittarius

The Milky Way center



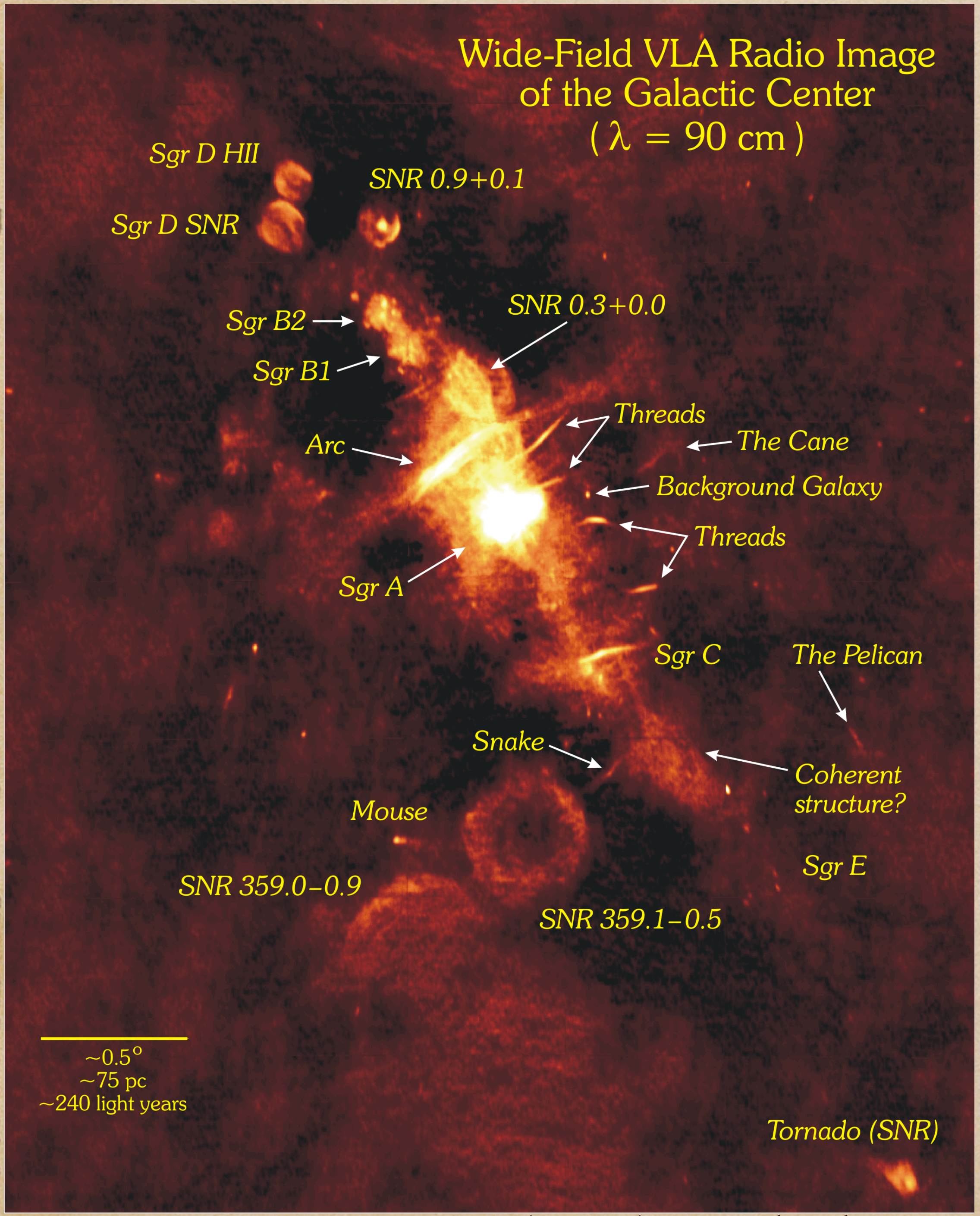
The Milky Way center





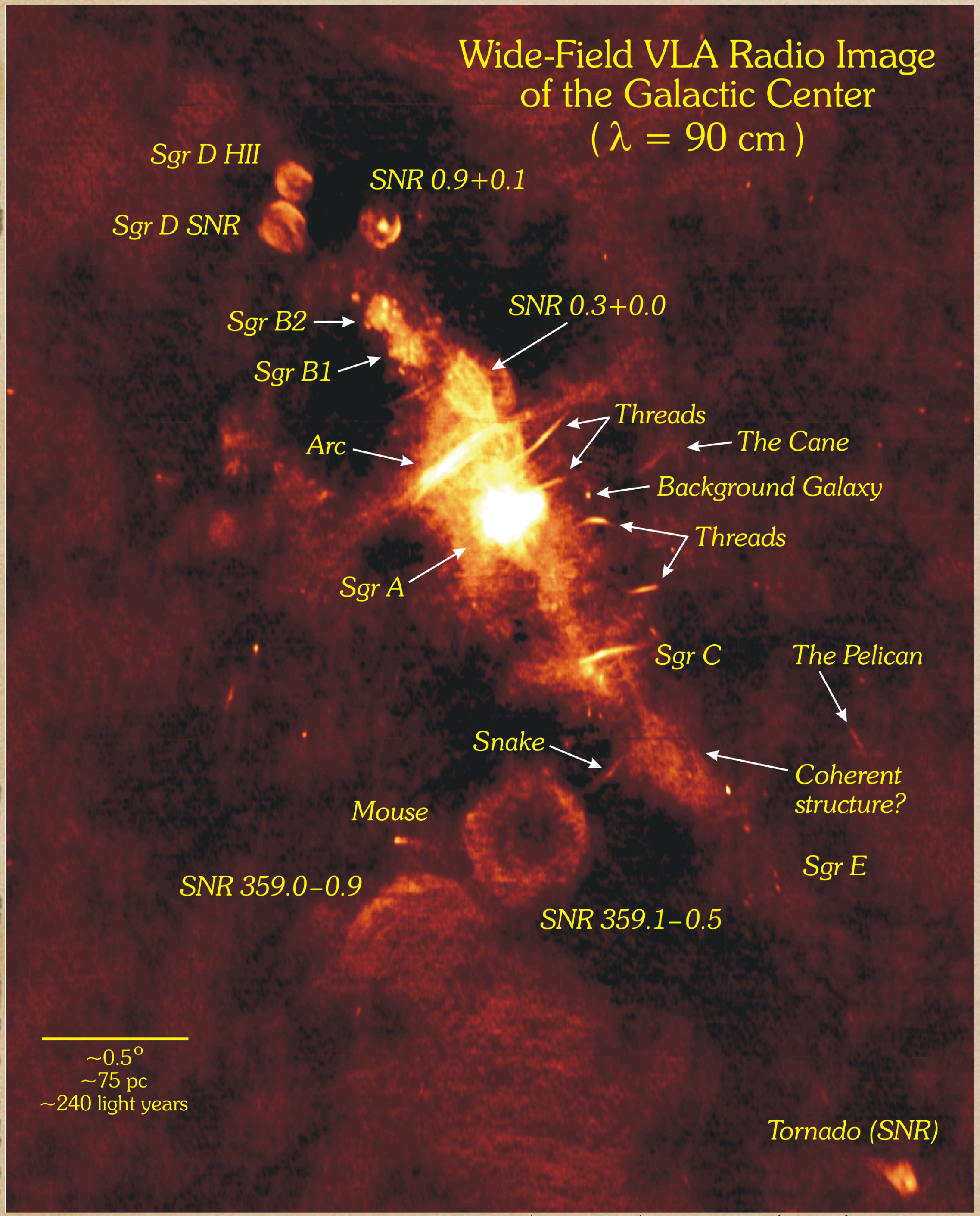
There is too much extinction in the optical wavelengths so Sagittarius A is only visible in other wavelengths, such as infrared, radio and X-rays.

The Milky Way center

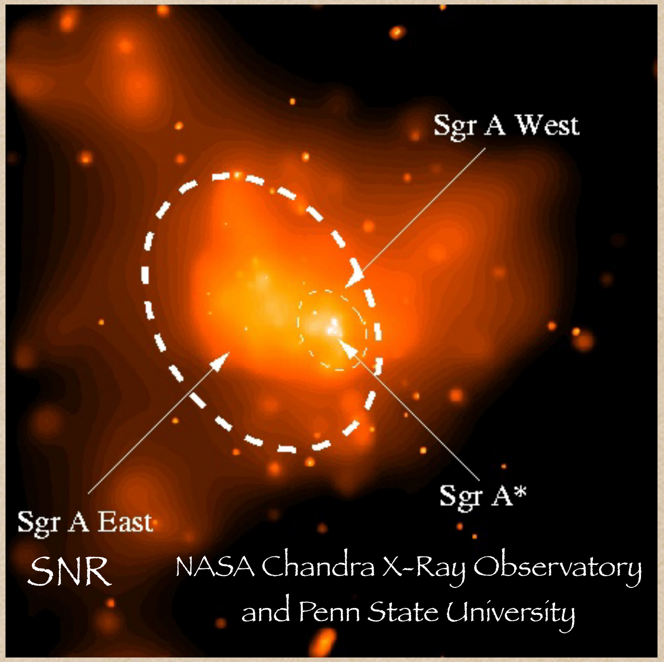


NRAO/AUI/NSF, N.E. Kassim et al., Naval Research Laboratory

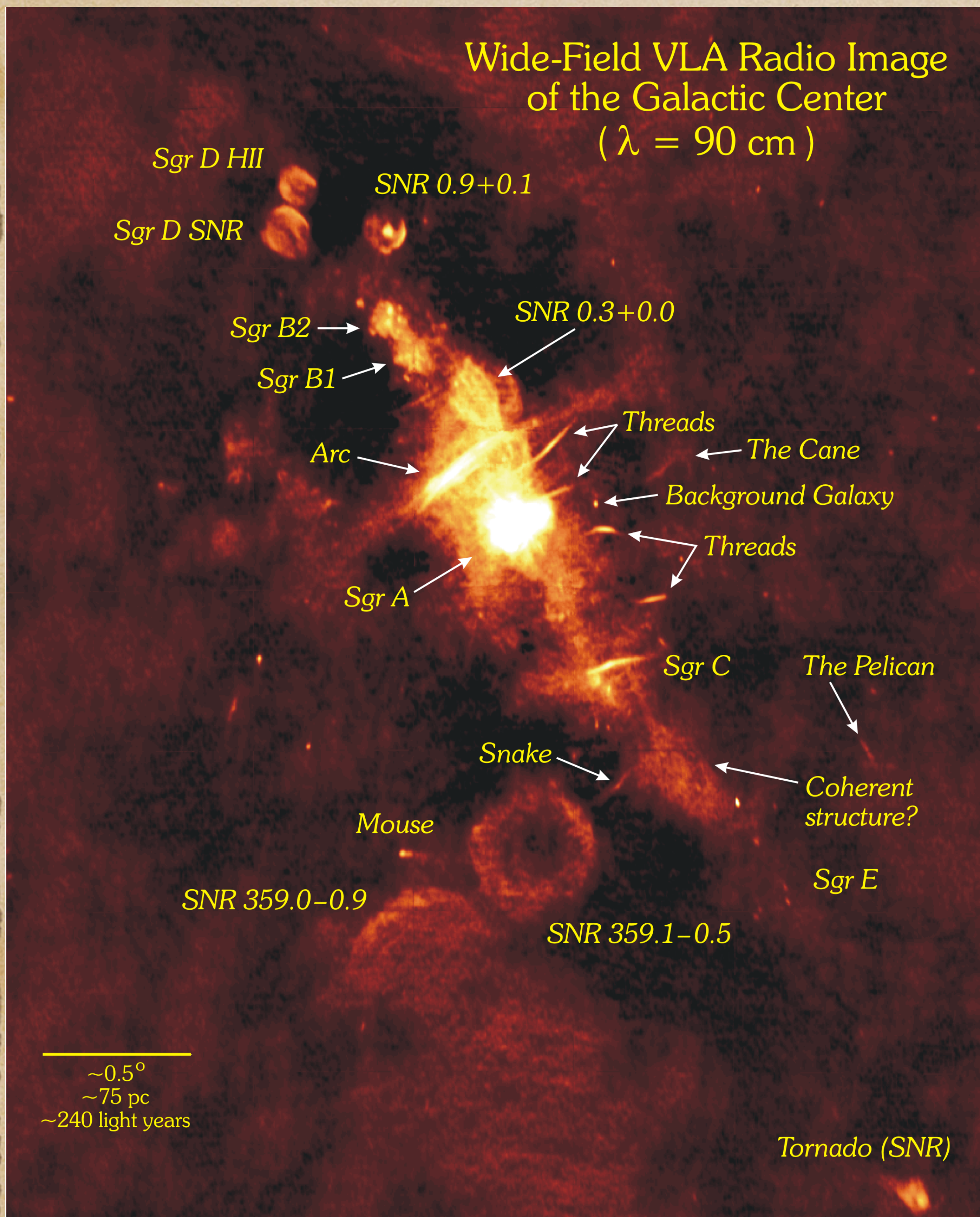
The Milky Way center



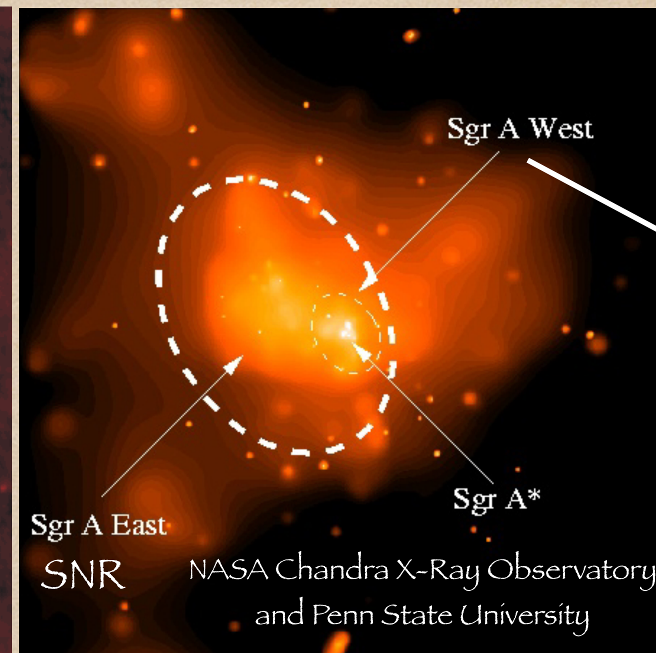
NRAO/AUI/NSF, N.E. Kassim et al., Naval Research Laboratory

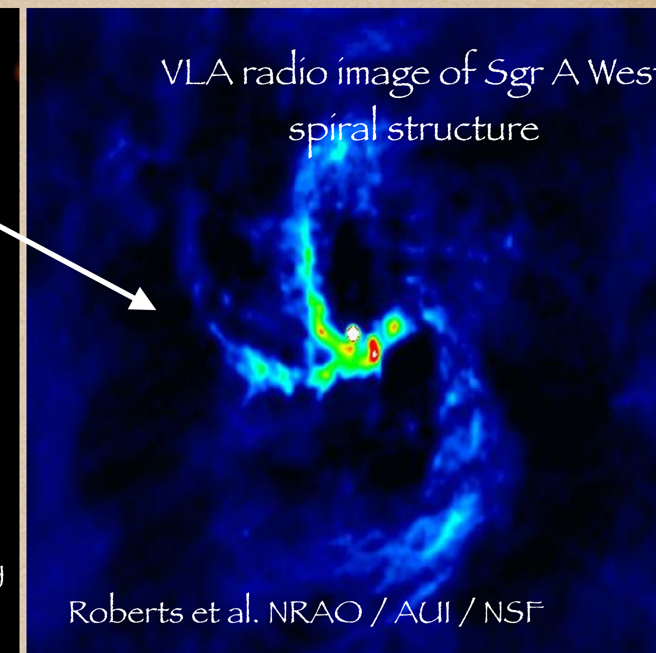


The Milky Way center

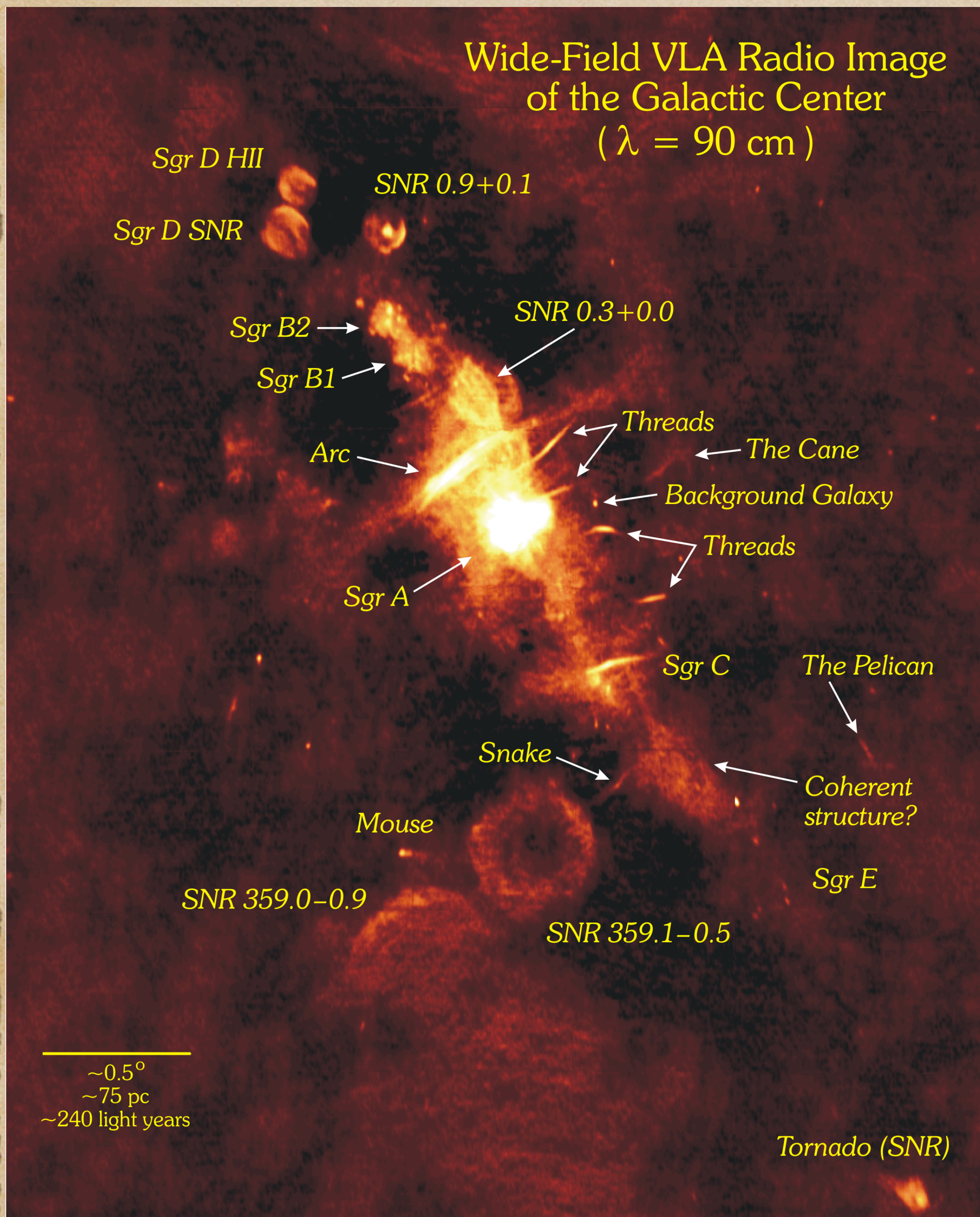


NRAO/AUI/NSF, N.E. Kassim et al., Naval Research Laboratory

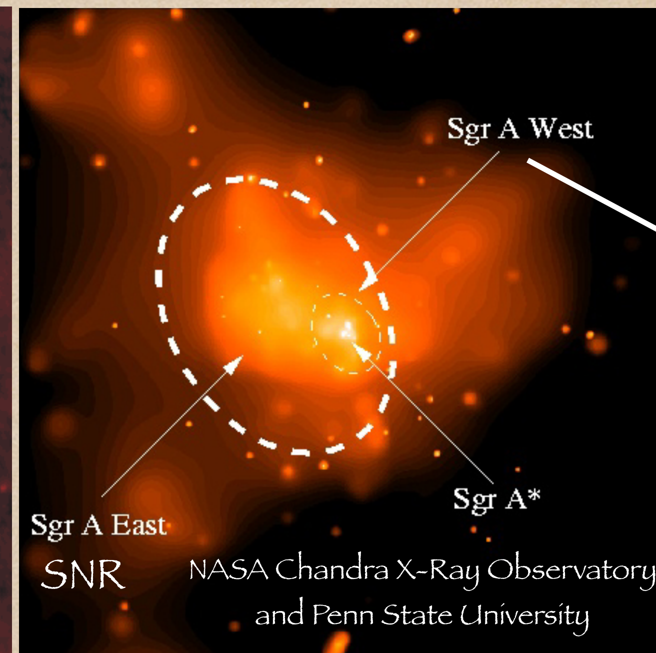


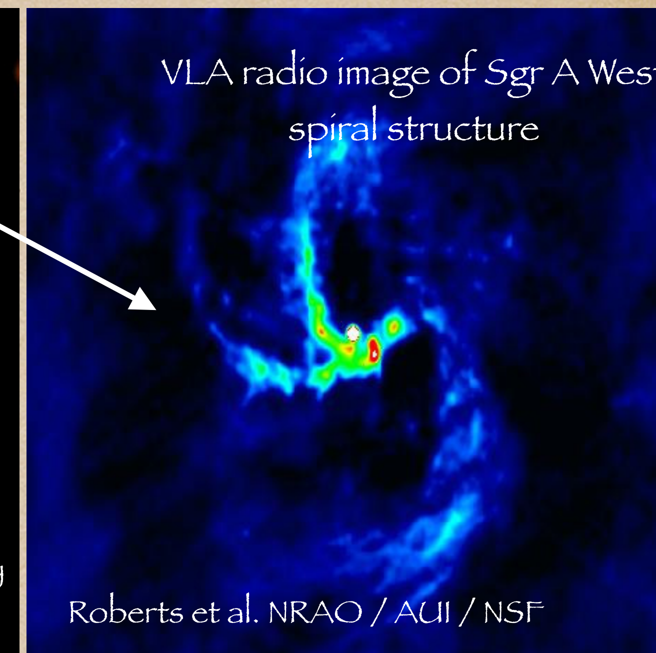


The Milky Way center



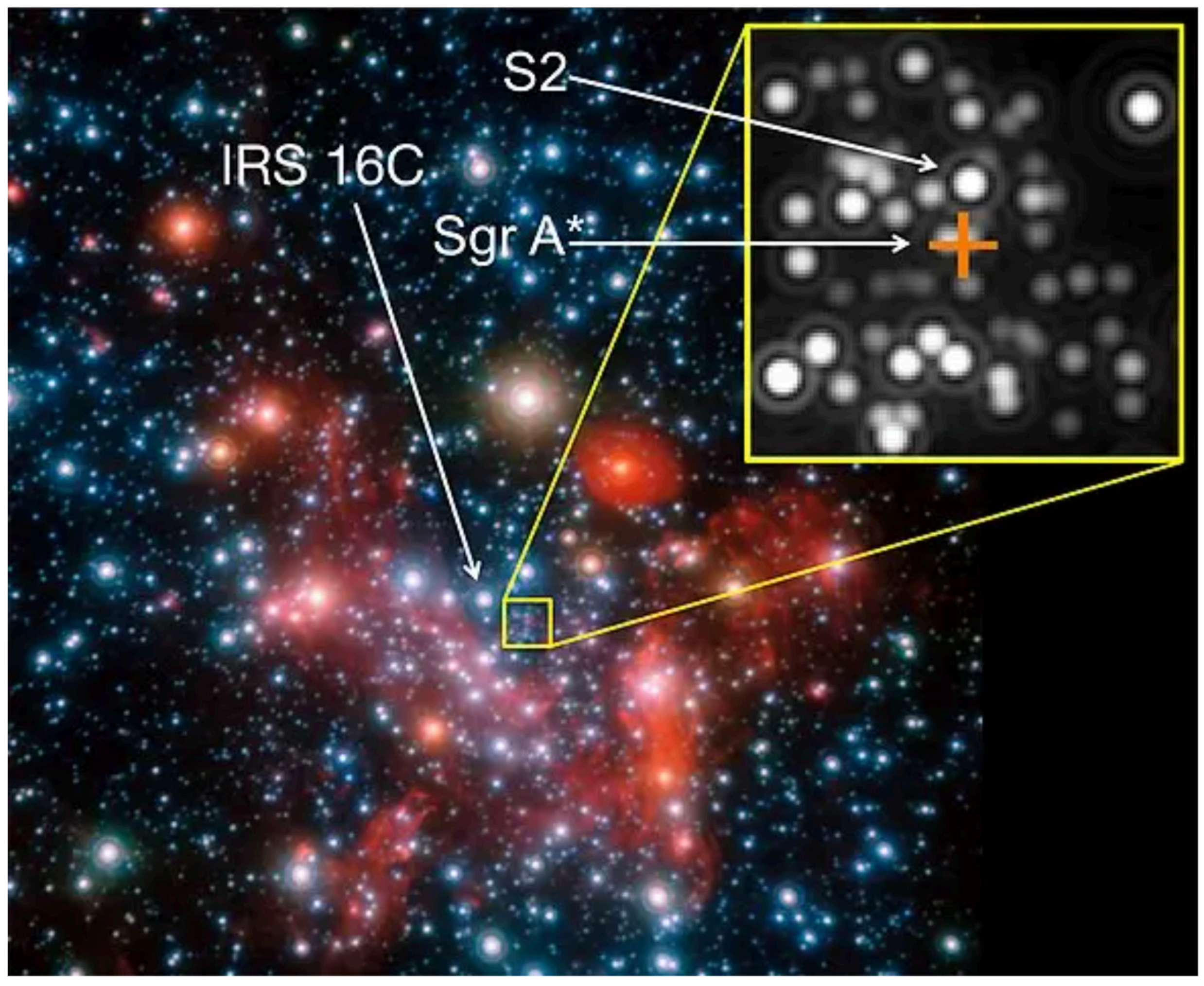
NRAO/AUI/NSF, N.E. Kassim et al., Naval Research Laboratory



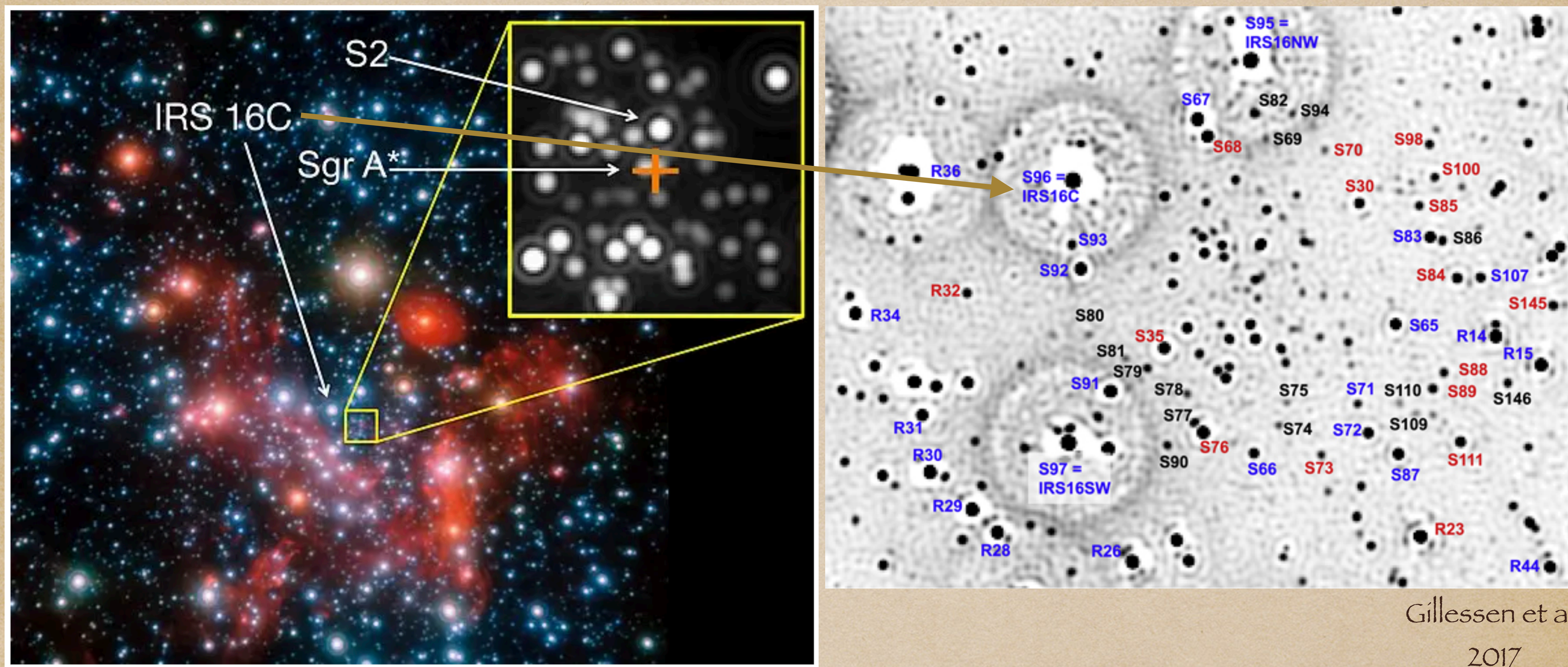


ESO, Gillessen et al. near-infrared image

The Milky Way center

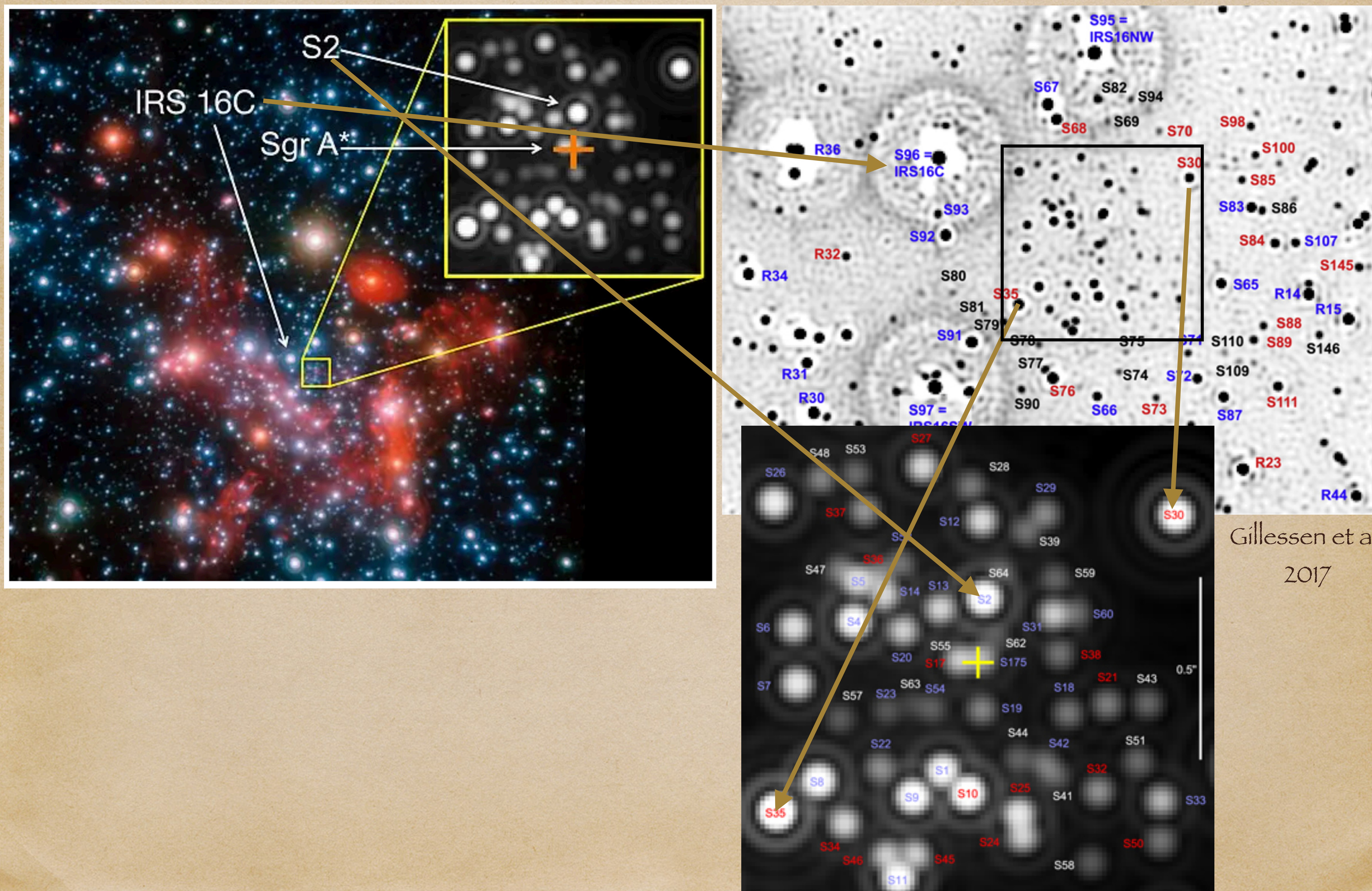


The Milky Way center



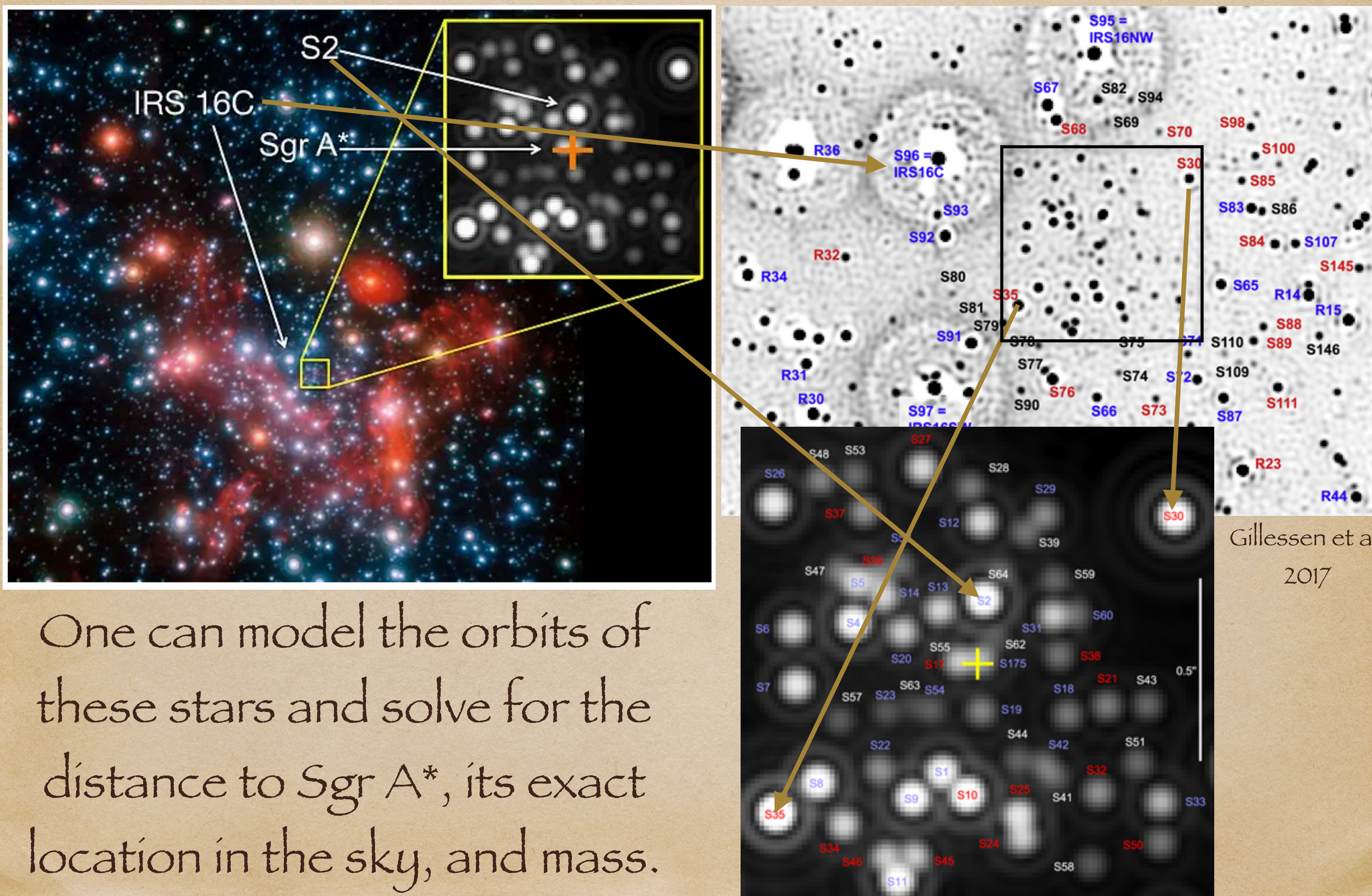
Gillessen et al.
2017

The Milky Way center



Gillessen et al. 2017

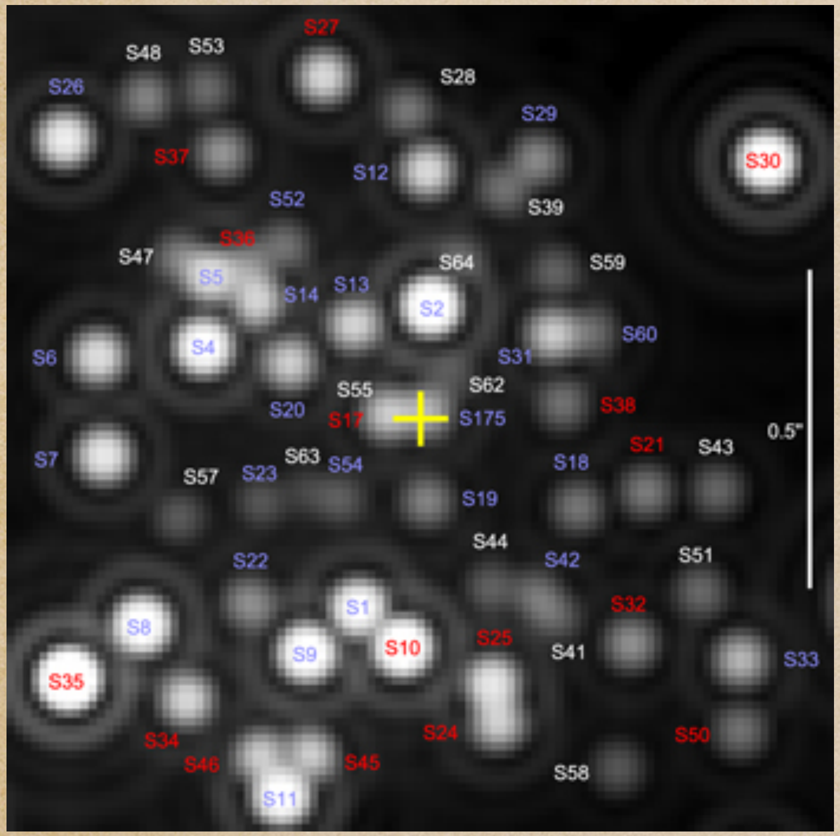
The Milky Way center



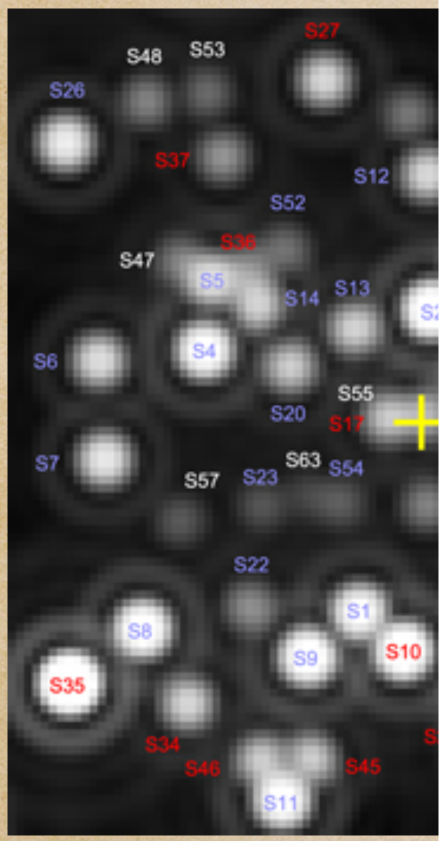
Gillessen et al. 2017

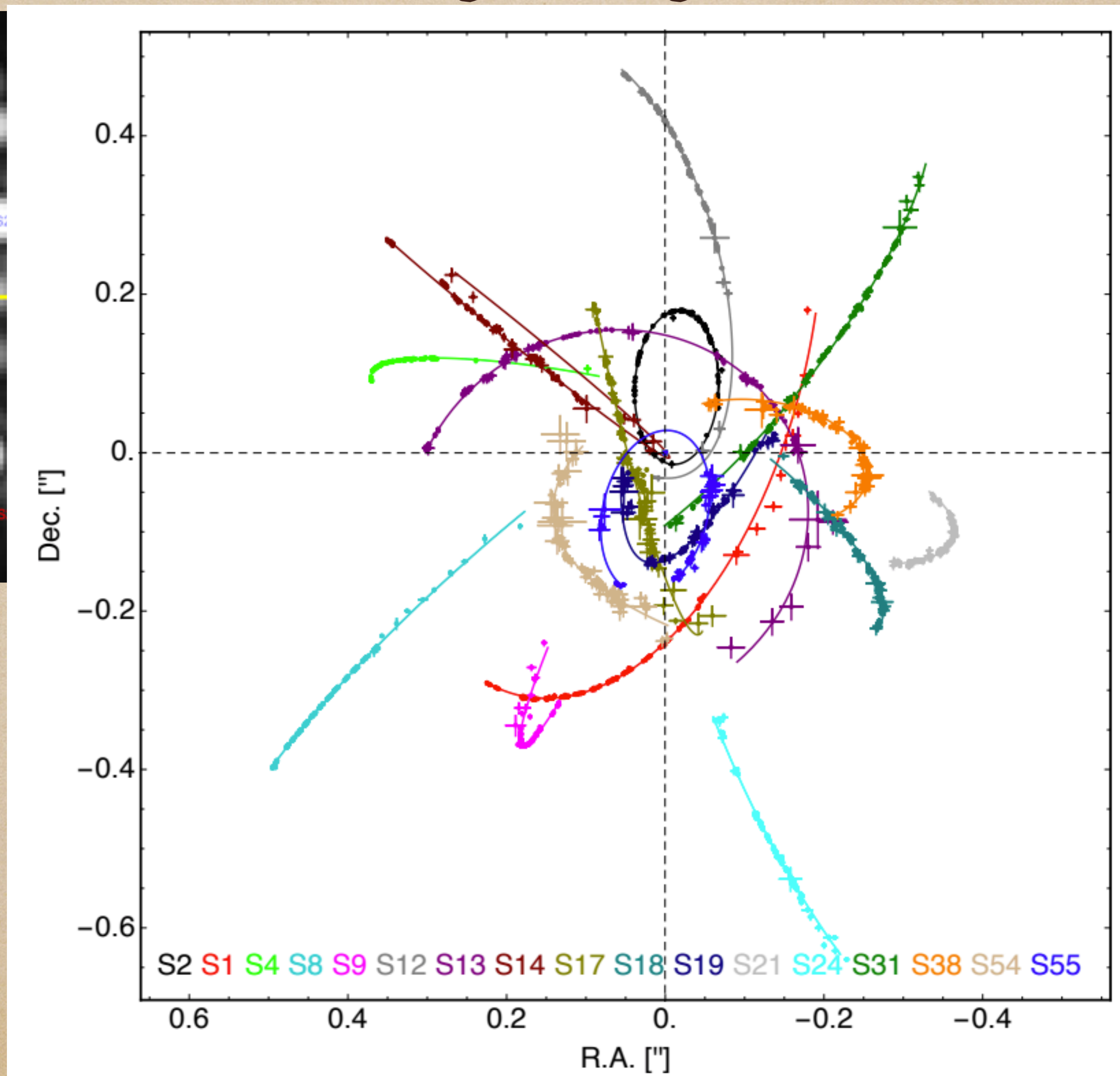
One can model the orbits of these stars and solve for the distance to Sgr A*, its exact location in the sky, and mass.

The Milky Way center

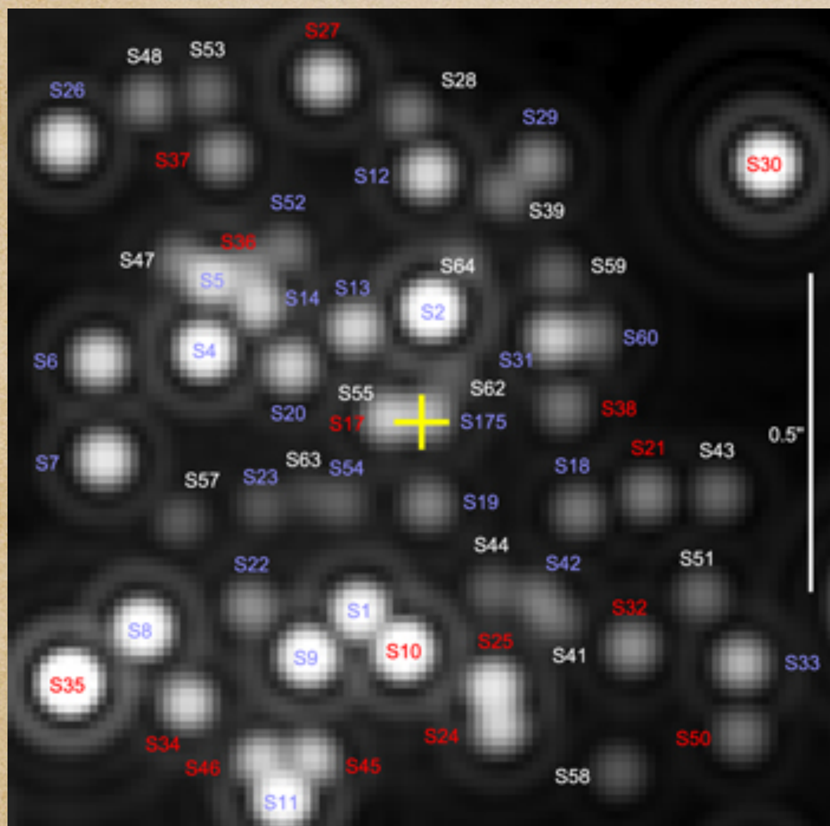


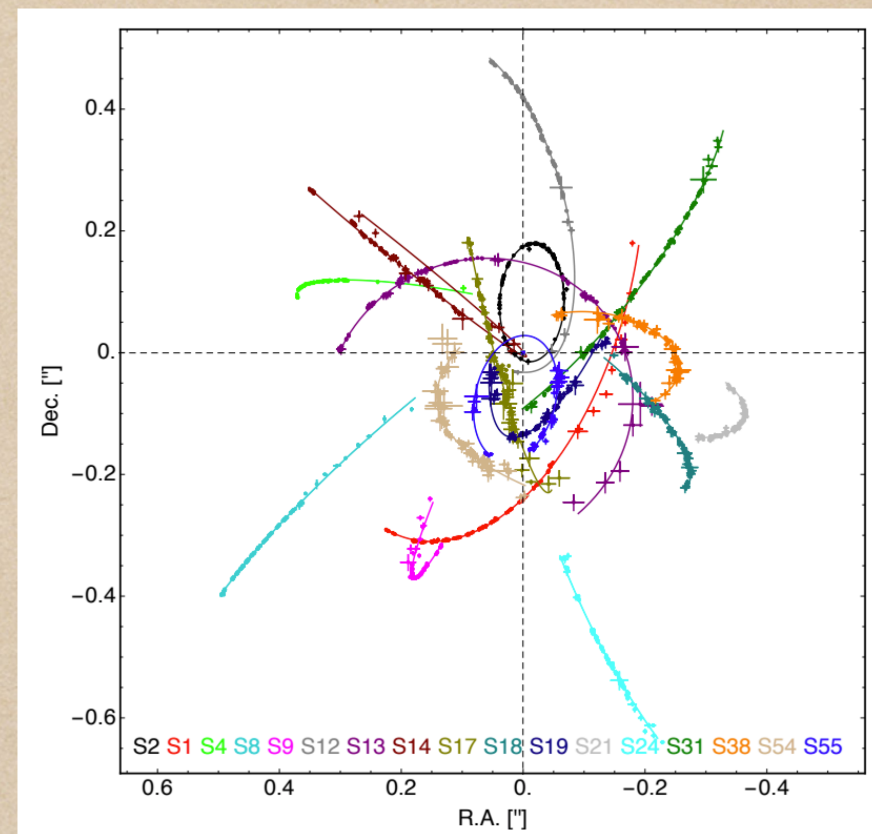
The Milky Way center





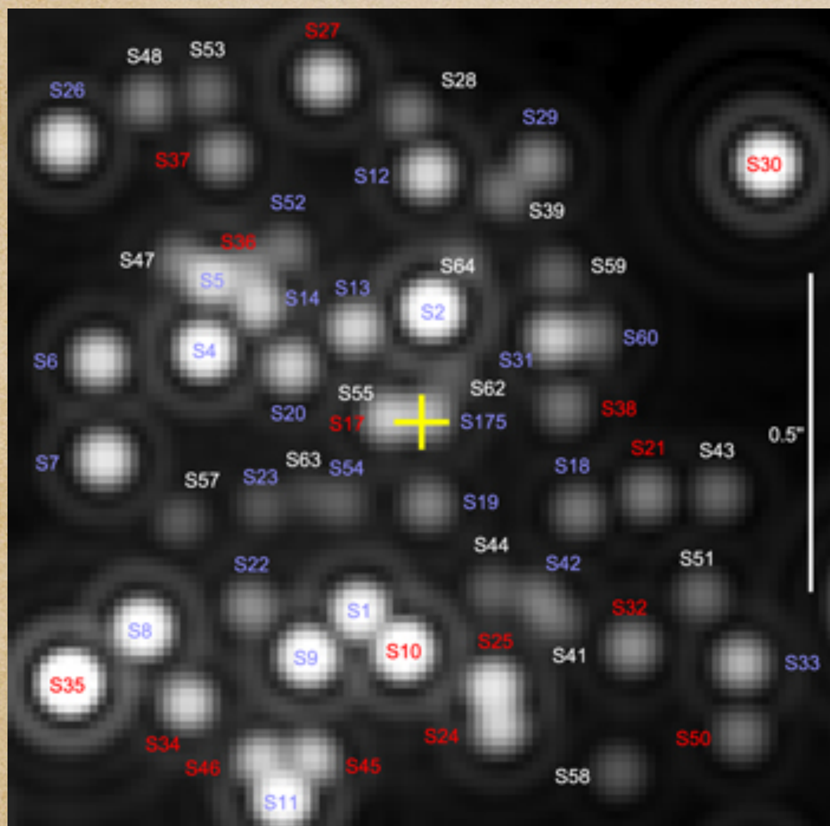
The Milky Way center

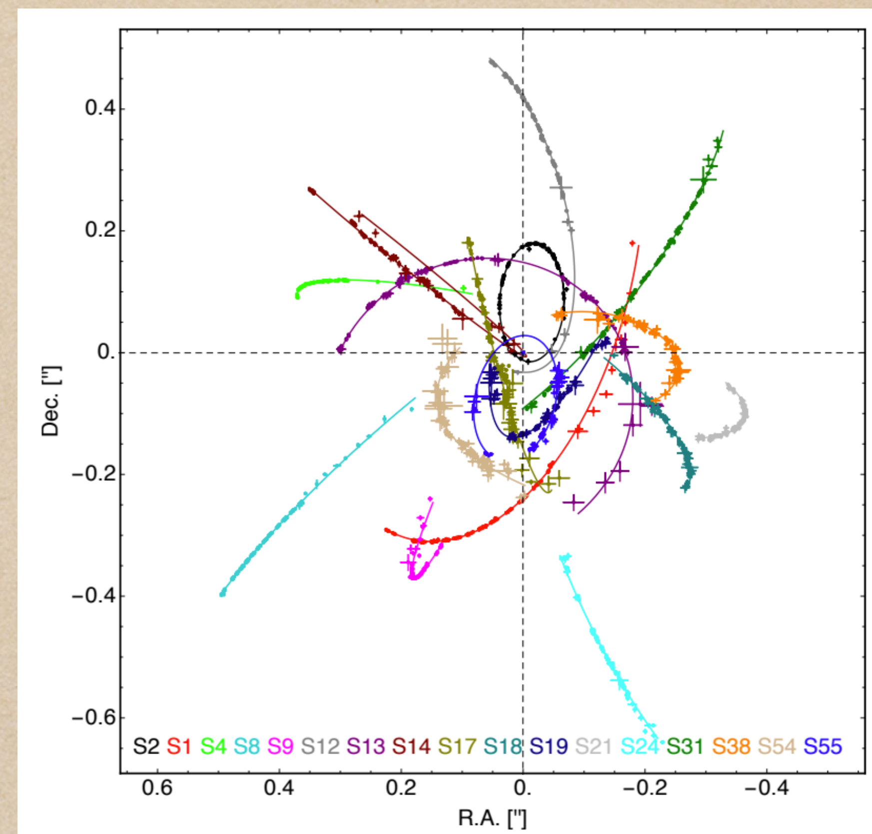




The distance to Sgr A* is determined to be $D = 8.3kpc$ while the S2 star on closest approach gets within $\sim 100AU$.

The Milky Way center



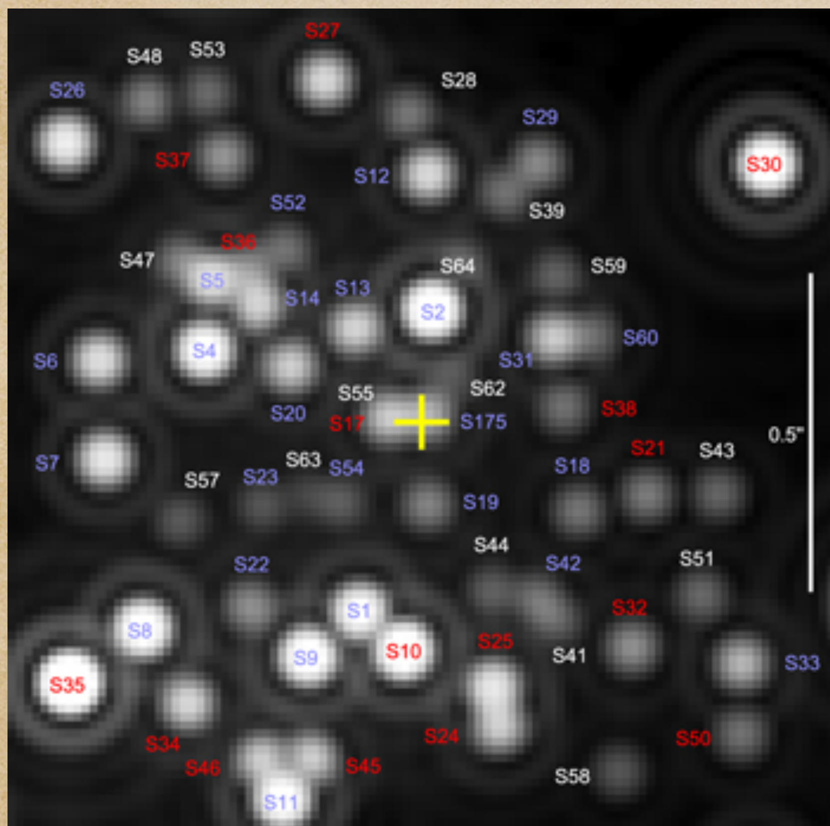


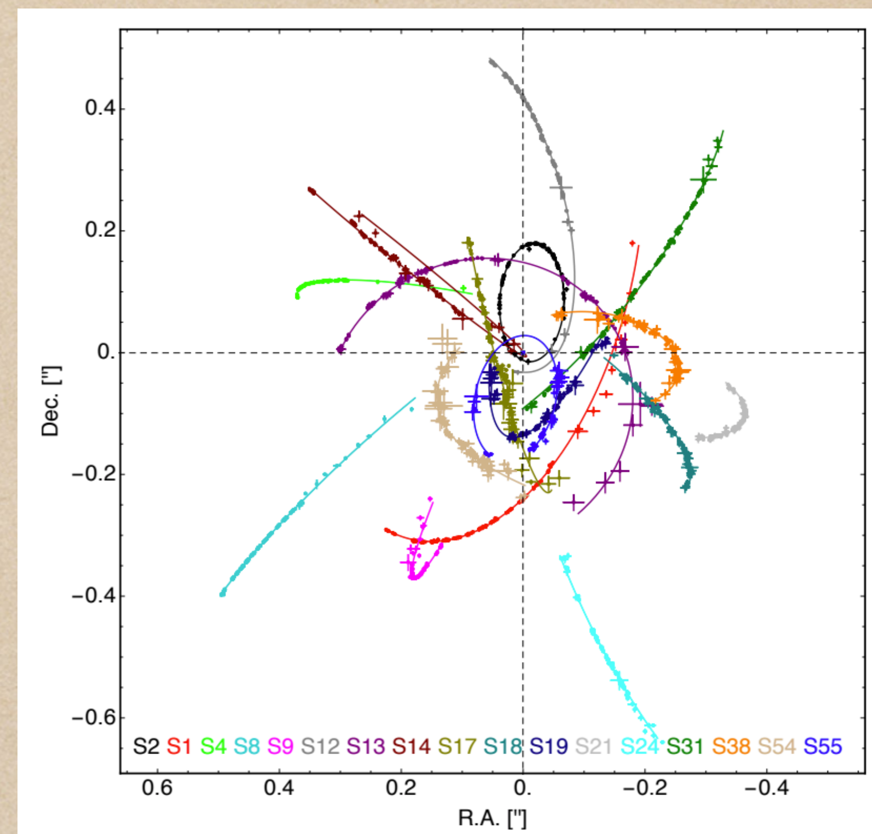
The distance to Sgr A* is determined to be $D = 8.3kpc$ while the S2 star on closest approach gets within $\sim 100AU$.

It is a simple exercise to estimate the mass of Sgr A* from Kepler's 3rd law and the size and periods of the orbits.

$$M = \frac{a^3}{T^2}$$

The Milky Way center





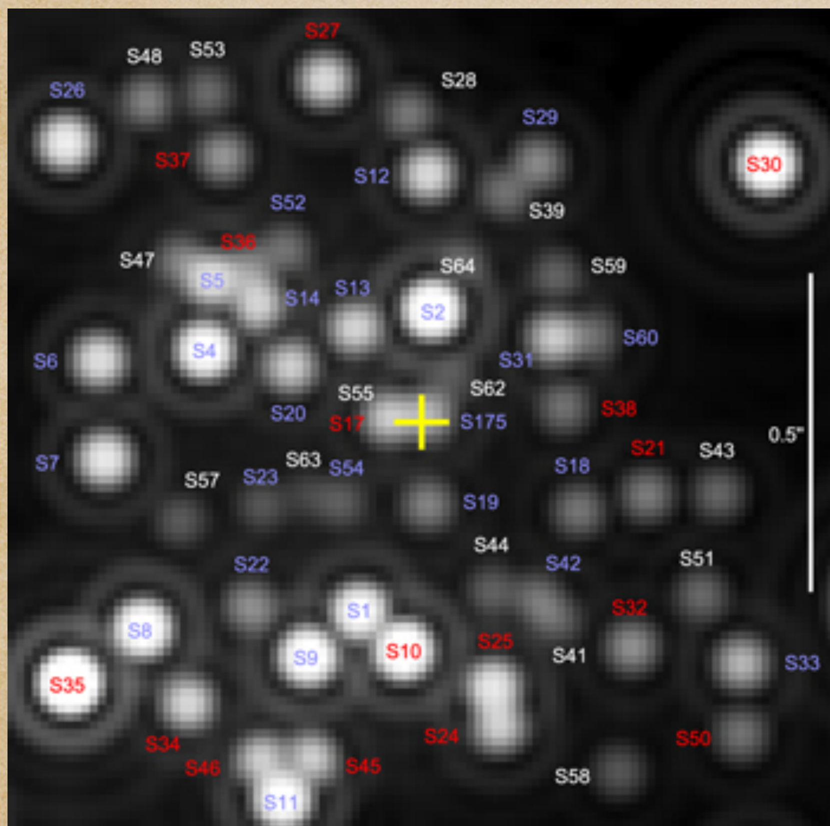
The distance to Sgr A* is determined to be $D = 8.3kpc$ while the S2 star on closest approach gets within $\sim 100AU$.

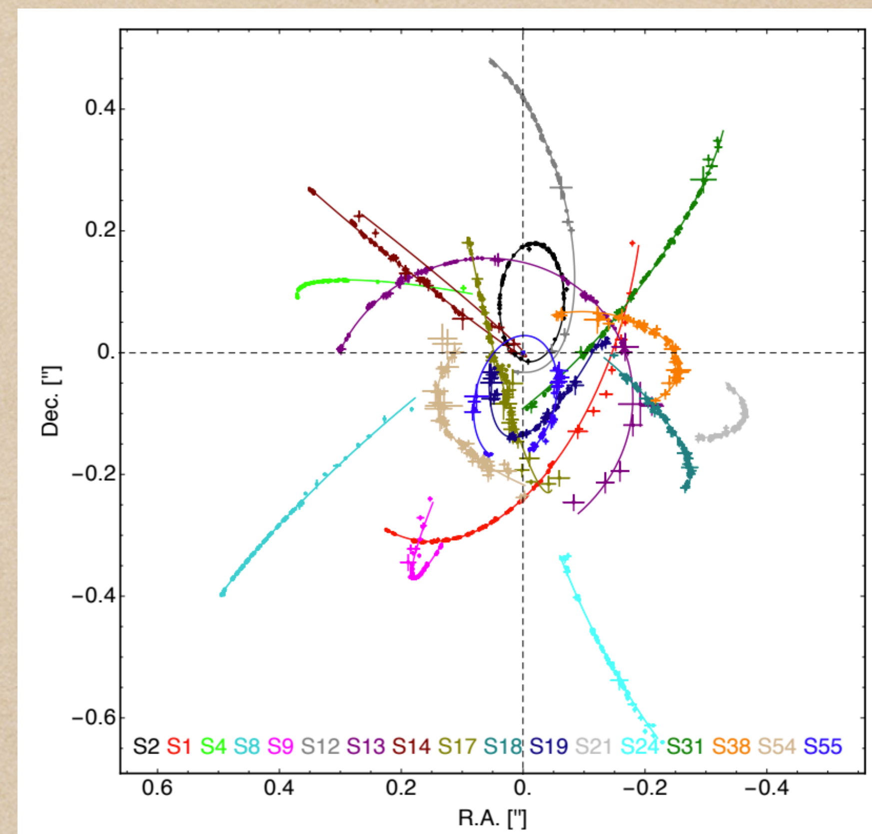
It is a simple exercise to estimate the mass of Sgr A* from Kepler's 3rd law and the size and periods of the orbits.

$$M = \frac{a^3}{T^2}$$

	$S1$	$S2$	$S4$	$S6$	$S8$	$S9$	$S22$	$S24$	$S33$	$S54$
a	0.595"	0.1255"	0.357"	0.6574"	0.4047"	0.2724"	1.31"	0.944"	0.657"	1.2"
$a(AU)$	4938	1041	2963	5456	3359	2261	10873	7835	5453	9960
T	$166yr$	$16yr$	$77yr$	$192yr$	$92yr$	$51yr$	$540yr$	$331yr$	$192yr$	$477yr$
M $(10^6 M_\odot)$	4.371	4.415	4.388	4.407	4.478	4.443	4.408	4.390	4.399	4.343

The Milky Way center





The distance to Sgr A* is determined to be $D = 8.3kpc$ while the S2 star on closest approach gets within $\sim 100AU$.

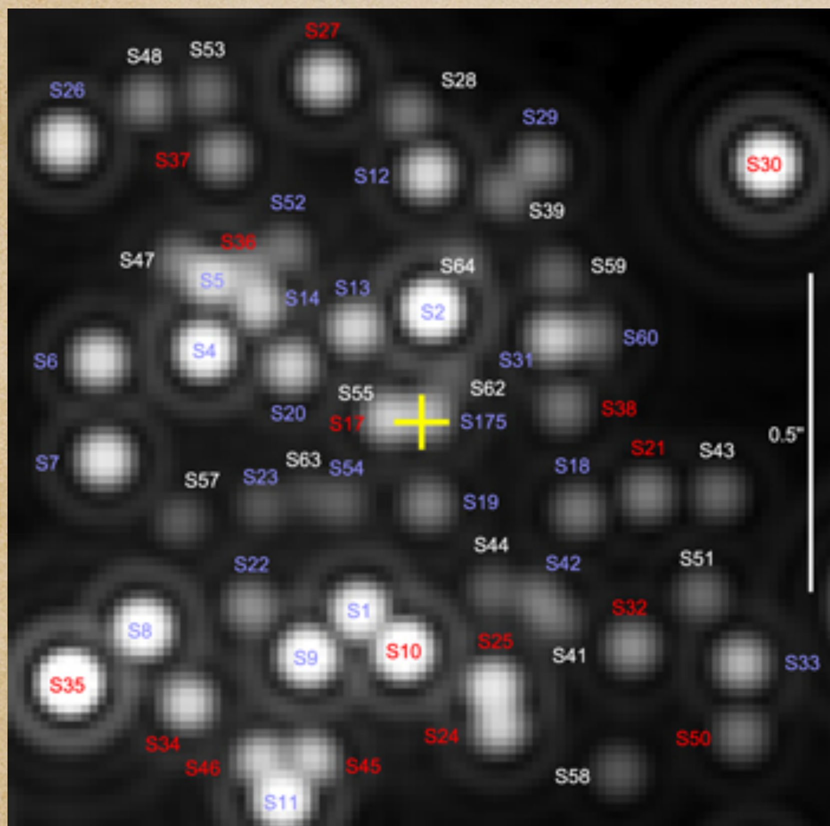
It is a simple exercise to estimate the mass of Sgr A* from Kepler's 3rd law and the size and periods of the orbits.

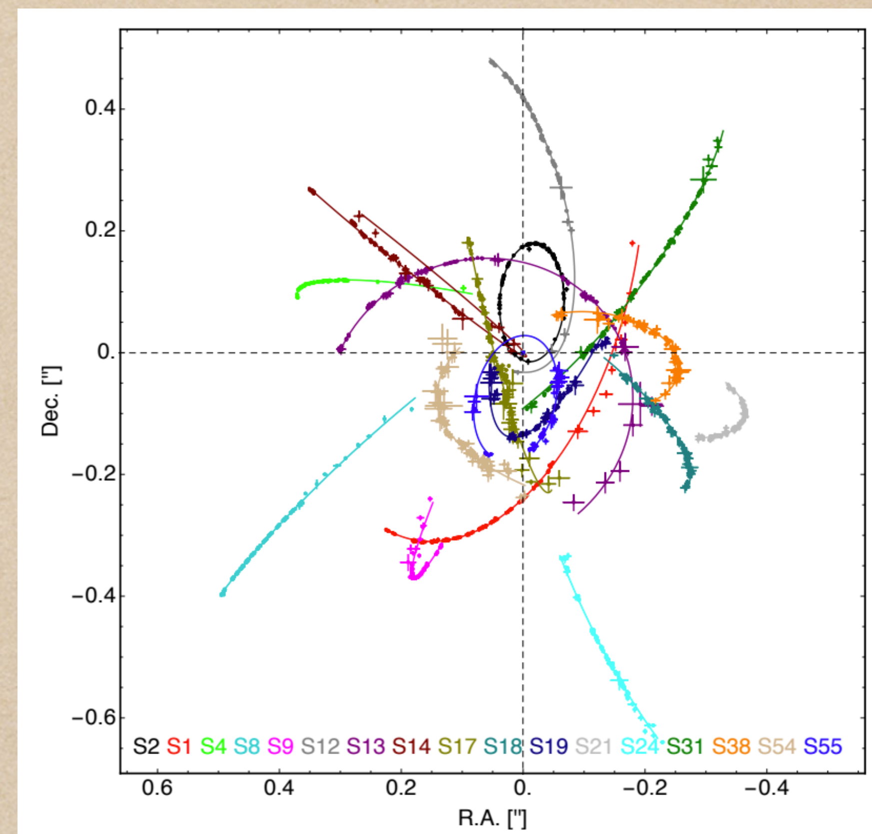
$$M = \frac{a^3}{T^2}$$

	$S1$	$S2$	$S4$	$S6$	$S8$	$S9$	$S22$	$S24$	$S33$	$S54$
a	0.595"	0.1255"	0.357"	0.6574"	0.4047"	0.2724"	1.31"	0.944"	0.657"	1.2"
$a(AU)$	4938	1041	2963	5456	3359	2261	10873	7835	5453	9960
T	$166yr$	$16yr$	$77yr$	$192yr$	$92yr$	$51yr$	$540yr$	$331yr$	$192yr$	$477yr$
M $(10^6 M_\odot)$	4.371	4.415	4.388	4.407	4.478	4.443	4.408	4.390	4.399	4.343

This is close to the properly measured value of $\sim 4.28 \times 10^6 M_\odot$

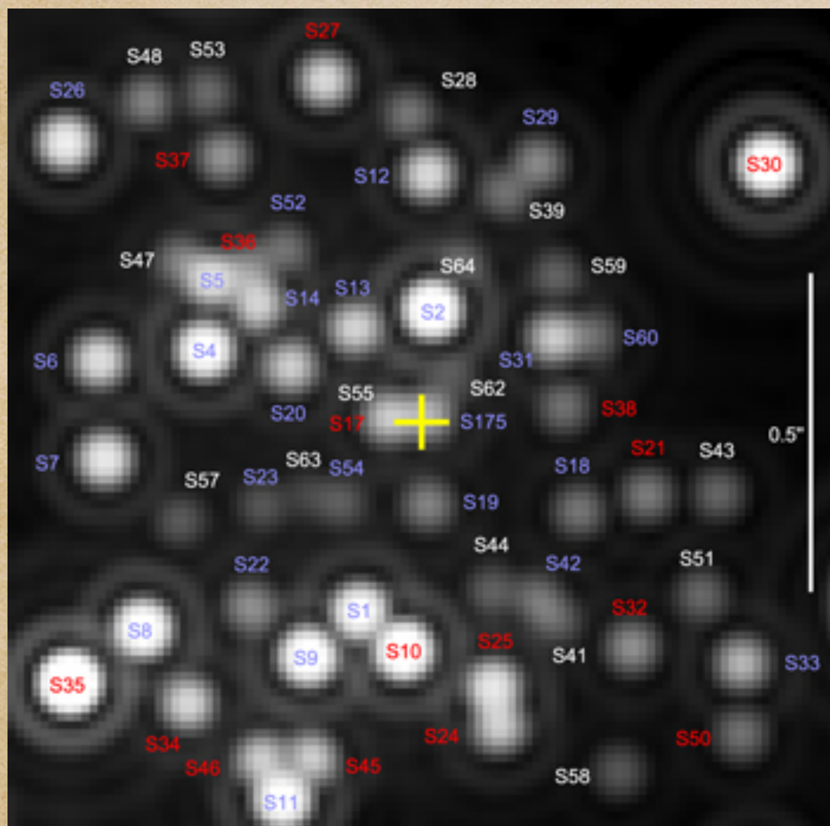
The Milky Way center

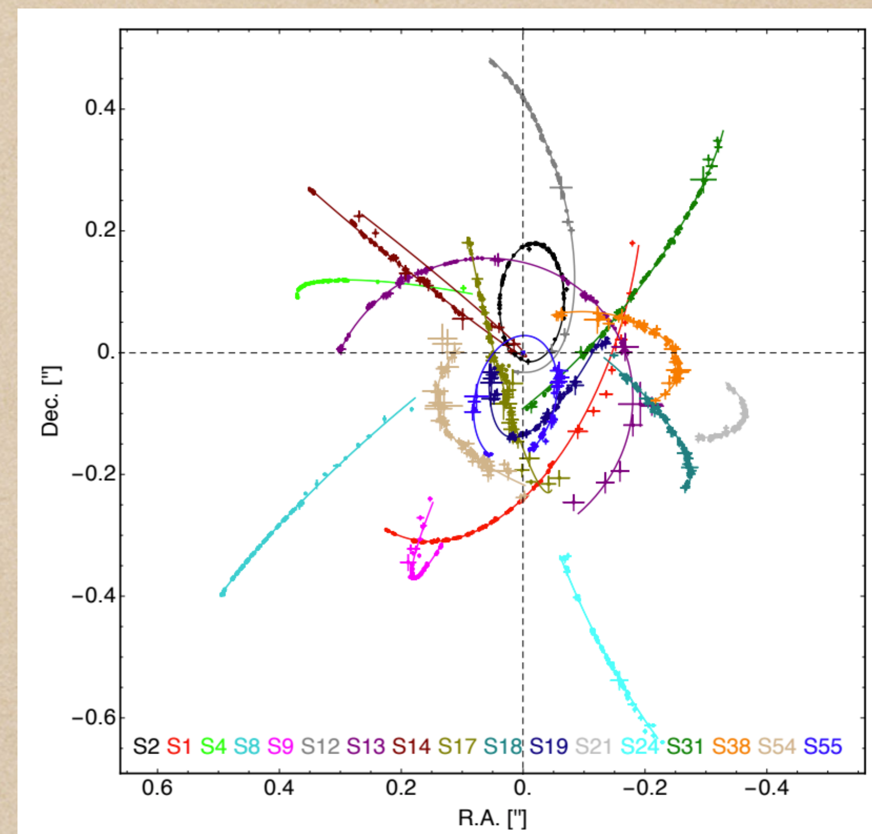




The S2 star on closest approach gets to $\sim 100 AU$, while other stars can get even closer.

The Milky Way center

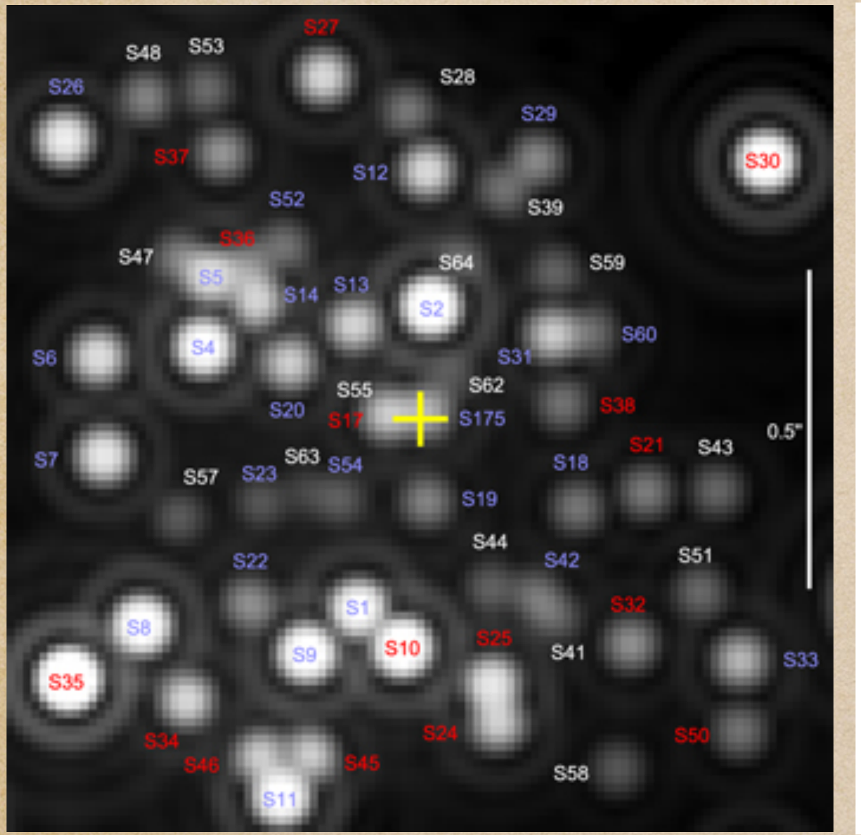


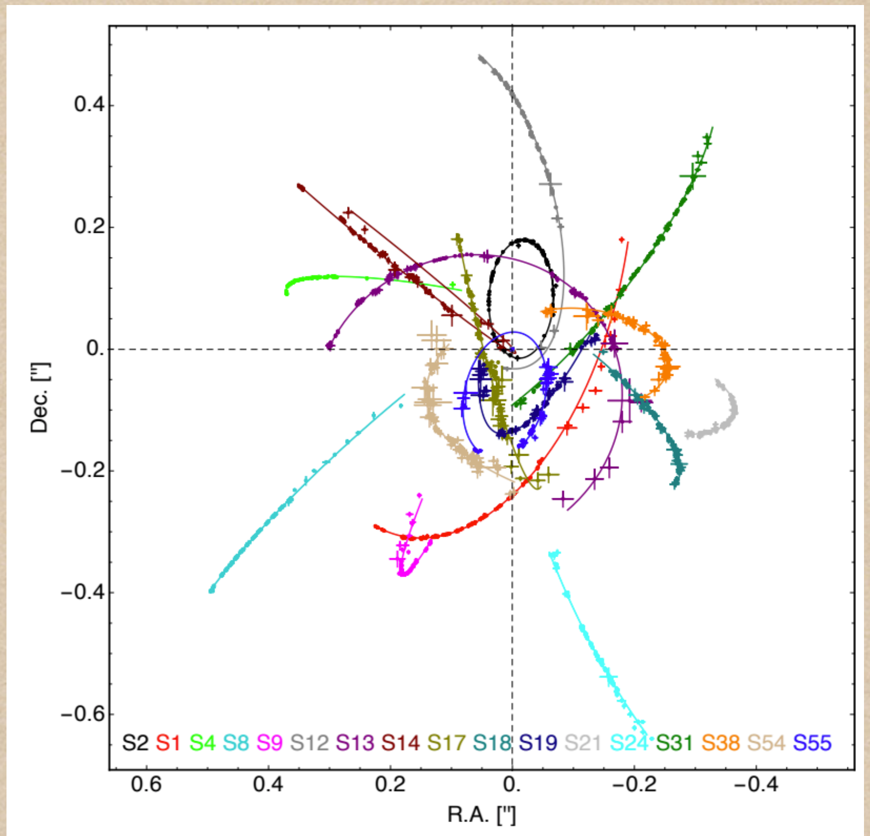


The S2 star on closest approach gets to $\sim 100AU$, while other stars can get even closer.

There is something with a mass of $\sim 4.28 \times 10^6 M_\odot$ that has a size that is less than 100 AU, that resides at the center of the Milky Way and can only be a Black Hole.

The Milky Way center





The S2 star on closest approach gets to $\sim 100 AU$, while other stars can get even closer.

There is something with a mass of $\sim 4.28 \times 10^6 M_\odot$ that has a size that is less than 100 AU, that resides at the center of the Milky Way and can only be a Black Hole.

The 2020 Nobel Prize in Physics "for the discovery of a supermassive compact object at the centre of our galaxy", was awarded to Andrea Ghez, Reinhard Genzel, and Roger Penrose.

What is a Black Hole

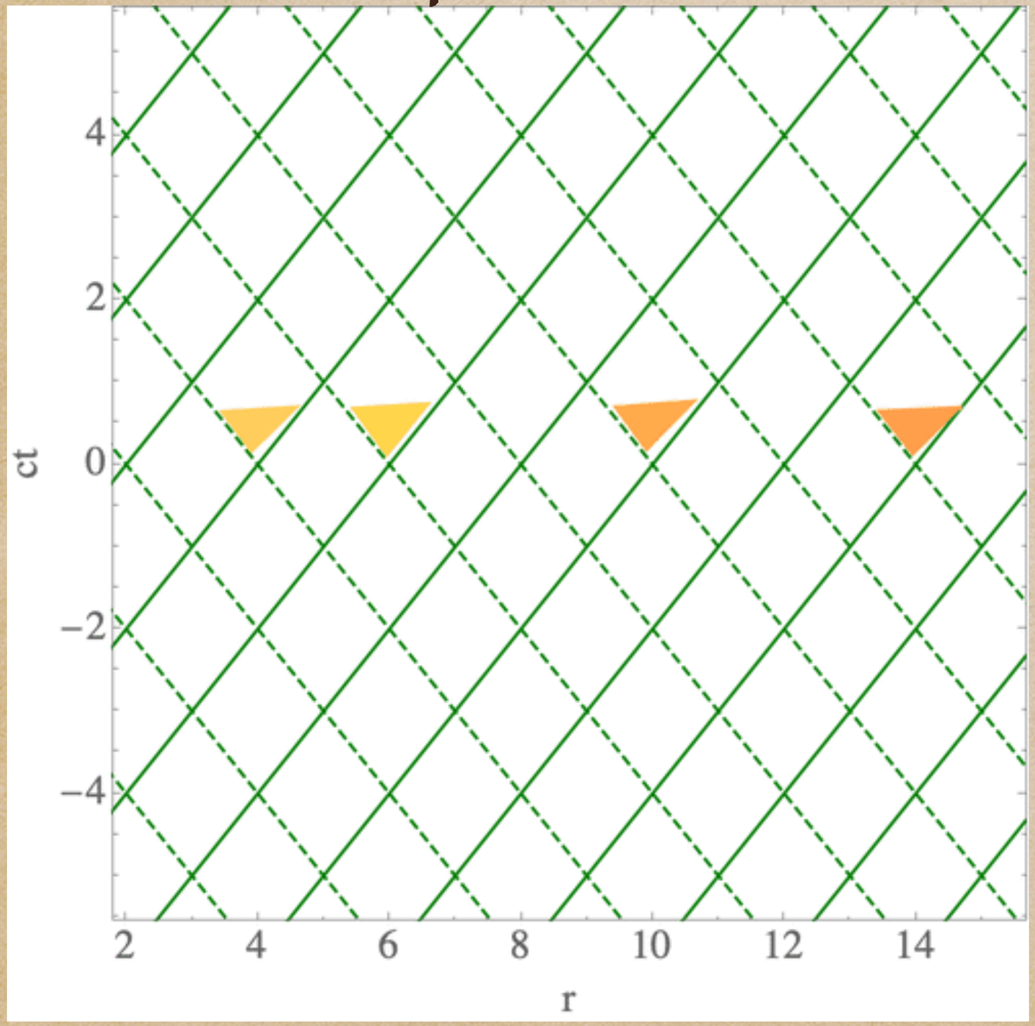
What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

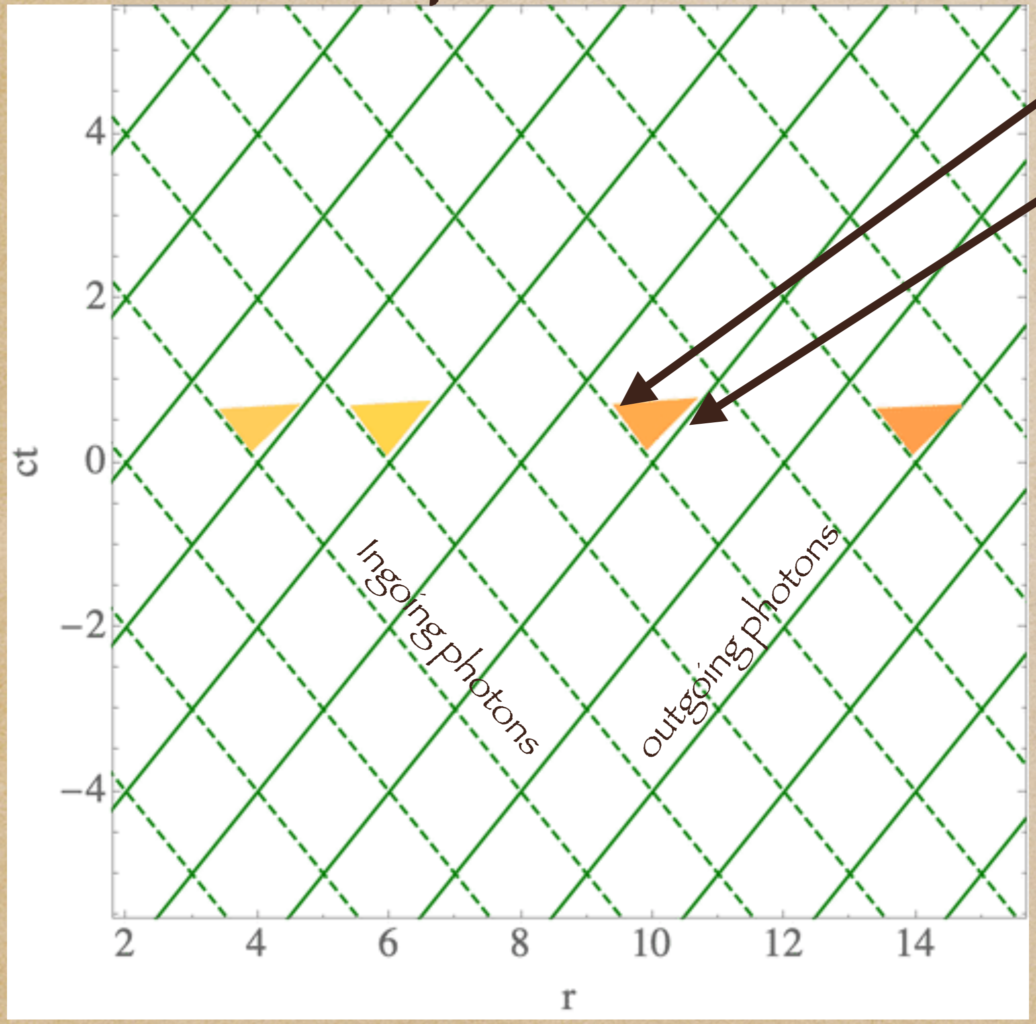
Flat spacetime



What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Flat spacetime

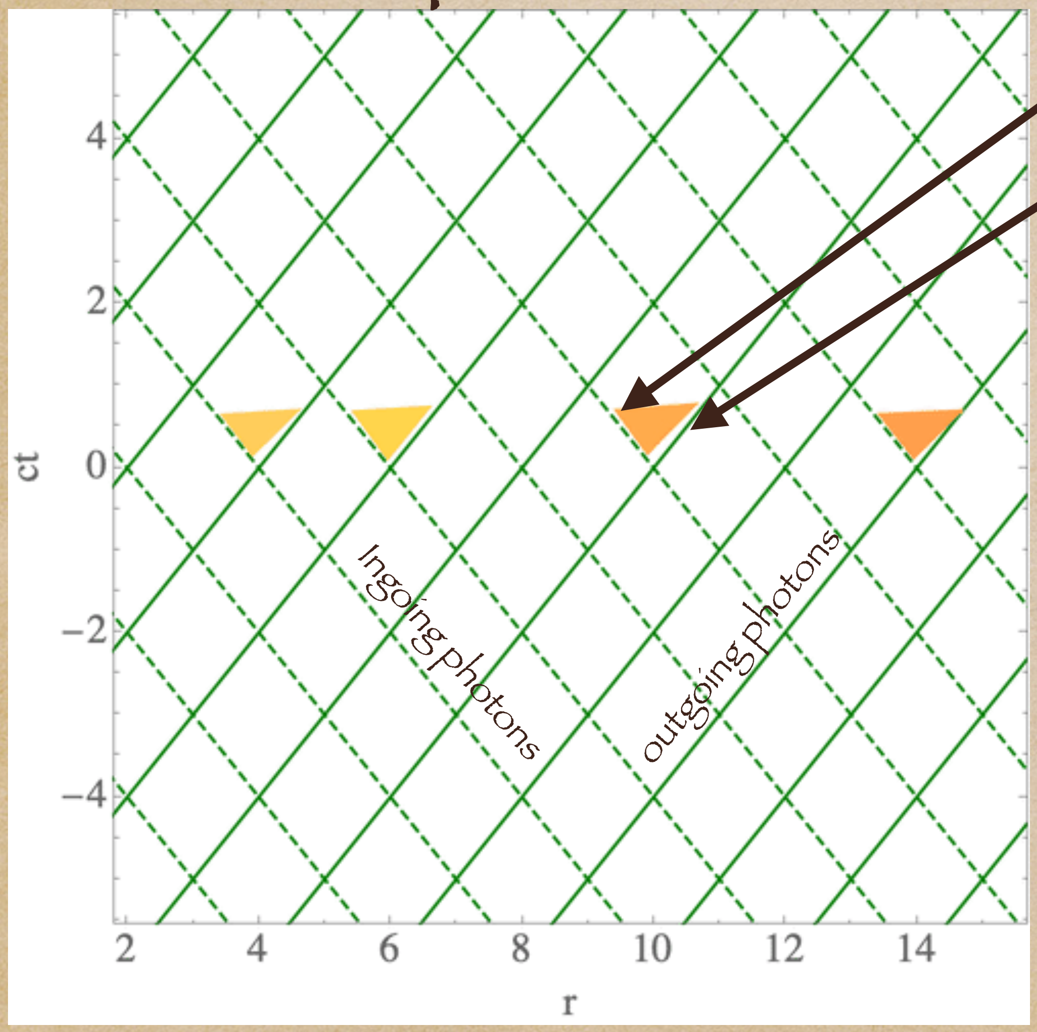


- Ingoing light-rays (dashed)
- Outgoing light-rays (solid)

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Flat spacetime

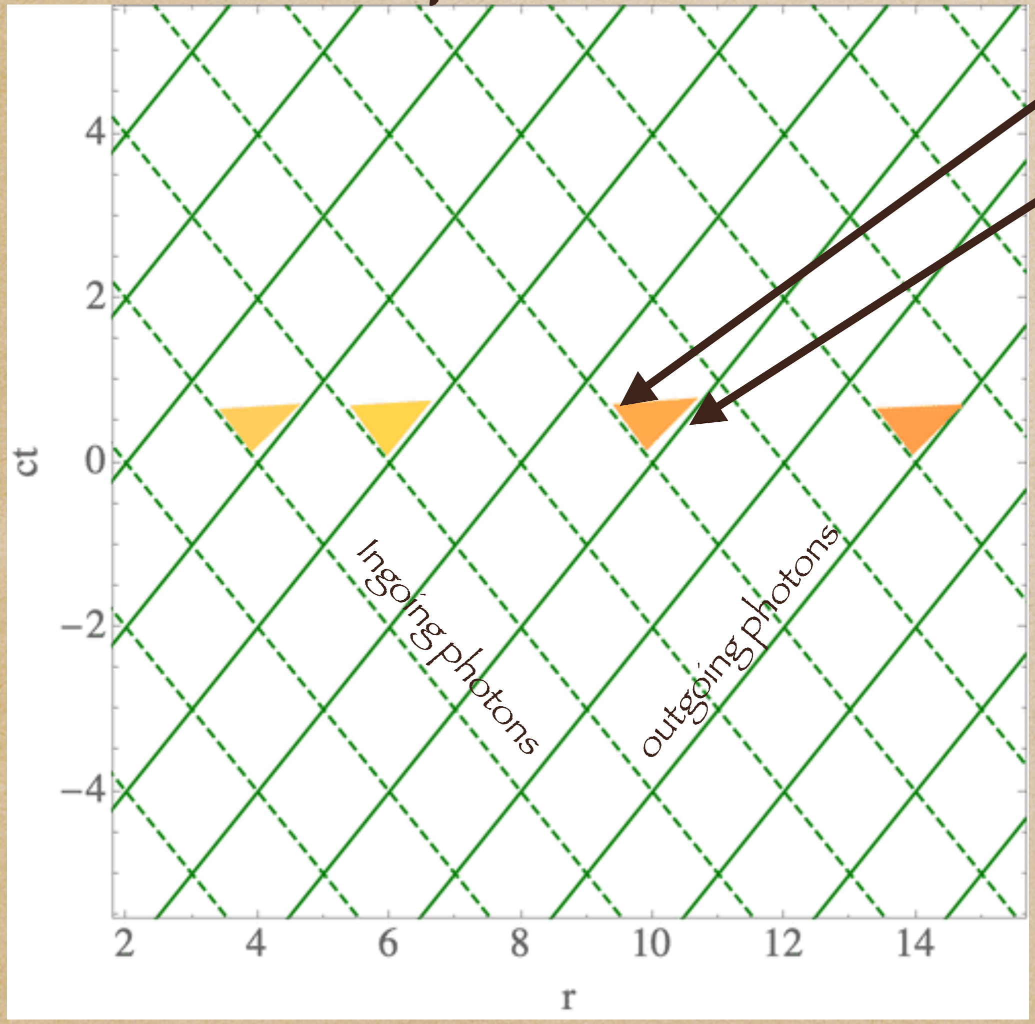


- Ingoing light-rays (dashed)
- Outgoing light-rays (solid)
  - These form the light-cones of the spacetime (orange)

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Flat spacetime

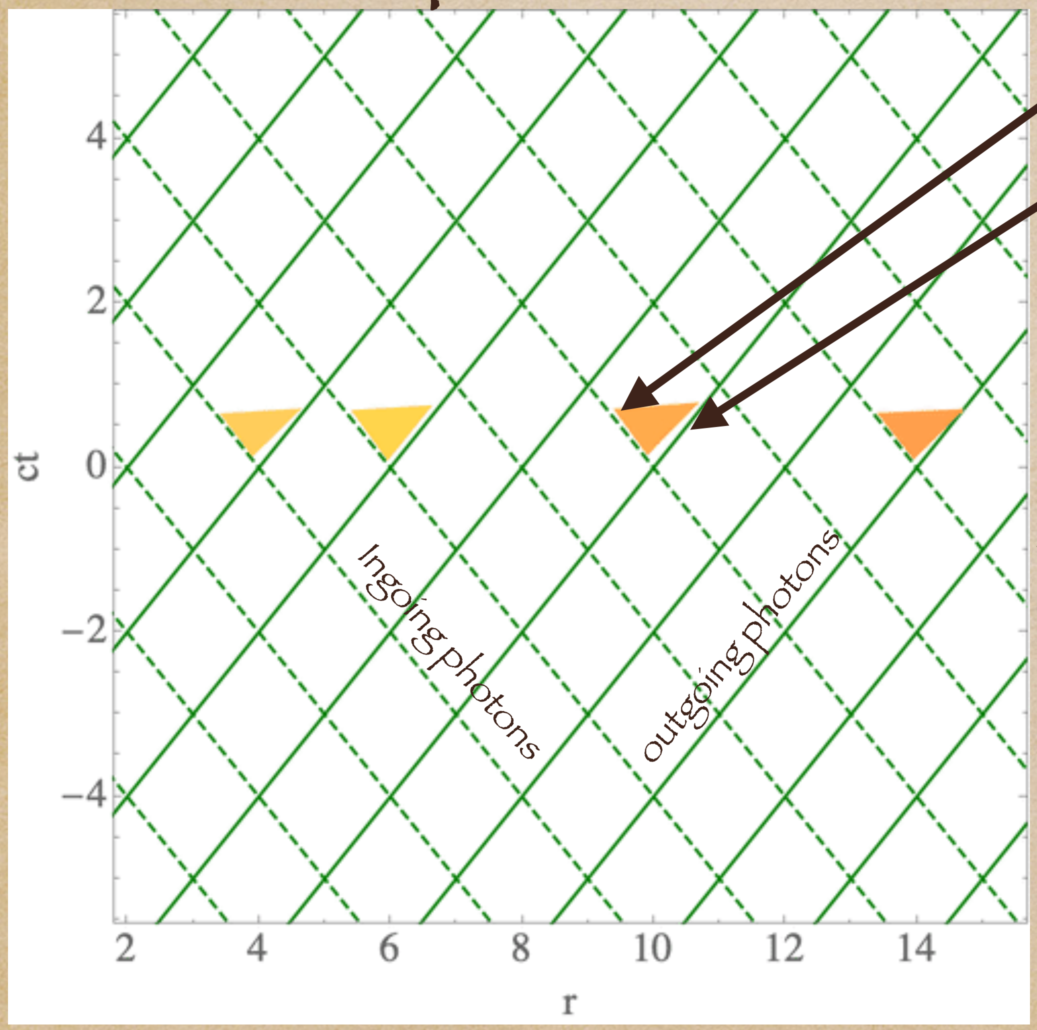


- Ingoing light-rays (dashed)
- Outgoing light-rays (solid)
    - These form the light-cones of the spacetime (orange)
- These light-cones define the causal structure of the spacetime and define the regions of the spacetime that can communicate with each other

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Flat spacetime



- Ingoing light-rays (dashed)
- Outgoing light-rays (solid)
  - These form the light-cones of the spacetime (orange)
- These light-cones define the causal structure of the spacetime and define the regions of the spacetime that can communicate with each other
- For a flat spacetime all regions are available

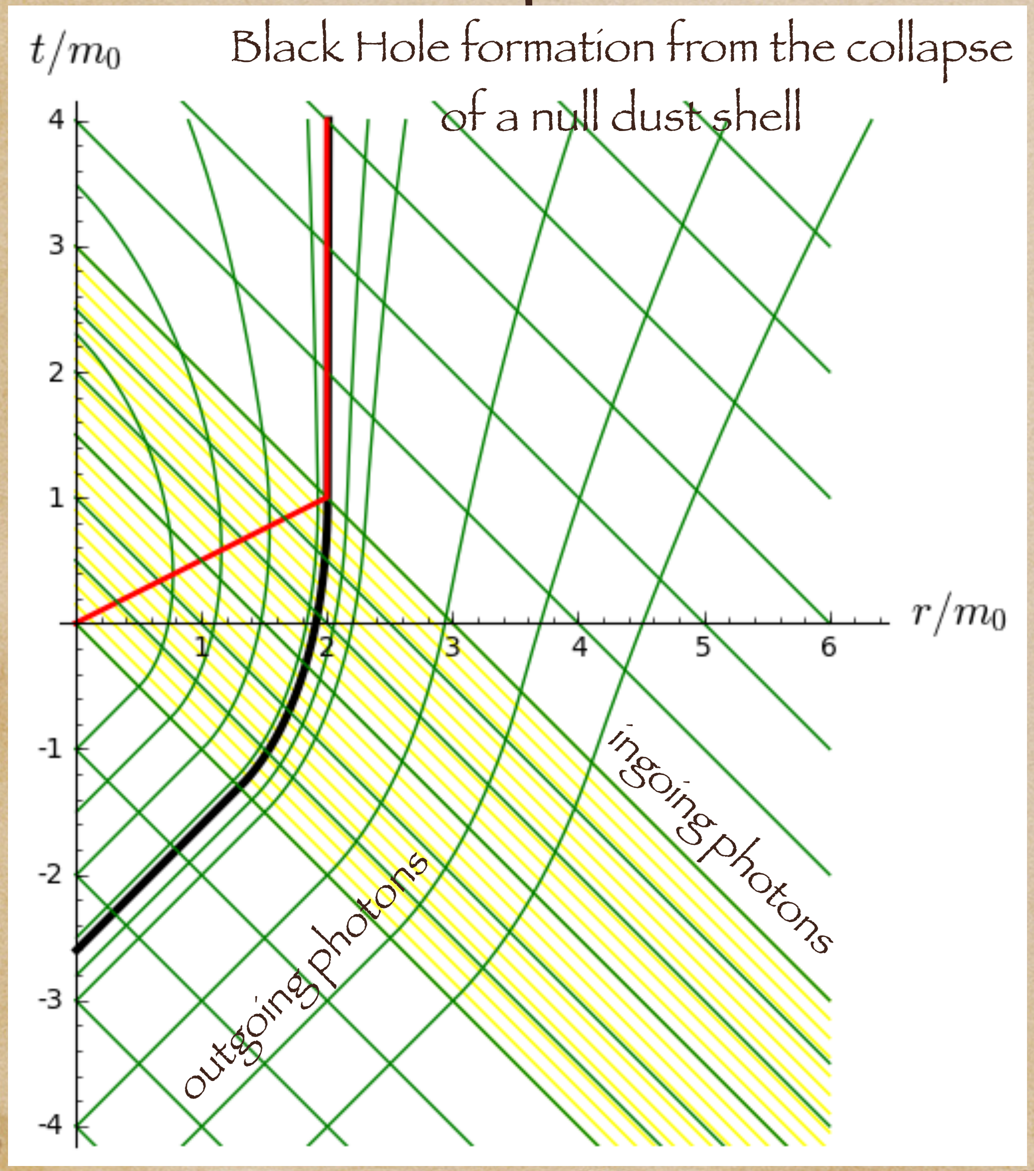
What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

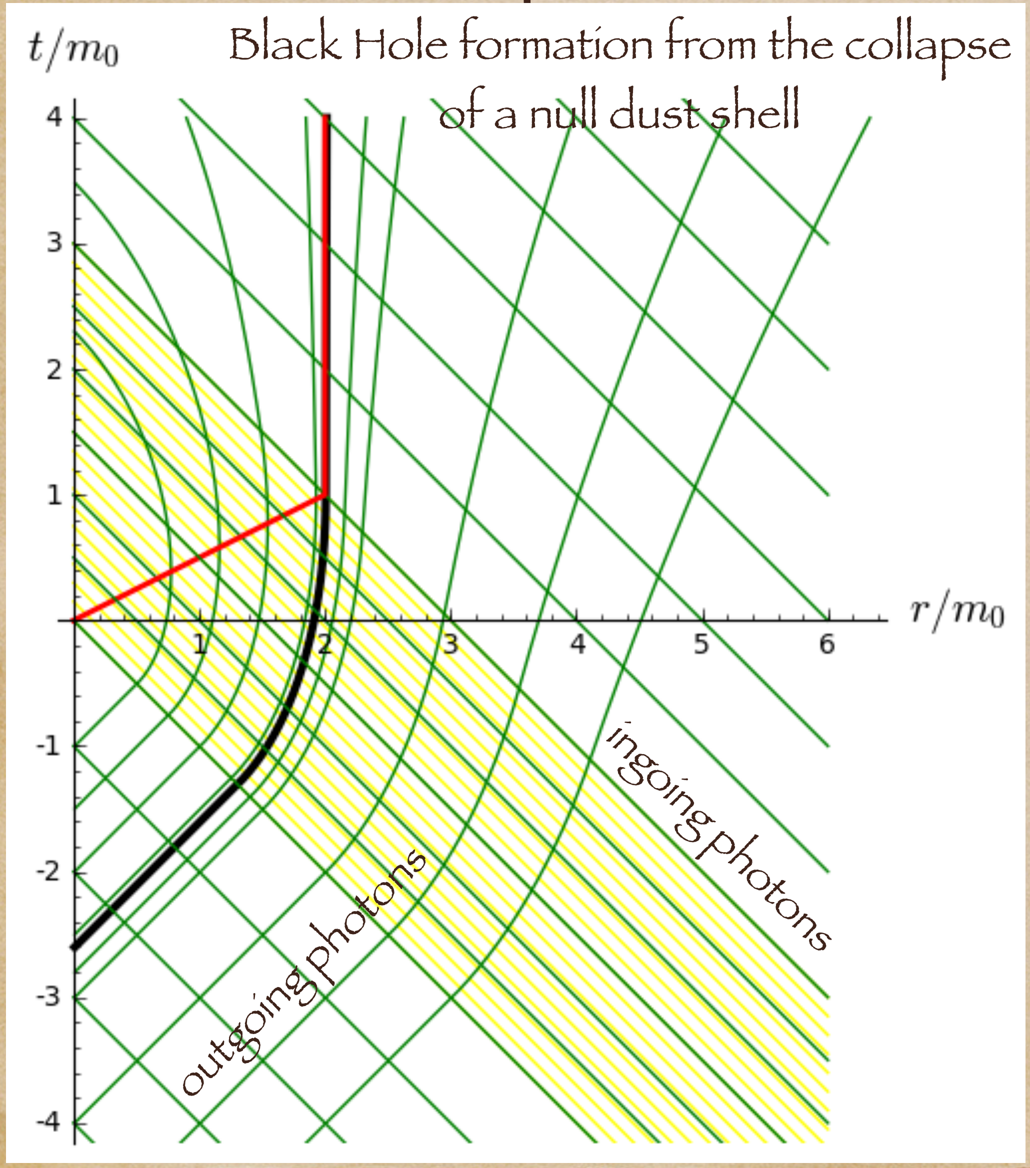
Curved spacetime



What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime

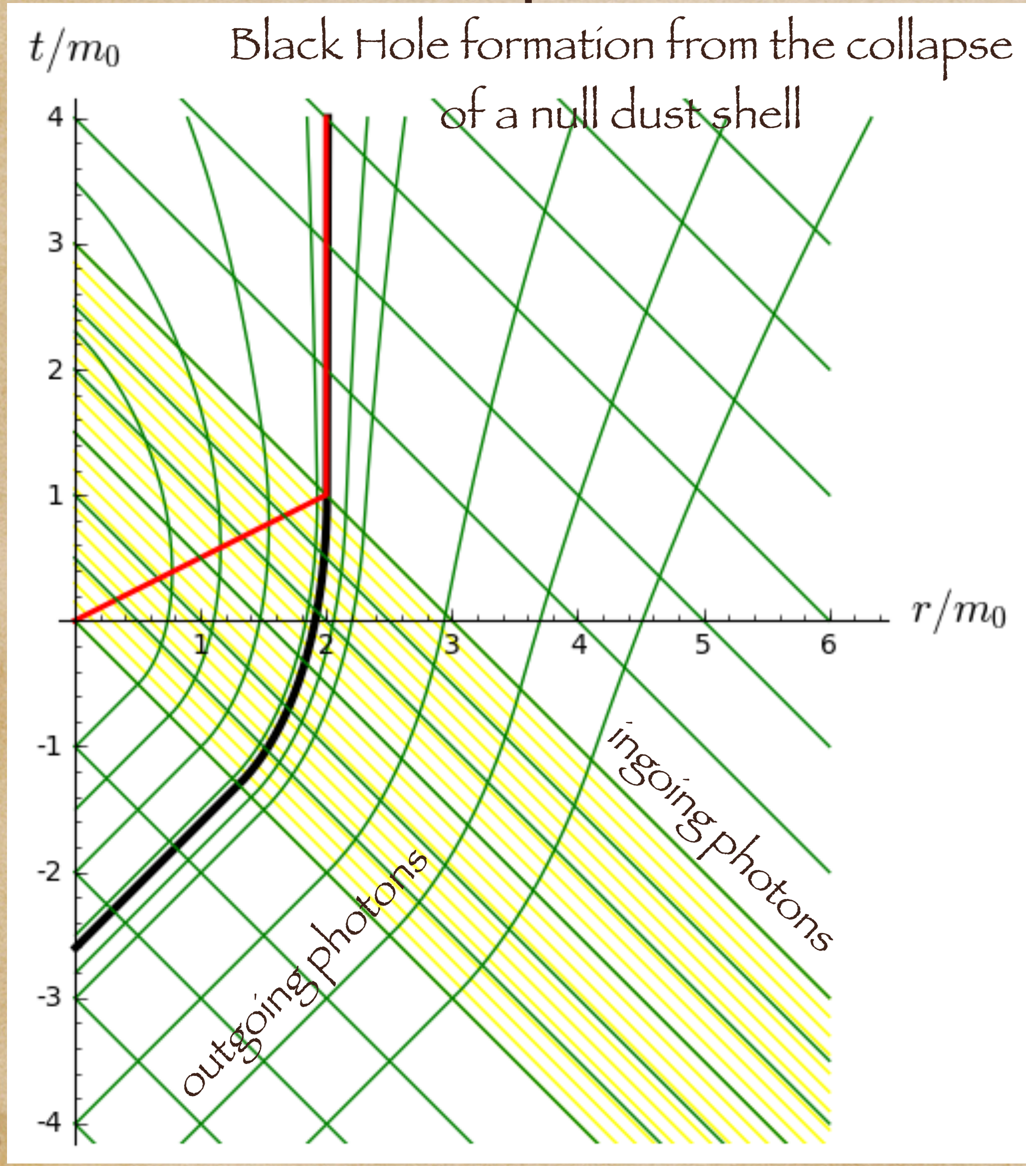


The presence of matter/energy curves spacetime

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



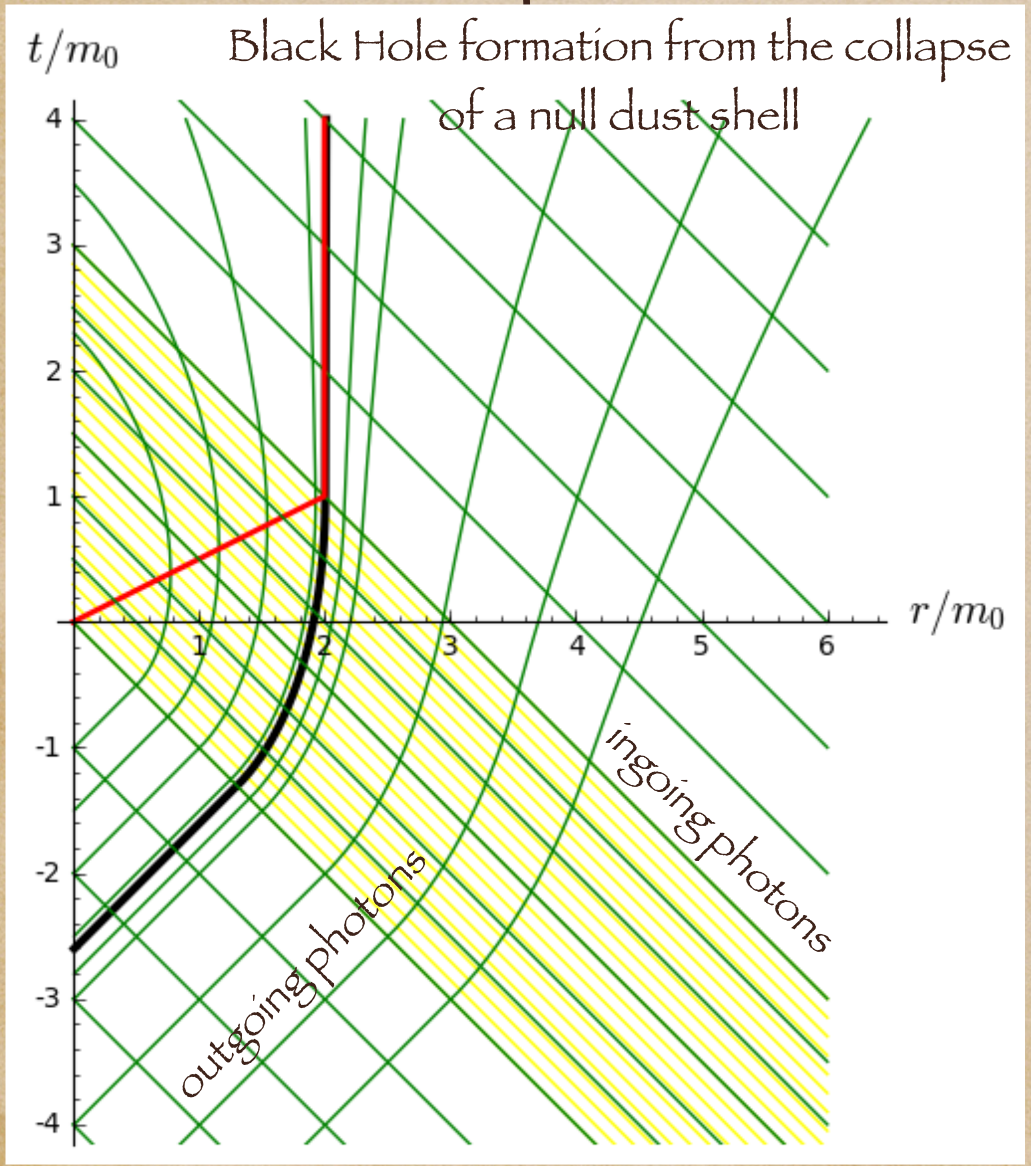
The presence of matter/energy curves spacetime

This results in bending of the light-rays

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy curves spacetime

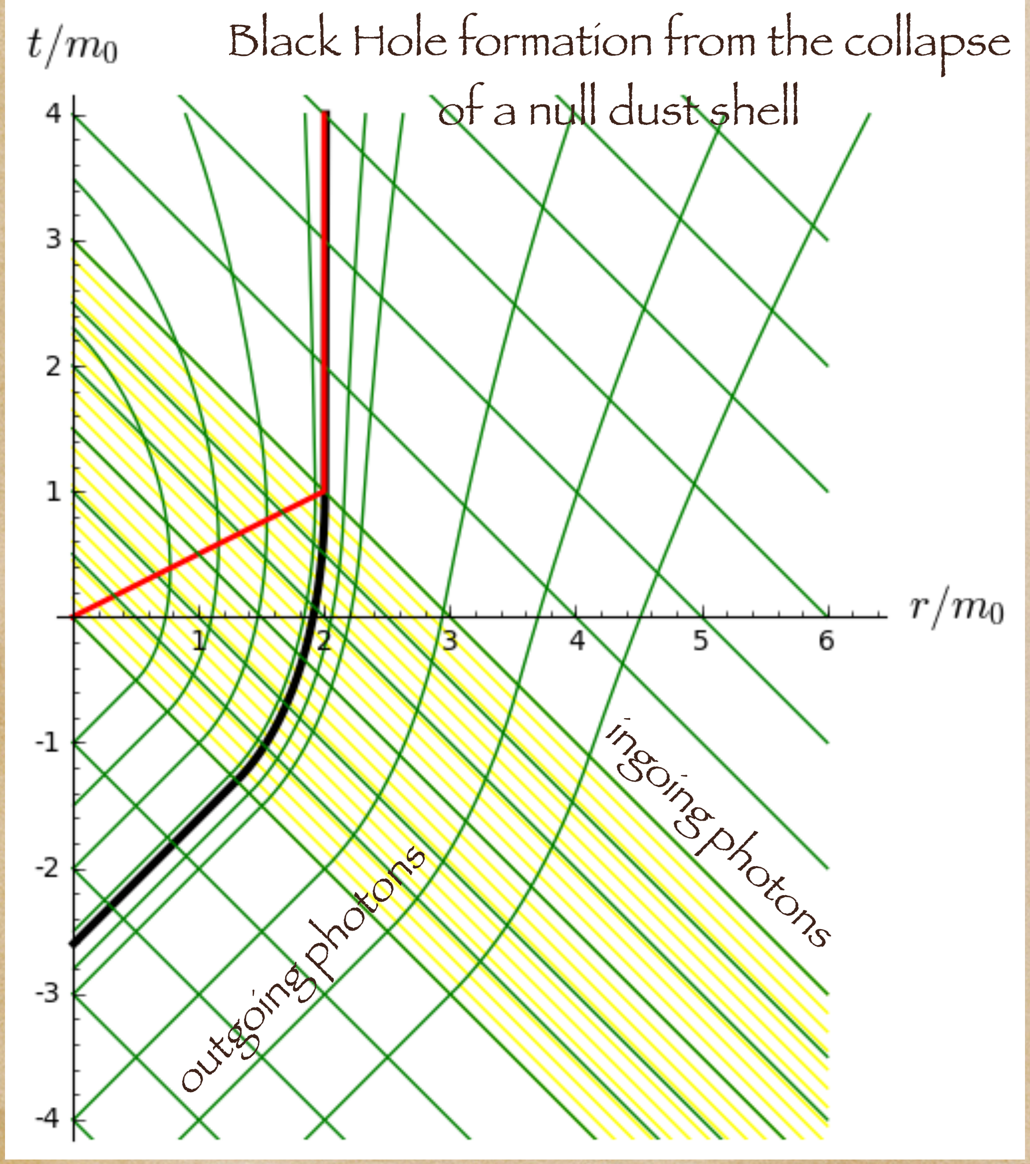
This results in bending of the light-rays

The bending tilts the light-cones and changes the causal structure of the spacetime

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy curves spacetime

This results in bending of the light-rays

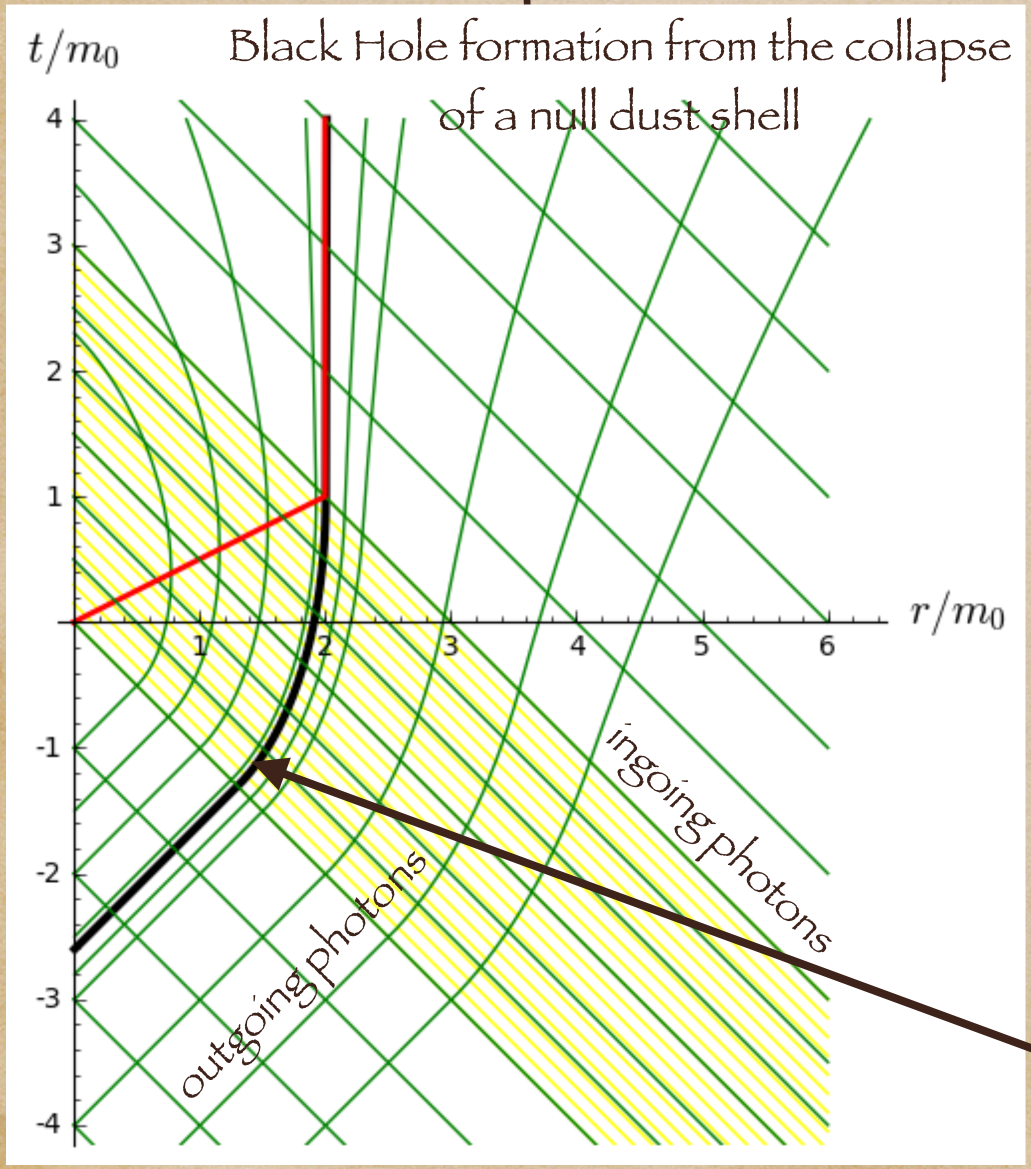
The bending tilts the light-cones and changes the causal structure of the spacetime

The black light-ray defines a boundary between the region where the light-cones can face outwards and the region where they only face inwards

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy curves spacetime

This results in bending of the light-rays

The bending tilts the light-cones and changes the causal structure of the spacetime

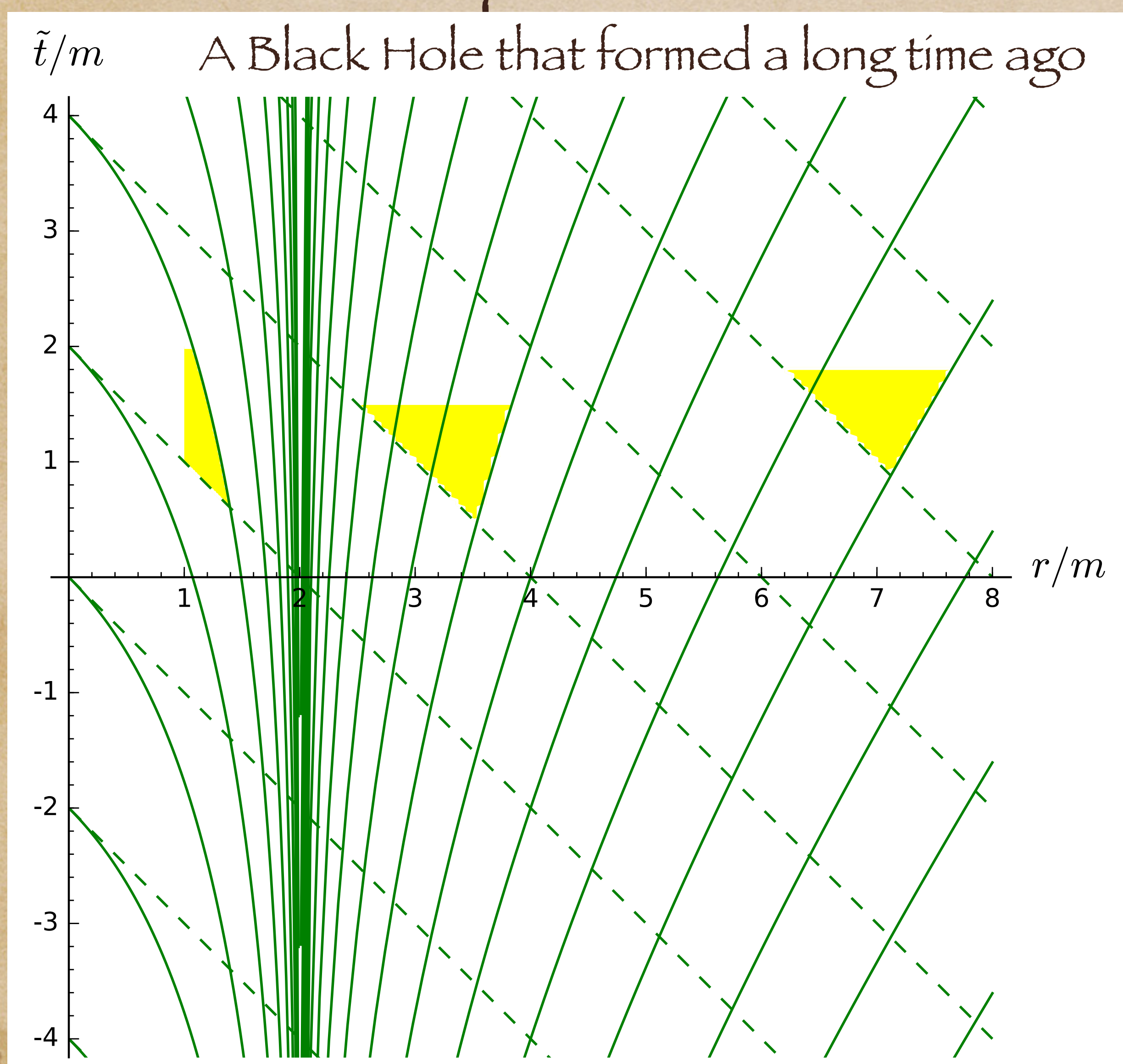
The black light-ray defines a boundary between the region where the light-cones can face outwards and the region where they only face inwards

This is the <u>Horizon</u> of a <u>Black Hole</u>

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy curves spacetime

s results in bending of the light-rays

e bending tilts the light-cones and

anges the causal structure of the spacetime

e black light-ray defines a boundary between the region where

light-cones can face outwards and

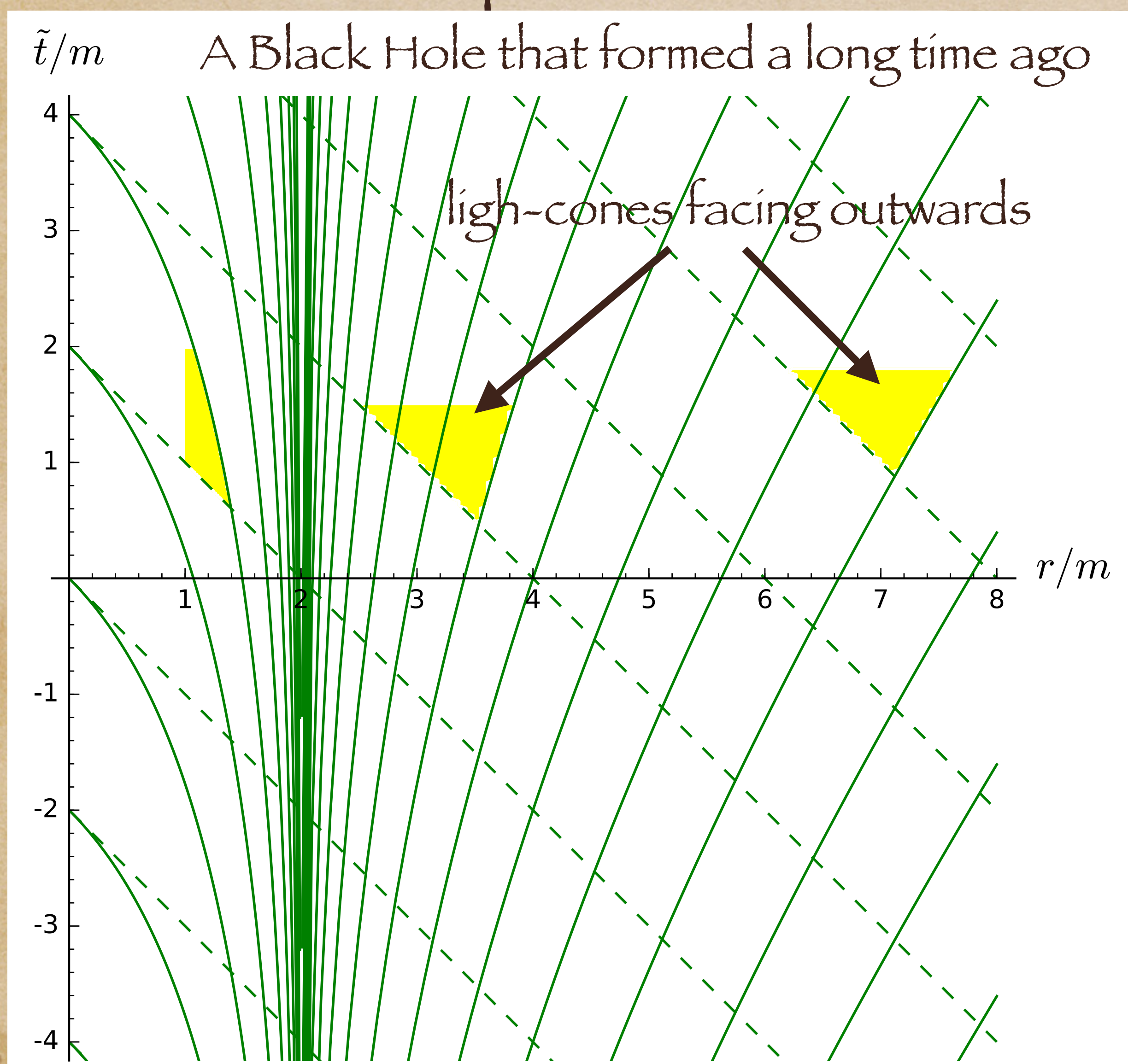
region where they only face inwards

This is the <u>Horizon</u> of a <u>Black Hole</u>

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy curves spacetime

s results in bending of the light-rays

e bending tilts the light-cones and

anges the causal structure of the spacetime

e black light-ray defines a boundary between the region where

light-cones can face outwards and

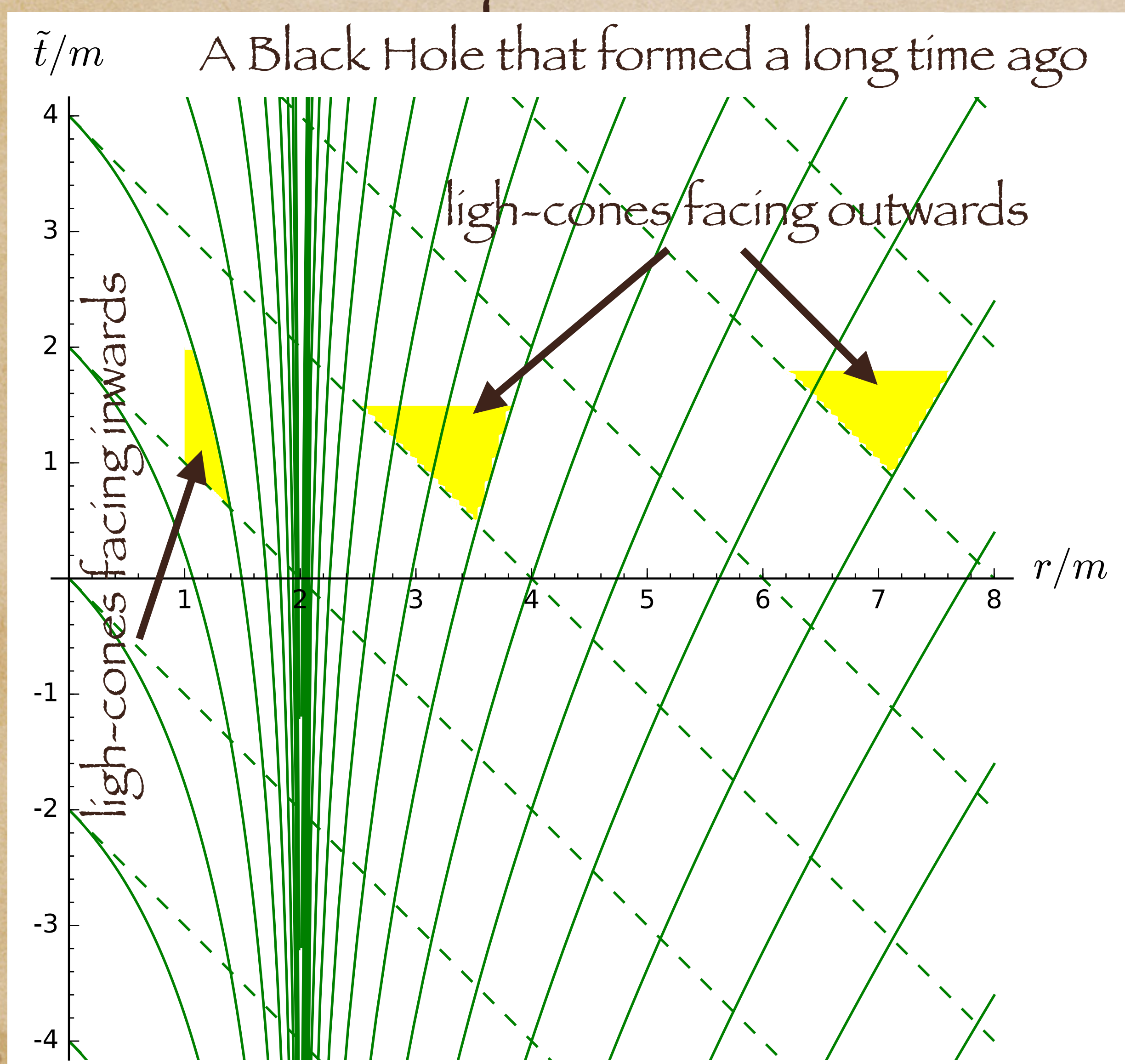
region where they only face inwards

This is the Horizon of a Black Hole

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy
curves spacetime

results in bending of the light-rays
e bending tilts the light-cones and
anges the causal structure of the
spacetime

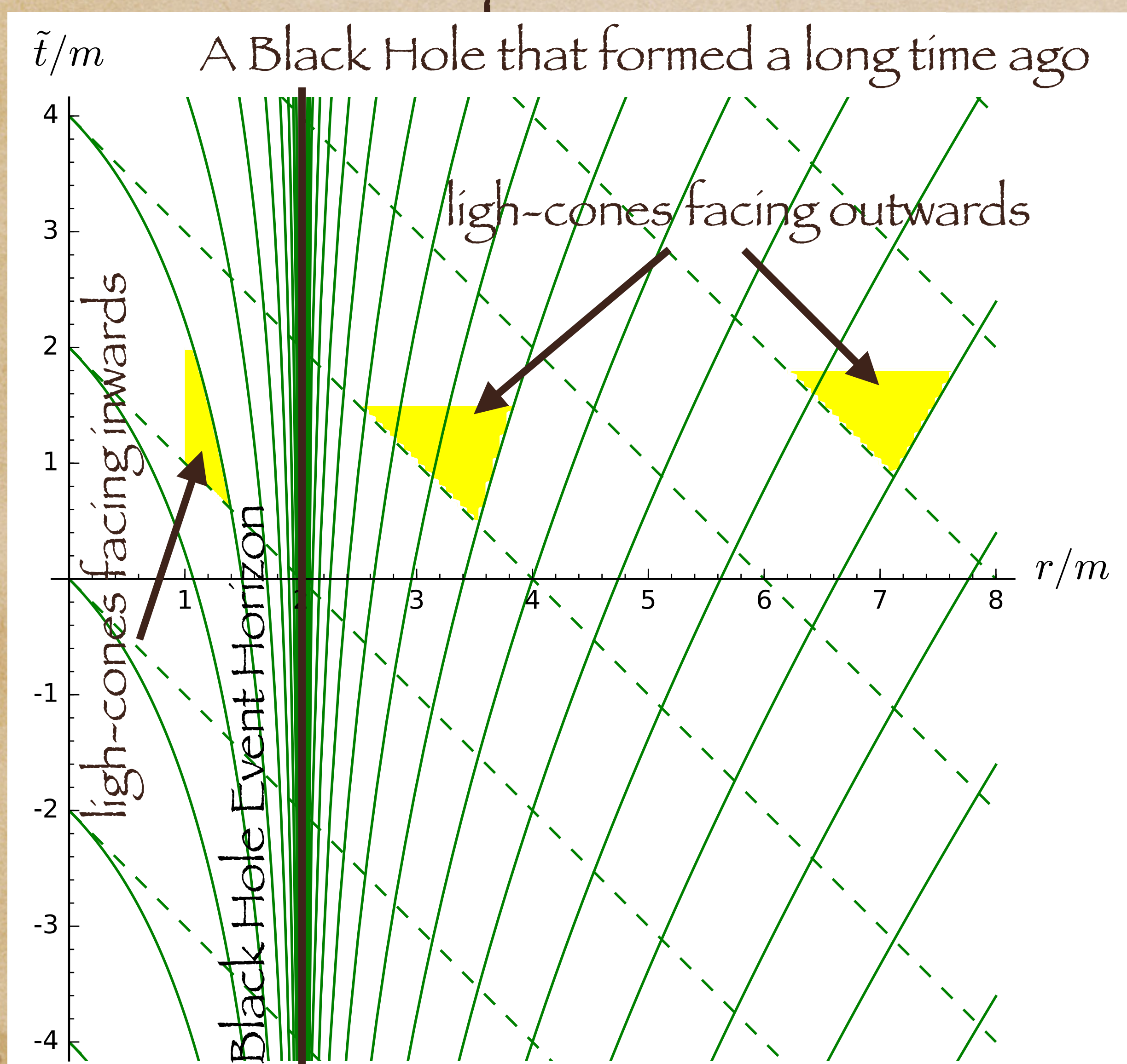
e black light-ray defines a boundary
between the region where
light-cones can face outwards and
region where they only face inwards

This is the <u>Horizon</u> of a <u>Black Hole</u>

What is a Black Hole

All we need to define a Black Hole is the behaviour of light rays (photon trajectories).

Curved spacetime



The presence of matter/energy curves spacetime

results in bending of the light-rays

e bending tilts the light-cones and

anges the causal structure of the spacetime

black light-ray defines a boundary between the region where

light-cones can face outwards and

region where they only face inwards

This is the <u>Horizon</u> of a <u>Black Hole</u>

Photon orbits around Black Holes

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Equations of motion:

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Equations of motion:

- Photons are null particles. One can use this condition to get the equations of motion, i.e., $ds^2 = g_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} = 0$

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Equations of motion:

- Photons are null particles. One can use this condition to get the equations of motion, i.e., $ds^2 = g_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} = 0$
- One could use the geodesics equation $p^a \nabla_a p^b = 0$ or $p^a \nabla_a p_b = 0$

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Equations of motion:

- Photons are null particles. One can use this condition to get the equations of motion, i.e., $ds^2 = g_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} = 0$
- One could use the geodesics equation $p^a \nabla_a p^b = 0$ or $p^a \nabla_a p_b = 0$
- One could alternatively produce the equations of motion from a Lagrangian, defined as $\mathscr{L} = \frac{1}{2} g_{ab} \dot{x}^a \dot{x}^b$

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Equations of motion:

- Photons are null particles. One can use this condition to get the equations of motion, i.e., $ds^2 = g_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} = 0$
- One could use the geodesics equation $p^a \nabla_a p^b = 0$ or $p^a \nabla_a p_b = 0$
- One could alternatively produce the equations of motion from a Lagrangian, defined as $\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b$

  which can then be used to write a Hamiltonian for the photons

  $\mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$ where the momenta are defined as $p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$

Photon orbits around Black Holes

In order to study the properties of light-rays in a given spacetime we need to setup the equations of motion for the photons.

Equations of motion:

- Photons are null particles. One can use this condition to get the equations of motion, i.e., $ds^2 = g_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} = 0$
- One could use the geodesics equation $p^a \nabla_a p^b = 0$ or $p^a \nabla_a p_b = 0$
- One could alternatively produce the equations of motion from a Lagrangian, defined as $\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a \dot{x}^b$

  which can then be used to write a Hamiltonian for the photons

  $\mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$ where the momenta are defined as $p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$

  The problem in this way becomes one of classical mechanics.

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Photon orbits around Black Holes

<u>Equations of motion in an axisymmetric spacetime:</u>

An axisymmetric spacetime looks the same as time passes or if we rotate it with respect to some specific axis. This means it has two <u>symmetries</u>, one w.r.t. <u>time translations</u> and one w.r.t. <u>rotations</u>.

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

An axisymmetric spacetime looks the same as time passes or if we rotate it with respect to some specific axis. This means it has two symmetries, one w.r.t. time translations and one w.r.t. rotations.

These symmetries are associated to two Killing vectors ξ, η

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

An axisymmetric spacetime looks the same as time passes or if we rotate it with respect to some specific axis. This means it has two symmetries, one w.r.t. time translations and one w.r.t. rotations.

These symmetries are associated to two Killing vectors ξ, η

One can choose t, ϕ coordinates that are adjusted to these symmetries. This means that the metric will be $\frac{\partial g_{ab}}{\partial t} = \frac{\partial g_{ab}}{\partial \phi} = 0$, while $\xi^a = \delta^a_t$ and $\eta^a = \delta^a_\phi$.

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

An axisymmetric spacetime looks the same as time passes or if we rotate it with respect to some specific axis. This means it has two symmetries, one w.r.t. time translations and one w.r.t. rotations.

These symmetries are associated to two Killing vectors ξ, η

One can choose t, ϕ coordinates that are adjusted to these symmetries. This means that the metric will be $\frac{\partial g_{ab}}{\partial t} = \frac{\partial g_{ab}}{\partial \phi} = 0$, while $\xi^a = \delta^a_t$ and $\eta^a = \delta^a_\phi$.

In addition, there are conserved quantities associated to these symmetries. Particles have a conserved energy $E = -\xi^a p_a = -p_t$ and a conserved angular momentum $L_z = \eta^a p_a = p_\phi$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r, \theta)$ where the coordinates are (t, ϕ, r, θ)

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$$ where we remind that $p_a = \dfrac{\partial \mathscr{L}}{\partial \dot{x}^a}$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a\dot{x}^a - \mathscr{L}$ where we remind that $p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$

The Hamiltonian therefore is $\mathscr{H} = -E\dot{t} + L_z\dot{\phi} + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 - \mathscr{L}$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$$ where we remind that $p_a = \dfrac{\partial \mathscr{L}}{\partial \dot{x}^a}$

The Hamiltonian therefore is $\mathscr{H} = -E\dot{t} + L_z\dot{\phi} + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 - \mathscr{L}$

where $E = -p_t, \quad L_z = p_\phi, \quad p_r = g_{rr}\dot{r}, \quad p_\theta = g_{\theta\theta}\dot{\theta}$.

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$$ where we remind that $$p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$$

The Hamiltonian therefore is $$\mathscr{H} = -E\dot{t} + L_z\dot{\phi} + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 - \mathscr{L}$$

where $E = -p_t,\quad L_z = p_\phi,\quad p_r = g_{rr}\dot{r},\quad p_\theta = g_{\theta\theta}\dot{\theta}$.

The Hamiltonian can finally become $$\mathscr{H} = \frac{1}{2}\left(g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}\right)$$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a\dot{x}^a - \mathscr{L}$ where we remind that $p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$

The Hamiltonian therefore is $\mathscr{H} = -E\dot{t} + L_z\dot{\phi} + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 - \mathscr{L}$

where $E = -p_t, \quad L_z = p_\phi, \quad p_r = g_{rr}\dot{r}, \quad p_\theta = g_{\theta\theta}\dot{\theta}$.

The Hamiltonian can finally become $\mathscr{H} = \frac{1}{2}\left(g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}\right)$

where $V_{\text{eff}} = -\frac{g_{\phi\phi} + 2bg_{t\phi} + b^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}$ and $b = \frac{L_z}{E}$ is an impact parameter.

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$$ where we remind that $p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$

The Hamiltonian therefore is $\mathscr{H} = -E\dot{t} + L_z\dot{\phi} + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 - \mathscr{L}$

where $E = -p_t, \quad L_z = p_\phi, \quad p_r = g_{rr}\dot{r}, \quad p_\theta = g_{\theta\theta}\dot{\theta}$.

The Hamiltonian can finally become $\mathscr{H} = \frac{1}{2}\left(g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}\right)$

where $V_{\text{eff}} = -\frac{g_{\phi\phi} + 2bg_{t\phi} + b^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}$ and $b = \frac{L_z}{E}$ is an impact parameter.

For photons, since $g_{ab}\dot{x}^a\dot{x}^b = 0$, we have that $\mathscr{H} = 0$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Returning to the formulation of the equations of motion for an axisymmetric spacetime, we have the metric $g_{ab}(r,\theta)$ where the coordinates are (t,ϕ,r,θ)

$$\mathscr{L} = \frac{1}{2} g_{ab}\dot{x}^a\dot{x}^b \Rightarrow \mathscr{H} = \sum p_a \dot{x}^a - \mathscr{L}$$ where we remind that $$p_a = \frac{\partial \mathscr{L}}{\partial \dot{x}^a}$$

The Hamiltonian therefore is $$\mathscr{H} = -E\dot{t} + L_z\dot{\phi} + g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 - \mathscr{L}$$

where $E = -p_t, \quad L_z = p_\phi, \quad p_r = g_{rr}\dot{r}, \quad p_\theta = g_{\theta\theta}\dot{\theta}$.

The Hamiltonian can finally become $$\mathscr{H} = \frac{1}{2}\left(g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}\right)$$

where $$V_{\text{eff}} = -\frac{g_{\phi\phi} + 2bg_{t\phi} + b^2 g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}$$ and $b = \frac{L_z}{E}$ is an impact parameter.

For photons, since $g_{ab}\dot{x}^a\dot{x}^b = 0$, we have that $\mathscr{H} = 0$

while motion is allowed only where $V_{\text{eff}} \leq 0$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Therefore for the metric $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Therefore for the metric $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2$

we have $$\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\rm eff}(r,\theta)\right] = \frac{1}{2}\left[\frac{p_r^2}{g_{rr}} + \frac{p_\theta^2}{g_{\theta\theta}} + V_{\rm eff}(r,\theta)\right]$$

Photon orbits around Black Holes

<u>Equations of motion in an axisymmetric spacetime:</u>

Therefore for the metric $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2$

we have $$\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}(r,\theta)\right] = \frac{1}{2}\left[\frac{p_r^2}{g_{rr}} + \frac{p_\theta^2}{g_{\theta\theta}} + V_{\text{eff}}(r,\theta)\right]$$

$$E = -\left(g_{tt}\frac{dt}{d\lambda} + g_{t\phi}\frac{d\phi}{d\lambda}\right), \quad L_z = g_{t\phi}\frac{dt}{d\lambda} + g_{\phi\phi}\frac{d\phi}{d\lambda}, \quad \mathscr{H} = 0, \quad b = L_z/E$$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Therefore for the metric $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2$

we have $$\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}(r,\theta)\right] = \frac{1}{2}\left[\frac{p_r^2}{g_{rr}} + \frac{p_\theta^2}{g_{\theta\theta}} + V_{\text{eff}}(r,\theta)\right]$$

$$E = -\left(g_{tt}\frac{dt}{d\lambda} + g_{t\phi}\frac{d\phi}{d\lambda}\right), \quad L_z = g_{t\phi}\frac{dt}{d\lambda} + g_{\phi\phi}\frac{d\phi}{d\lambda}, \quad \mathscr{H} = 0, \quad b = L_z/E$$

and from Hamilton's canonical equations $\dot{x}^a = \dfrac{\partial \mathscr{H}}{\partial p_a}, \quad \dot{p}_a = -\dfrac{\partial \mathscr{H}}{\partial x^a},$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Therefore for the metric $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2$

we have $$\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}(r,\theta)\right] = \frac{1}{2}\left[\frac{p_r^2}{g_{rr}} + \frac{p_\theta^2}{g_{\theta\theta}} + V_{\text{eff}}(r,\theta)\right]$$

$$E = -\left(g_{tt}\frac{dt}{d\lambda} + g_{t\phi}\frac{d\phi}{d\lambda}\right), \quad L_z = g_{t\phi}\frac{dt}{d\lambda} + g_{\phi\phi}\frac{d\phi}{d\lambda}, \quad \mathscr{H} = 0, \quad b = L_z/E$$

and from Hamilton's canonical equations $\dot{x}^a = \dfrac{\partial \mathscr{H}}{\partial p_a}, \quad \dot{p}_a = -\dfrac{\partial \mathscr{H}}{\partial x^a},$

$$\dot{r} = \frac{p_r}{g_{rr}}, \qquad \dot{p}_r = -\frac{\partial \mathscr{H}}{\partial r}, \quad \dot{t} = \frac{Eg_{\phi\phi} + Lg_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_t = 0,$$

$$\dot{\theta} = \frac{p_\theta}{g_{\theta\theta}}, \qquad \dot{p}_\theta = -\frac{\partial \mathscr{H}}{\partial \theta}, \quad \dot{\phi} = -\frac{Lg_{tt} + Eg_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_\phi = 0.$$

Photon orbits around Black Holes

Equations of motion in an axisymmetric spacetime:

Therefore for the metric $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2$

we have $\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\rm eff}(r,\theta)\right] = \frac{1}{2}\left[\frac{p_r^2}{g_{rr}} + \frac{p_\theta^2}{g_{\theta\theta}} + V_{\rm eff}(r,\theta)\right]$

$$E = -\left(g_{tt}\frac{dt}{d\lambda} + g_{t\phi}\frac{d\phi}{d\lambda}\right), \quad L_z = g_{t\phi}\frac{dt}{d\lambda} + g_{\phi\phi}\frac{d\phi}{d\lambda}, \quad \mathscr{H} = 0, \quad b = L_z/E$$

and from Hamilton's canonical equations $\dot{x}^a = \frac{\partial \mathscr{H}}{\partial p_a}, \quad \dot{p}_a = -\frac{\partial \mathscr{H}}{\partial x^a},$

$$\dot{r} = \frac{p_r}{g_{rr}}, \qquad \dot{p}_r = -\frac{\partial \mathscr{H}}{\partial r}, \quad \dot{t} = \frac{E g_{\phi\phi} + L g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_t = 0,$$

$$\dot{\theta} = \frac{p_\theta}{g_{\theta\theta}}, \qquad \dot{p}_\theta = -\frac{\partial \mathscr{H}}{\partial \theta}, \quad \dot{\phi} = -\frac{L g_{tt} + E g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_\phi = 0.$$

These are supplemented by appropriate initial conditions ($\mathscr{H} = 0$).

Photon orbits around Black Holes

Photon orbits in an axisymmetric spacetime:

The equations of motion seem complicated, but things are simpler than they look at first glance.

$$\dot{r} = \frac{p_r}{g_{rr}}, \qquad \dot{p}_r = -\frac{\partial \mathscr{H}}{\partial r}, \quad \dot{t} = \frac{E g_{\phi\phi} + L g_{t\phi}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}, \quad \dot{p}_t = 0,$$

$$\dot{\theta} = \frac{p_\theta}{g_{\theta\theta}}, \qquad \dot{p}_\theta = -\frac{\partial \mathscr{H}}{\partial \theta}, \quad \dot{\phi} = -\frac{L g_{tt} + E g_{t\phi}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}, \quad \dot{p}_\phi = 0.$$

Photon orbits around Black Holes

Photon orbits in an axisymmetric spacetime:

The equations of motion seem complicated, but things are simpler than they look at first glance. Looking at the Hamiltonian, everything is a function of only r, θ

$$\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}(r,\theta)\right]$$

$$\dot{r} = \frac{p_r}{g_{rr}}, \qquad \dot{p}_r = -\frac{\partial \mathscr{H}}{\partial r}, \quad \dot{t} = \frac{E g_{\phi\phi} + L g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_t = 0,$$

$$\dot{\theta} = \frac{p_\theta}{g_{\theta\theta}}, \qquad \dot{p}_\theta = -\frac{\partial \mathscr{H}}{\partial \theta}, \quad \dot{\phi} = -\frac{L g_{tt} + E g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_\phi = 0.$$

Photon orbits around Black Holes

Photon orbits in an axisymmetric spacetime:

The equations of motion seem complicated, but things are simpler than they look at first glance. Looking at the Hamiltonian,

$$\mathcal{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\text{eff}}(r,\theta)\right]$$

everything is a function of only r, θ in addition to having two constants of motion. This allows us to split the system in two "independent" parts, the r, θ part and the t, ϕ part.

$$\dot{r} = \frac{p_r}{g_{rr}}, \qquad \dot{p}_r = -\frac{\partial \mathcal{H}}{\partial r},$$

$$\dot{\theta} = \frac{p_\theta}{g_{\theta\theta}}, \qquad \dot{p}_\theta = -\frac{\partial \mathcal{H}}{\partial \theta},$$

$$\dot{t} = \frac{E g_{\phi\phi} + L g_{t\phi}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}, \quad \dot{p}_t = 0,$$

$$\dot{\phi} = -\frac{L g_{tt} + E g_{t\phi}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}, \quad \dot{p}_\phi = 0.$$

Photon orbits around Black Holes

<u>Photon orbits in an axisymmetric spacetime:</u>

The equations of motion seem complicated, but things are simpler than they look at first glance. Looking at the Hamiltonian,

$$\mathscr{H} = \frac{1}{2}\left[g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 + V_{\rm eff}(r,\theta)\right]$$

everything is a function of only r, θ in addition to having two constants of motion. This allows us to split the system in two "independent" parts, the r, θ part and the t, ϕ part.

$$\dot{r} = \frac{p_r}{g_{rr}}, \qquad \dot{p}_r = -\frac{\partial \mathscr{H}}{\partial r},$$

$$\dot{\theta} = \frac{p_\theta}{g_{\theta\theta}}, \qquad \dot{p}_\theta = -\frac{\partial \mathscr{H}}{\partial \theta},$$

$$\dot{t} = \frac{E g_{\phi\phi} + L g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_t = 0,$$

$$\dot{\phi} = -\frac{L g_{tt} + E g_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad \dot{p}_\phi = 0.$$

The r, θ motion can also be studied with the help of the $V_{\rm eff}(r,\theta)$.

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

In the Schwarzschild case,
due to spherical symmetry,
we can work on the equatorial
plane without loss of generality.
The problem is practically 1D

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

In the Schwarzschild case, due to spherical symmetry, we can work on the equatorial plane without loss of generality. The problem is practically 1D

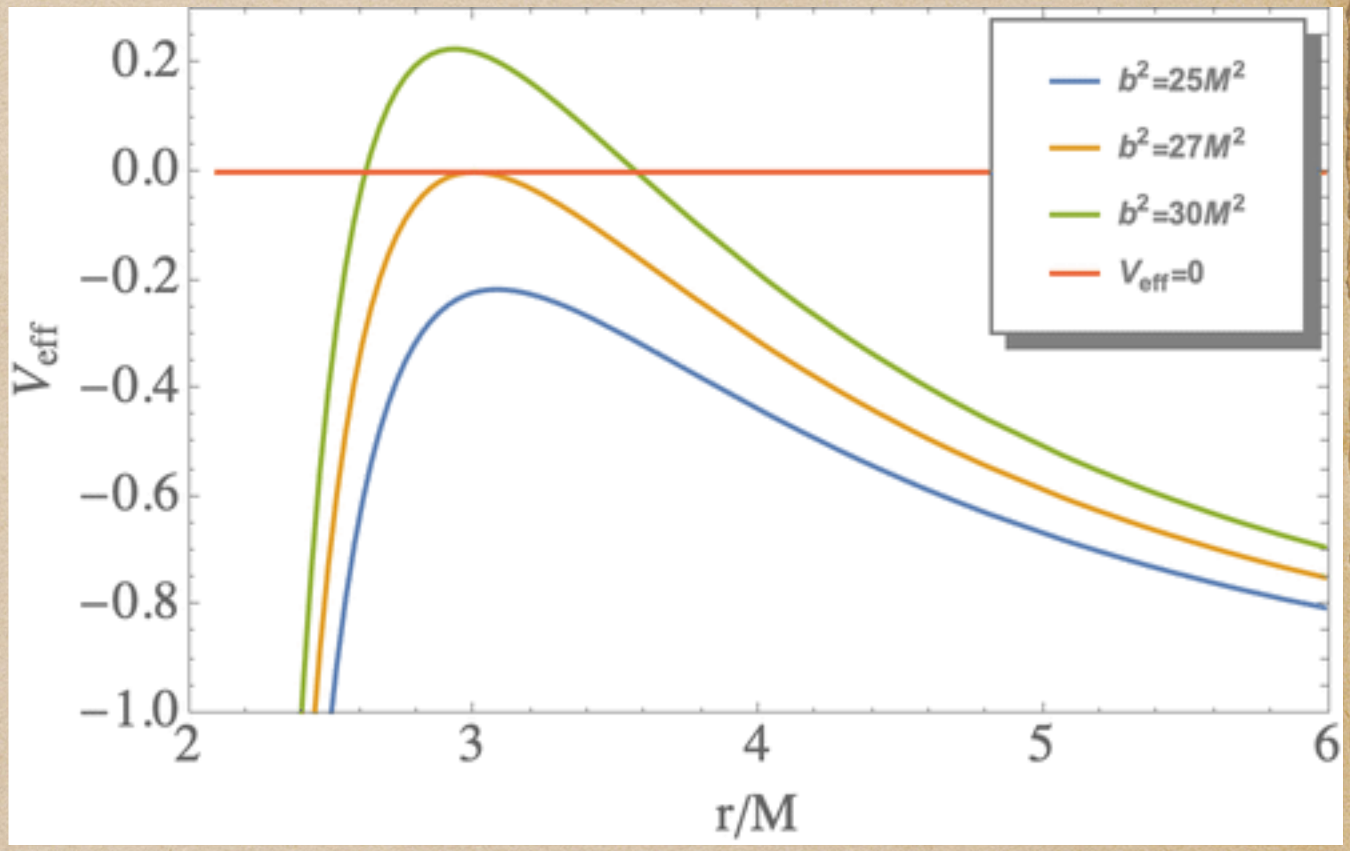
$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

In the Schwarzschild case, due to spherical symmetry, we can work on the equatorial plane without loss of generality. The problem is practically 1D

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$

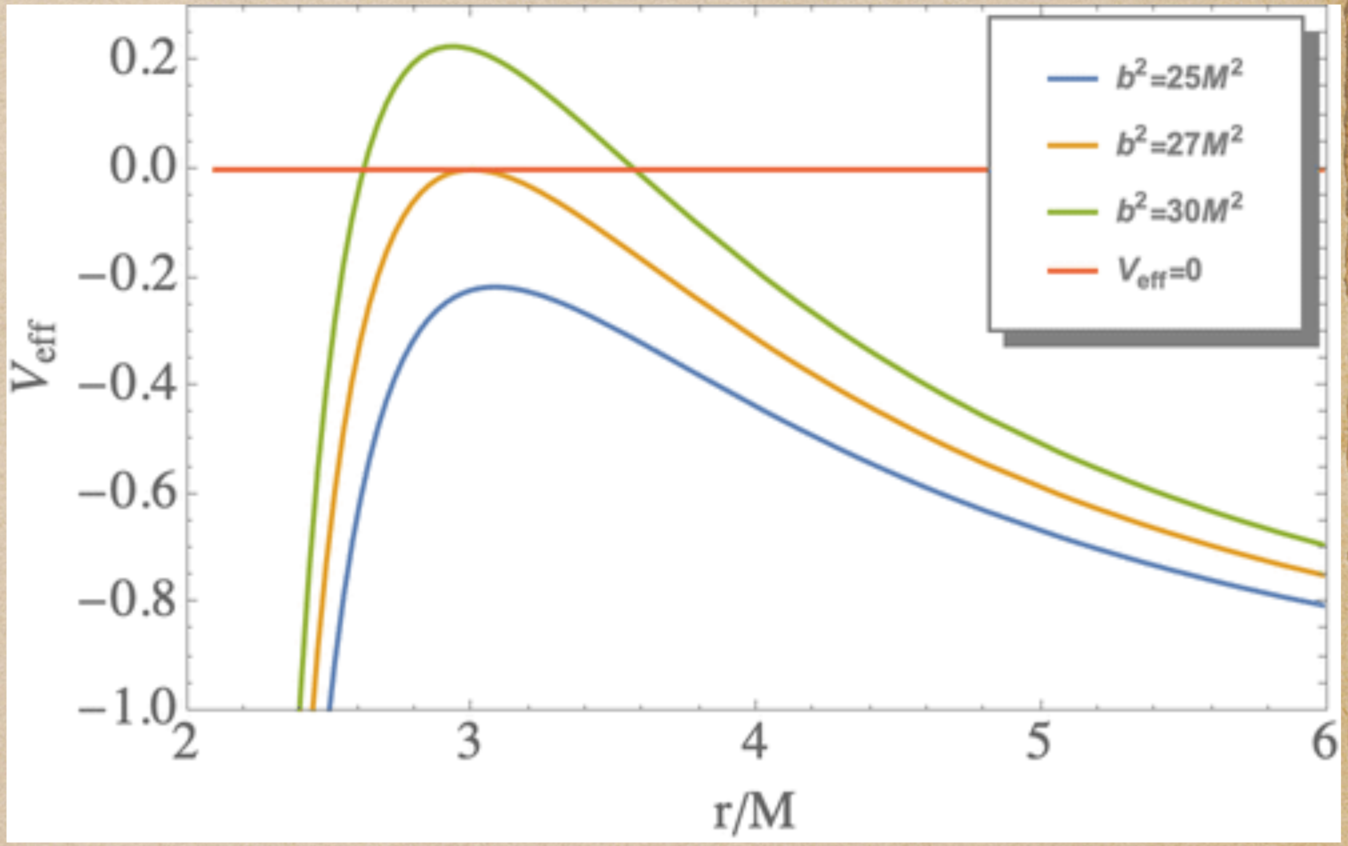


Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

In the Schwarzschild case, due to spherical symmetry, we can work on the equatorial plane without loss of generality. The problem is practically 1D

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$



The figure shows $V_{\rm eff}$ for 3 values of b. The allowed region is for $V_{\rm eff} \leq 0$.

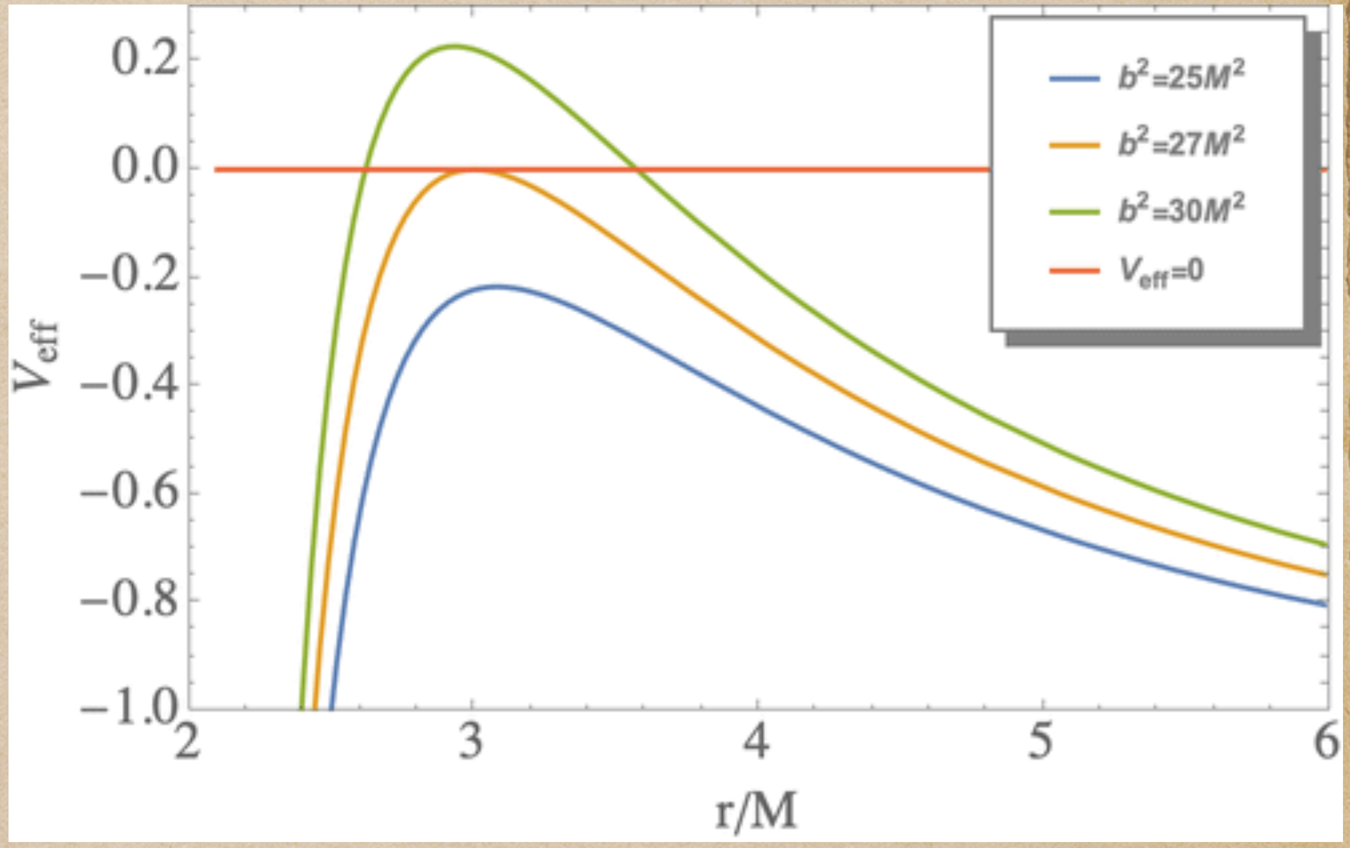
Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$

In the Schwarzschild case, due to spherical symmetry, we can work on the equatorial plane without loss of generality. The problem is practically 1D

The figure shows $V_{\rm eff}$ for 3 values of b. The allowed region is for $V_{\rm eff} \leq 0$.



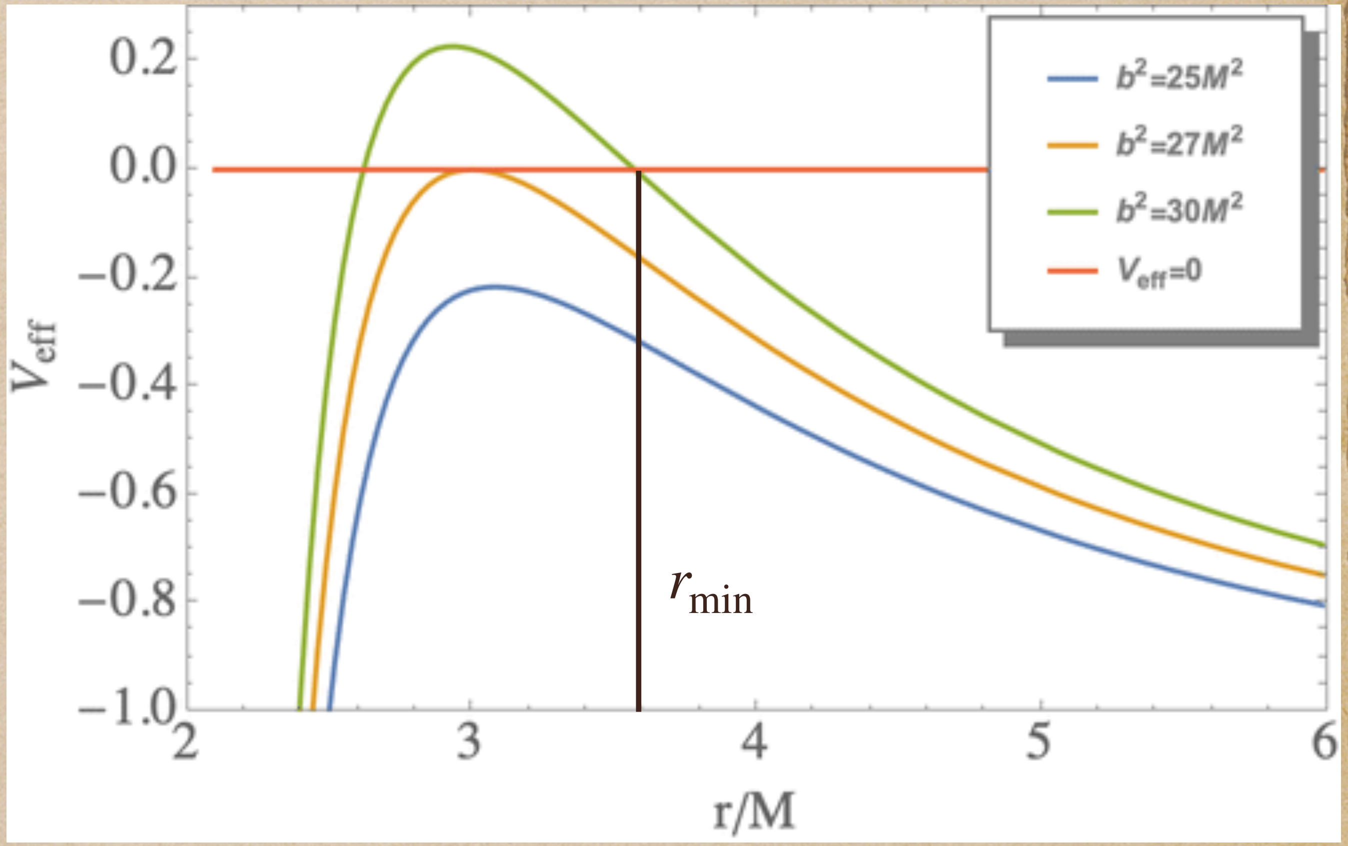
If the impact parameter is $b < 3\sqrt{3}M$ the photon falls into the Black Hole.

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

In the Schwarzschild case, due to spherical symmetry, we can work on the equatorial plane without loss of generality. The problem is practically 1D

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$



The figure shows $V_{\rm eff}$ for 3 values of b. The allowed region is for $V_{\rm eff} \leq 0$.

If the impact parameter is $b < 3\sqrt{3}M$ the photon falls into the Black Hole. If $b > 3\sqrt{3}M$ the photon is scattered getting down to a minimum radius $r_{\rm min}$.

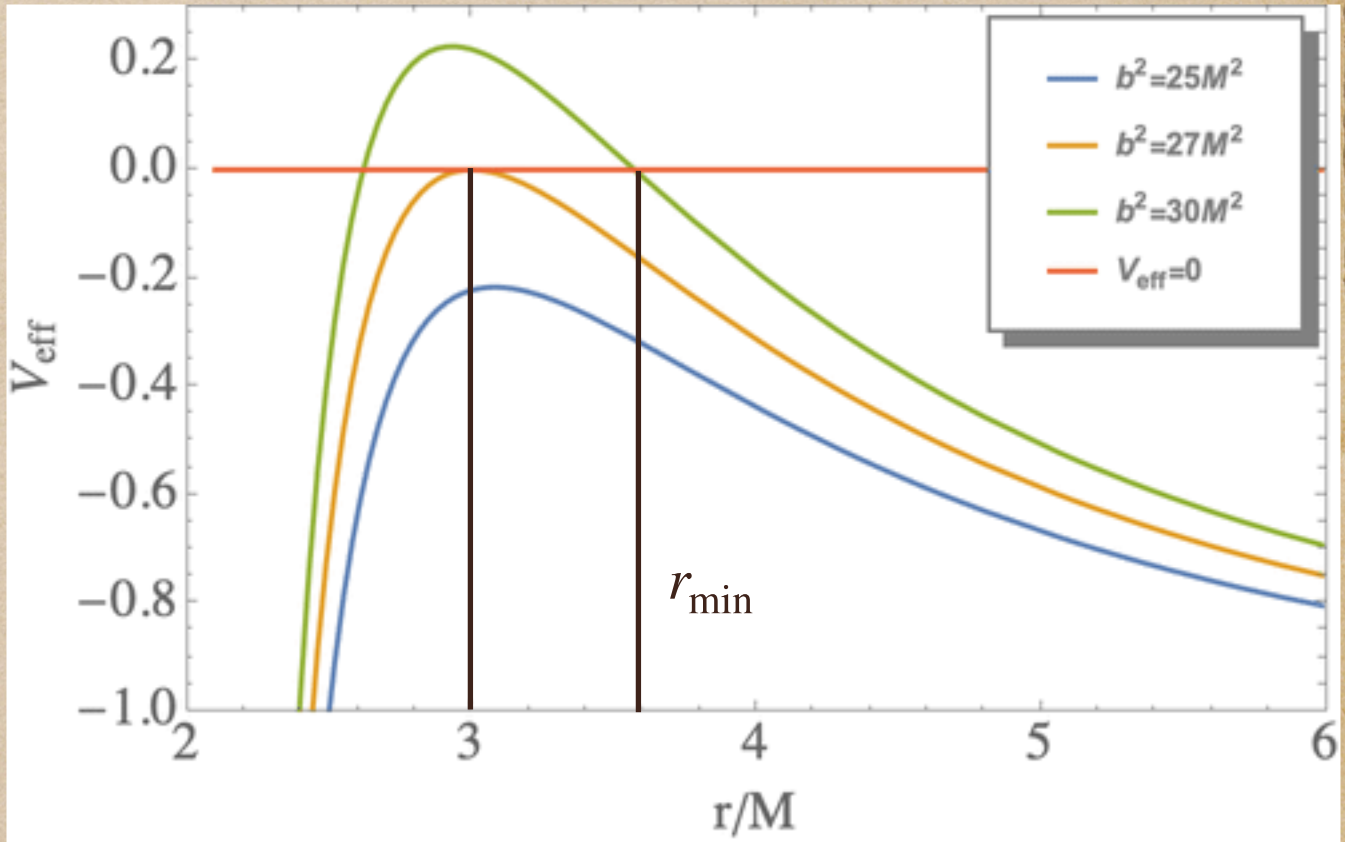
Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

$$V_{\text{eff}} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$

In the Schwarzschild case, due to spherical symmetry, we can work on the equatorial plane without loss of generality. The problem is practically 1D

The figure shows V_{eff} for 3 values of b. The allowed region is for $V_{\text{eff}} \leq 0$.



If the impact parameter is $b < 3\sqrt{3}M$ the photon falls into the Black Hole. If $b > 3\sqrt{3}M$ the photon is scattered getting down to a minimum radius r_{min}.

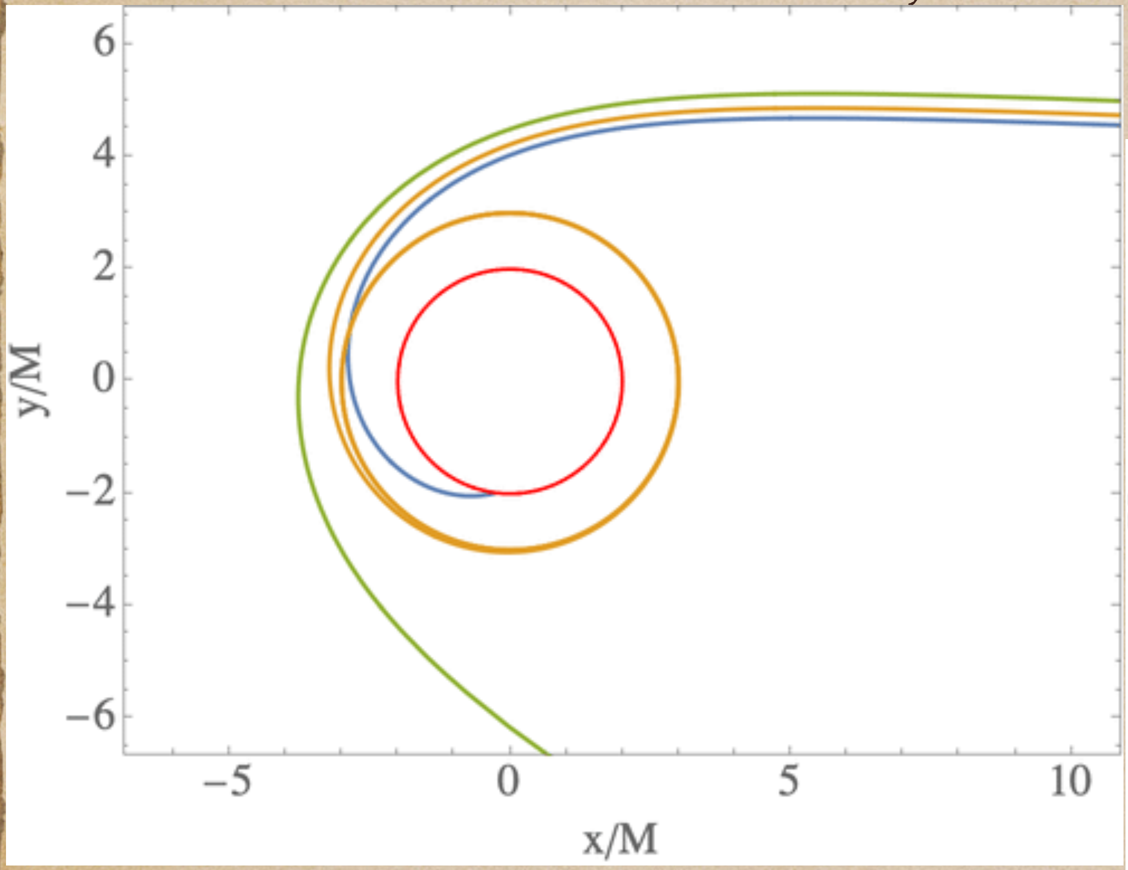
If $b = 3\sqrt{3}M$ then the photon reaches $r_{\text{min}} = 3M$, the photon orbit.

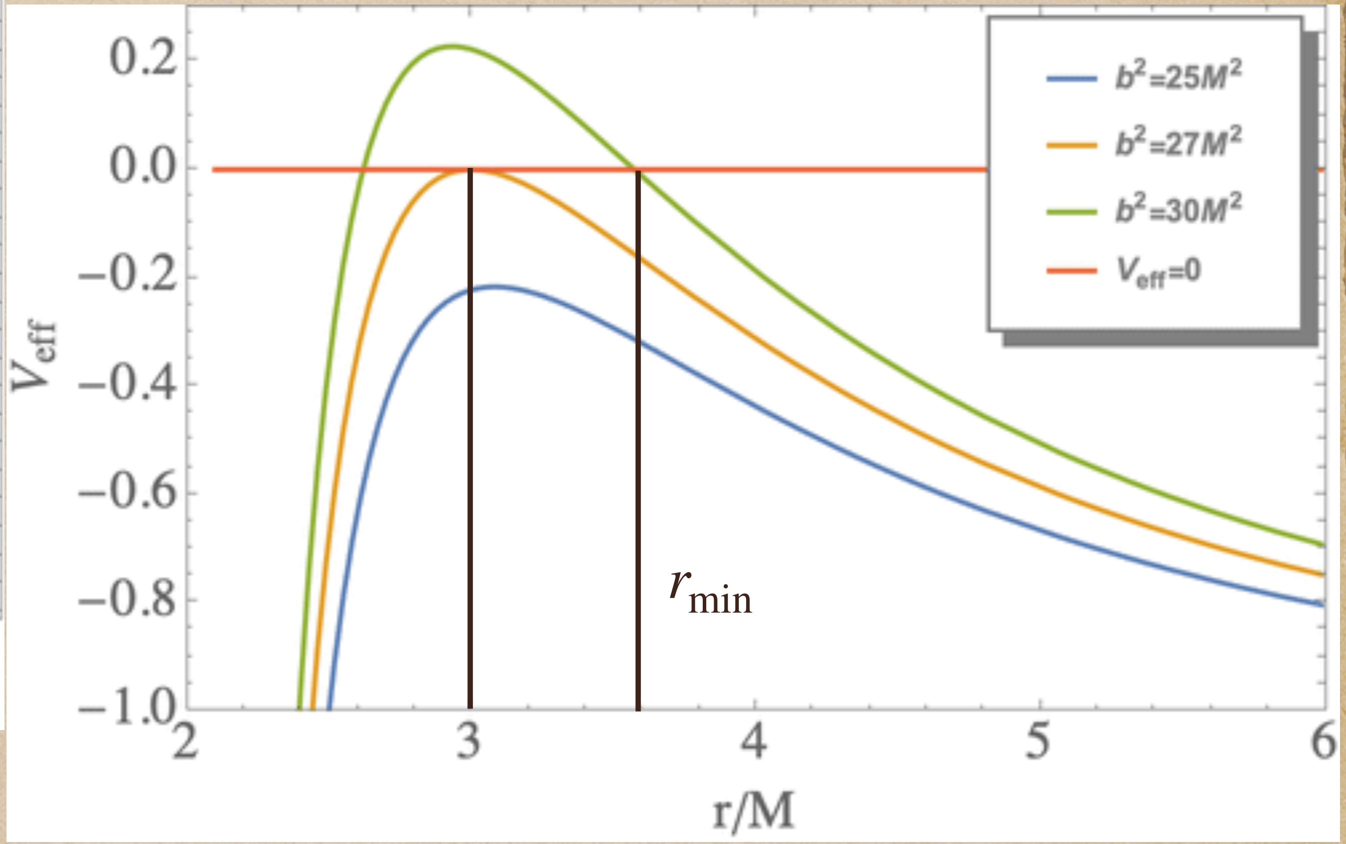
Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

In the Schwarzschild case,

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1-2M/r)}{r^2(1-2M/r)}$$





If the impact parameter is $b < 3\sqrt{3}M$ the photon falls into the Black Hole. If $b > 3\sqrt{3}M$ the photon is scattered getting down to a minimum radius $r_{\rm min}$.

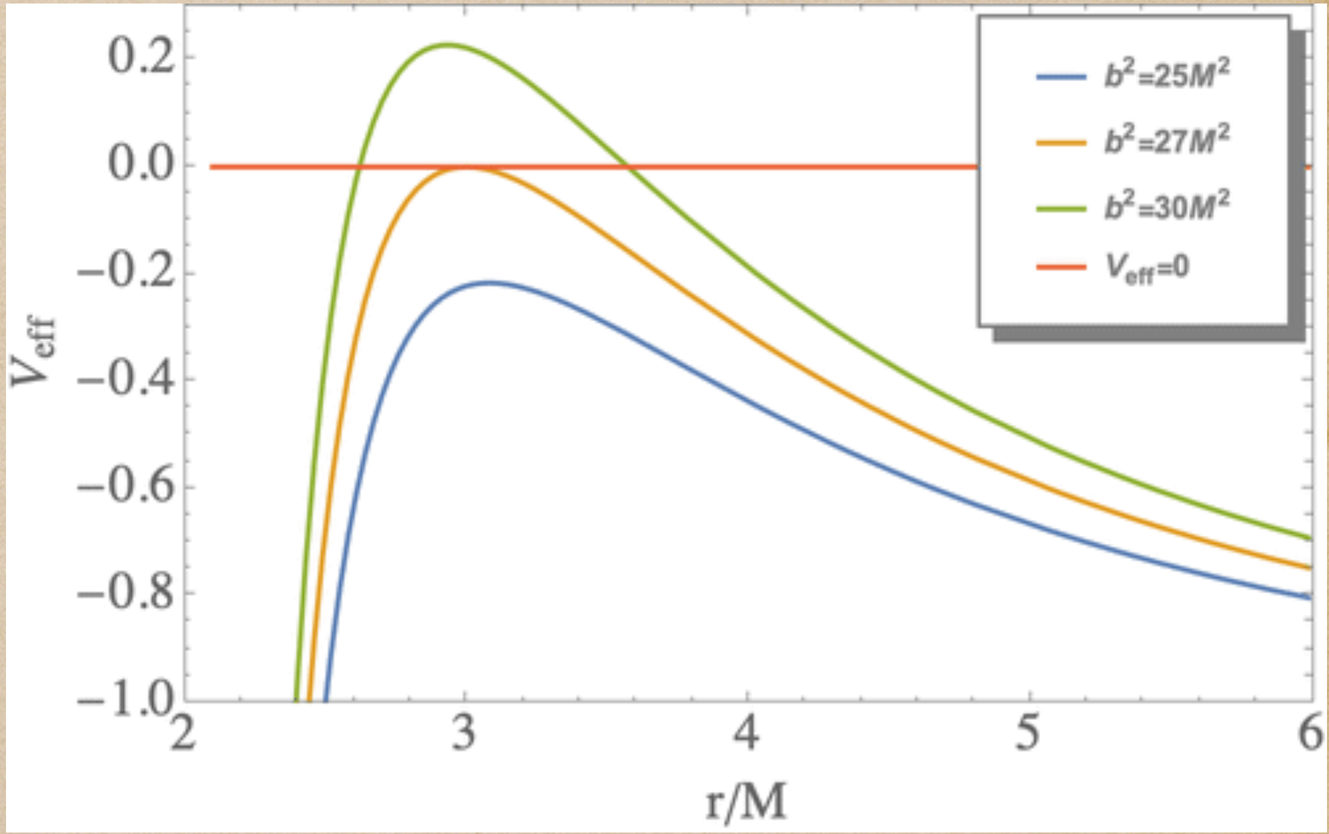
If $b = 3\sqrt{3}M$ then the photon reaches $r_{\rm min} = 3M$, the photon orbit.

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

The figure shows $V_{\rm eff}$ for 3 values of b. The allowed region is for $V_{\rm eff} \leq 0$.

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$

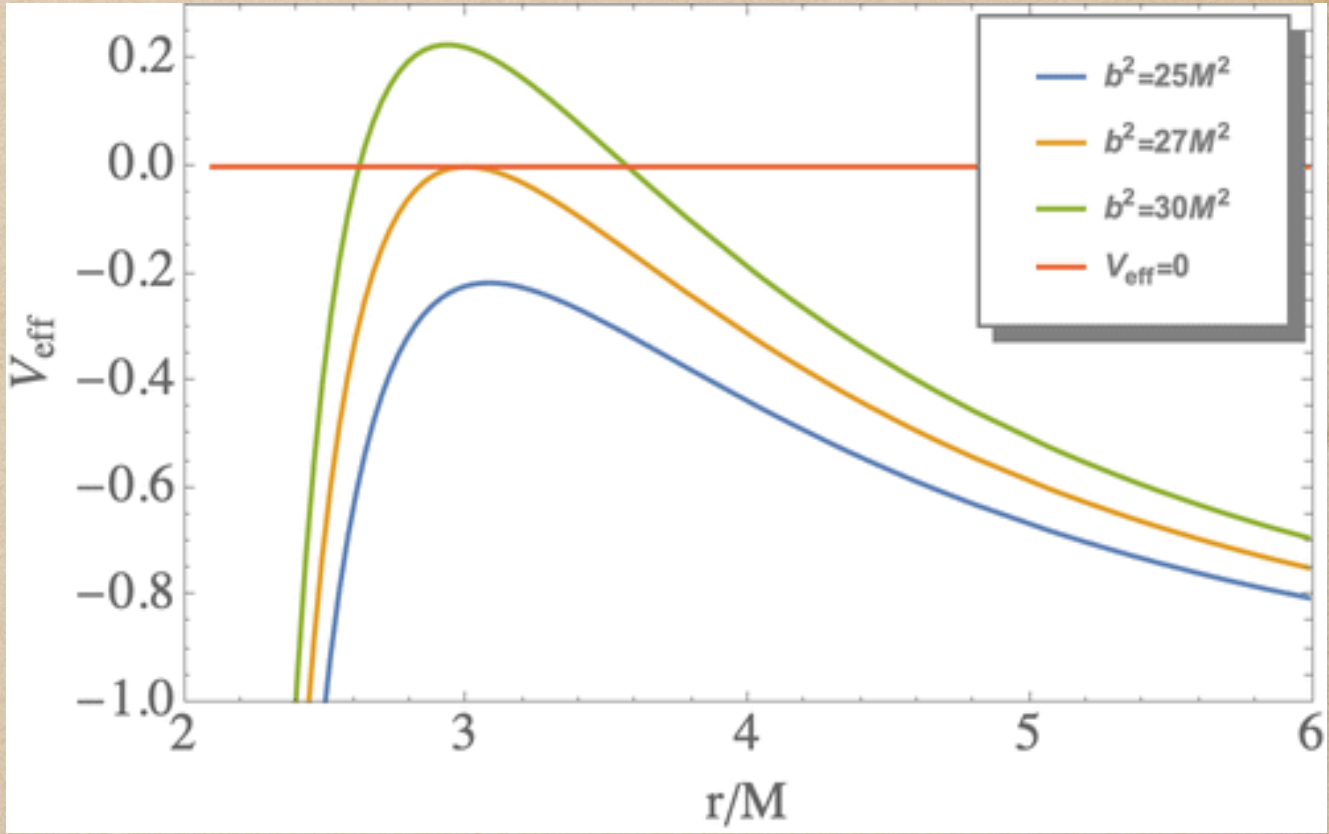


Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$

The figure shows $V_{\rm eff}$ for 3 values of b. The allowed region is for $V_{\rm eff} \leq 0$.



The photon orbit is the unstable circular orbit at the peak of the potential determined by the conditions,

$$V_{\rm eff} = 0, \quad \frac{dV_{eff}}{dr} = 0$$

Photon orbits around Black Holes

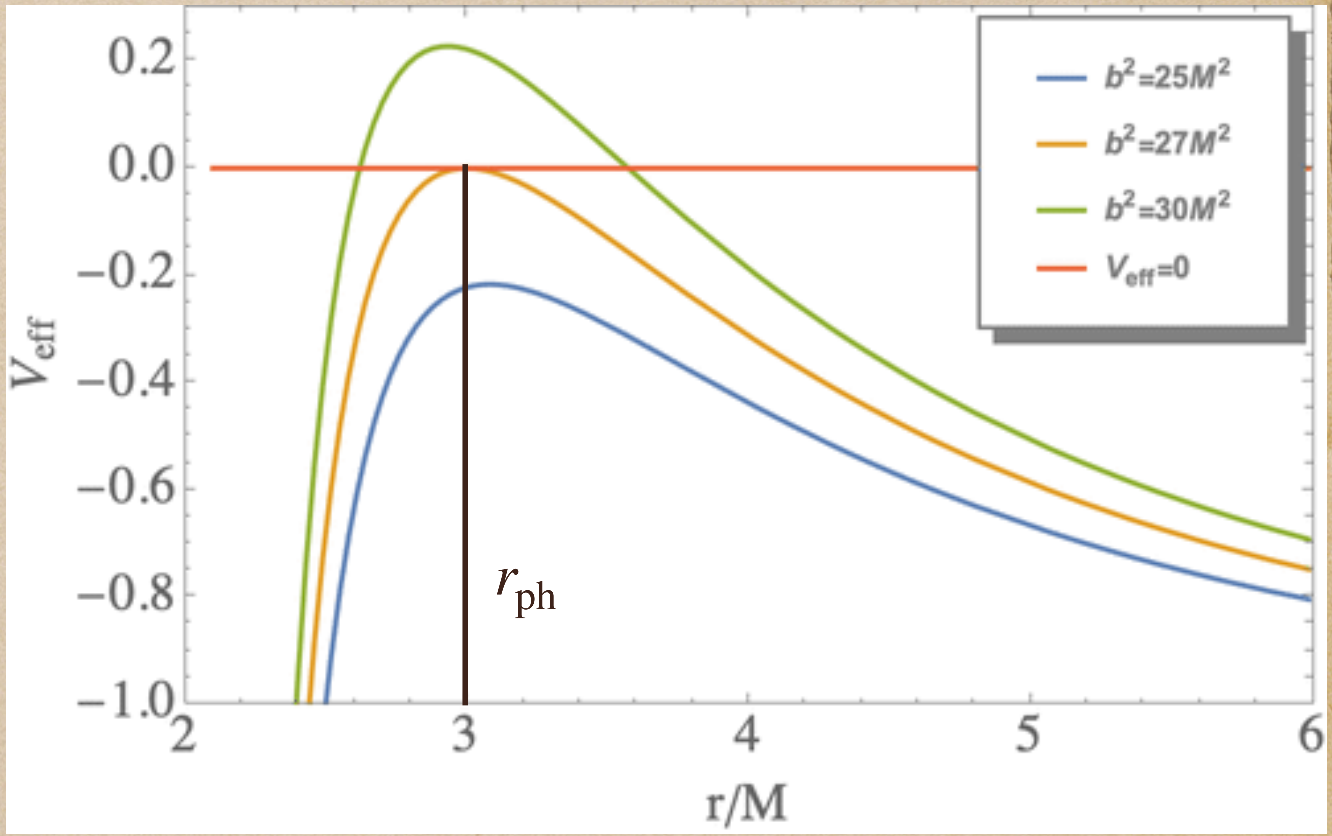
A simple application - the Schwarzschild spacetime:

The figure shows $V_{\rm eff}$ for 3 values of b. The allowed region is for $V_{\rm eff} \le 0$.

$$V_{\rm eff} = \frac{g_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1-2M/r)}{r^2(1-2M/r)}$$

The photon orbit is the unstable circular orbit at the peak of the potential determined by the conditions,

$$V_{\rm eff} = 0, \quad \frac{dV_{eff}}{dr} = 0$$

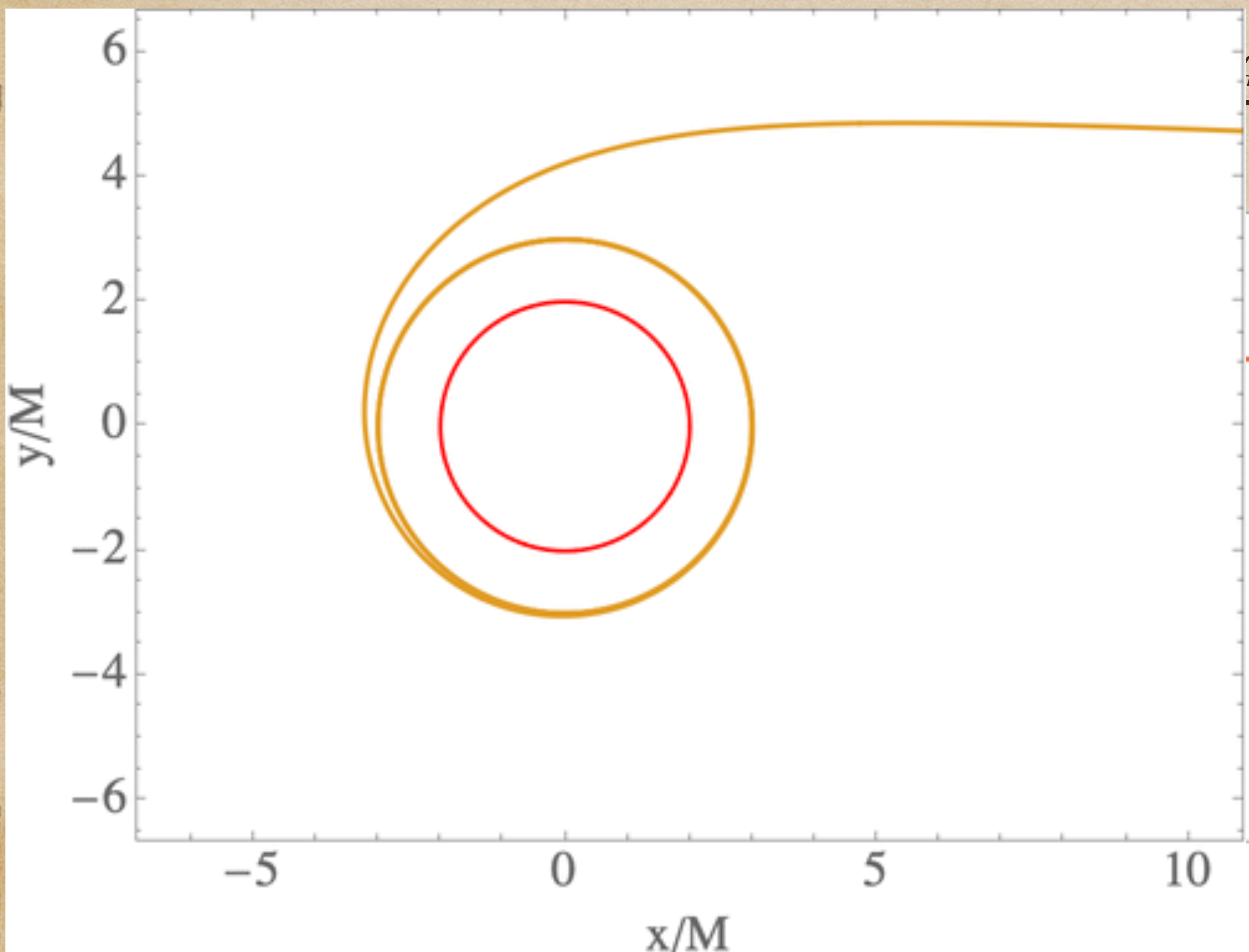


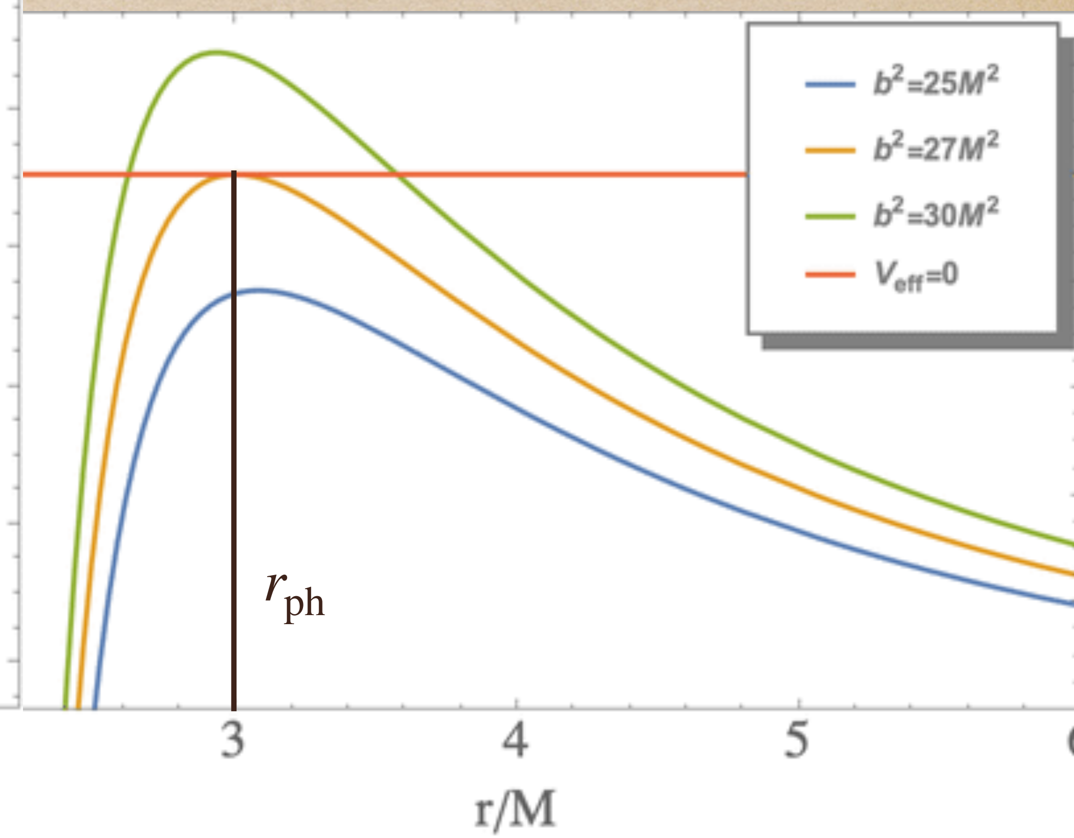
In the Schwarzschild case this gives $b_{\rm ph} = 3\sqrt{3}M$ and $r_{\rm ph} = 3M$.

Photon orbits around Black Holes

A simple application - the Schwarzschild spacetime:

$$\frac{\ldots_{\phi\phi} + b^2 g_{tt}}{g_{\phi\phi} g_{tt}} = -\frac{r^2 - b^2(1 - 2M/r)}{r^2(1 - 2M/r)}$$





In the Schwarzschild case this gives $b_{ph} = 3\sqrt{3}M$ and $r_{ph} = 3M$.

A photon with this impact parameter will asymptotically get to $r_{ph} = 3M$.

Photon orbits around Black Holes

A more general case - the Kerr spacetime:

The case of the Kerr rotating Black Hole is a little more complicated.

Photon orbits around Black Holes

A more general case - the Kerr spacetime:

The case of the Kerr rotating Black Hole is a little more complicated.
We no longer have spherical symmetry. We only have axisymmetry.

Photon orbits around Black Holes

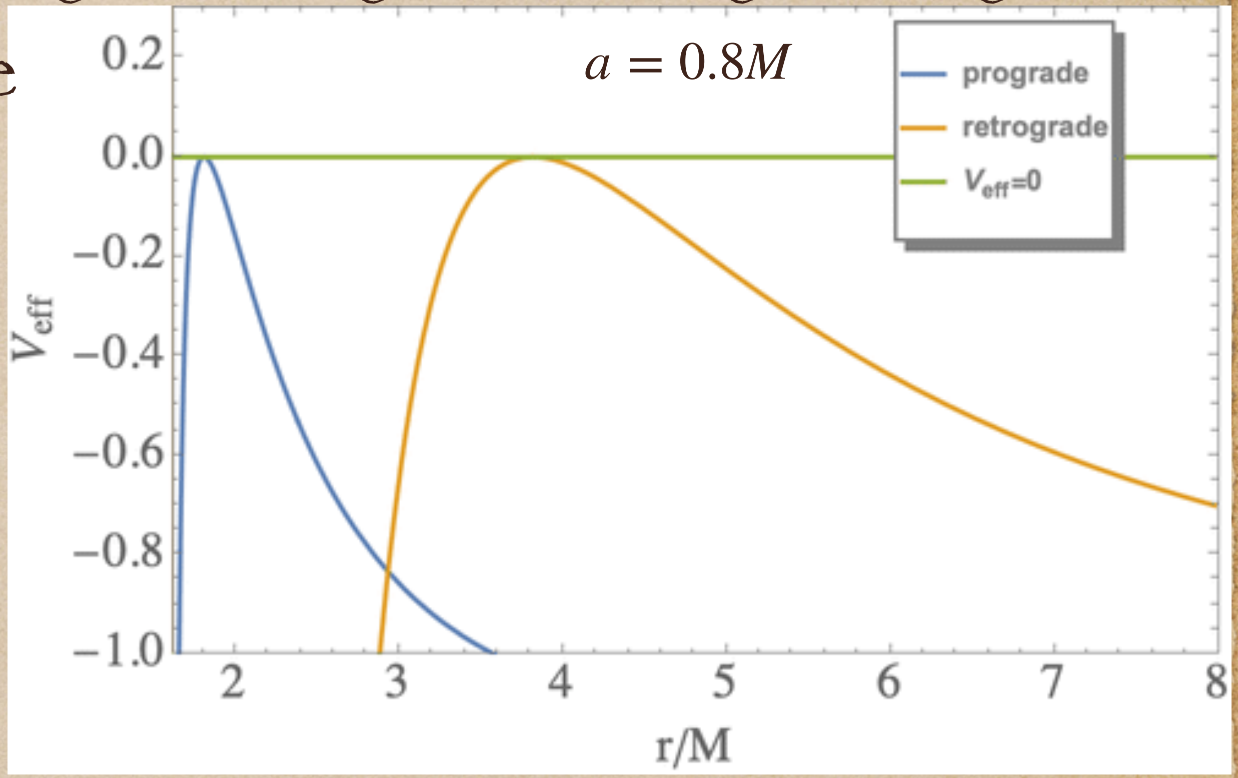
A more general case - the Kerr spacetime:

The case of the Kerr rotating Black Hole is a little more complicated. We no longer have spherical symmetry. We only have axisymmetry. On the equatorial plane $\theta = \pi/2$ we can treat things as 1D. We can still write a V_{eff} and find the unstable circular photon orbits.

Photon orbits around Black Holes

A more general case - the Kerr spacetime:

The case of the Kerr rotating Black Hole is a little more complicated. We no longer have spherical symmetry. We only have axisymmetry. On the equatorial plane $\theta = \pi/2$ we can treat things as 1D. We can still write a V_{eff} and find the unstable circular photon orbits.



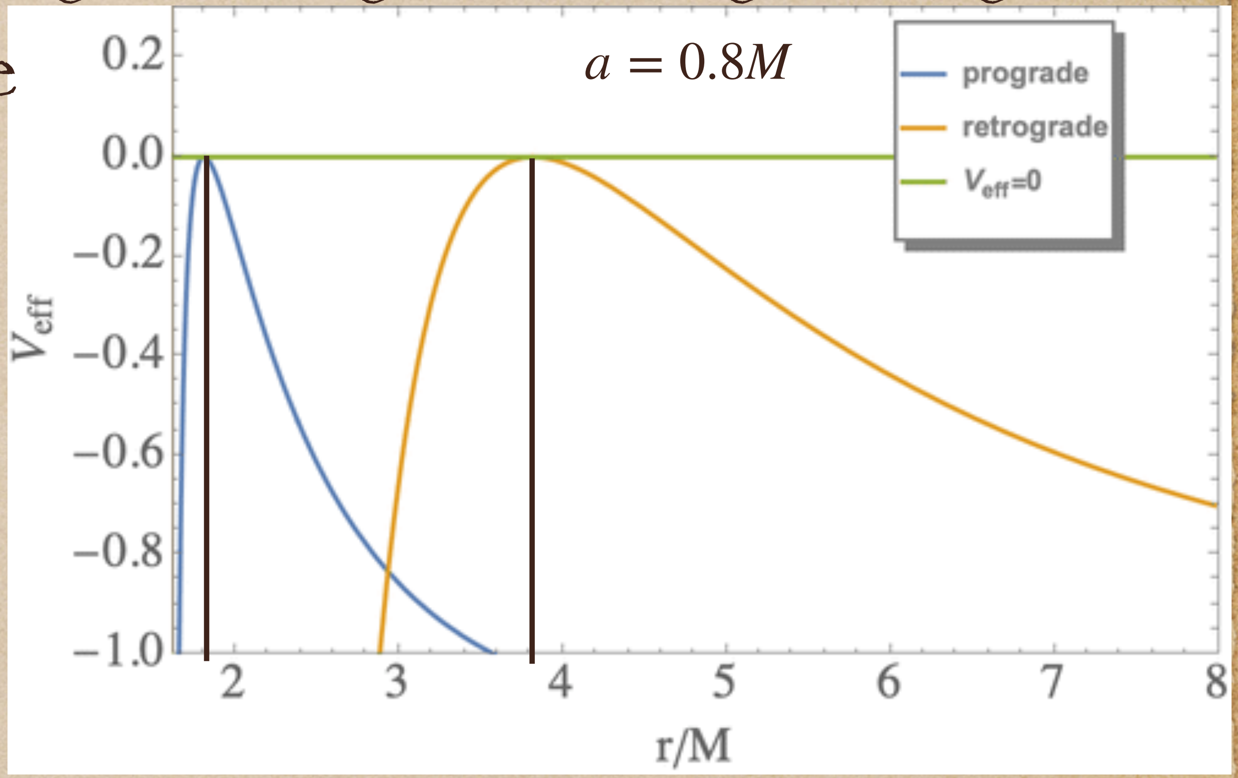
Photon orbits around Black Holes

A more general case - the Kerr spacetime:

The case of the Kerr rotating Black Hole is a little more complicated. We no longer have spherical symmetry. We only have axisymmetry.

On the equatorial plane $\theta = \pi/2$ we can treat things as 1D. We can still write a $V_{\rm eff}$ and find the unstable circular photon orbits.

In this case though, we will have two such orbits, one co-rotating and one counter-rotating with the BH.



Photon orbits around Black Holes

A more general case - the Kerr spacetime:

The case of the Kerr rotating Black Hole is a little more complicated.

ymmetry. We only have axisymmetry.

/2 we

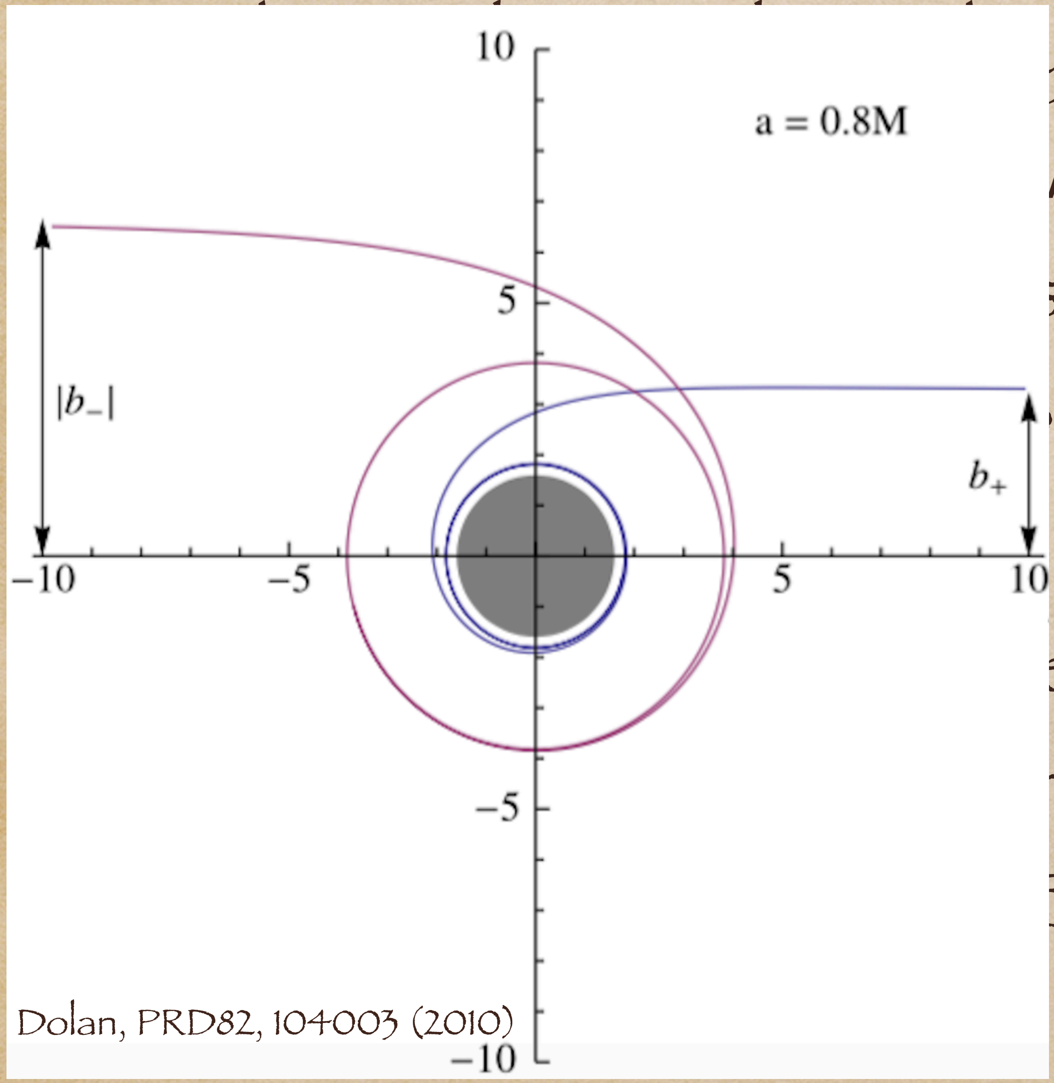
still

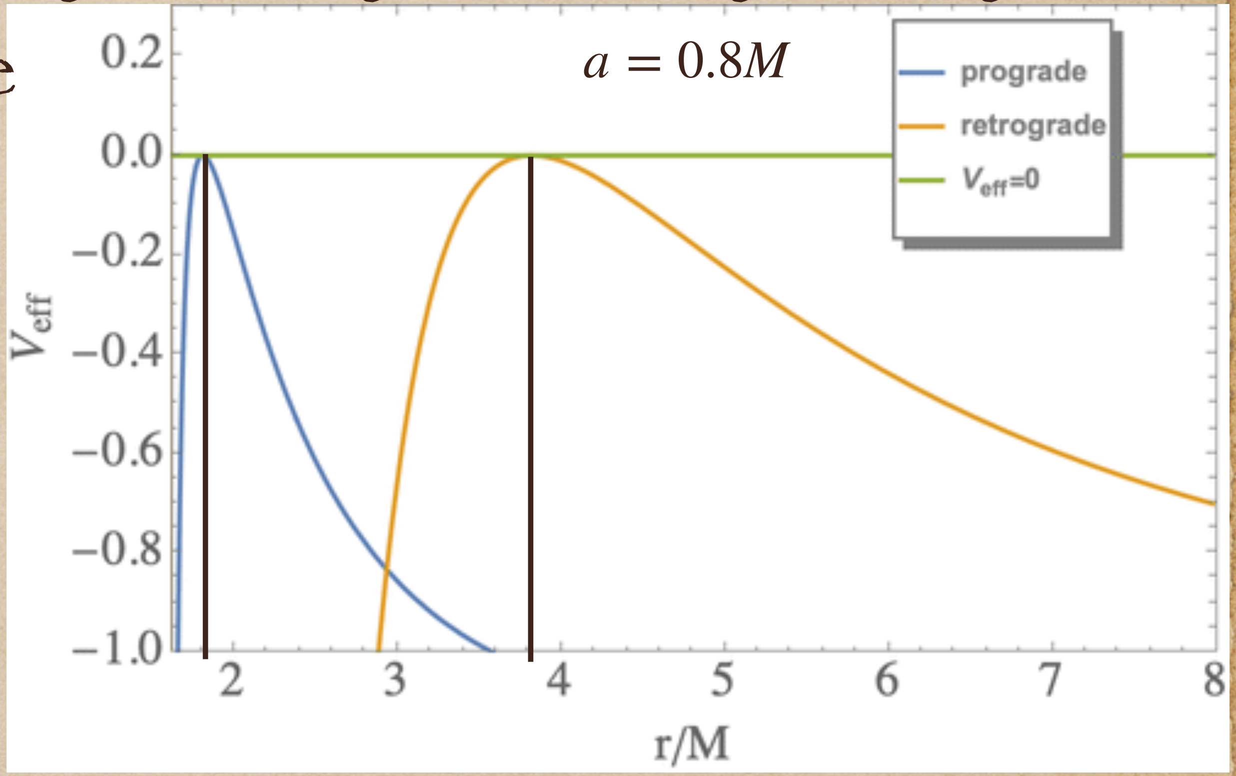
le

e two

nd

3H.





Photon orbits around Black Holes

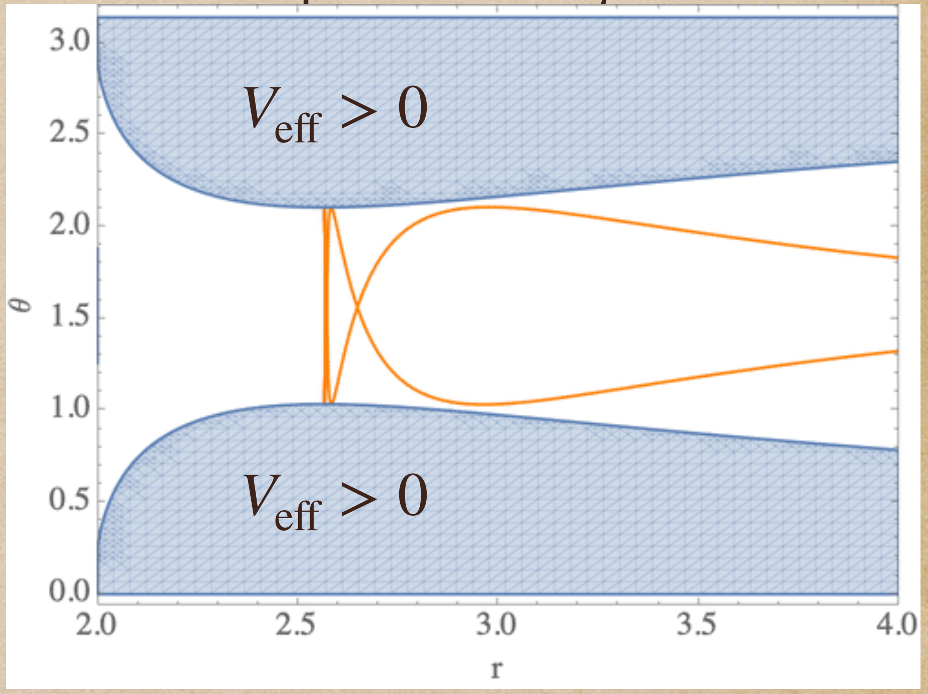
A more general case - the Kerr spacetime:

And one can have even more general orbits off the equatorial plane described by a more general $V_{\text{eff}}(r, \theta)$

Photon orbits around Black Holes

A more general case - the Kerr spacetime:

And one can have even more general orbits off the equatorial plane described by a more general $V_{\text{eff}}(r,\theta)$

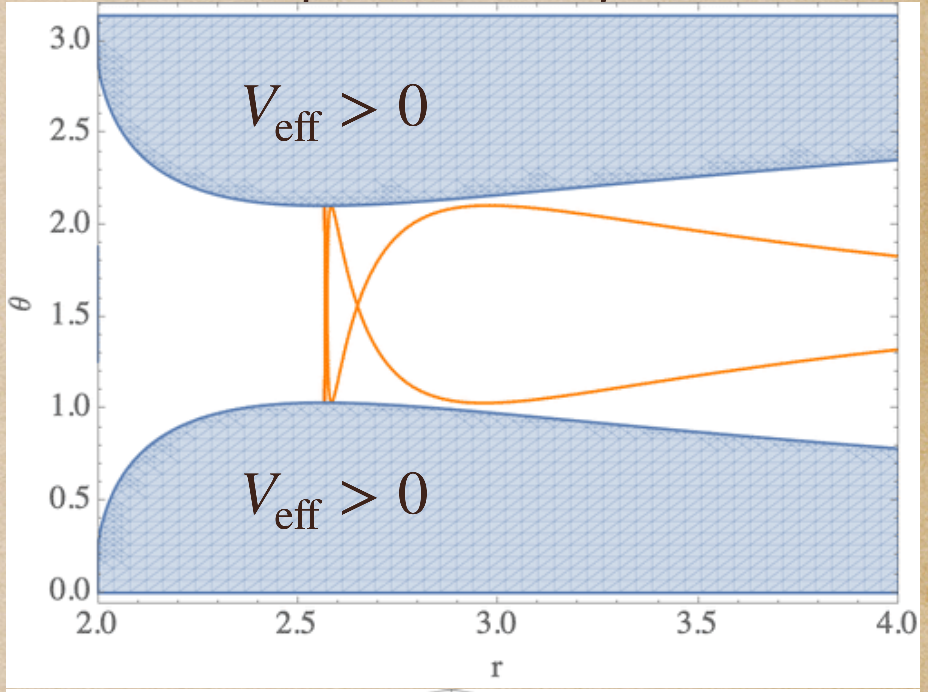


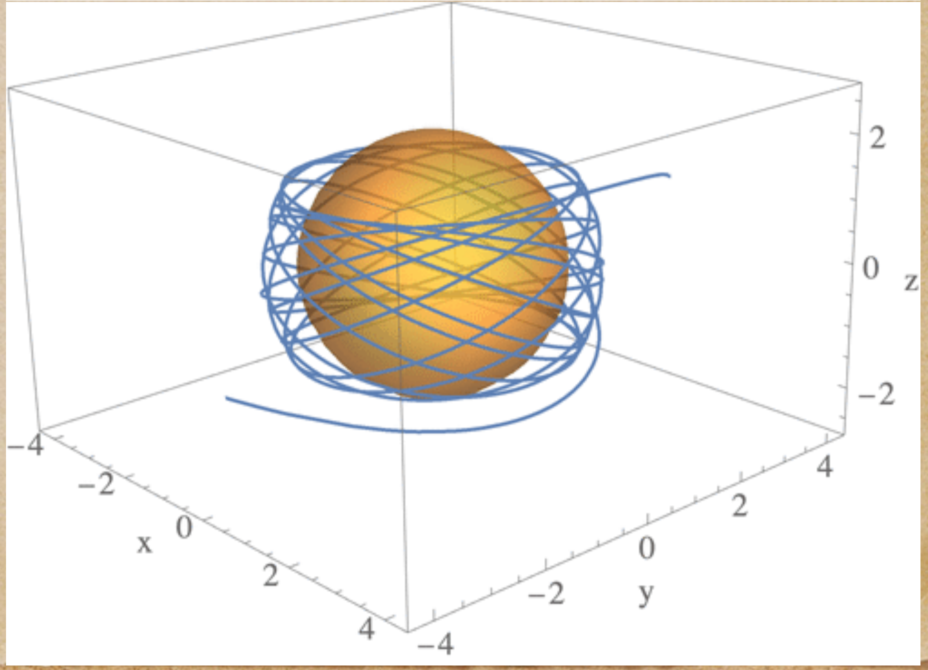
Photon orbits around Black Holes

<u>A more general case - the Kerr spacetime:</u>

And one can have even more general orbits off the equatorial plane described by a more general $V_{\text{eff}}(r, \theta)$

These are no longer circular orbits. Instead they are spherical photon orbits.





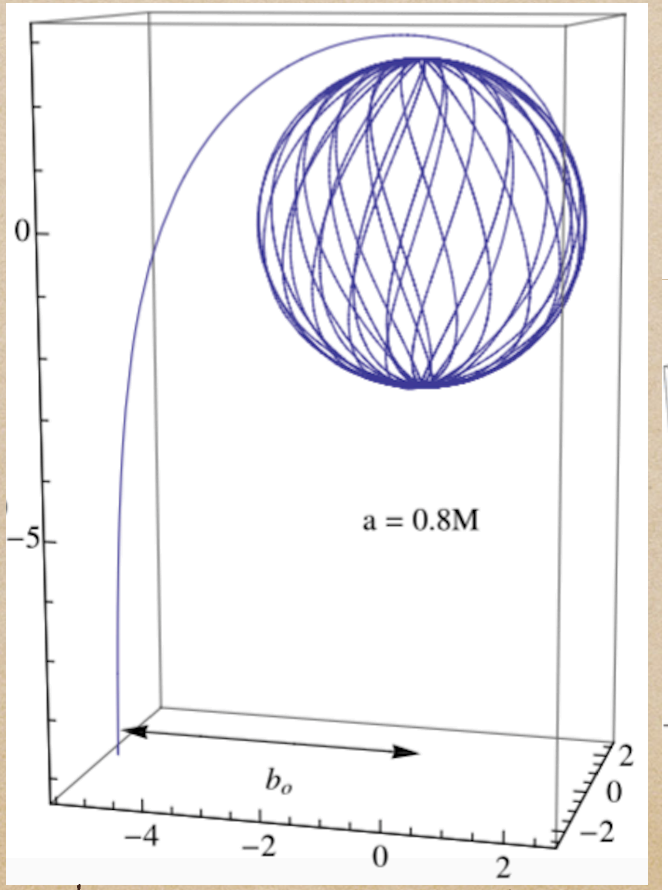
Photon orbits around Black Holes

<u>A more general case - the Kerr spacetime:</u>

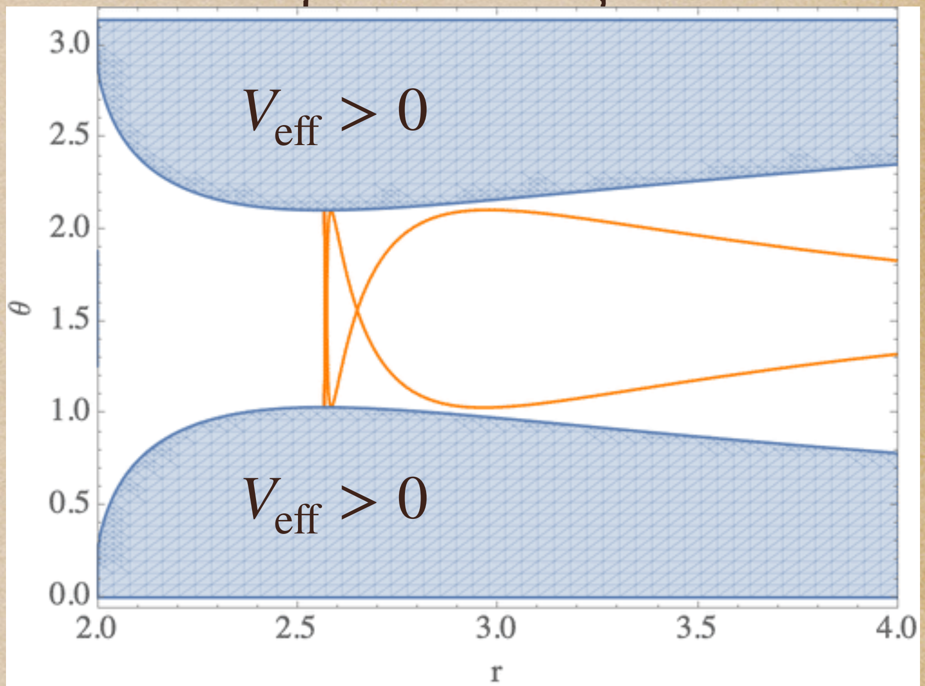
And one can have even more general orbits off the equatorial plane described by a more general $V_{\text{eff}}(r,\theta)$

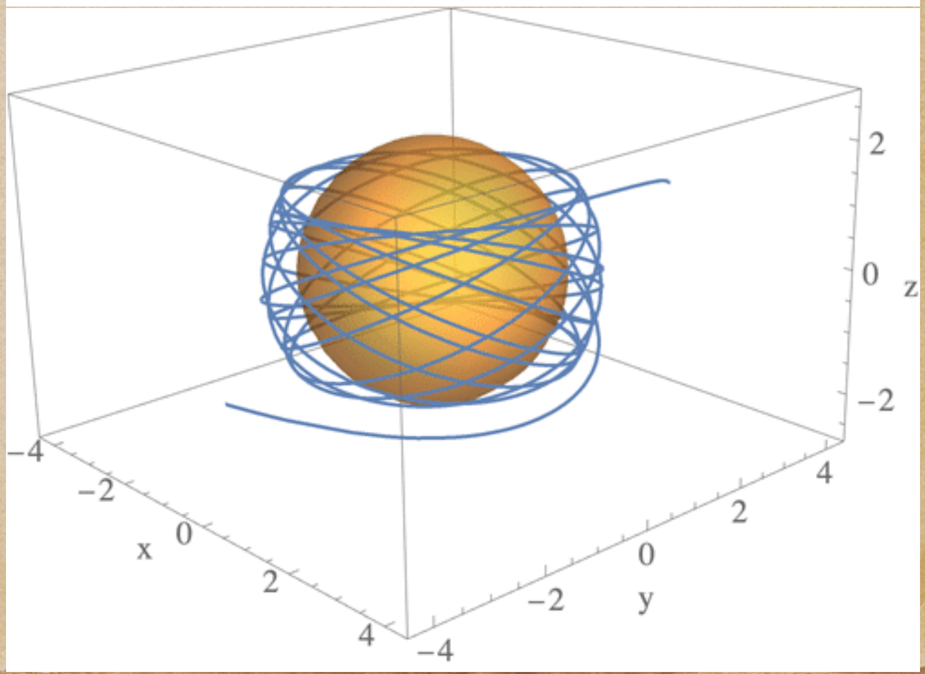
These are no longer circular orbits. Instead they are spherical photon orbits.

The most extreme case of those being the polar orbits, that form a full sphere.



Dolan, PRD82, 104003 (2010)





What is the shadow of a Black Hole?

A brief history of imaging Black Holes:

What is the shadow of a Black Hole?

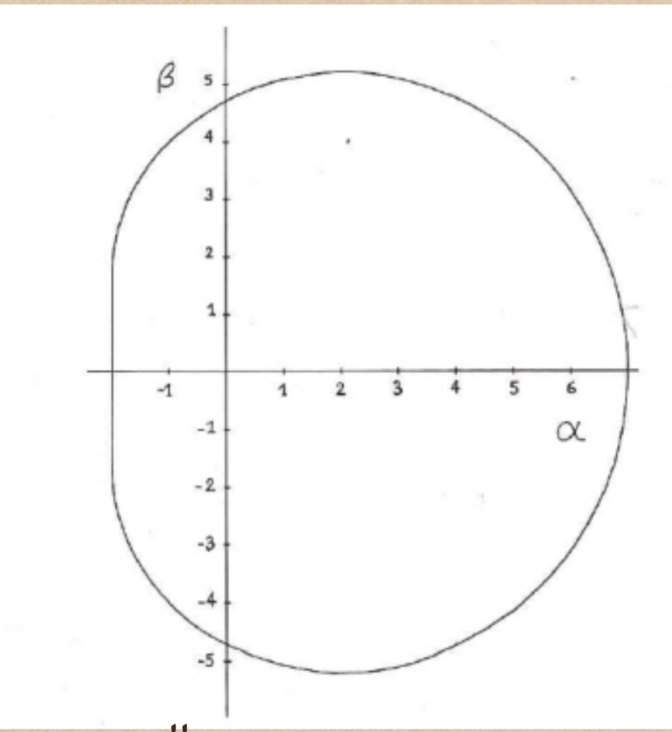
A brief history of imaging Black Holes:

It was first James Bardeen in the early 1970s that gave the mathematical definition of the shadow of a BH in terms of the two impact parameters (α, β), where

$\alpha \rightarrow p_\phi / p_t = -b$ and

$\beta \rightarrow p_\theta / p_t$

J. Bardeen



maximally rotating BH

What is the shadow of a Black Hole?

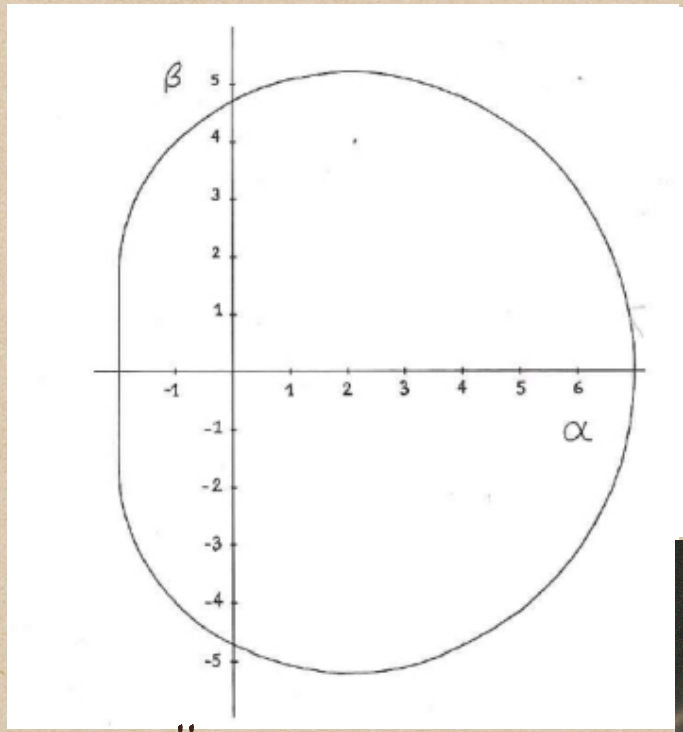
A brief history of imaging Black Holes:

It was first James Bardeen in the early 1970s that gave the mathematical definition of the shadow of a BH in terms of the two impact parameters (α, β), where

$\alpha \to p_\phi / p_t = -b$ and

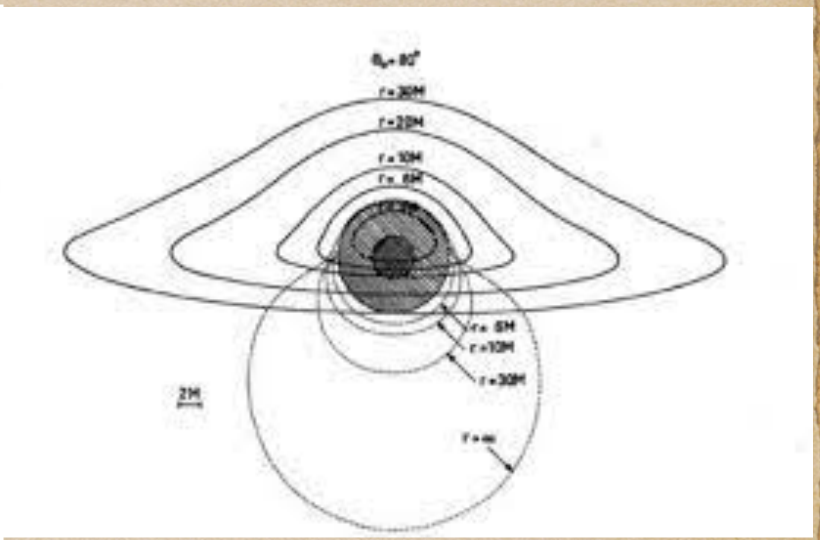
$\beta \to p_\theta / p_t$

J. Bardeen



maximally rotating BH

Jean-Pierre Luminet

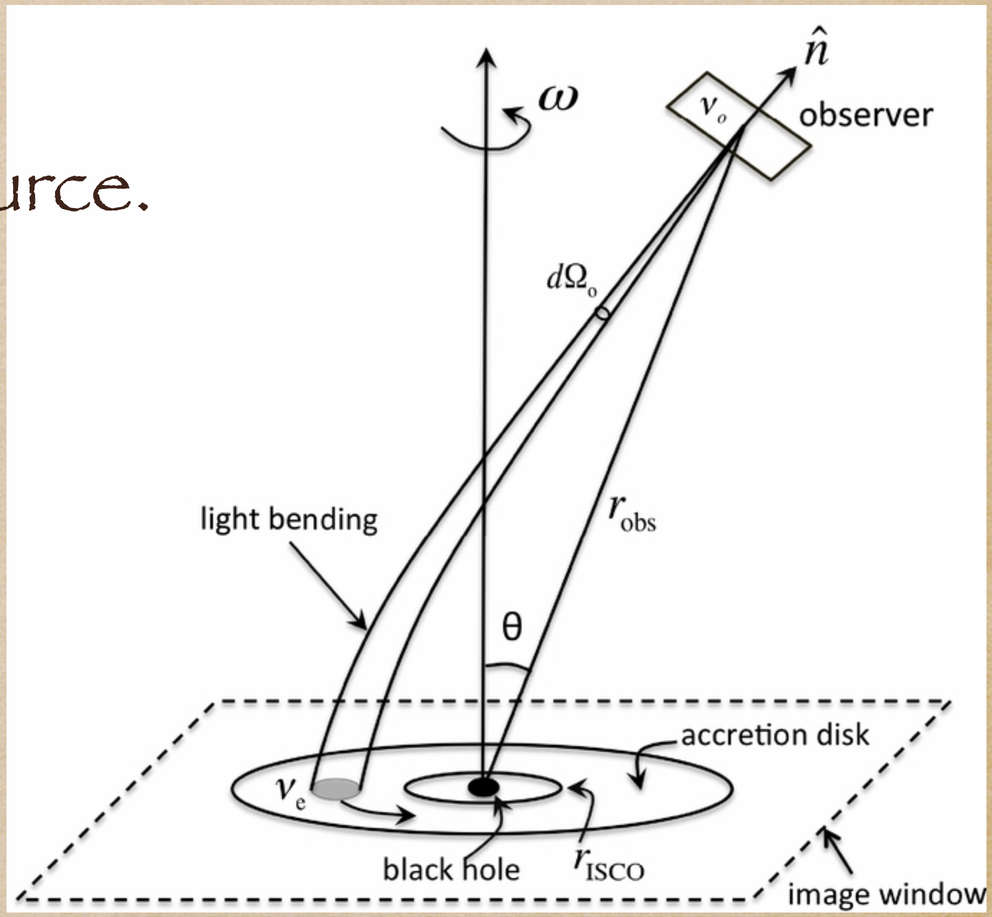


While in the late 1970s Jean-Pierre Luminet gave the first image of an accretion disc around a Black Hole.

What is the shadow of a Black Hole?

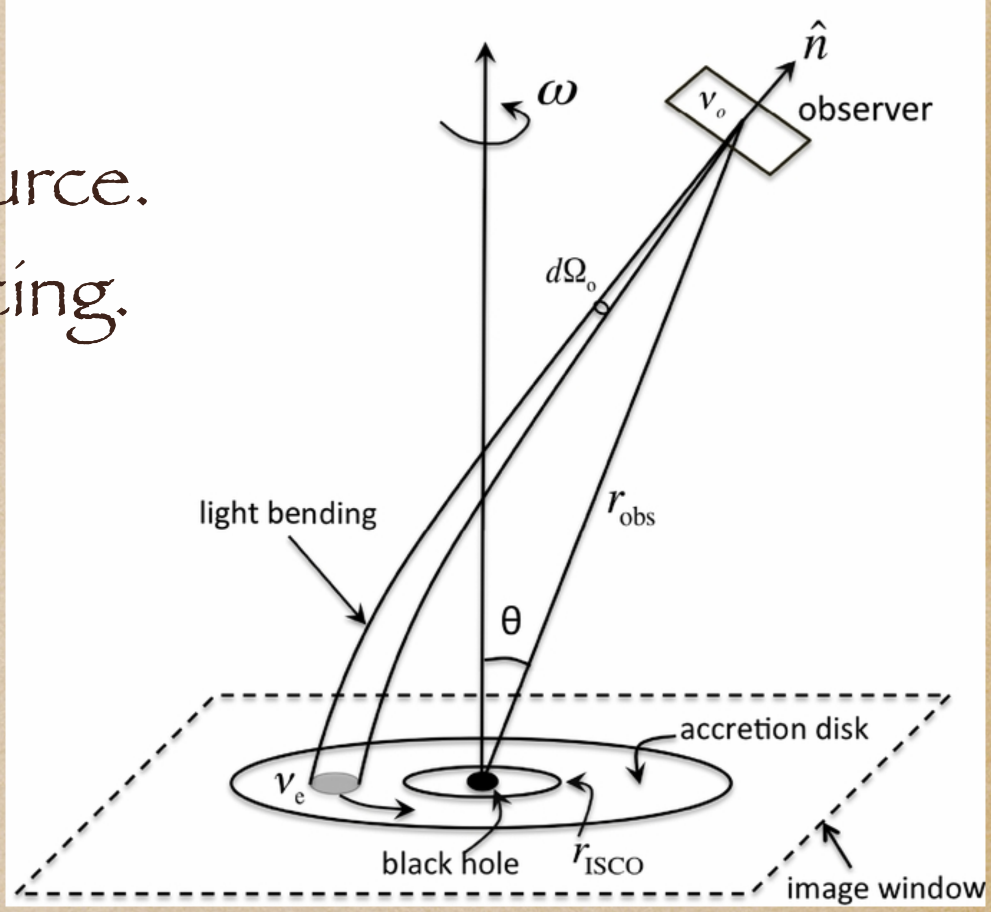
What is the shadow of a Black Hole?

We can think of imaging a Black Hole in two equivalent ways:
1) Light from a source comes to the observer from the direction of the BH , or 2) we cast a photon trajectory towards the BH and see if it hits a light source.



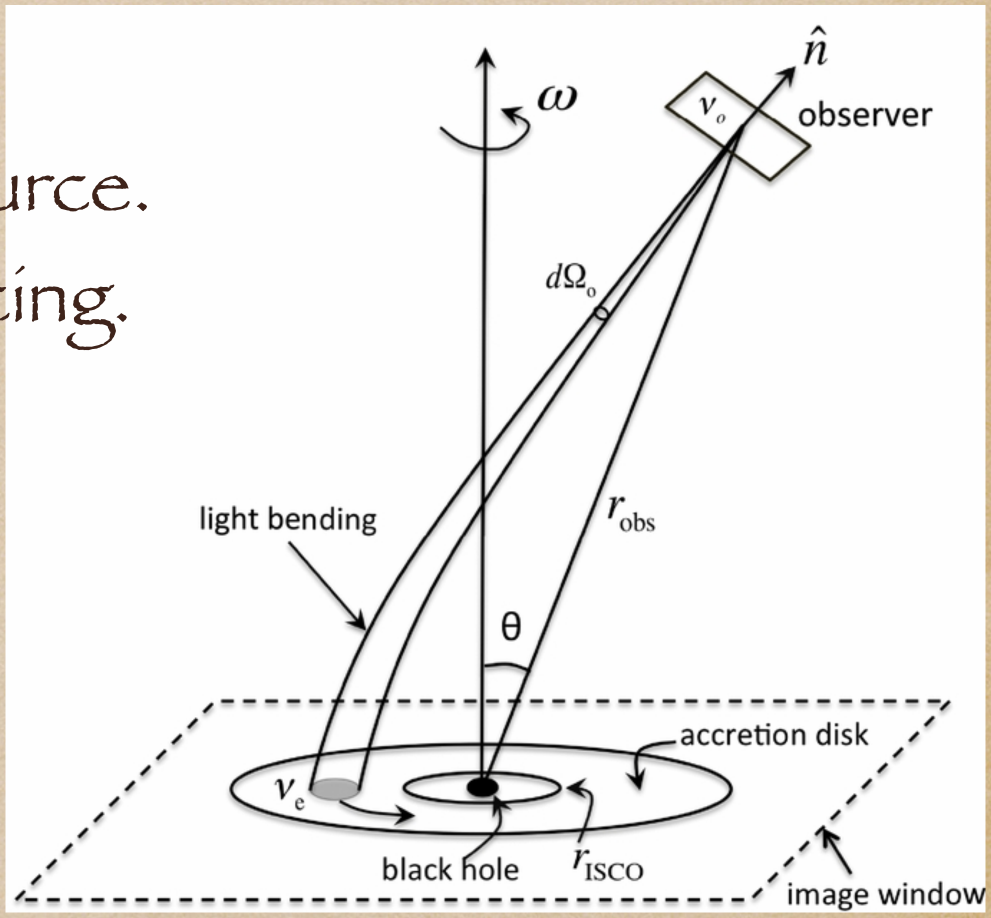
What is the shadow of a Black Hole?

We can think of imaging a Black Hole in two equivalent ways:
1) Light from a source comes to the observer from the direction of the BH , or 2) we cast a photon trajectory towards the BH and see if it hits a light source.
This is the technique of backward ray-tracing.



What is the shadow of a Black Hole?

We can think of imaging a Black Hole in two equivalent ways:
 1) Light from a source comes to the observer from the direction of the BH , or 2) we cast a photon trajectory towards the BH and see if it hits a light source. This is the technique of backward ray-tracing.

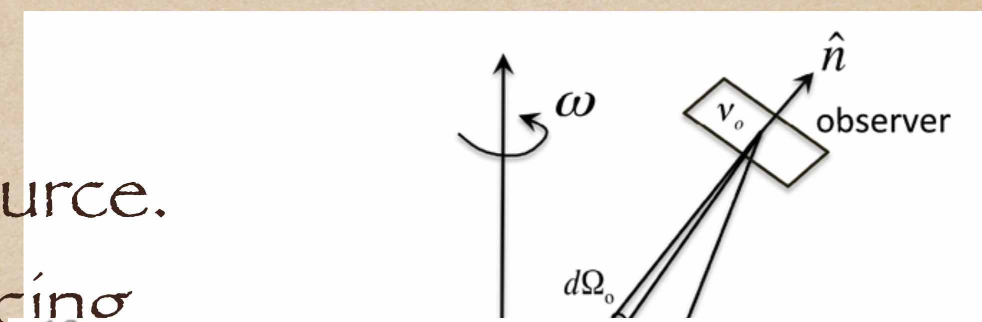


In this way, we can determine whether a photon is lost inside the horizon of a BH and therefore forms part of the shadow or the photon is simply scattered and thus provides a possible source of light.

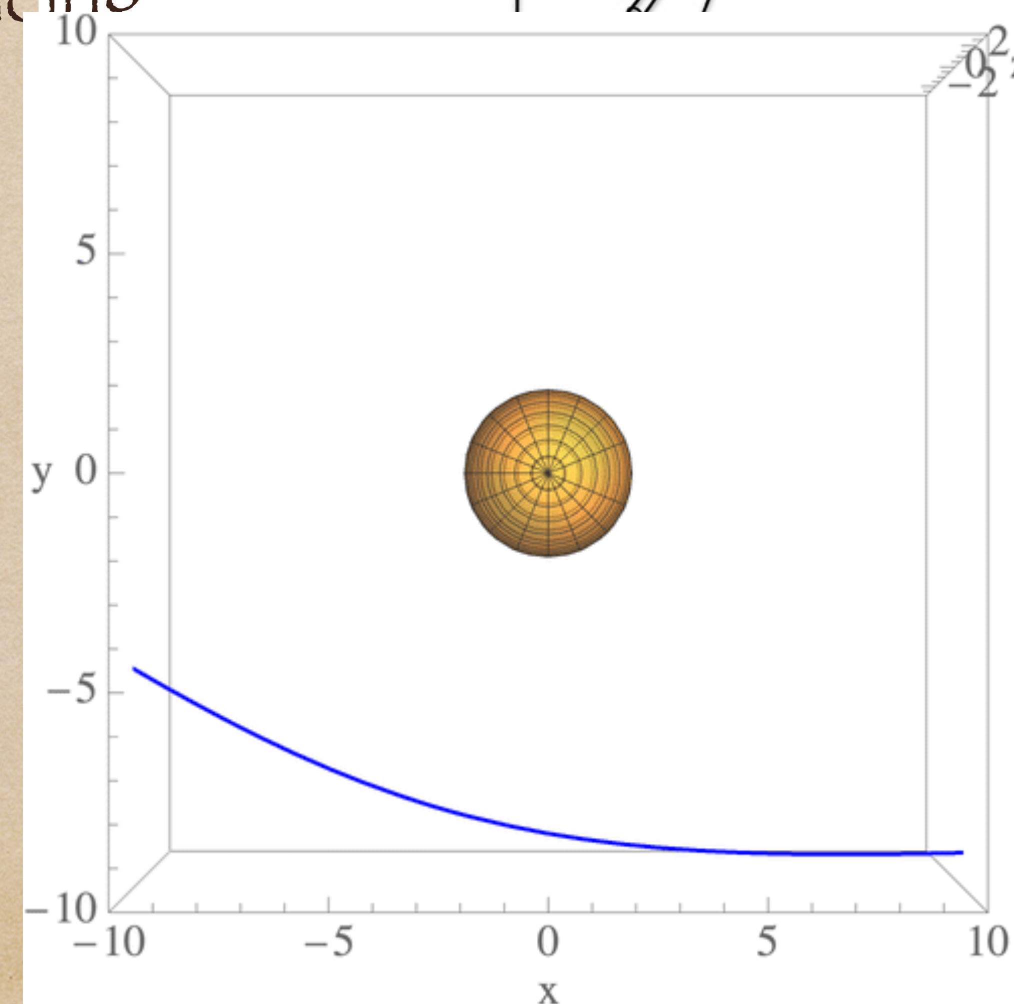
What is the shadow of a Black Hole?

We can think of imaging a Black Hole in two equivalent ways:

1) Light from a source comes to the observer from the direction of the BH , or 2) we cast a photon trajectory towards the BH and see if it hits a light source. This is the technique of backward ray-tracing

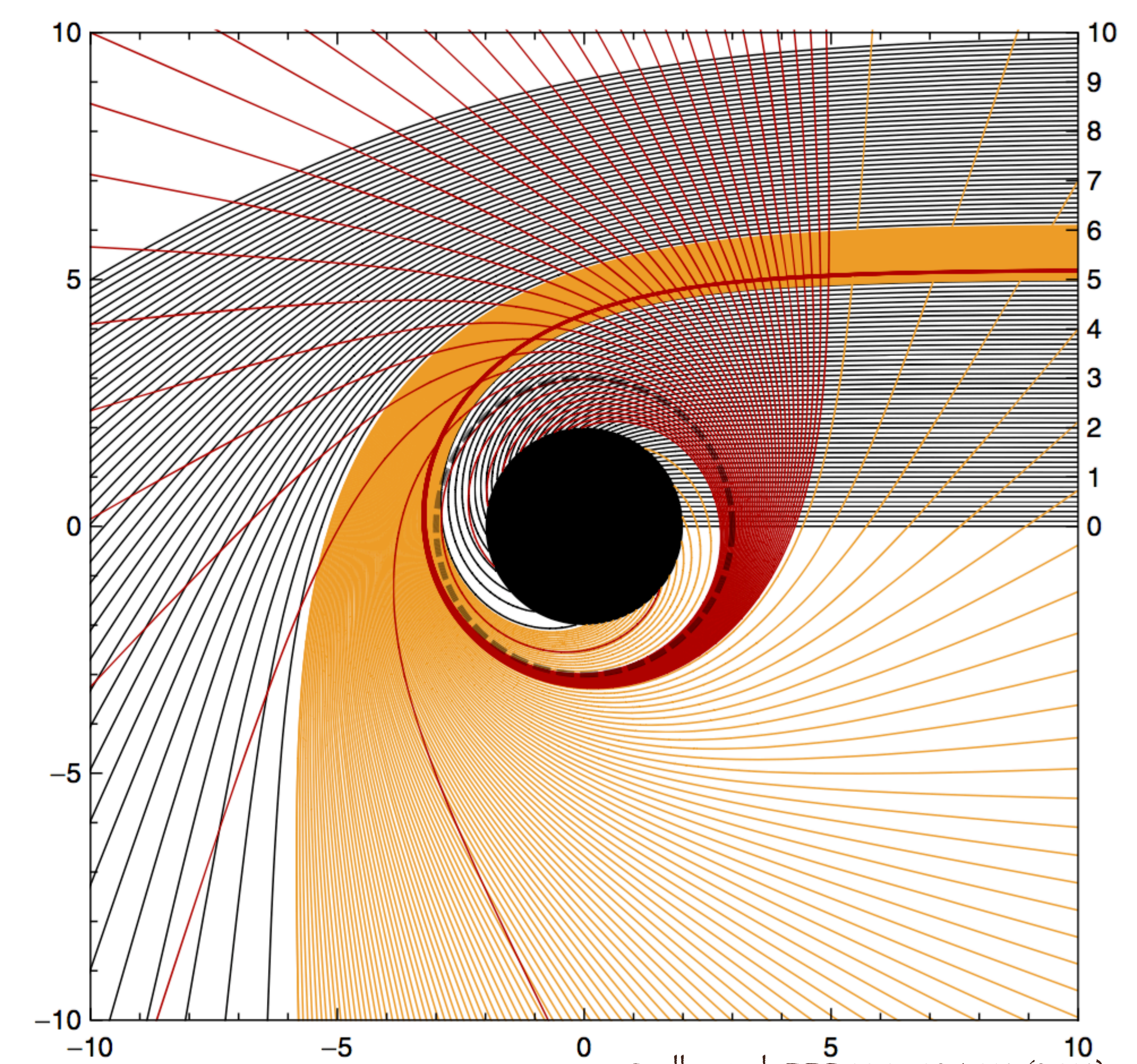


In this way, we can determine whether a photon is lost inside the horizon of a BH and therefore forms part of the shadow or the photon is simply scattered and thus provides a possible source of light.



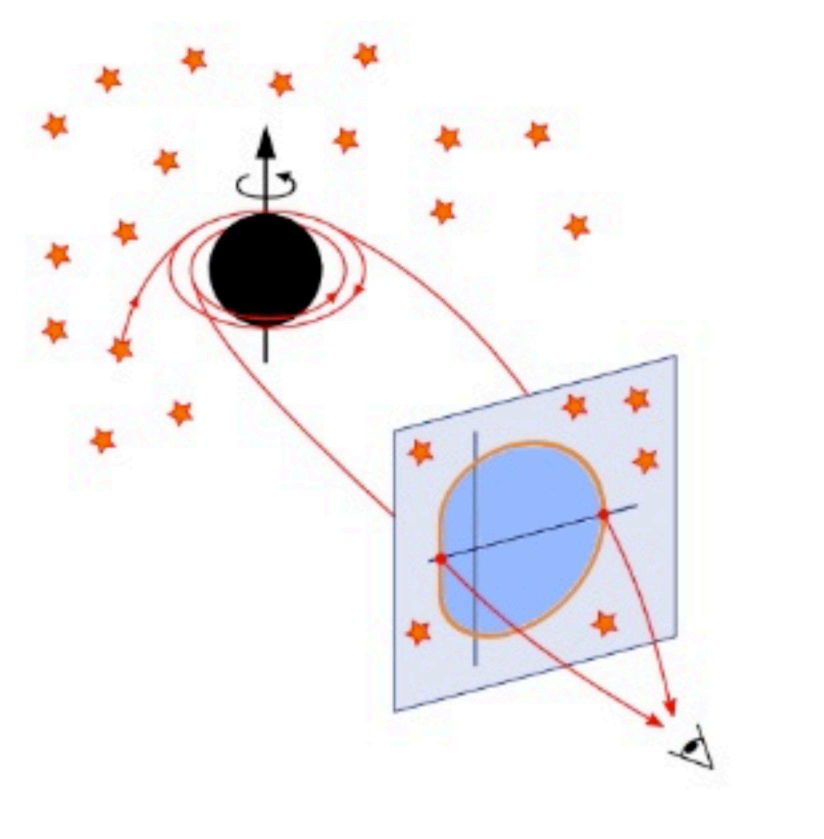
What is the shadow of a Black Hole?

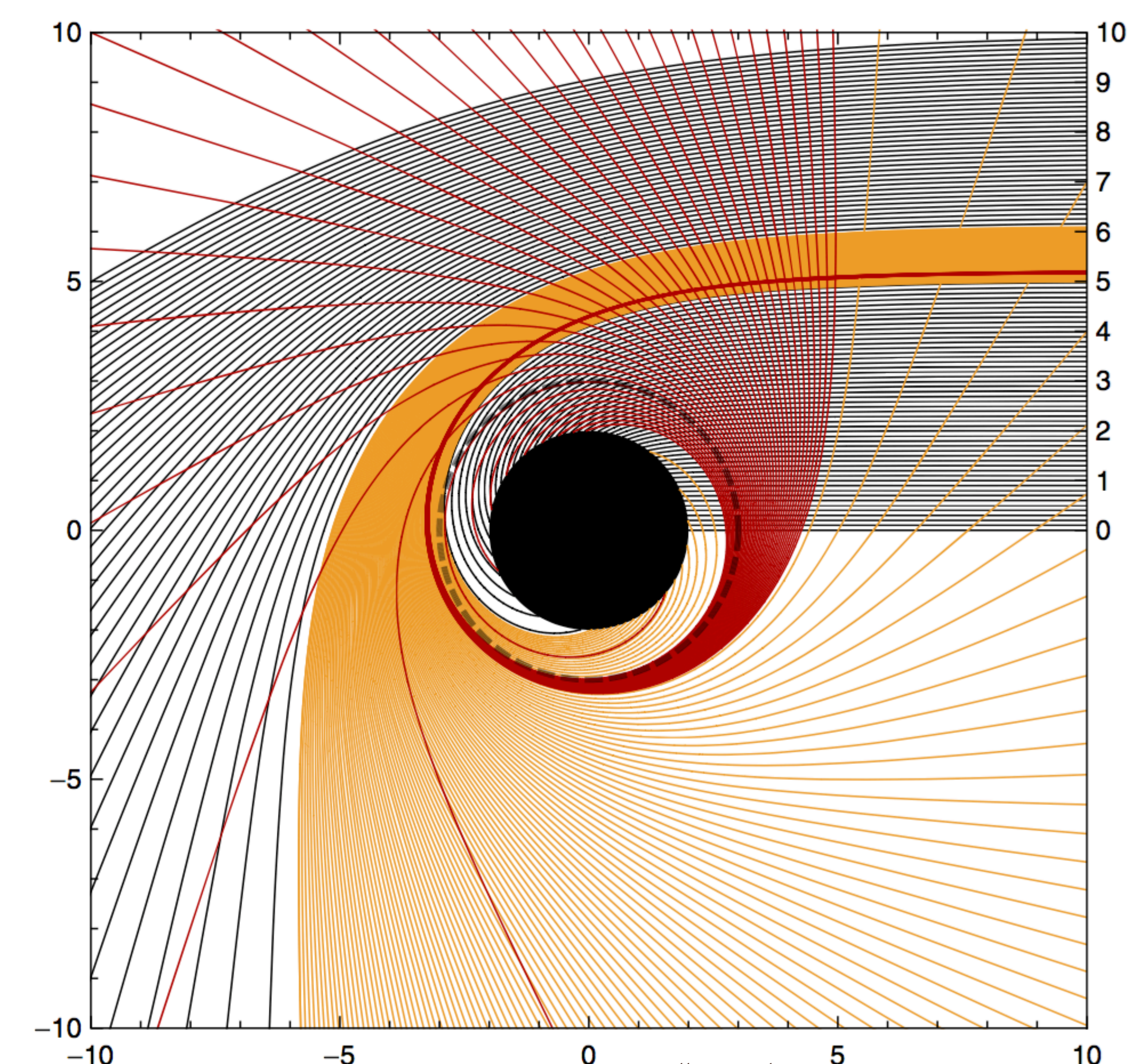
What is the shadow of a Black Hole?



Gralla et al. PRD100, 024018 (2019)

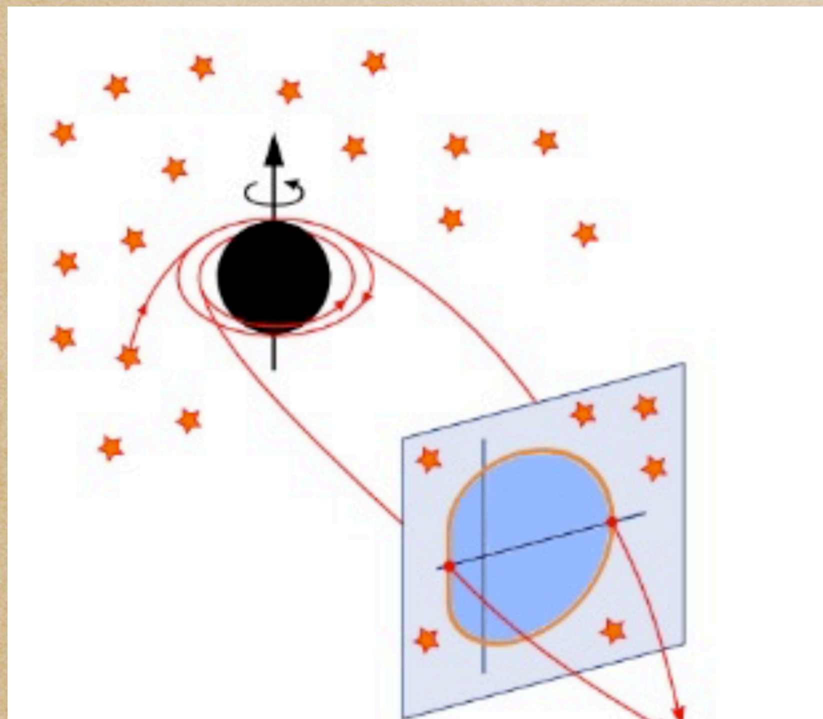
What is the shadow of a Black Hole?

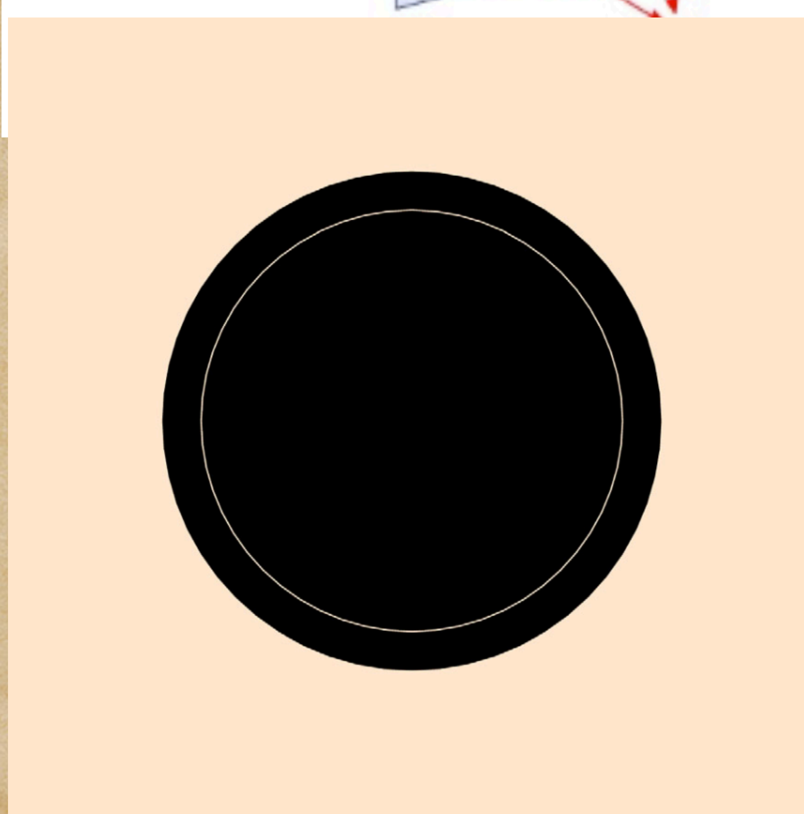


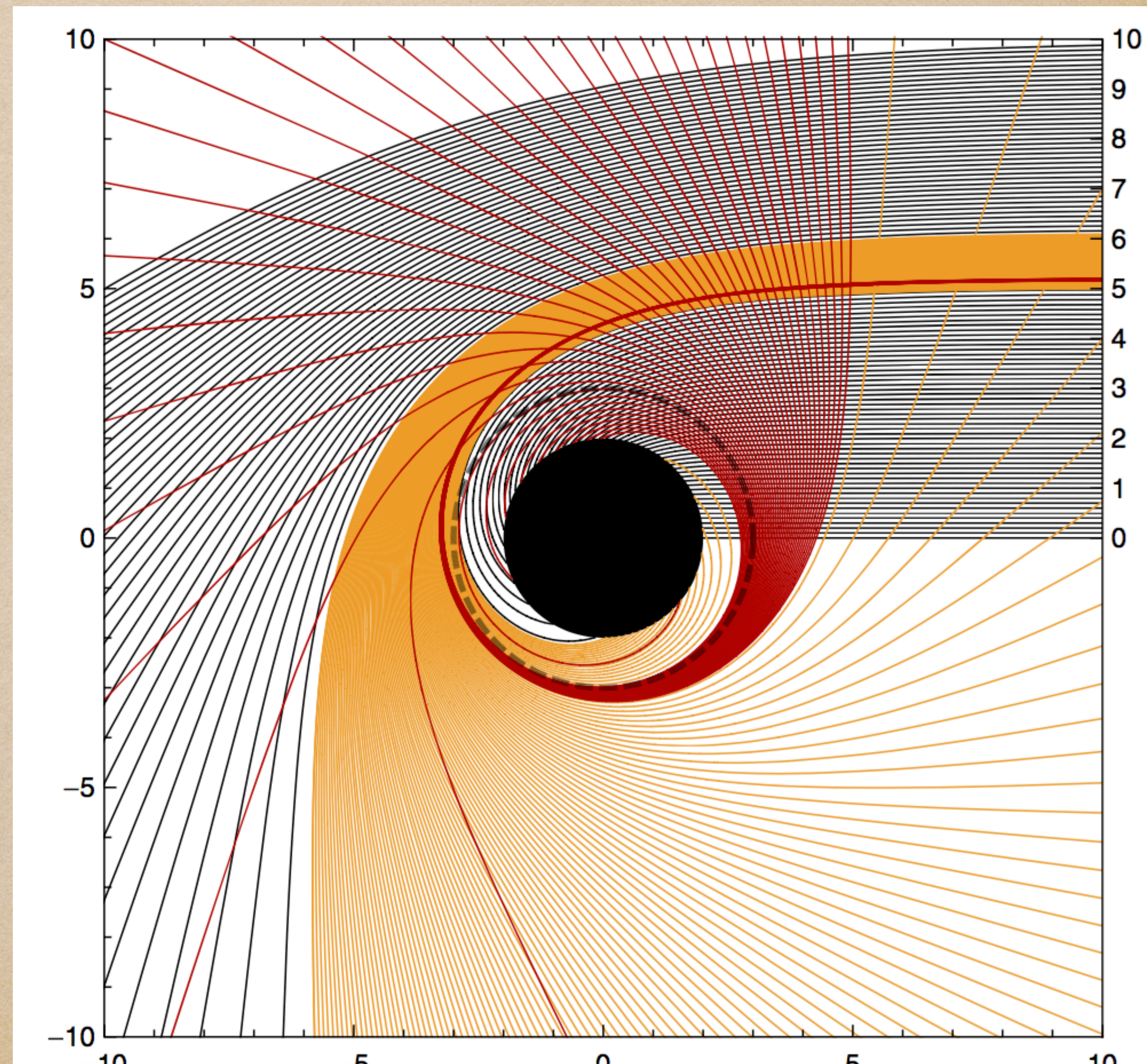


Gralla et al. PRD100, 024018 (2019)

What is the shadow of a Black Hole?



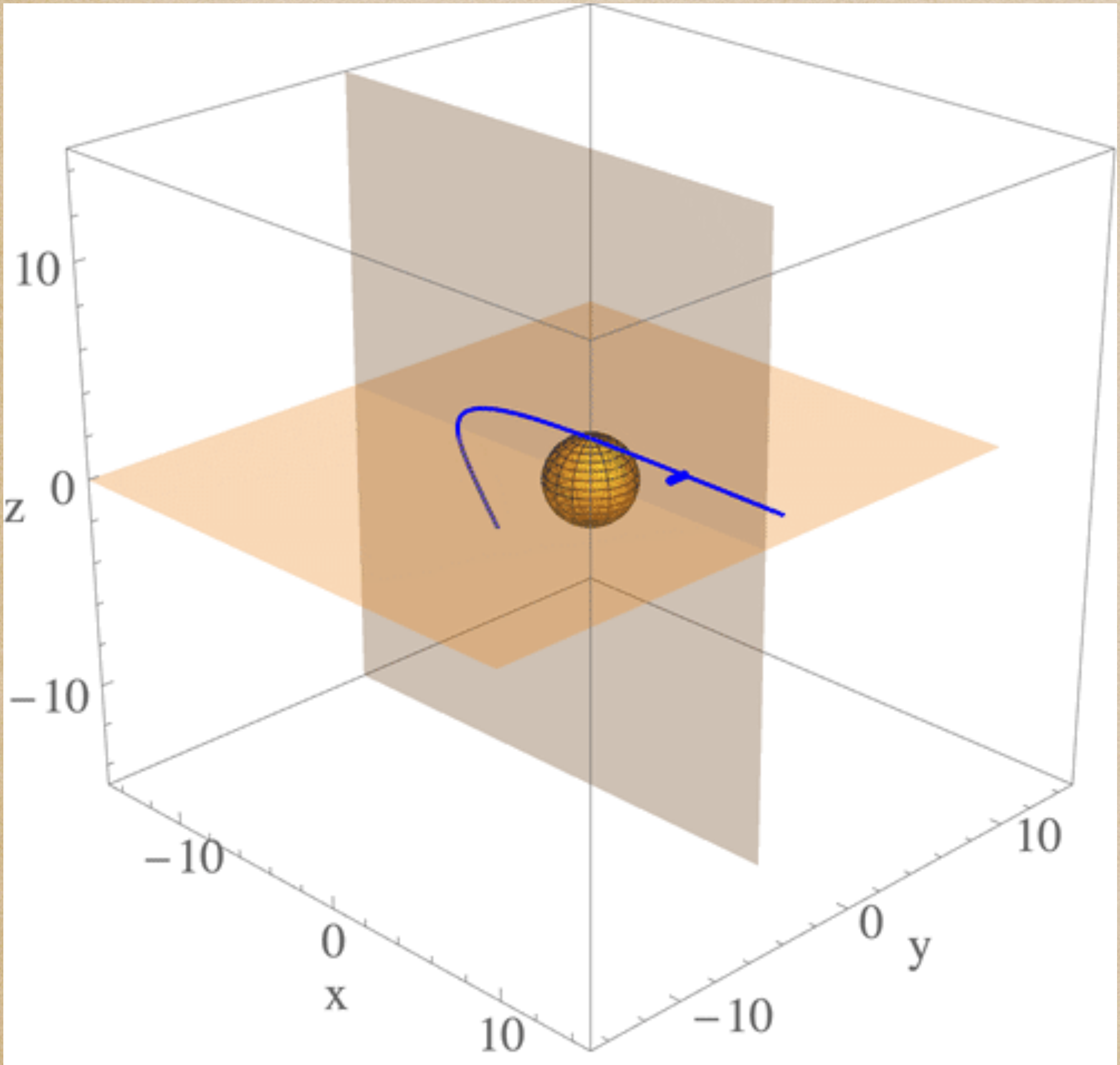


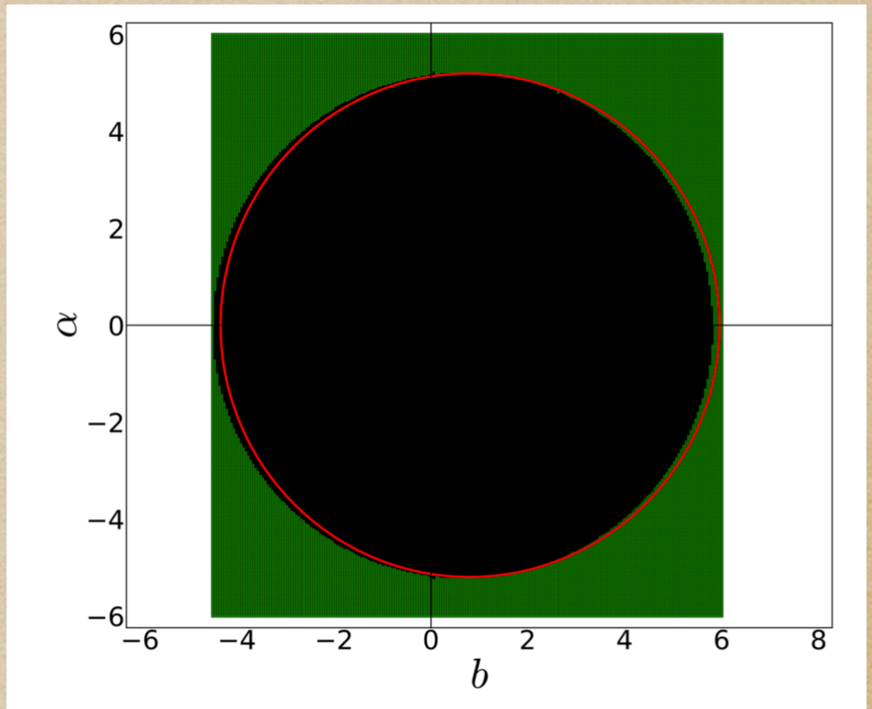


Gralla et al. PRD100, 024018 (2019)

What is the shadow of a Black Hole?

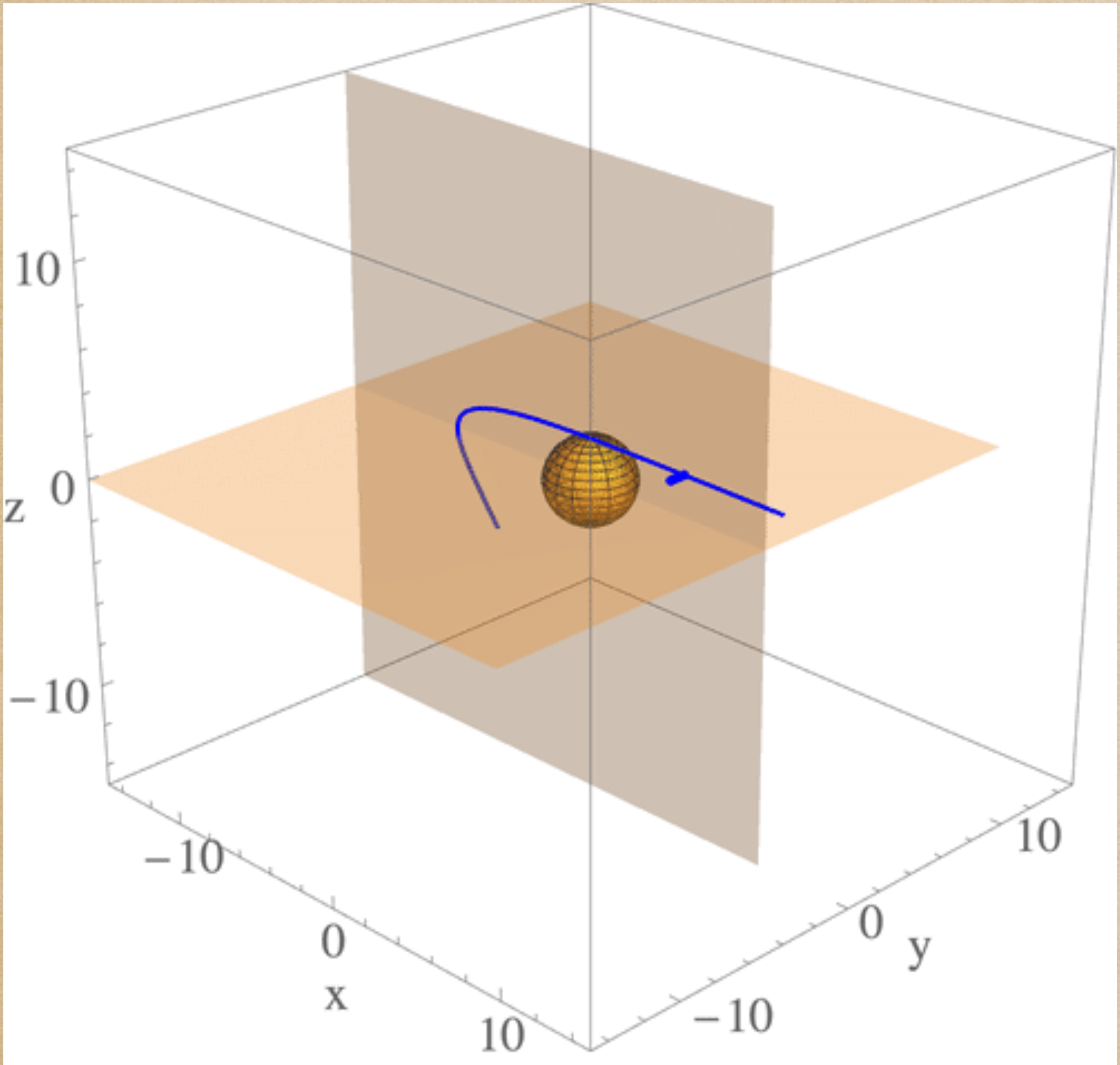
The mathematical shadow:

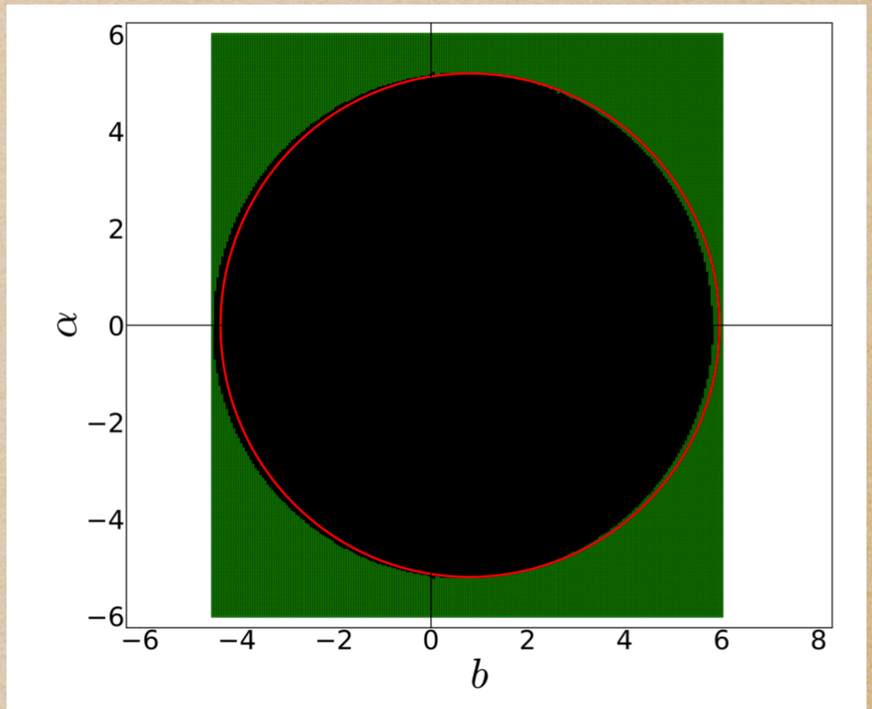




What is the shadow of a Black Hole?

The mathematical shadow:



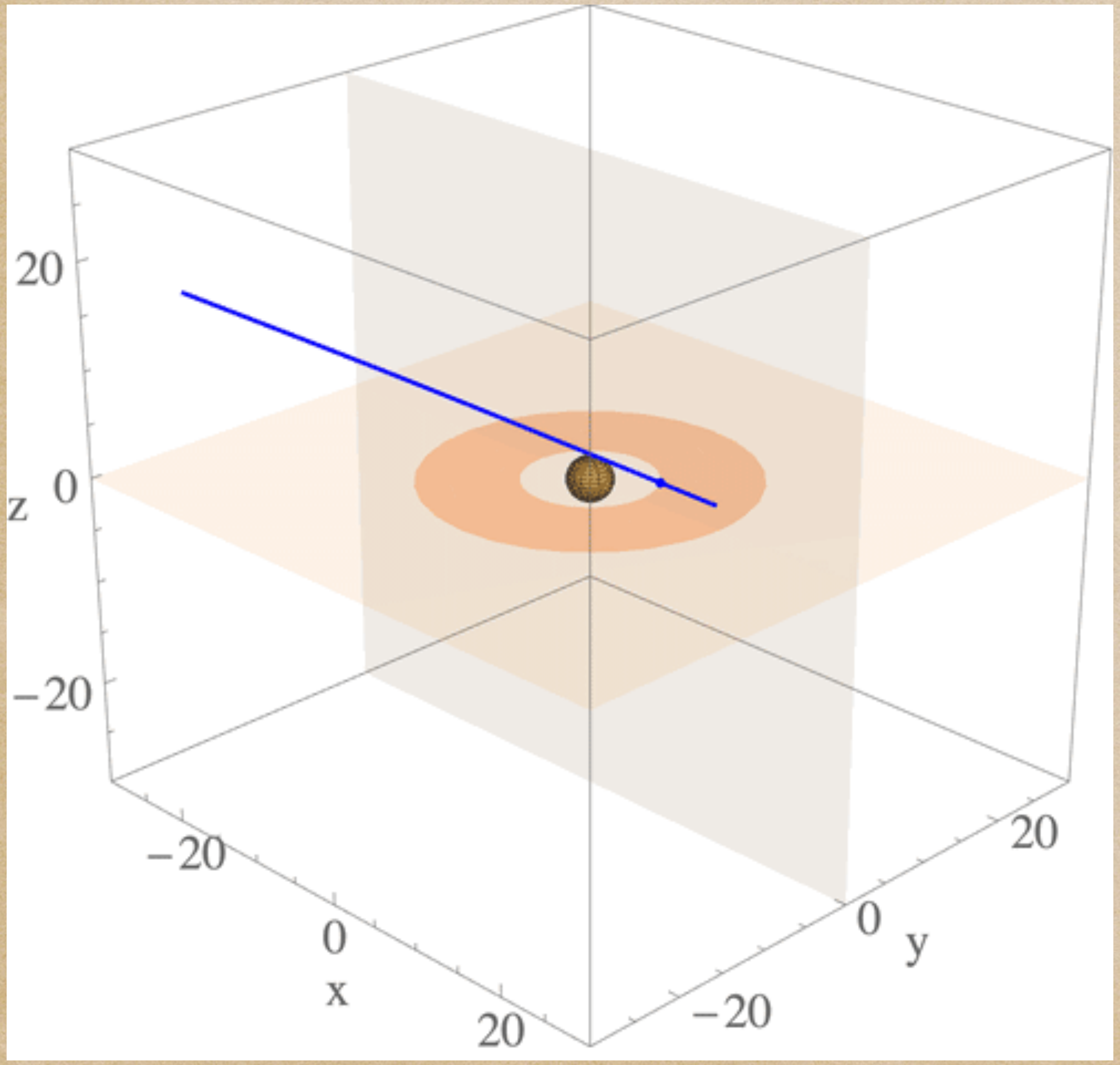


What is the shadow of a Black Hole?

The image of a thin accretion disc:

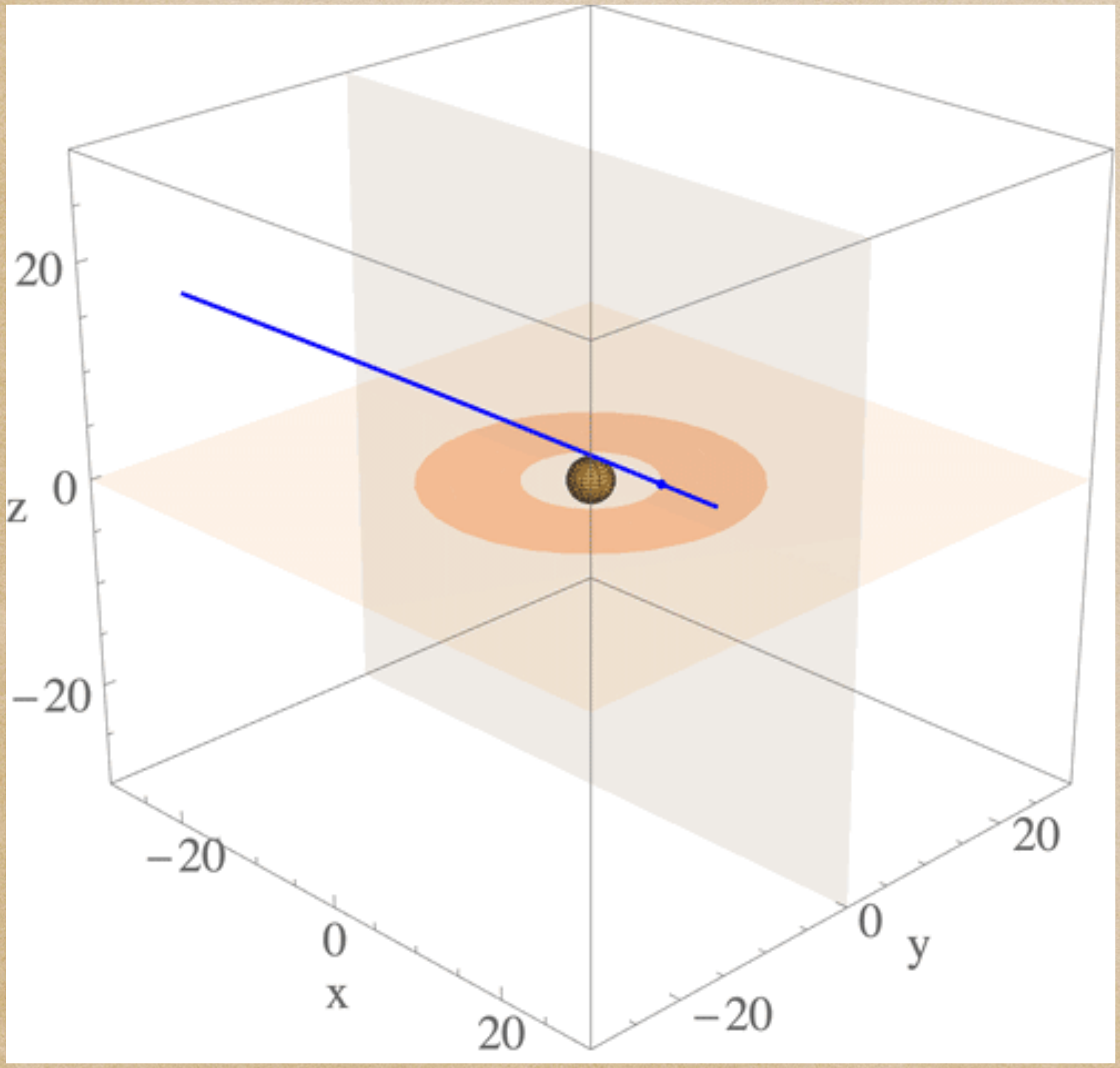
What is the shadow of a Black Hole?

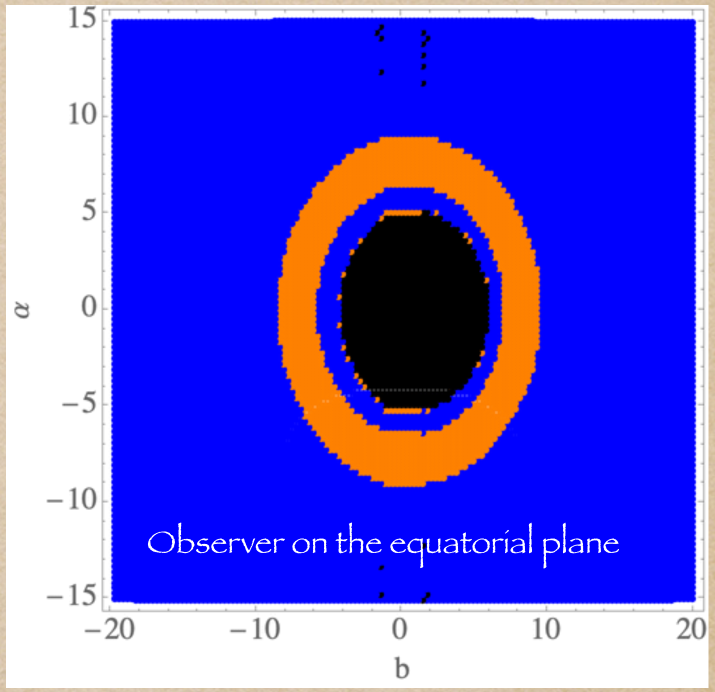
The image of a thin accretion disc:



What is the shadow of a Black Hole?

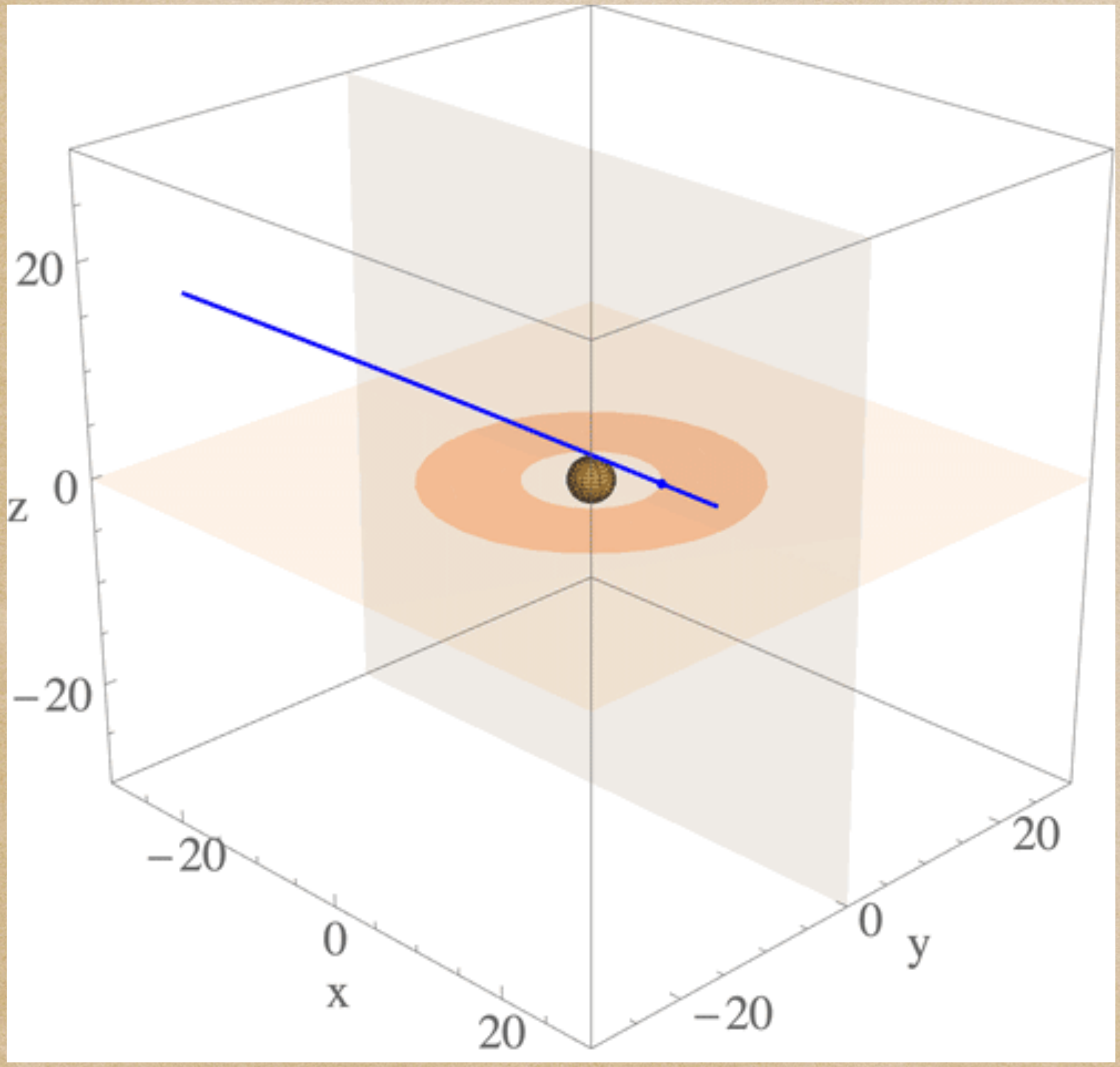
The image of a thin accretion disc:

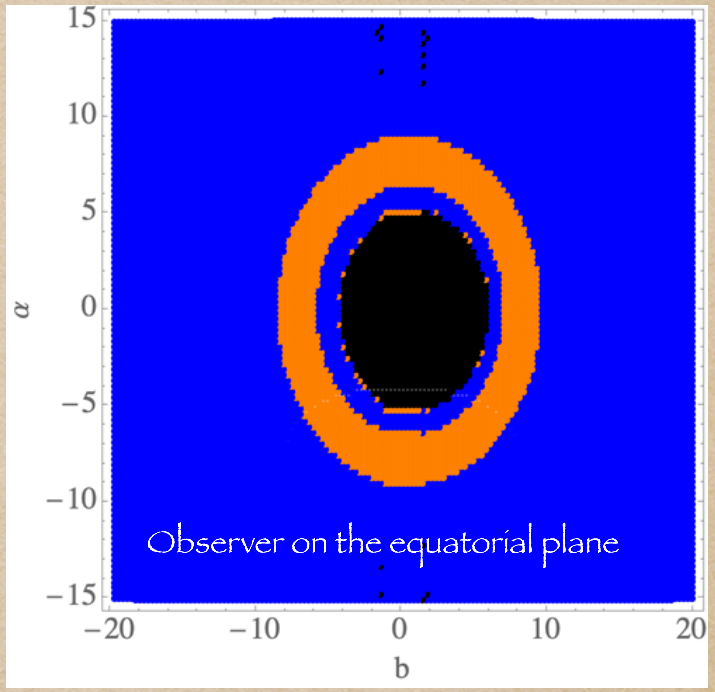


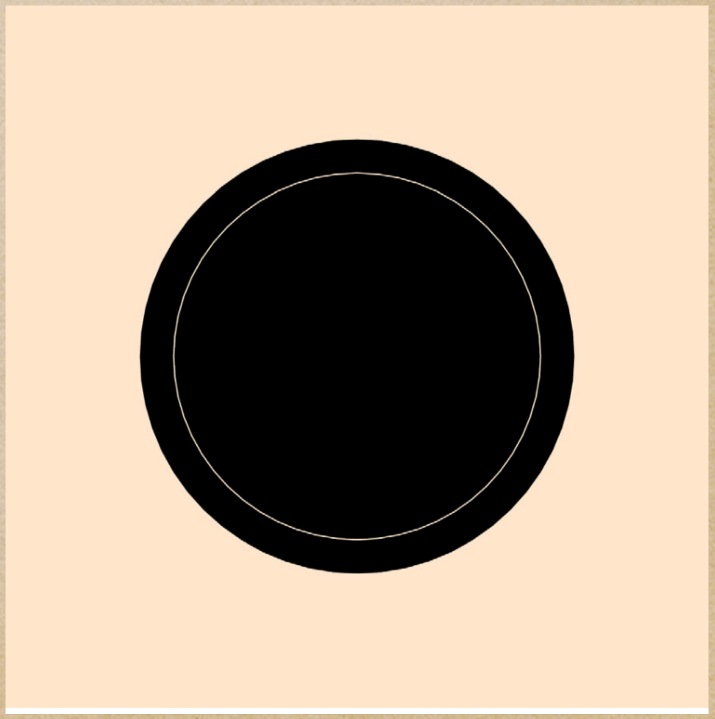


What is the shadow of a Black Hole?

The image of a thin accretion disc:

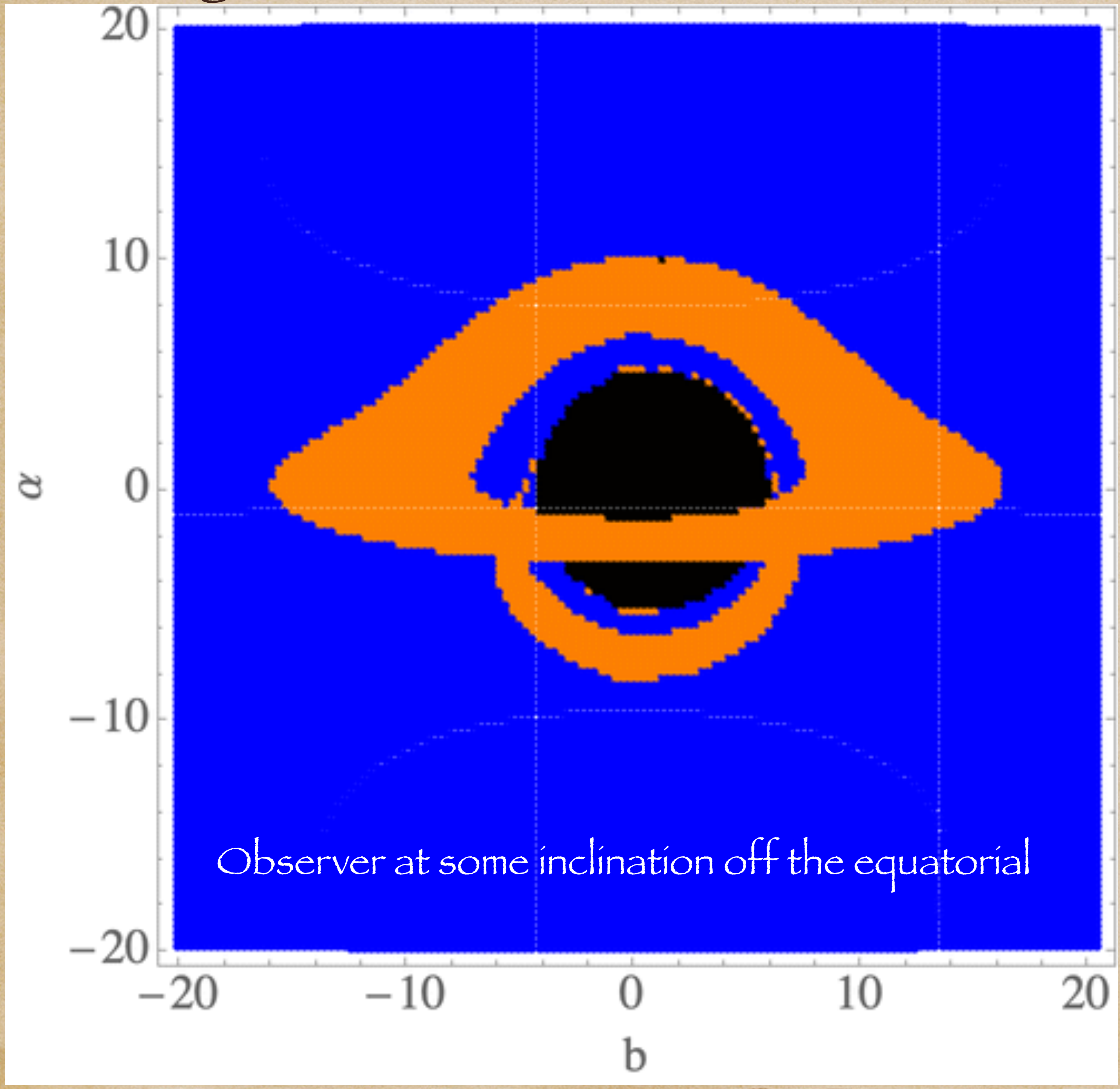


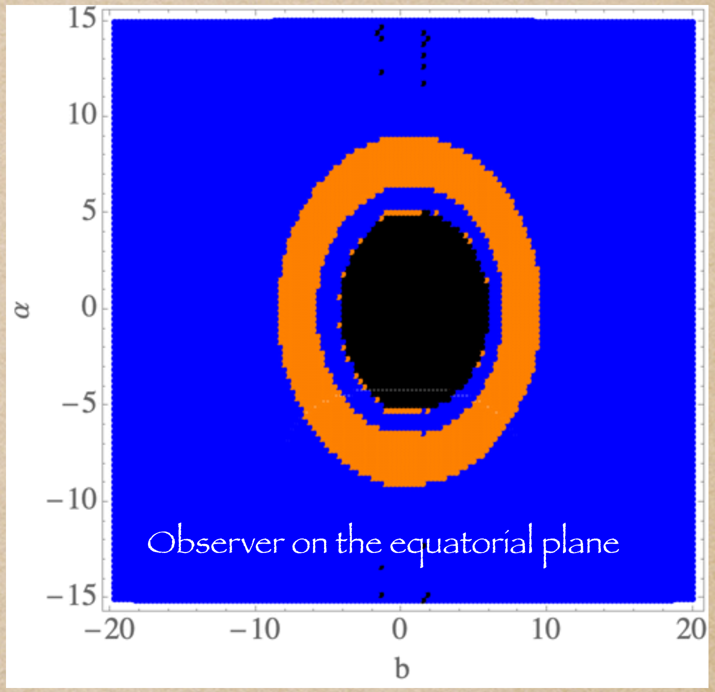


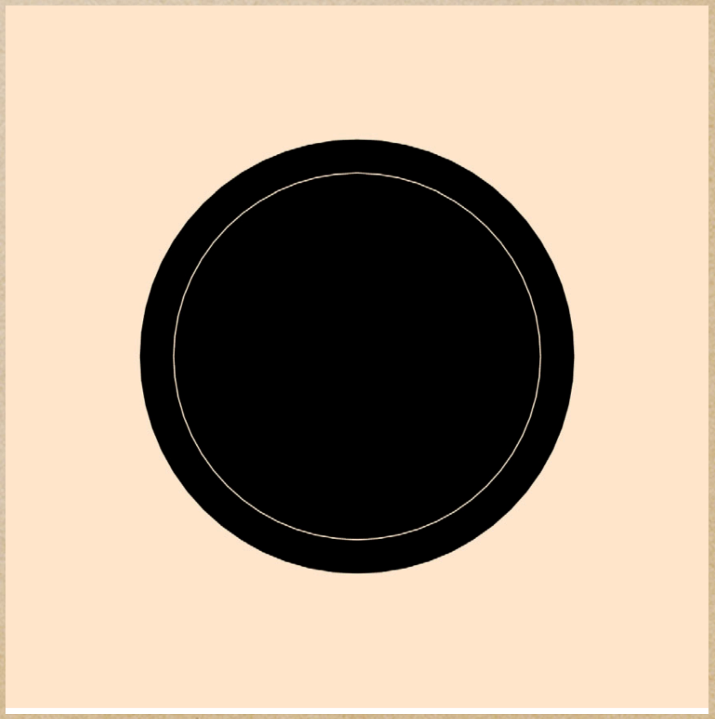


What is the shadow of a Black Hole?

The image of a thin accretion disc:

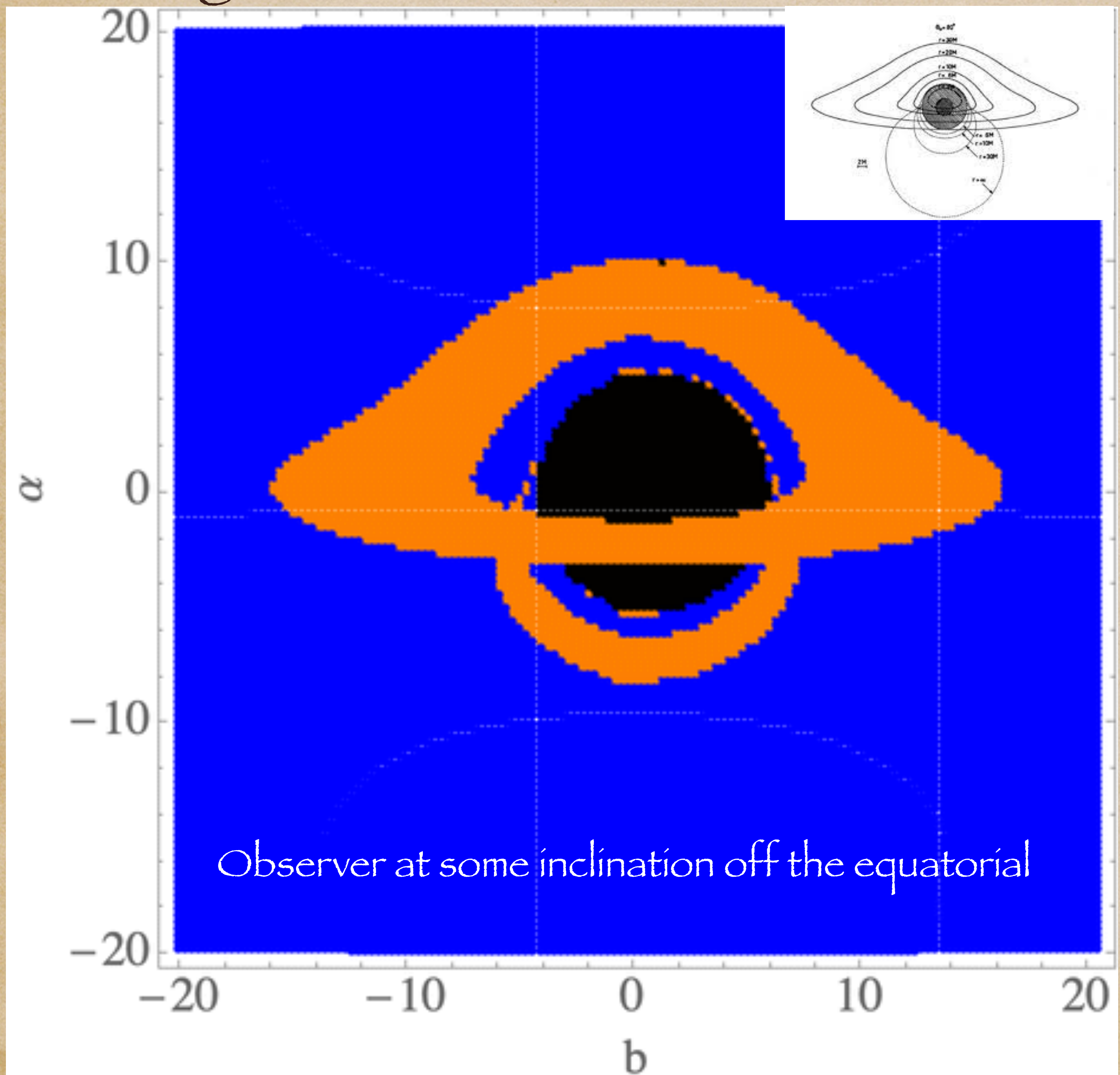


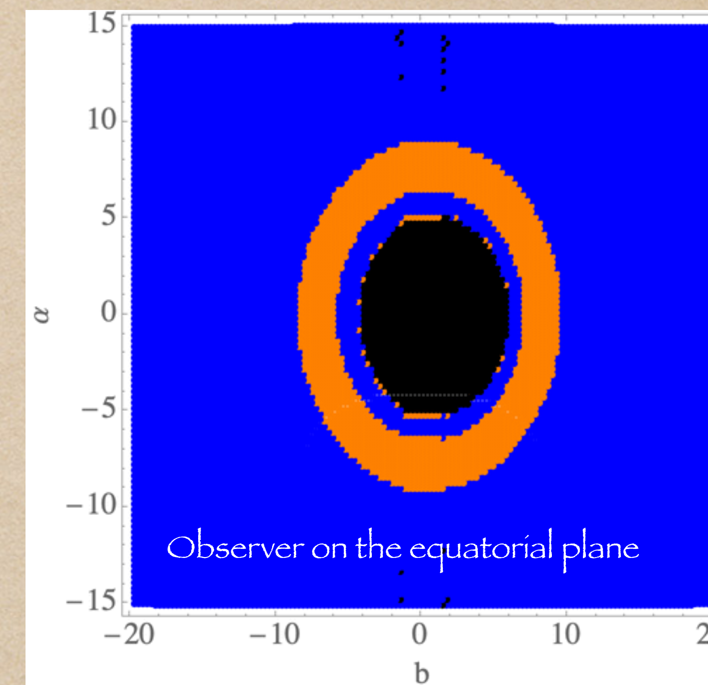


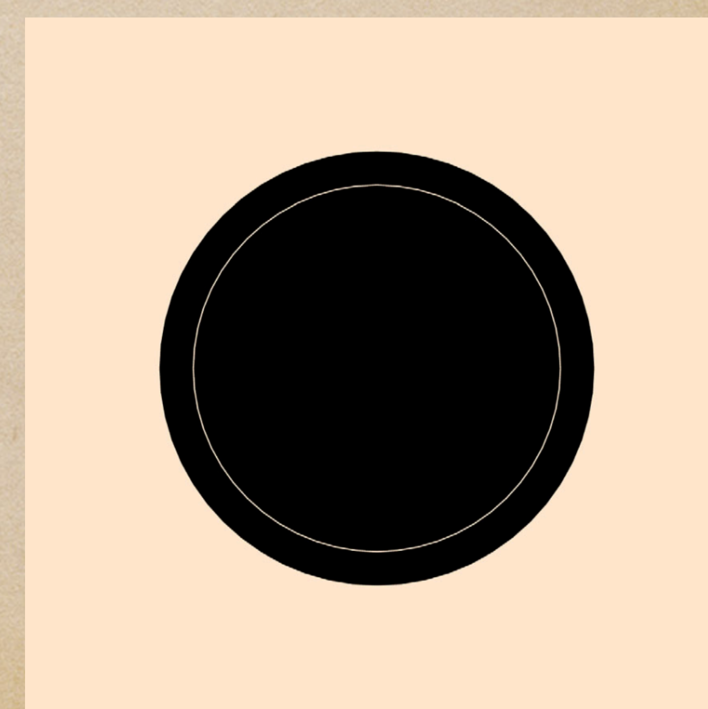


What is the shadow of a Black Hole?

The image of a thin accretion disc:

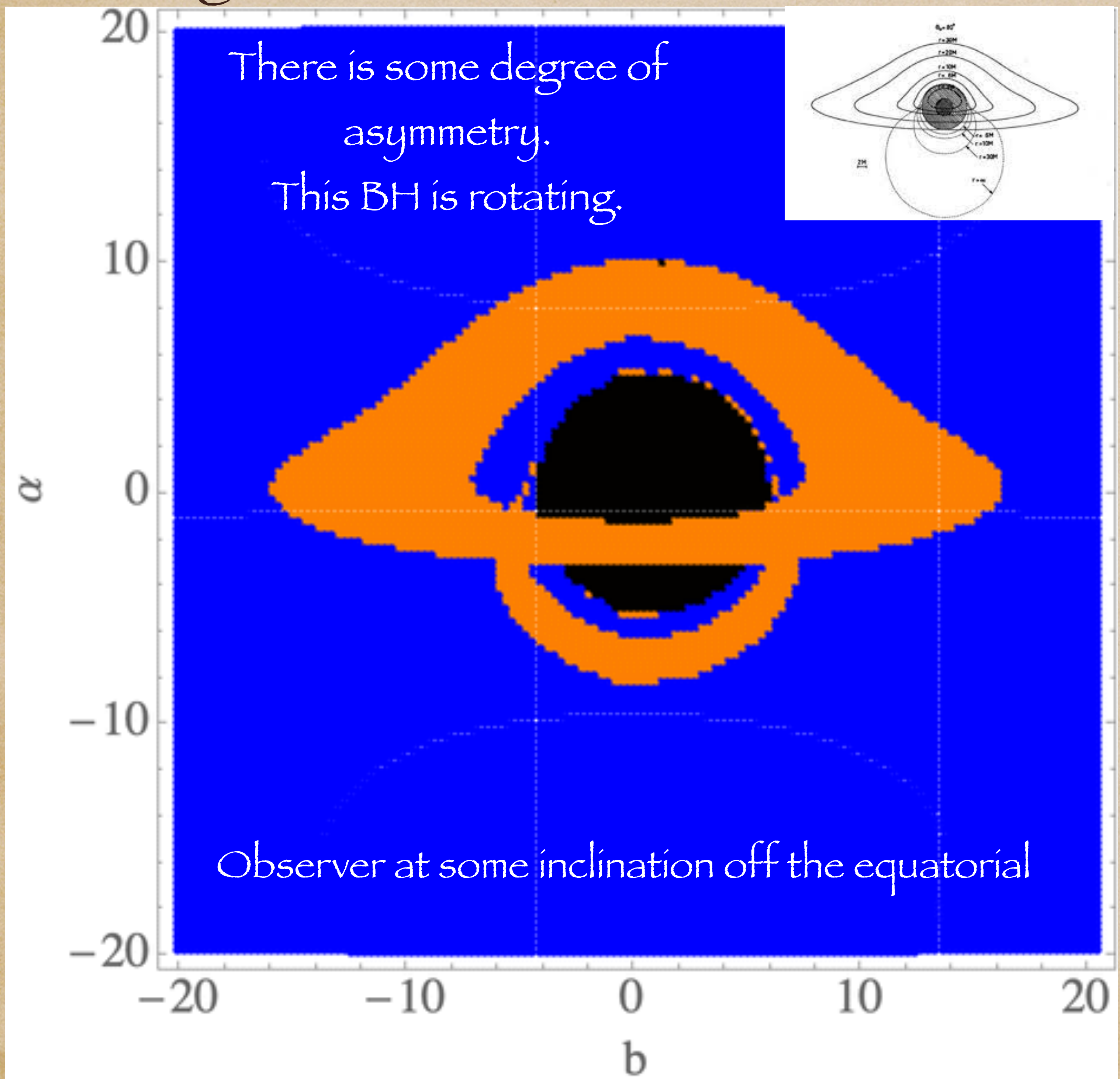


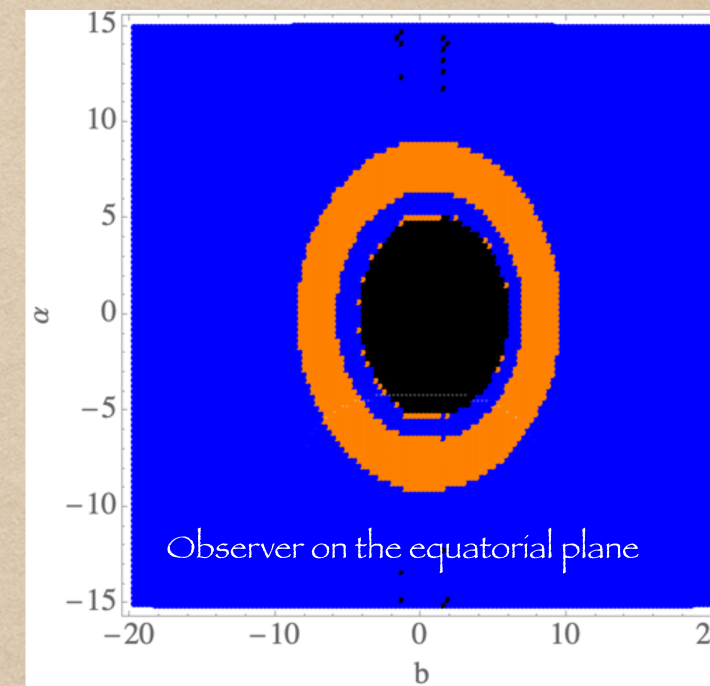


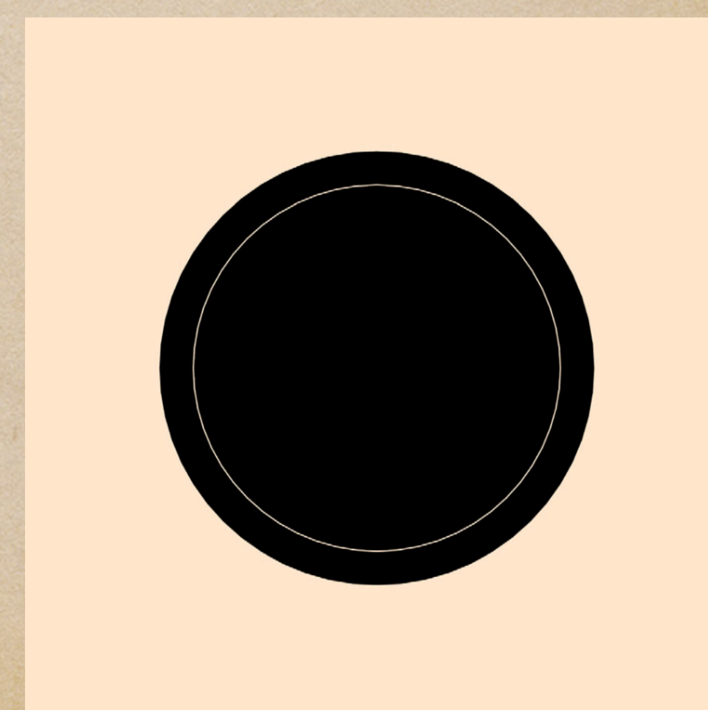


What is the shadow of a Black Hole?

The image of a thin accretion disc:

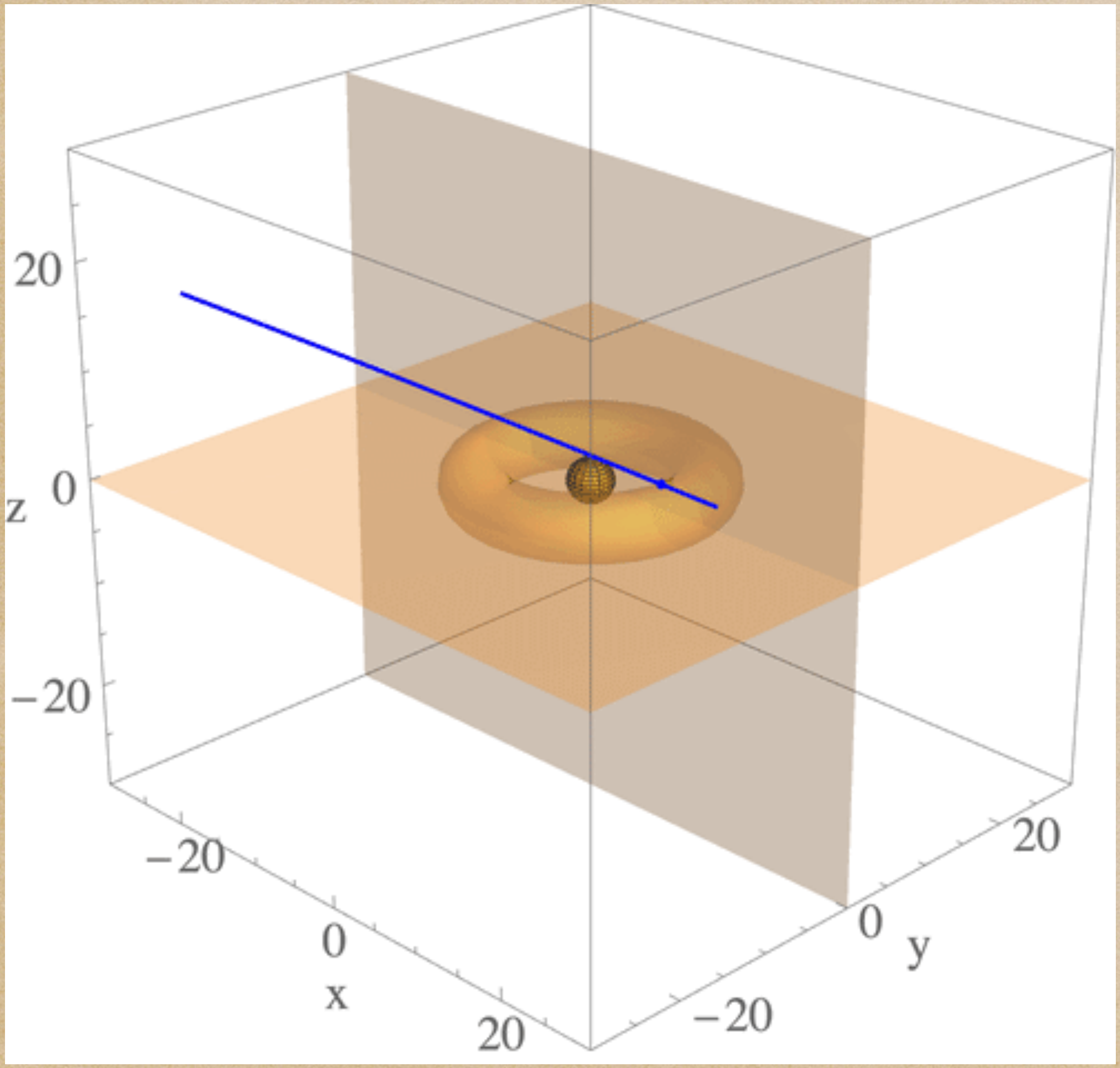


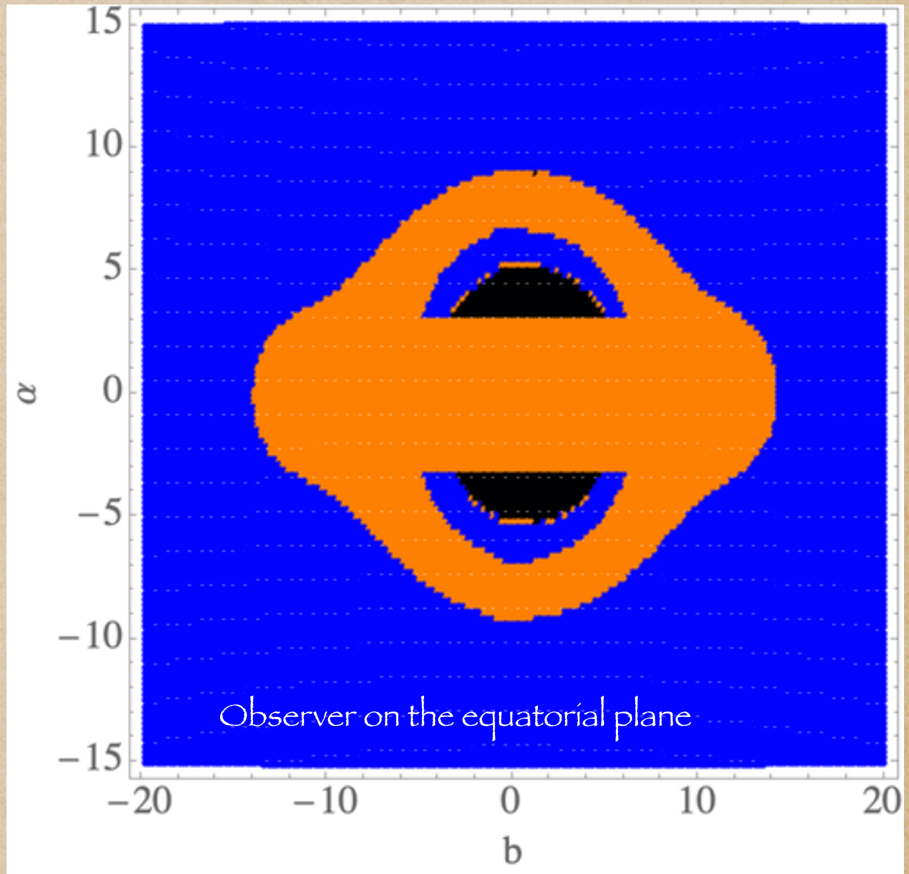




What is the shadow of a Black Hole?

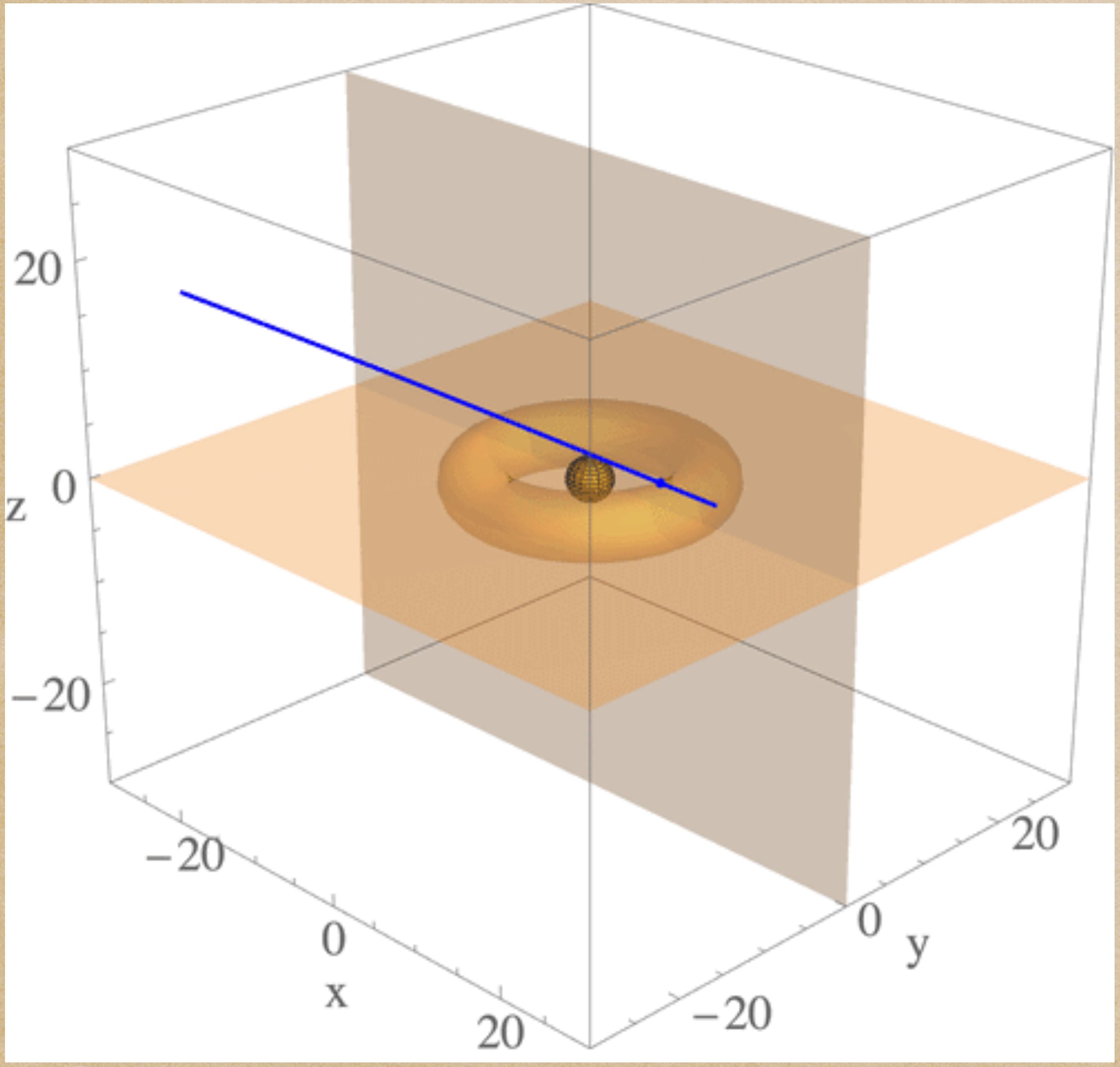
The image of a thick accretion disc:

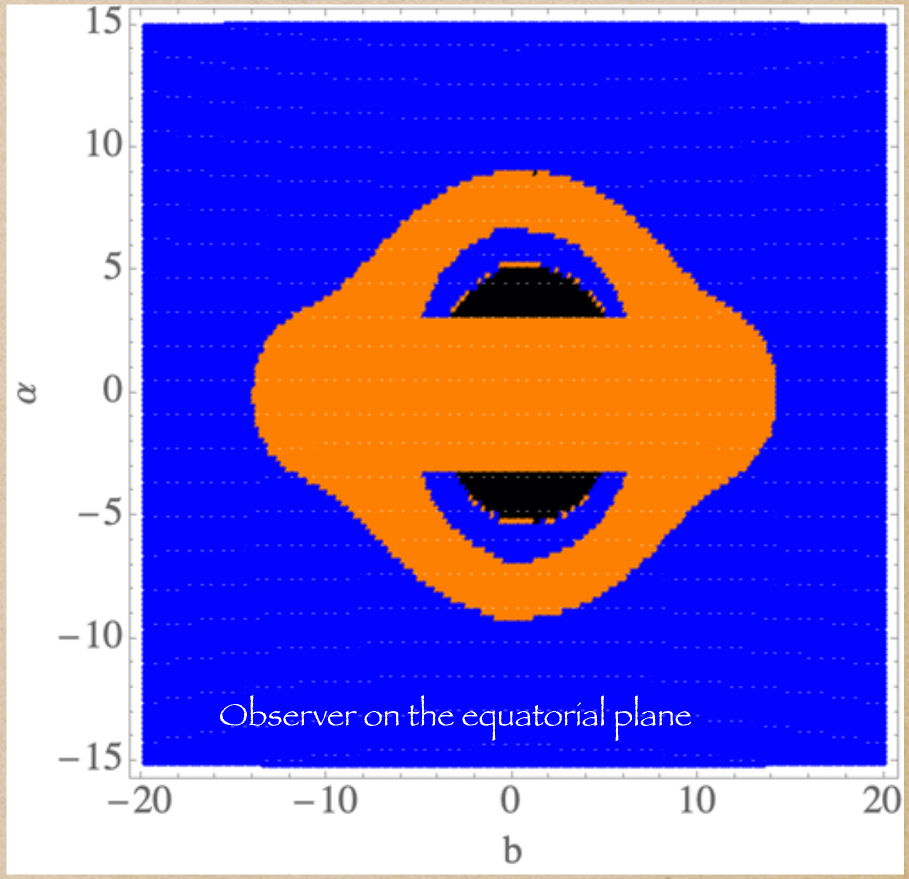




What is the shadow of a Black Hole?

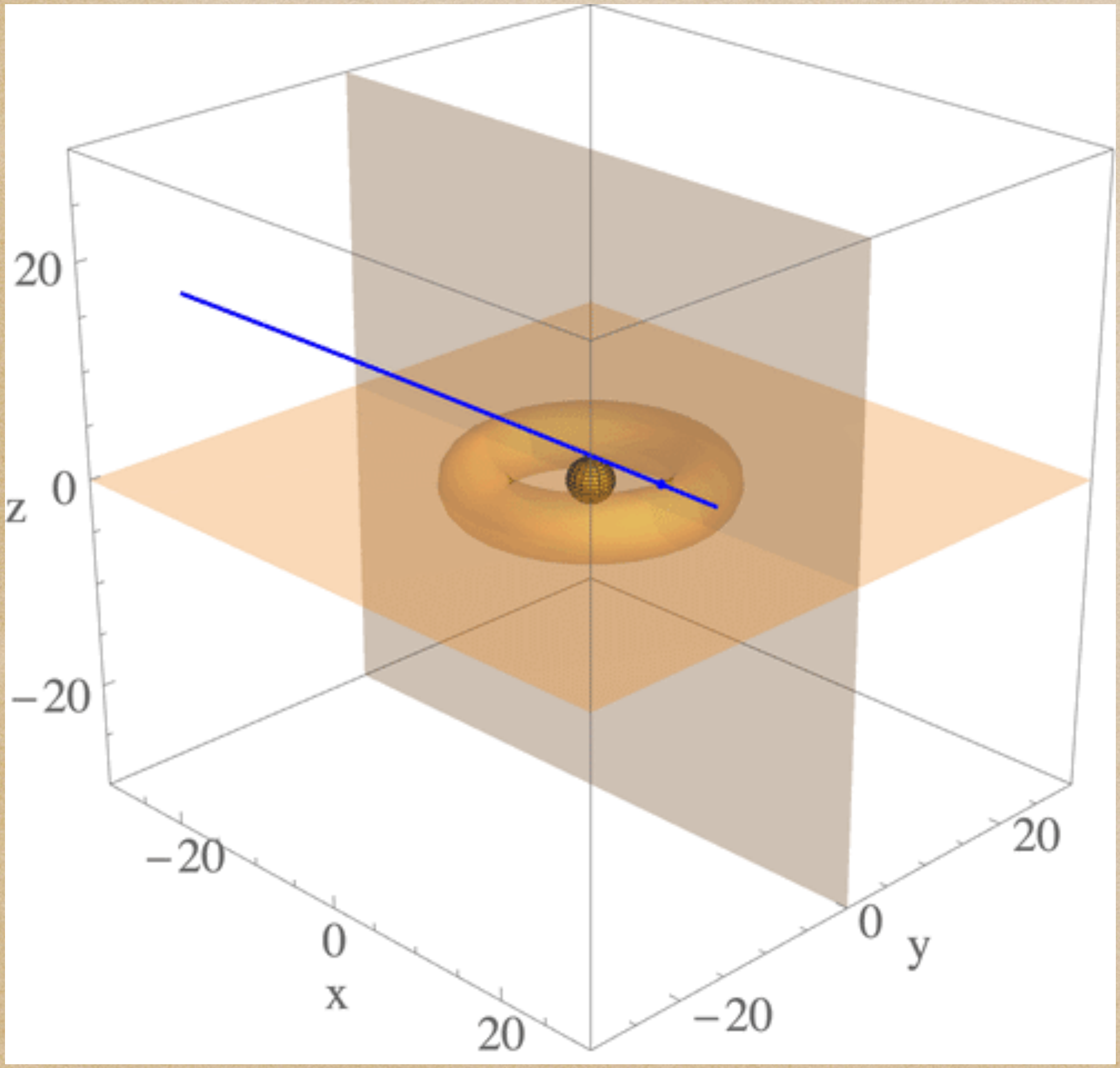
The image of a thick accretion disc:

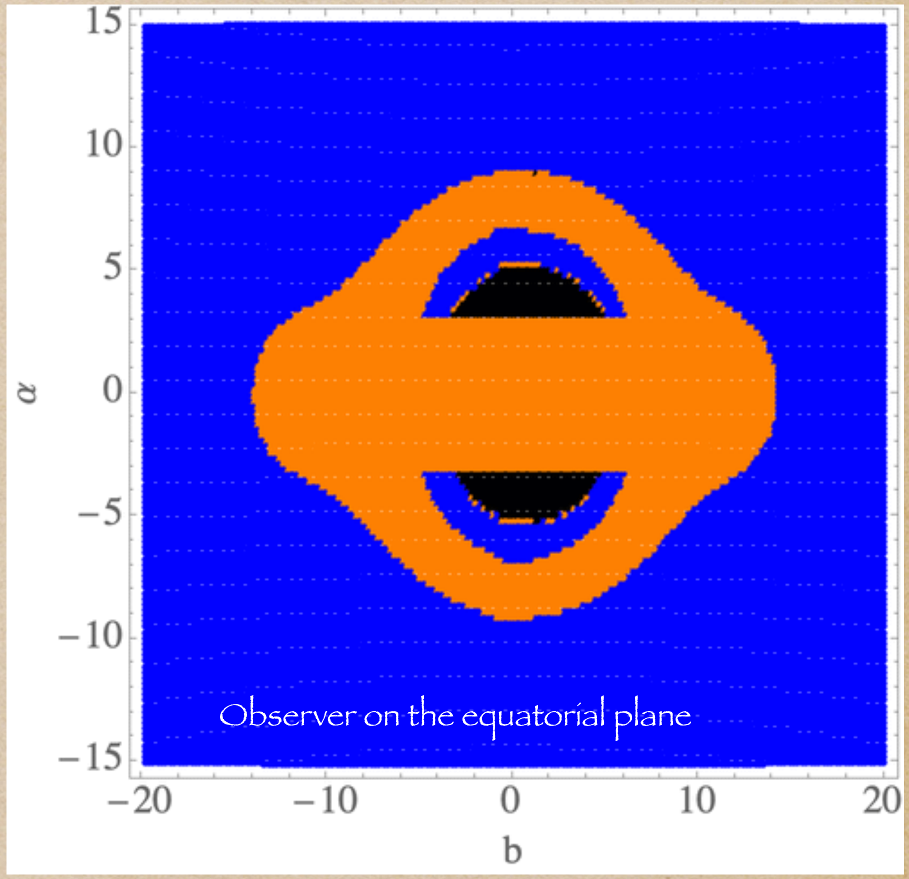




What is the shadow of a Black Hole?

The image of a thick accretion disc:





This BH is also rotating.

What is the shadow of a Black Hole?

Summary on shadows:

What is the shadow of a Black Hole?

Summary on shadows:

- The BH shadow is the result of the combination of an absorbing surface (the horizon) and the existence of the unstable spherical photon orbits.

What is the shadow of a Black Hole?

Summary on shadows:

- The BH shadow is the result of the combination of an absorbing surface (the horizon) and the existence of the unstable spherical photon orbits.
- The size of the shadow is determined by the impact parameter of these spherical photon orbits, and is essentially the angular size of these orbits as seen at infinity.

What is the shadow of a Black Hole?

Summary on shadows:

- The BH shadow is the result of the combination of an absorbing surface (the horizon) and the existence of the unstable spherical photon orbits.
- The size of the shadow is determined by the impact parameter of these spherical photon orbits, and is essentially the angular size of these orbits as seen at infinity.
- For a non-rotating Black Hole that shadow is circular and its size is equal to $3\sqrt{3}M \simeq 5.2M$.

What is the shadow of a Black Hole?

Summary on shadows:

- The BH shadow is the result of the combination of an absorbing surface (the horizon) and the existence of the unstable spherical photon orbits.
- The size of the shadow is determined by the impact parameter of these spherical photon orbits, and is essentially the angular size of these orbits as seen at infinity.
- For a non-rotating Black Hole that shadow is circular and its size is equal to $3\sqrt{3}M \simeq 5.2M$.
- Rotating Kerr Black Holes have an approximately circular shadow for a wide range of rotations, that is also close to $5.2M$.

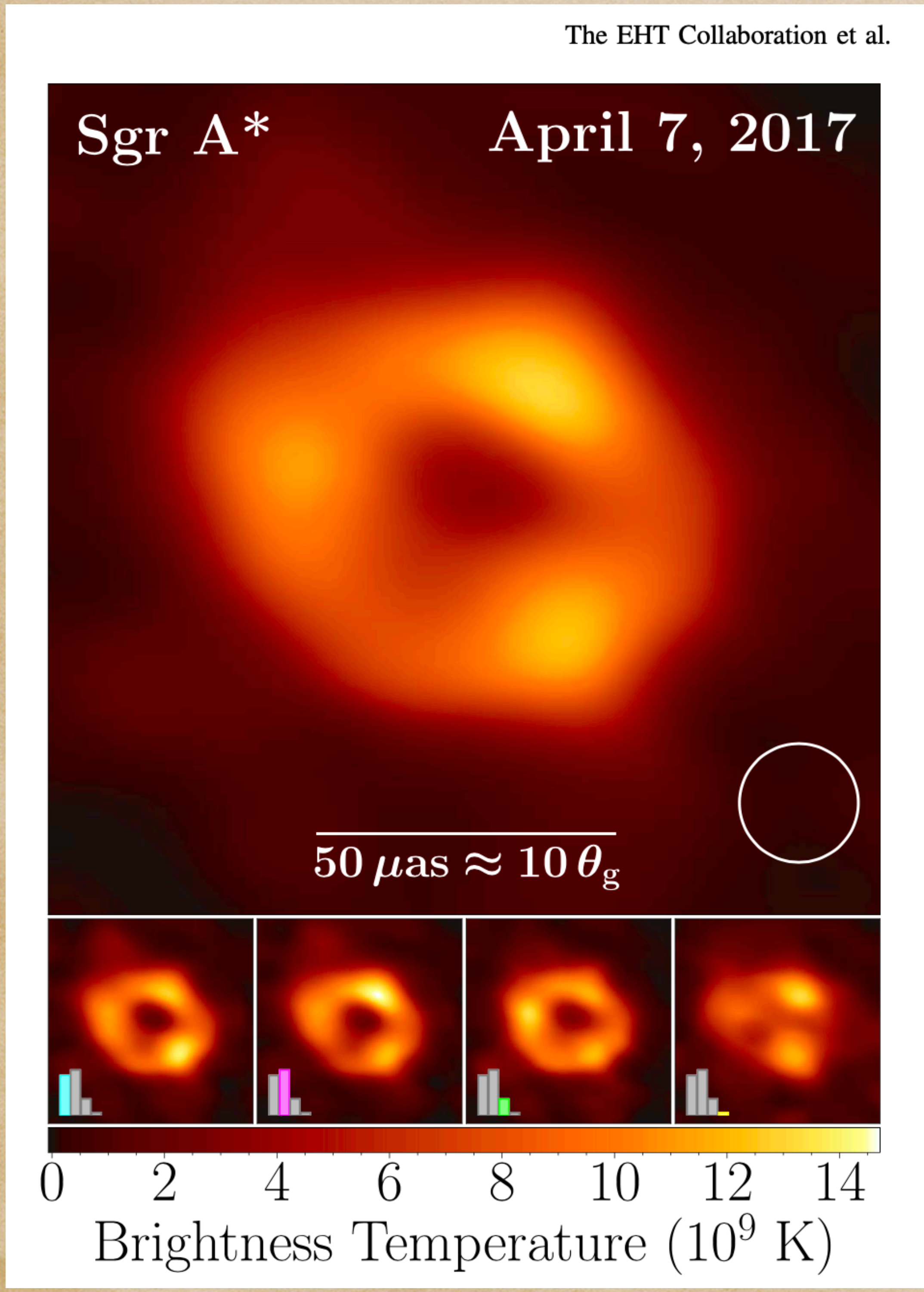
What is the shadow of a Black Hole?

Summary on shadows:

- The BH shadow is the result of the combination of an absorbing surface (the horizon) and the existence of the unstable spherical photon orbits.
- The size of the shadow is determined by the impact parameter of these spherical photon orbits, and is essentially the angular size of these orbits as seen at infinity.
- For a non-rotating Black Hole that shadow is circular and its size is equal to $3\sqrt{3}M \simeq 5.2M$.
- Rotating Kerr Black Holes have an approximately circular shadow for a wide range of rotations, that is also close to $5.2M$.
- In the end, the image of a BH also depends on the source of light and is not just the mathematical shadow.

The shadow of Sgr A*

The shadow of Sgr A*



The shadow of Sgr A*

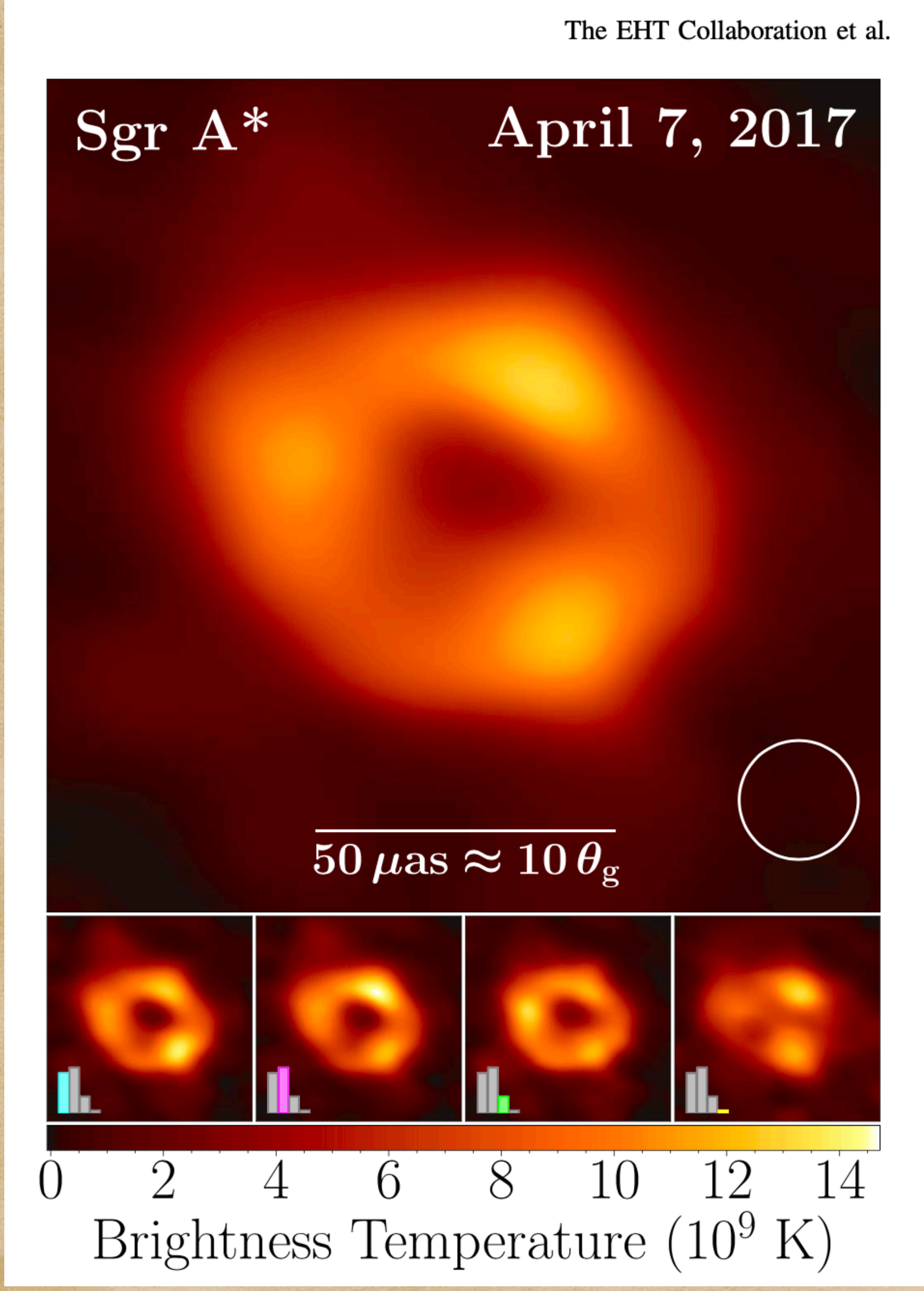


Table 1
Measured Parameters of Sgr A*

Parameter	EHT Estimate
Emission ring:[a]	
Diameter, d	51.8 ± 2.3 μas
Fractional width, W/d	∼30–50
Orientation, η	…
Brightness asymmetry, A	∼0.04%–0.3%
Angular gravitational radius,[a] θ_g	$4.8^{+1.4}_{-0.7}$ μas
Black hole mass,[b] M	$4.0^{+1.1}_{-0.6} \times 10^6 M_\odot$
Angular shadow diameter,[c] d_{sh}	48.7 ± 7.0 μas
Schwarzschild shadow deviation,[c] δ	$-0.08^{+0.09}_{-0.09}$ (VLTI)
	$-0.04^{+0.09}_{-0.10}$ (Keck)
Parameter	**Previous Estimate**
Angular gravitational radius, θ_g:	
Stellar orbits (VLTI)[d]	5.125 ± 0.009 ± 0.020 μas
Stellar orbits (Keck)[e]	4.92 ± 0.03 ± 0.01 μas
Black hole distance, D:	
Stellar orbits (VLTI)[d]	8277 ± 9 ± 33 pc
Stellar orbits (Keck)[e]	7935 ± 50 ± 32 pc
Masers (cm VLBI)	8150 ± 150 pc
Black hole mass, M:	
Stellar orbits (VLTI)[d]	$(4.297 \pm 0.013) \times 10^6 M_\odot$
Stellar orbits (Keck)[e]	$(3.951 \pm 0.047) \times 10^6 M_\odot$

The shadow of Sgr A*

Table 1

The Astrophysical Journal Letters, 930:L12 (21pp), 2022 May 10

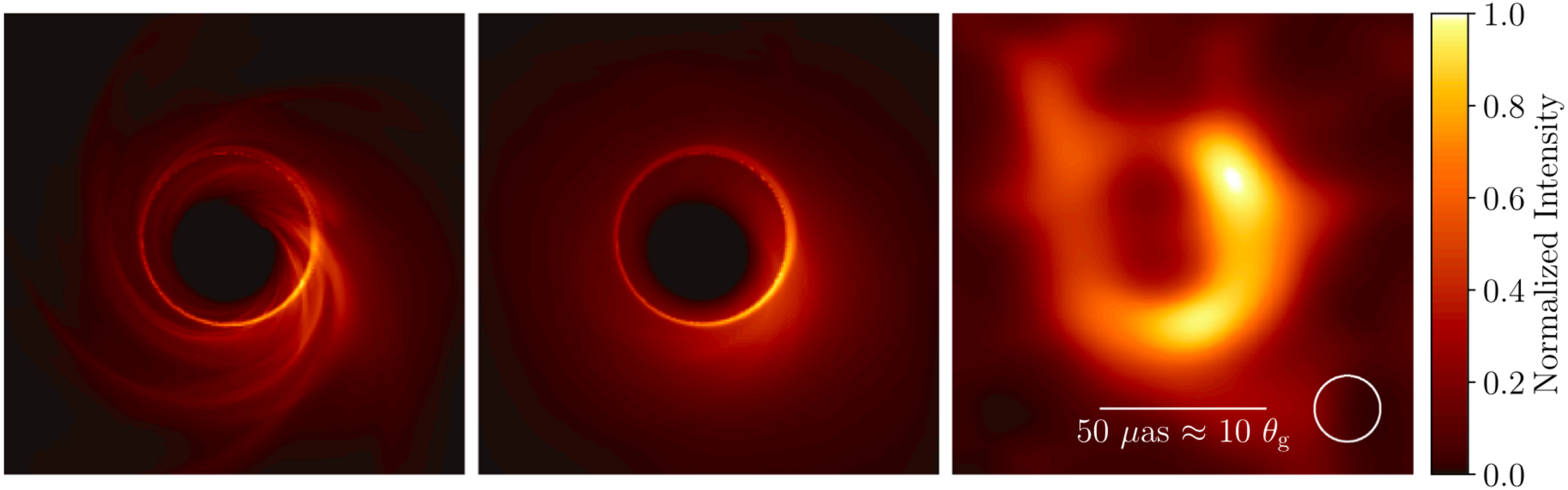
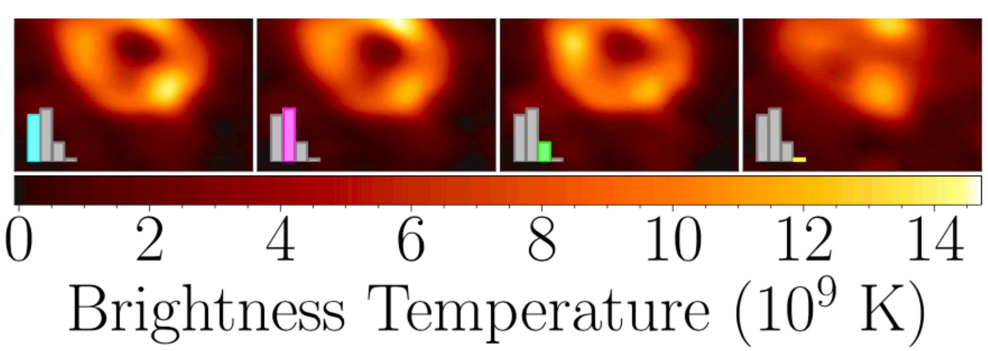


Figure 5. Simulated images of Sgr A*. Left: a single snapshot image of a numerical simulation of Sgr A* that passes 10 out of the 11 observational criteria described in Paper V. Middle: the average of this simulation with time sampling that matches the EHT observational cadence on April 7. Right: representative image reconstruction using synthetic visibilities generated from the simulation in the adjacent panels (see Appendix H in Paper III). This image has been averaged across methodologies and reconstructed morphologies, as in Figure 3. Each panel is shown on a linear brightness scale that is normalized to its peak.



Stellar orbits (Keck)[e]	7935 ± 50 ± 32 pc
Masers (cm VLBI)	8150 ± 150 pc
Black hole mass, M:	
Stellar orbits (VLTI)[d]	$(4.297 \pm 0.013) \times 10^6\ M_\odot$
Stellar orbits (Keck)[e]	$(3.951 \pm 0.047) \times 10^6\ M_\odot$

The Black Hole in Sgr A*

Table 1
Measured Parameters of Sgr A*

Parameter	EHT Estimate
Emission ring:[a]	
Diameter, d	$51.8 \pm 2.3\ \mu$as
Fractional width, W/d	~30–50
Orientation, η	…
Brightness asymmetry, A	~0.04%–0.3%
Angular gravitational radius,[a] θ_g	$4.8^{+1.4}_{-0.7}\ \mu$as
Black hole mass,[b] M	$4.0^{+1.1}_{-0.6} \times 10^6\ M_\odot$
Angular shadow diameter,[c] d_{sh}	$48.7 \pm 7.0\ \mu$as
Schwarzschild shadow deviation,[c] δ	$-0.08^{+0.09}_{-0.09}$ (VLTI)
	$-0.04^{+0.09}_{-0.10}$ (Keck)
Parameter	**Previous Estimate**
Angular gravitational radius, θ_g:	
Stellar orbits (VLTI)[d]	$5.125 \pm 0.009 \pm 0.020\ \mu$as
Stellar orbits (Keck)[e]	$4.92 \pm 0.03 \pm 0.01\ \mu$as
Black hole distance, D:	
Stellar orbits (VLTI)[d]	$8277 \pm 9 \pm 33$ pc
Stellar orbits (Keck)[e]	$7935 \pm 50 \pm 32$ pc
Masers (cm VLBI)	8150 ± 150 pc
Black hole mass, M:	
Stellar orbits (VLTI)[d]	$(4.297 \pm 0.013) \times 10^6\ M_\odot$
Stellar orbits (Keck)[e]	$(3.951 \pm 0.047) \times 10^6\ M_\odot$

The Black Hole in Sgr A*

- We have very strong evidence from astrometry for a very massive very compact object at the center of the Milky Way.

Table 1
Measured Parameters of Sgr A*

Parameter	EHT Estimate
Emission ring:[a]	
Diameter, d	$51.8 \pm 2.3\ \mu$as
Fractional width, W/d	$\sim$30–50
Orientation, η	…
Brightness asymmetry, A	$\sim$0.04%–0.3%
Angular gravitational radius,[a] θ_g	$4.8^{+1.4}_{-0.7}\ \mu$as
Black hole mass,[b] M	$4.0^{+1.1}_{-0.6} \times 10^6\ M_\odot$
Angular shadow diameter,[c] d_{sh}	$48.7 \pm 7.0\ \mu$as
Schwarzschild shadow deviation,[c] δ	$-0.08^{+0.09}_{-0.09}$ (VLTI)
	$-0.04^{+0.09}_{-0.10}$ (Keck)

Parameter	Previous Estimate
Angular gravitational radius, θ_g:	
Stellar orbits (VLTI)[d]	$5.125 \pm 0.009 \pm 0.020\ \mu$as
Stellar orbits (Keck)[e]	$4.92 \pm 0.03 \pm 0.01\ \mu$as
Black hole distance, D:	
Stellar orbits (VLTI)[d]	$8277 \pm 9 \pm 33$ pc
Stellar orbits (Keck)[e]	$7935 \pm 50 \pm 32$ pc
Masers (cm VLBI)	8150 ± 150 pc
Black hole mass, M:	
Stellar orbits (VLTI)[d]	$(4.297 \pm 0.013) \times 10^6\ M_\odot$
Stellar orbits (Keck)[e]	$(3.951 \pm 0.047) \times 10^6\ M_\odot$

The Black Hole in Sgr A*

- We have very strong evidence from astrometry for a very massive very compact object at the center of the Milky Way.
- The EHT telescope has provided further evidence for this very massive compact object, in good agreement with astrometry.

Table 1
Measured Parameters of Sgr A*

Parameter	EHT Estimate
Emission ring:[a]	
Diameter, d	51.8 ± 2.3 μas
Fractional width, W/d	$\sim$30–50
Orientation, η	…
Brightness asymmetry, A	$\sim$0.04%–0.3%
Angular gravitational radius,[a] θ_g	$4.8^{+1.4}_{-0.7}$ μas
Black hole mass,[b] M	$4.0^{+1.1}_{-0.6} \times 10^6 M_\odot$
Angular shadow diameter,[c] d_{sh}	48.7 ± 7.0 μas
Schwarzschild shadow deviation,[c] δ	$-0.08^{+0.09}_{-0.09}$ (VLTI)
	$-0.04^{+0.09}_{-0.10}$ (Keck)
Parameter	**Previous Estimate**
Angular gravitational radius, θ_g:	
Stellar orbits (VLTI)[d]	$5.125 \pm 0.009 \pm 0.020$ μas
Stellar orbits (Keck)[e]	$4.92 \pm 0.03 \pm 0.01$ μas
Black hole distance, D:	
Stellar orbits (VLTI)[d]	$8277 \pm 9 \pm 33$ pc
Stellar orbits (Keck)[e]	$7935 \pm 50 \pm 32$ pc
Masers (cm VLBI)	8150 ± 150 pc
Black hole mass, M:	
Stellar orbits (VLTI)[d]	$(4.297 \pm 0.013) \times 10^6 M_\odot$
Stellar orbits (Keck)[e]	$(3.951 \pm 0.047) \times 10^6 M_\odot$

The Black Hole in Sgr A*

- We have very strong evidence from astrometry for a very massive very compact object at the center of the Milky Way.
- The EHT telescope has provided further evidence for this very massive compact object, in good agreement with astrometry.
- There are indications that the observed compact object may have a horizon, but this is not definitive yet, as it is very hard to prove.

Table 1
Measured Parameters of Sgr A*

Parameter	EHT Estimate
Emission ring:[a]	
Diameter, d	51.8 ± 2.3 μas
Fractional width, W/d	~30–50
Orientation, η	…
Brightness asymmetry, A	~0.04%–0.3%
Angular gravitational radius,[a] θ_g	$4.8^{+1.4}_{-0.7}$ μas
Black hole mass,[b] M	$4.0^{+1.1}_{-0.6} \times 10^6 M_\odot$
Angular shadow diameter,[c] d_{sh}	48.7 ± 7.0 μas
Schwarzschild shadow deviation,[c] δ	$-0.08^{+0.09}_{-0.09}$ (VLTI)
	$-0.04^{+0.09}_{-0.10}$ (Keck)
Parameter	**Previous Estimate**
Angular gravitational radius, θ_g:	
Stellar orbits (VLTI)[d]	$5.125 \pm 0.009 \pm 0.020$ μas
Stellar orbits (Keck)[e]	$4.92 \pm 0.03 \pm 0.01$ μas
Black hole distance, D:	
Stellar orbits (VLTI)[d]	$8277 \pm 9 \pm 33$ pc
Stellar orbits (Keck)[e]	$7935 \pm 50 \pm 32$ pc
Masers (cm VLBI)	8150 ± 150 pc
Black hole mass, M:	
Stellar orbits (VLTI)[d]	$(4.297 \pm 0.013) \times 10^6 M_\odot$
Stellar orbits (Keck)[e]	$(3.951 \pm 0.047) \times 10^6 M_\odot$