MHD with Physics Informed Neural Networks (PINNs)

Ioannis Contopoulos

Research Center for Astronomy, Academy of Athens

5th Summer School of Hel.A.S.

September 16, 2024, Ioannina

Outline

- Introduction to PINNs
- First PINN applications in MHD
- Pulsar Magnetospheres
- Solution of the Pulsar Equation with PINNs
- We can do better…
- Solution of the 3D Pulsar Equation with PINNs
- Future PINN applications

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Introduction to PINNs

Physics Informed NNs (PINNs)

ELSEVIER

Journal of Computational Physics

Volume 378, 1 February 2019, Pages 686-707

Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations

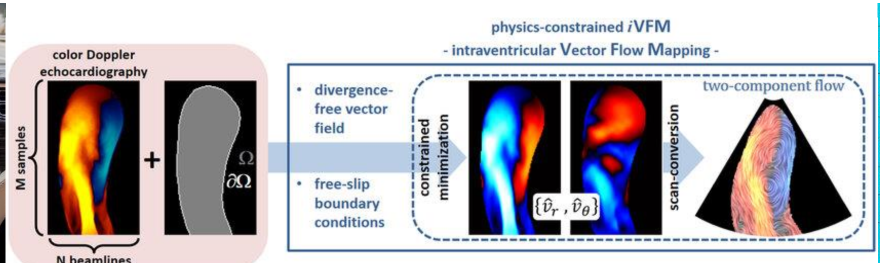
M. Raissi [a], P. Perdikaris [b], G.E. Karniadakis [a]

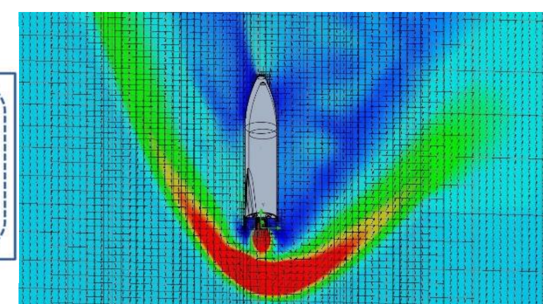
Neural Networks (NNs)

Complex models that can be trained to approximate any function or operation

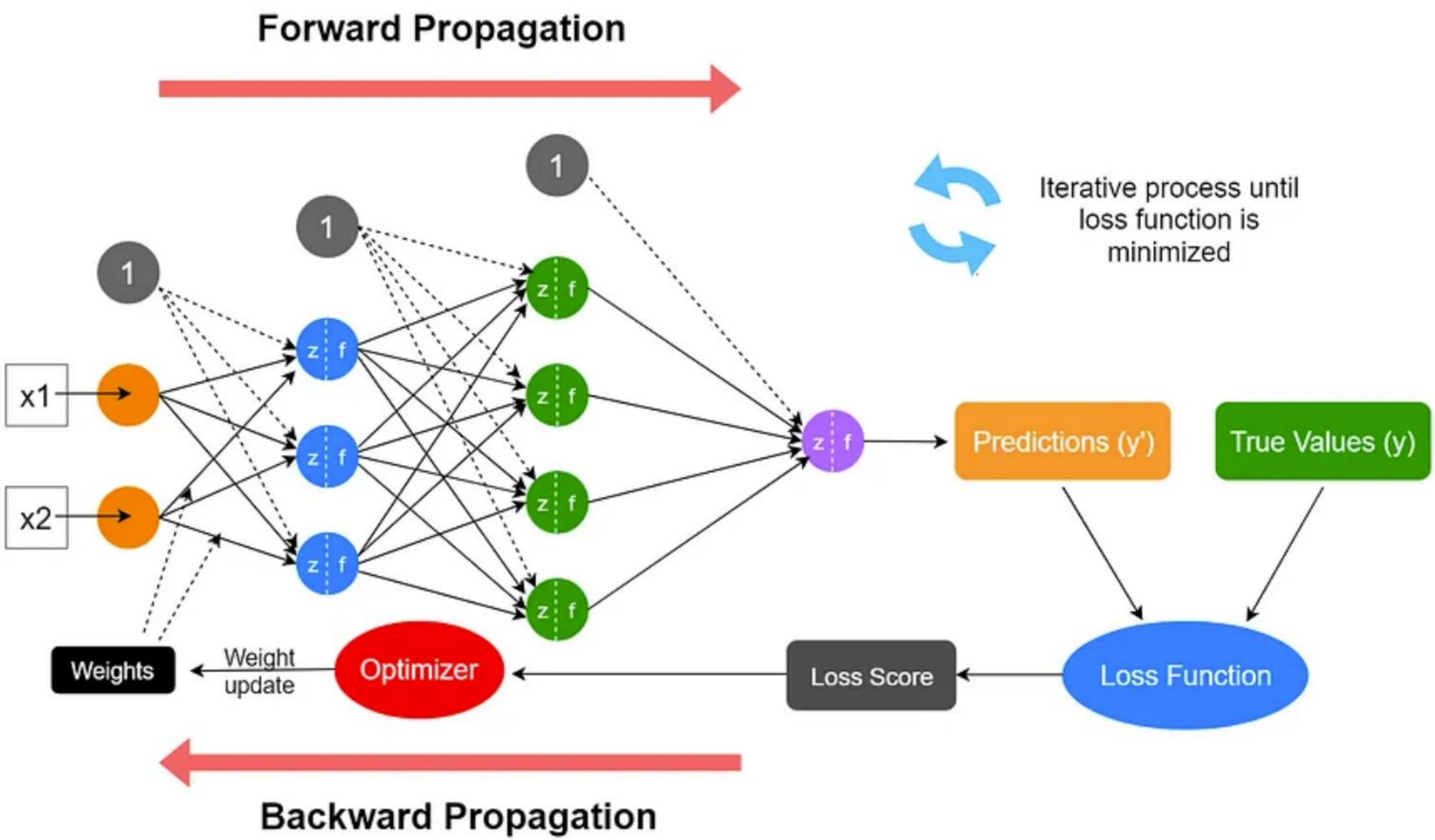
Physics Informed NNs (PINNs)

– Unknown physics/equations
– High cost of data acquisition
– High-dimensional data
– Missing data, complex noise processes
– Multiple formats (images, time-series, scattered data)

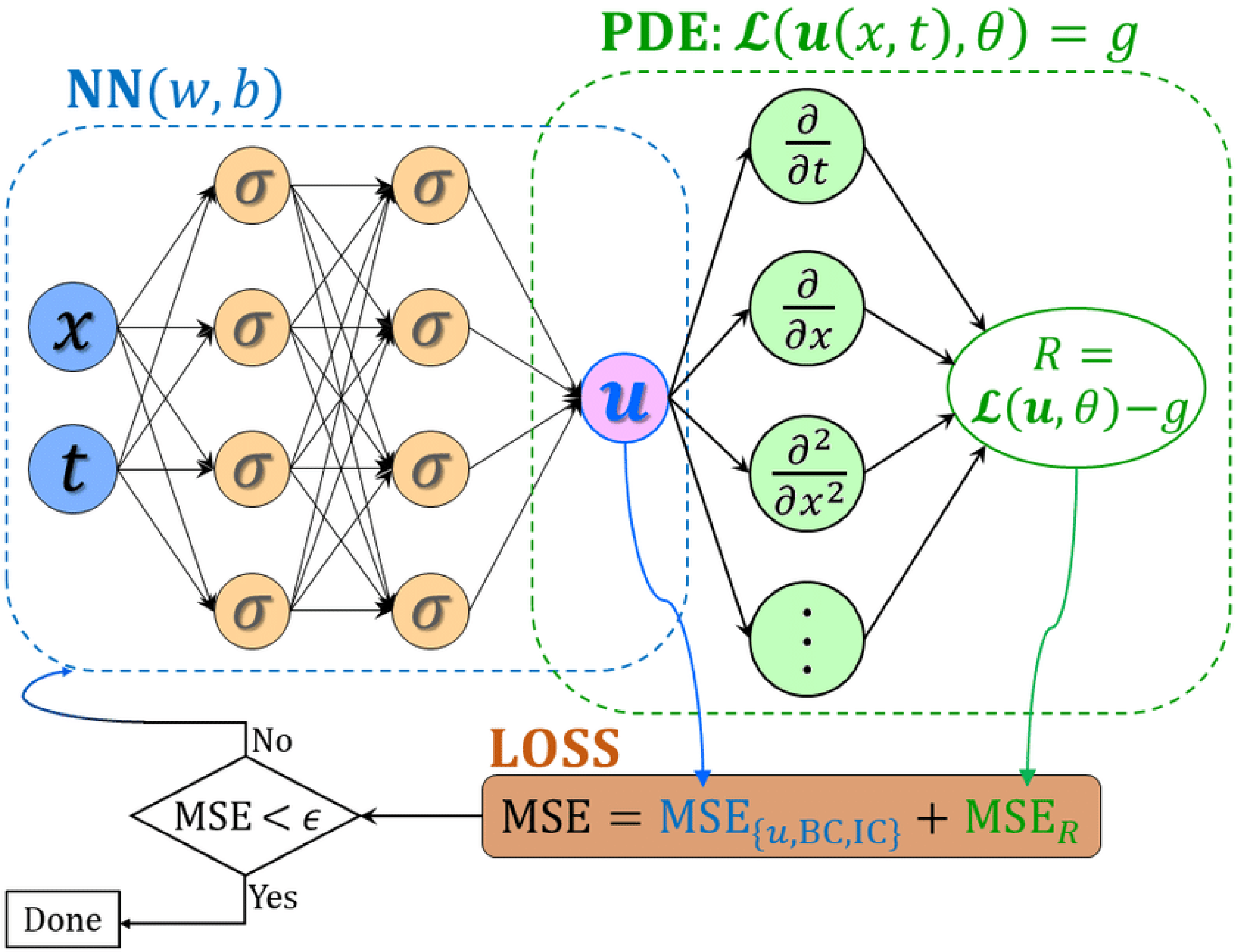


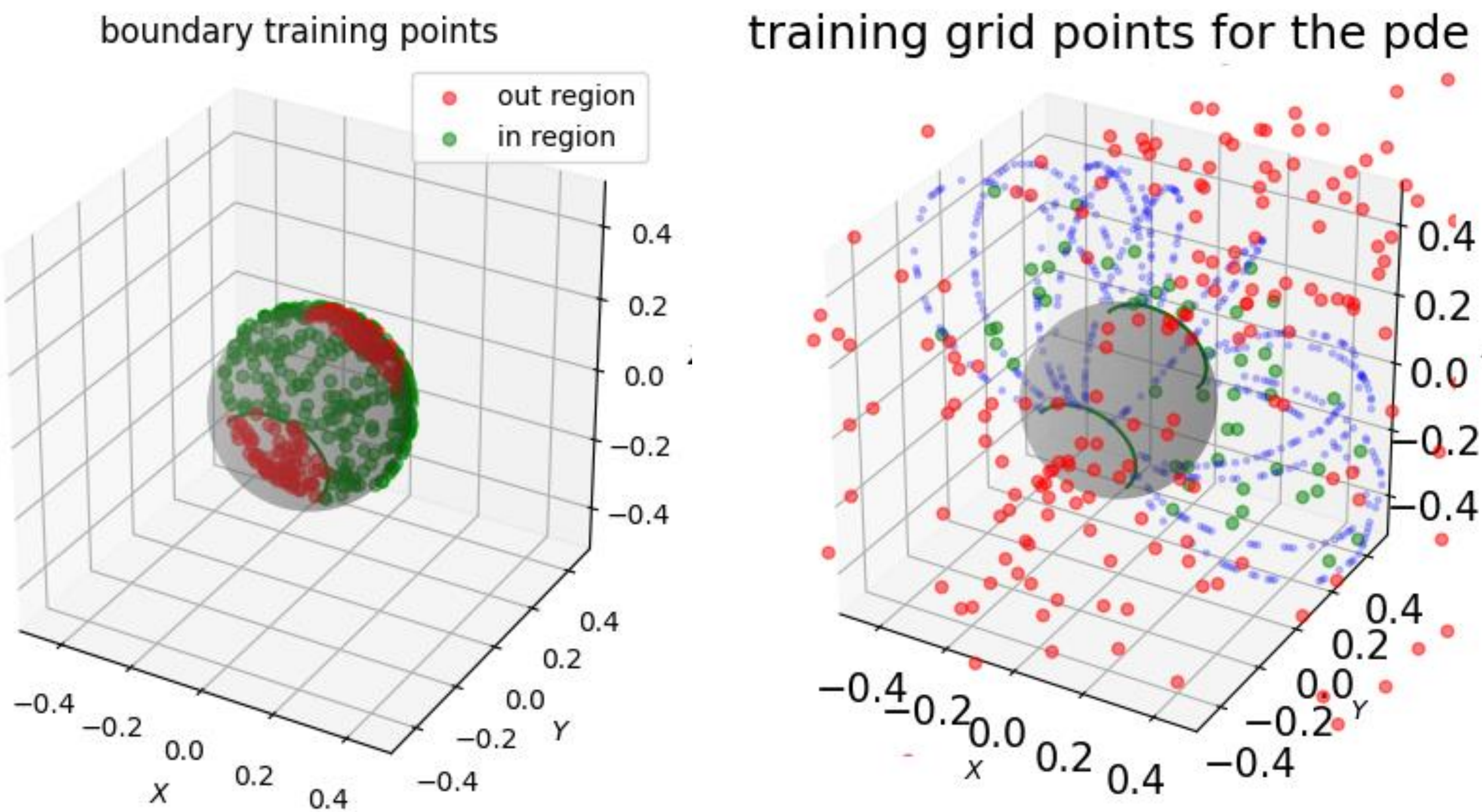


Neural Networks (NNs)



Physics Informed NNs (PINNs)




boundary training points
out region
in region
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4
X
0.4
0.2
0.0
−0.2
−0.4
Y
training grid points for the pde
0.4
0.2
0.0
−0.2
−0.4
Z
−0.4
−0.2
0.0
0.2
0.4
X
0.4
0.2
0.0
−0.2
−0.4
Y

Review Article | Published: 24 May 2021

Physics-informed machine learning

George Em Karniadakis, Ioannis G. Kevrekidis, Lu Lu, Paris Perdikaris, Sifan Wang & Liu Yang

models and approximations must be accompanied by guarantees and error bounds of their predictions.

The advent of ML has provided new hope (and also hype) for the solution of PDEs in physics and fluid dynamics. Although ML has shown tremendous success in problems related to pattern recognition and process automation, there have not been similar advances in solving challenging PDEs. For example, several state-of-the-art methods solve rather simplified problems. In a fluid dynamics context, I am not aware of a ML approach that can reliably simulate flows past a cylinder or vortex merging at Reynolds numbers above 5,000, even in 2D. Moreover, there are large costs associated with the training phase of these algorithms, which need attention. However, there is hope that solving real-life physical problems with missing, gappy or noisy boundary conditions is the area where ML approaches have an advantage. The resolution to these challenges lies not in the domain of the learning algorithms alone but at their interface with classical numerical methods. Lessons learned in CoS can be valuable in accelerating AI discoveries in this context.

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

First PINN applications in MHD

PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **524**, 32–42 (2023) https://doi.org/10.1093/mnras/stad1810
Advance Access publication 2023 June 16

Modelling force-free neutron star magnetospheres using physics-informed neural networks

Jorge F. Urbán,[1] Petros Stefanou,[1,2★] Clara Dehman[3,4] and José A. Pons[1]

$$\boldsymbol{J} \times \boldsymbol{B}/c = 0$$

$$(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = 0$$

$$\boldsymbol{B} = \nabla\mathcal{P} \times \nabla\phi + \mathcal{T}\nabla\phi$$

$$\nabla\mathcal{P} \times \nabla\mathcal{T} = 0$$

$$\Delta_{\mathrm{GS}}\mathcal{P} + G(\mathcal{P}) = 0$$

PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **524,** 32–42 (2023)
Advance Access publication 2023 June 16

https://doi.org/10.1093/mnras/stad1810

Modelling force-free neutron star magnetospheres using physics-informed neural networks

$$\mathcal{T}(\mathcal{P}) = s_1\mathcal{P} + s_2\mathcal{P}^2$$

$$\mathcal{P}(q = 1, \mu) = \left(1 - \mu^2\right) \sum_{l=1}^{l_{\max}} \frac{b_l}{l} P_l'(\mu)$$

$$\mathcal{P}(\boldsymbol{x}; \Omega) = f_b(\boldsymbol{x}) + h_b(\boldsymbol{x})\mathcal{N}(\boldsymbol{x}; \Omega)$$

$$f_b(q, \mu) = q^n \left(1 - \mu^2\right) \sum_{l=1}^{l_{\max}} \frac{b_l}{l} P_l'(\mu)$$

PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **524,** 32–42 (2023) https://doi.org/10.1093/mnras/stad1810
Advance Access publication 2023 June 16

Modelling force-free neutron star magnetospheres using physics-informed neural networks

$$\mathcal{T}(\mathcal{P}) = s_1\mathcal{P} + s_2\mathcal{P}^2 \qquad \mathcal{P}(q=1,\mu) = \left(1-\mu^2\right)\sum_{l=1}^{l_{\max}} \frac{b_l}{l} P_l'(\mu)$$

$$\mathcal{P}(\boldsymbol{x};\Omega) = f_b(\boldsymbol{x}) + h_b(\boldsymbol{x})\mathcal{N}(\boldsymbol{x};\Omega)$$

$$f_b(q,\mu) = q^n\left(1-\mu^2\right)\sum_{l=1}^{l_{\max}} \frac{b_l}{l} P_l'(\mu)$$

We have performed a detailed study to measure the influence of various hyperparameters of our model. In particular, we have considered the following:

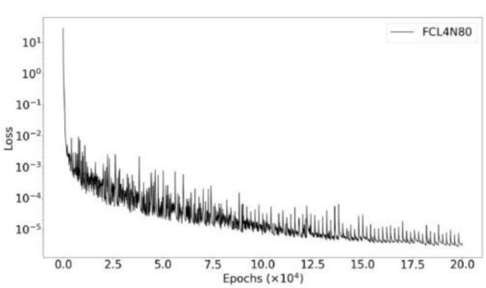
(i) Changes of the parametrization of the boundary (q^n power).
(ii) Number of neurons at each layer.
(iii) Number of hidden layers.
(iv) Resnet versus FC architectures.

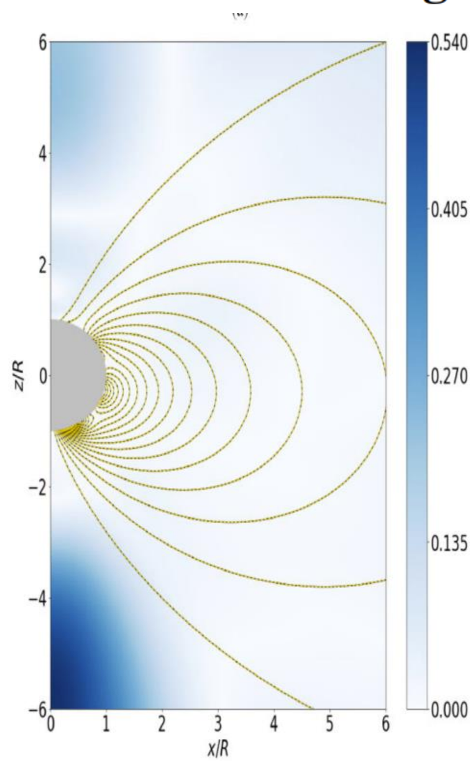
PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **524**, 32–42 (2023) https://doi.org/10.1093/mnras/stad1810
Advance Access publication 2023 June 16

Modelling force-free neutron star magnetospheres using physics-informed neural networks





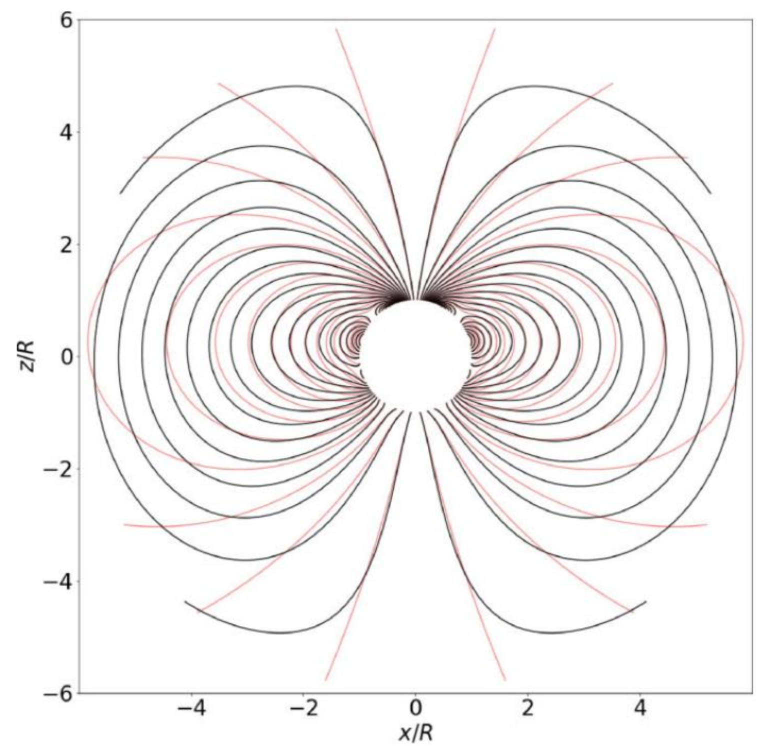


Figure 7. Field lines for the current-free (red) and FF (black) cases. The multipole coefficients at the surface are $b_{l>1} = 0.5$ for both. For the FF case, the coefficients in expression (9) for $\mathcal{T}(\mathcal{P})$ are $s_1 = 0.2$, $s = 0.4$.

PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **524,** 32–42 (2023)
Advance Access publication 2023 June 16

https://doi.org/10.1093/mnras/stad1810

Modelling force-free neutron star magnetospheres using physics-informed neural networks

5 APPLICATION TO THE MAGNETOTHERMAL EVOLUTION OF NEUTRON STARS

Our astrophysical scenario of interest is the long-term evolution of magnetic fields in NSs. The evolution of the system is governed by two coupled equations: the heat diffusion equation and the induction equation

(q,μ, b2, b3, b4, b5, b6, b7,s1,s2)

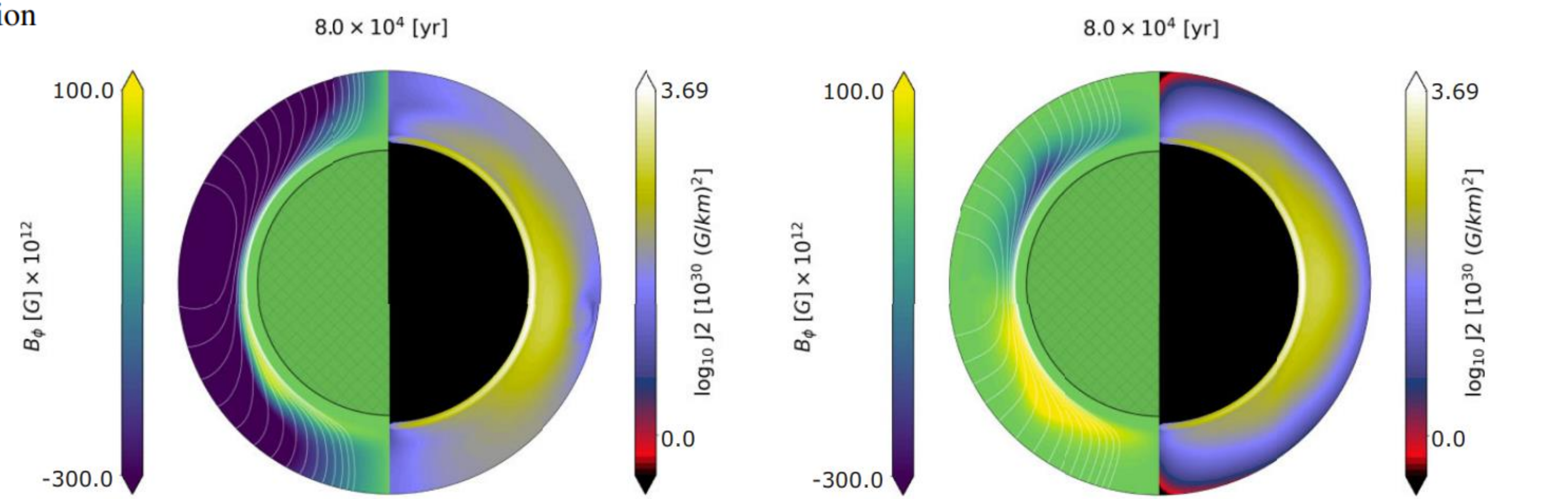


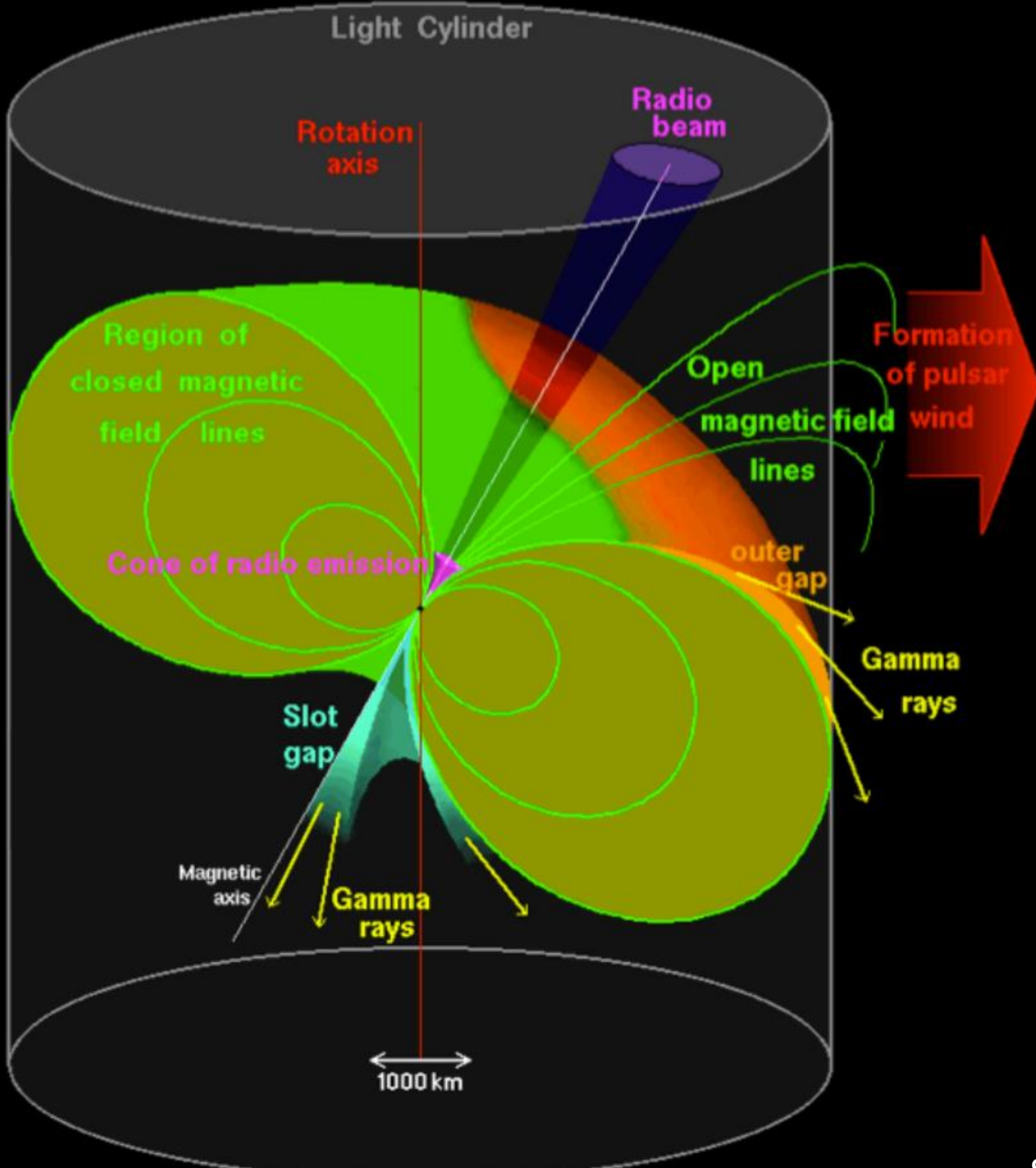
Figure 10. Same as Fig. 9. A snapshot of the magnetic field evolution and the electric current at 80 kyr. Left-hand panel: FF BCs. Right-hand panel: vacuum

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

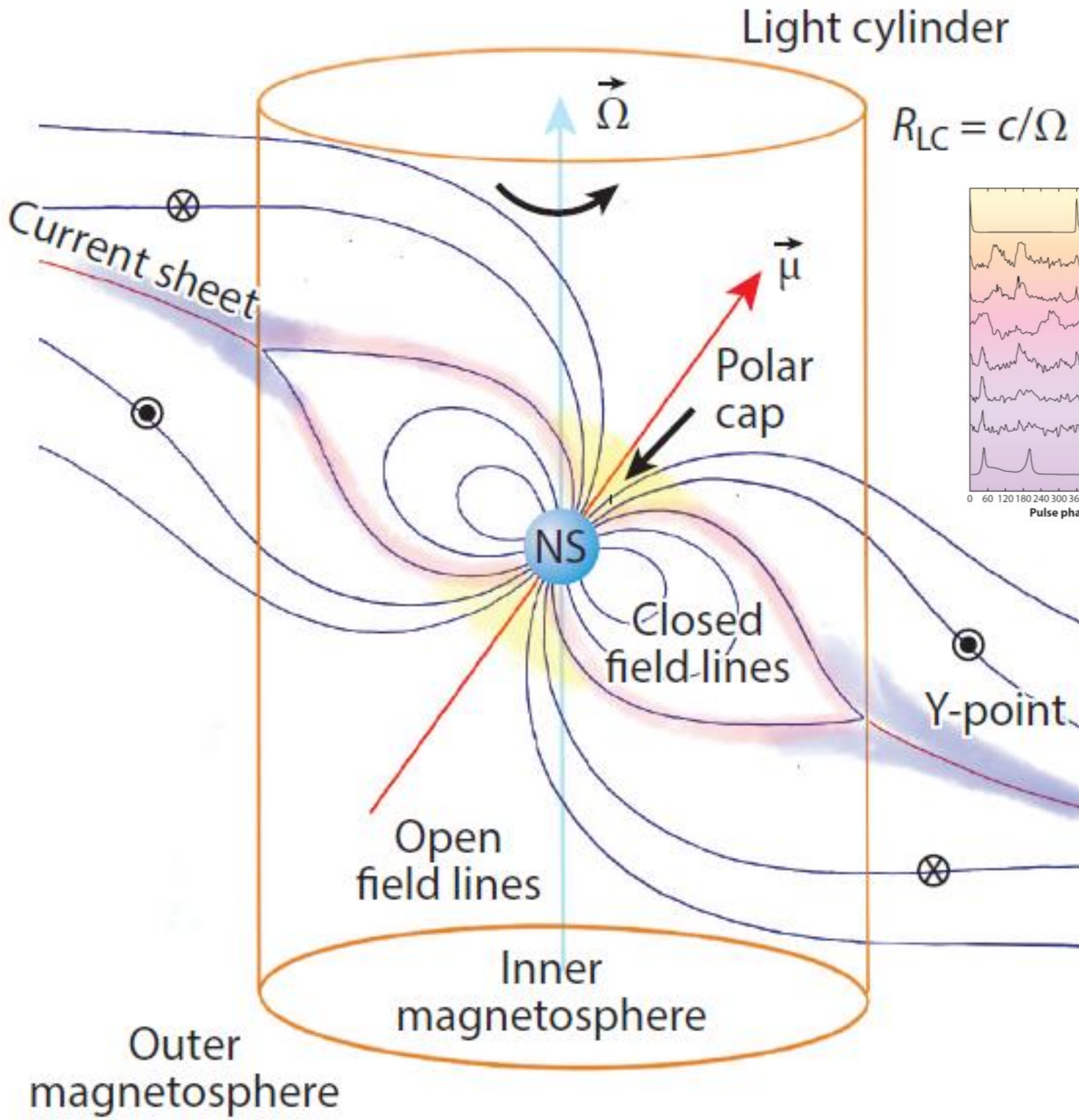
Pulsar Magnetospheres

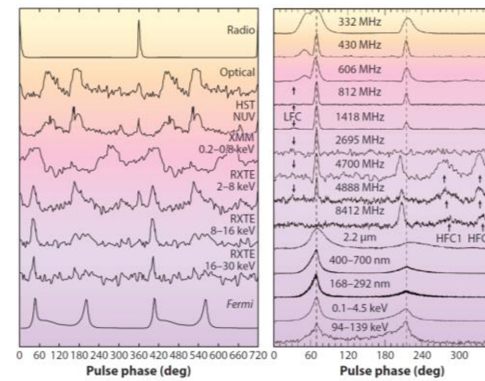
ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

What is the reference steady-state solution for Pulsars?



Spitkovsky 2005





Philippov, Kramer 2023

Pulsar Magnetospheres today

- The Y-point at 75% to 85% of the light cylinder (in all global PIC simulations of the last 10 years)
- Current sheets cannot be treated with the ideal force-free formalism (e.g. the pulsar equation)
- The whole magnetosphere disappears via dissipation in the equatorial current sheet within a few hundred light cylinders
- Simulations relax to one solution: how about mode switching?
- Issues with resolution of global PIC simulations

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Solution of the Pulsar Equation with PINNs

Steady-state in 2D: pulsar equation

$$\rho_e \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}/c = 0$$

$$B_r \equiv \frac{1}{r^2 \sin\theta} \frac{\partial \Psi}{\partial \theta} ,$$

$$B_\theta \equiv -\frac{1}{r \sin\theta} \frac{\partial \Psi}{\partial r} ,$$

$$B_\phi \equiv \frac{I(\Psi)}{r \sin\theta} ,$$

$$\left(1 - r^2 \sin^2\theta\right) \left[\frac{\partial^2 \Psi}{\partial r^2} - \frac{\partial \Psi}{\partial \theta} \frac{\cos\theta}{r^2 \sin\theta} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2}\right] - 2r \sin\theta \left[\frac{\partial \Psi}{\partial \theta} \frac{\cos\theta}{r} + \frac{\partial \Psi}{\partial r} \sin\theta\right] + I I'(\Psi) = 0$$

Steady-state in 2D: pulsar equation

$$\rho_e \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}/c = 0$$

$$B_r \equiv \frac{1}{r^2 \sin\theta} \frac{\partial \Psi}{\partial \theta} ,$$

$$B_\theta \equiv -\frac{1}{r \sin\theta} \frac{\partial \Psi}{\partial r} ,$$

$$B_\phi \equiv \frac{I(\Psi)}{r \sin\theta} ,$$

$$\left(1-\beta^2\right) \Delta_{\mathrm{GS}} P + 2\beta^2 q^2 \left(q \partial_q P + \mu \partial_\mu P\right) + G(P) = 0,$$

where $G(P) = TT'$ and Δ_{GS} is the Grad–Shafranov operator

$$\Delta_{\mathrm{GS}} \equiv q^2 \partial_q \left(q^2 \partial_q\right) + \left(1-\mu^2\right) q^2 \partial_{\mu\mu} .$$

PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Royal Astronomical Society

MNRAS **526,** 1504–1511 (2023) https://doi.org/10.1093/mnras/stad2840

Advance Access publication 2023 September 18

Solving the pulsar equation using physics-informed neural networks

Petros Stefanou,[1,2★] Jorge F. Urbán[1] and José A. Pons[1]

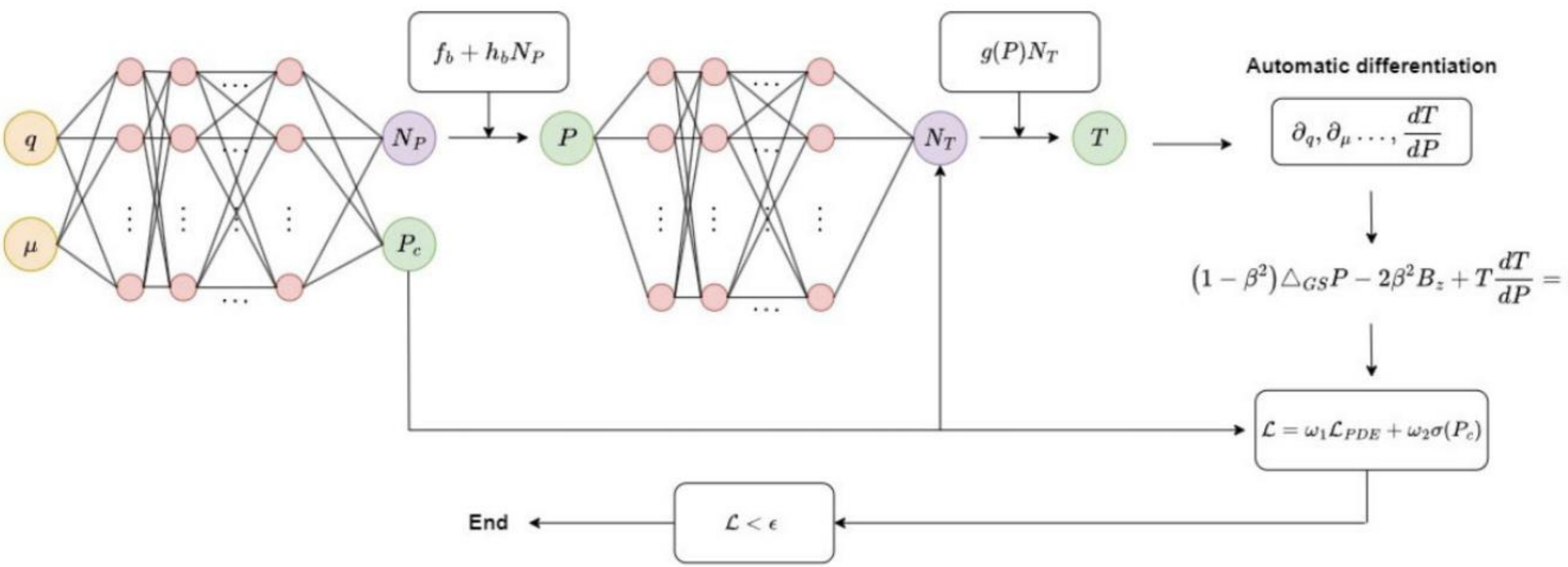


Figure 1. A sketch of the network structure. Two sub-networks are employed to ensure that $P = P(q, \mu)$ and $T = T(P)$.

PINN applications in MHD

Monthly Notices *of the* ROYAL ASTRONOMICAL SOCIETY

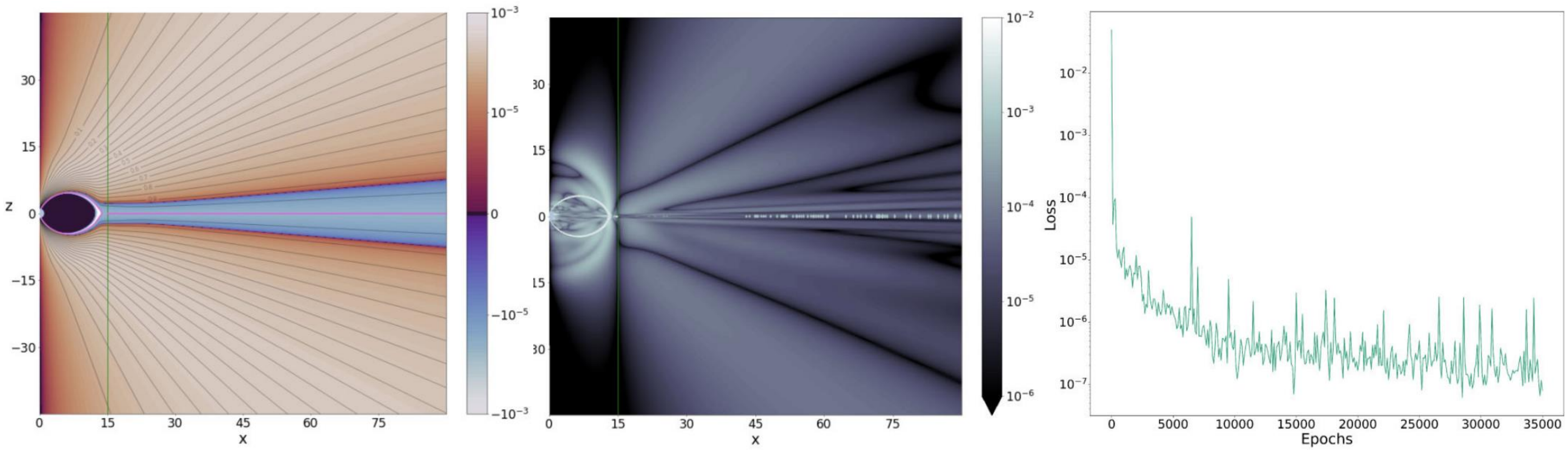
Royal Astronomical Society

MNRAS **526,** 1504–1511 (2023) https://doi.org/10.1093/mnras/stad2840

Advance Access publication 2023 September 18

Solving the pulsar equation using physics-informed neural networks

Petros Stefanou,[1,2★] Jorge F. Urbán[1] and José A. Pons[1]



PINN applications in MHD

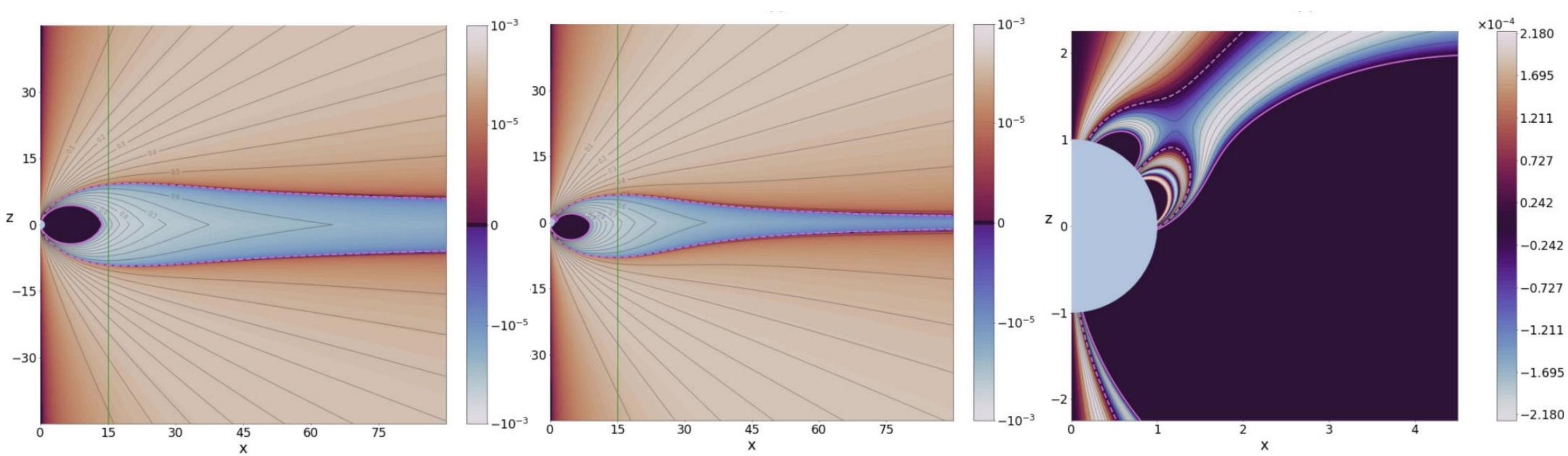
Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Royal Astronomical Society

MNRAS **526,** 1504–1511 (2023) https://doi.org/10.1093/mnras/stad2840
Advance Access publication 2023 September 18

Solving the pulsar equation using physics-informed neural networks

Petros Stefanou,[1,2★] Jorge F. Urbán[1] and José A. Pons[1]



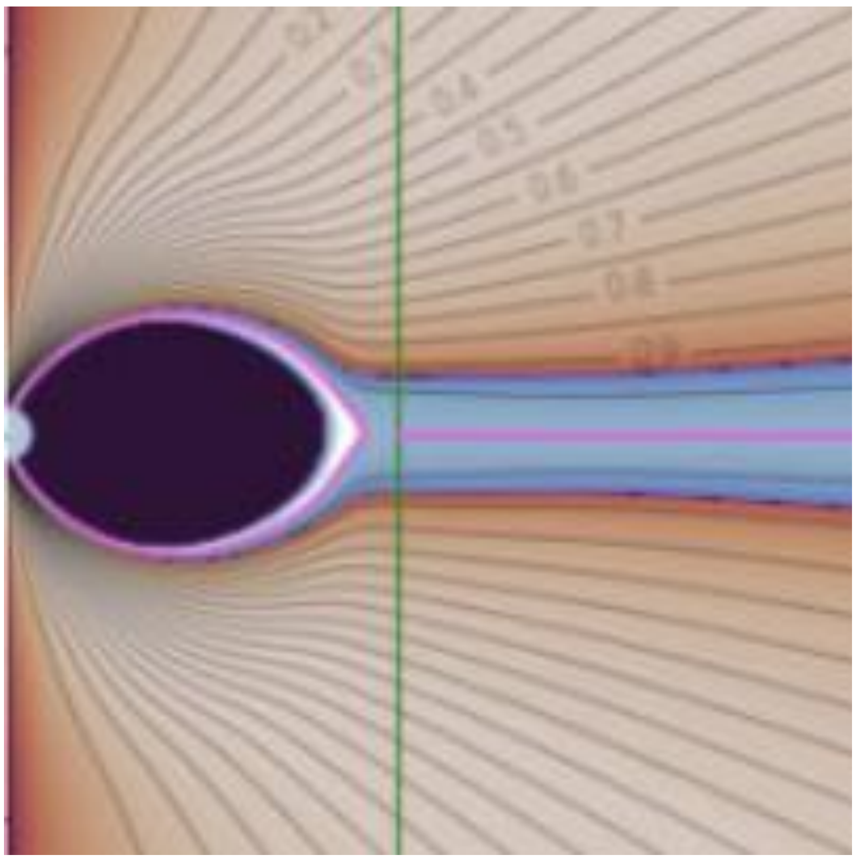
PINN applications in MHD

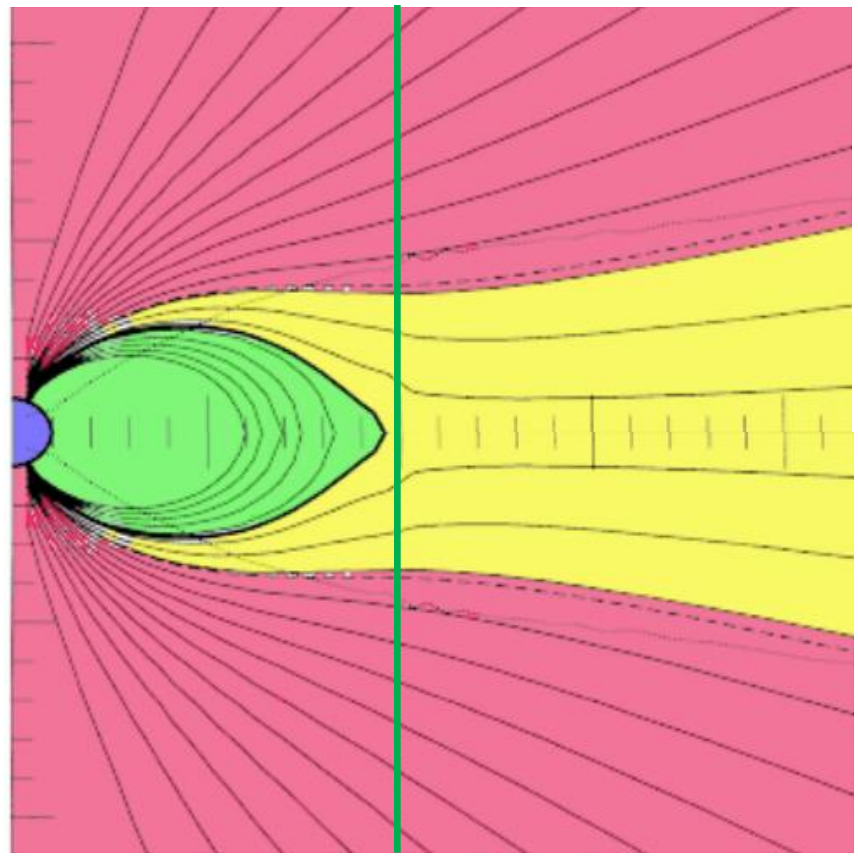
Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **526,** 1504–1511 (2023)
Advance Access publication 2023 September 18

https://doi.org/10.1093/mnras/stad2840

Solving the pulsar equation using physics-informed neural networks





ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

We can do better…

PINN applications in MHD

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
Royal Astronomical Society
MNRAS **528,** 3141–3152 (2024)
https://doi.org/10.1093/mnras/stae192
Advance Access publication 2024 January 17

The pulsar magnetosphere with machine learning: methodology

I. Dimitropoulos,[1] I. Contopoulos,[2★] V. Mpisketzis[3] and E. Chaniadakis[4]
[1]*Department of Physics, University of Patras, Rio 26504, Greece*
[2]*Research Center for Astronomy and Applied Mathematics, Academy of Athens, Athens 11527, Greece*
[3]*Department of Physics, National and Kapodistrian University of Athens, Athens 15784, Greece*
[4]*Department of Informatics and Telecommunications, National and Kapodistrian University of Athens, Athens 15784, Greece*

universe
MDPI

Article

A New Solution of the Pulsar Equation

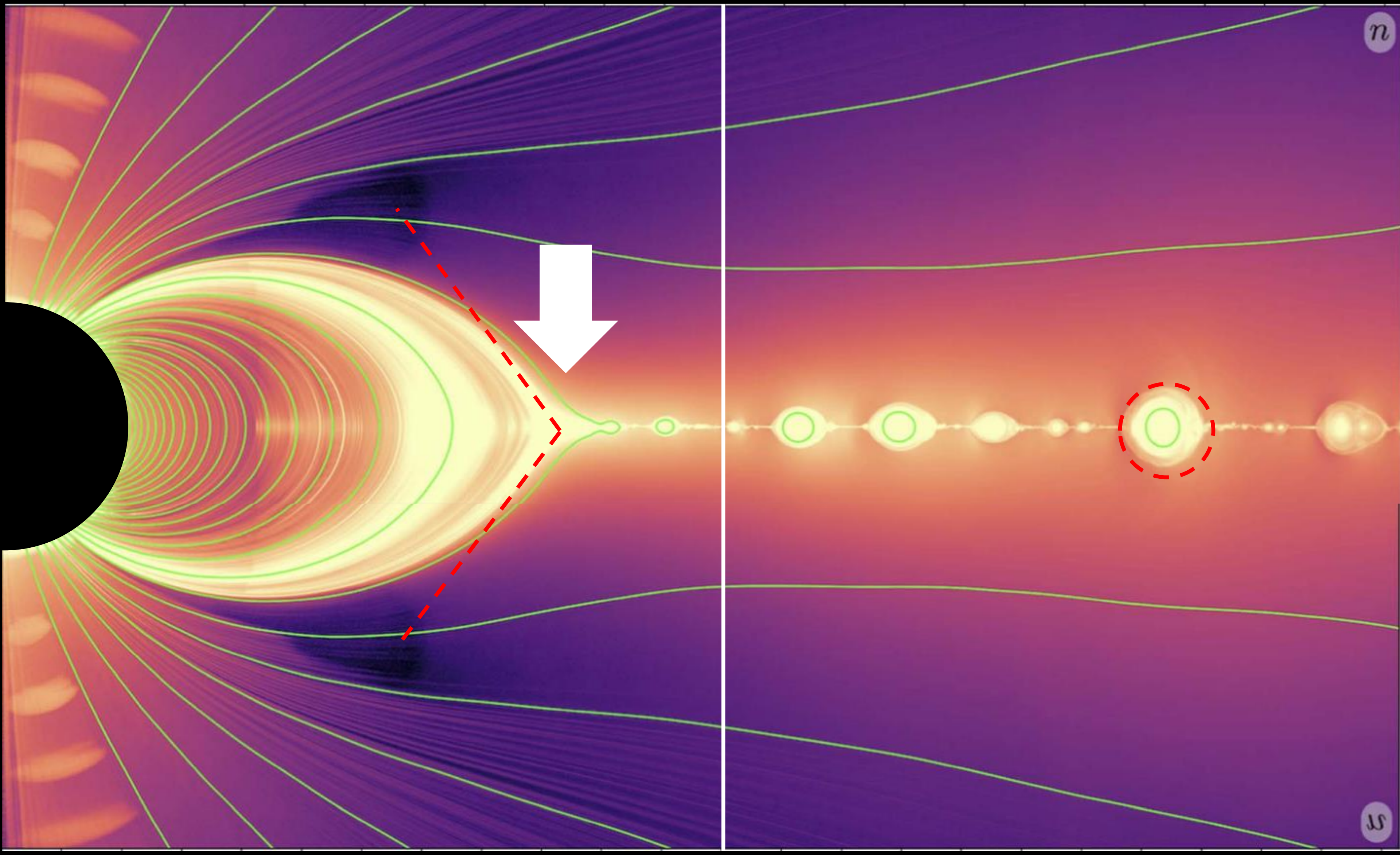
Ioannis Contopoulos [1,*], Ioannis Dimitropoulos [1,2], Dimitris Ntotsikas [2] and Konstantinos N. Gourgouliatos [2]

1 Research Center for Astronomy and Applied Mathematics, Academy of Athens, 11527 Athens, Greece; johndim888@gmail.com
2 Department of Physics, University of Patras, 26504 Rio, Greece; d.ntotsikas@ac.upatras.gr (D.N.); kngourg@upatras.gr (K.N.G.)
* Correspondence: icontop@academyofathens.gr

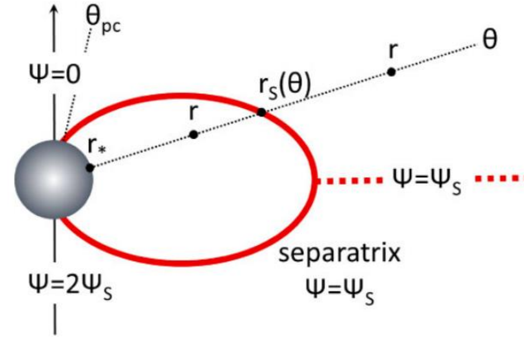
Old methodology

- Solve the 2D pulsar equation (steady-state)
  - Not yet in 3D but doable
- Start with a dipole and set the star in rotation at t=0 ➔ the FFE/MHD/PIC simulation relaxes to the steady-state (does it?)
- Issues with Current Sheets
  - Numerical vs physical dissipation
  - Internal structure (non ideal, finite thickness)
- Y-point at ~80% of the light cylinder

Bransgrove, Beloborodov, Levin 2023



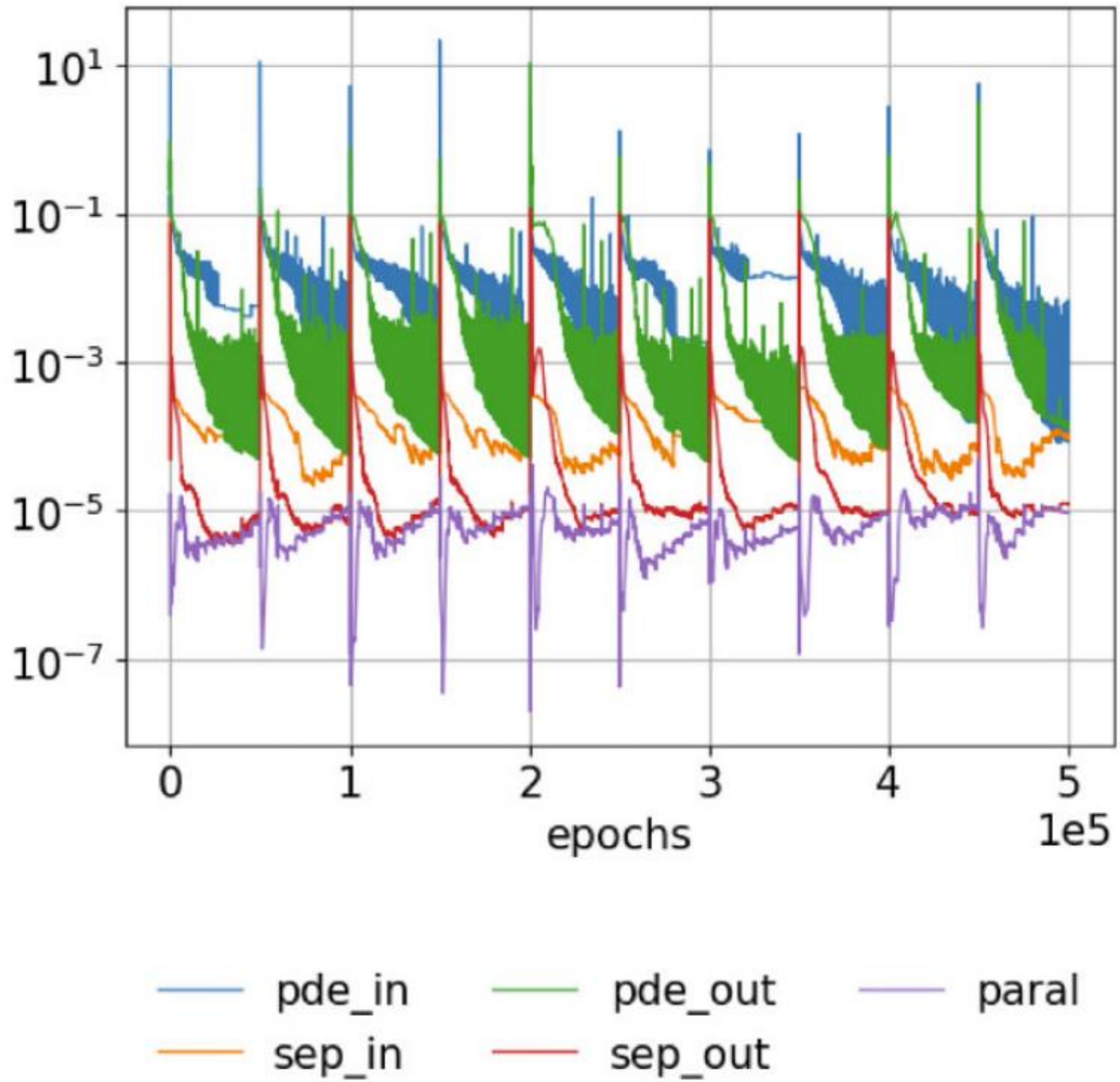
New methodology



- Choose $\Psi_S = \Psi_{open}$ (or θ_{pc})
- Separate closed (IN) and open (OUT) regions
  - Monopole OUT ➔equatorial CS disappears!
- Solve IN, solve OUT (e.g. meshless with PINNs)
- Check for pressure imbalance across separatrix
- Adjust separatrix

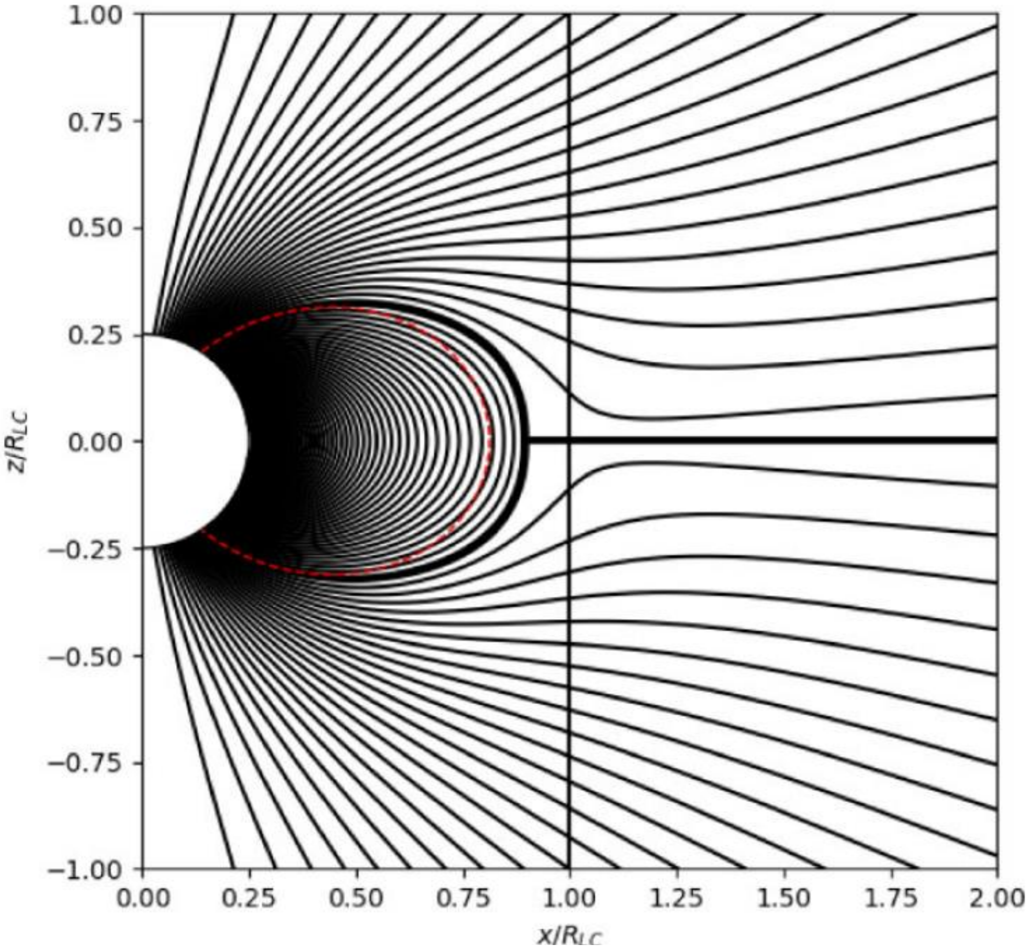
➔Solution with pressure balance across separatrix

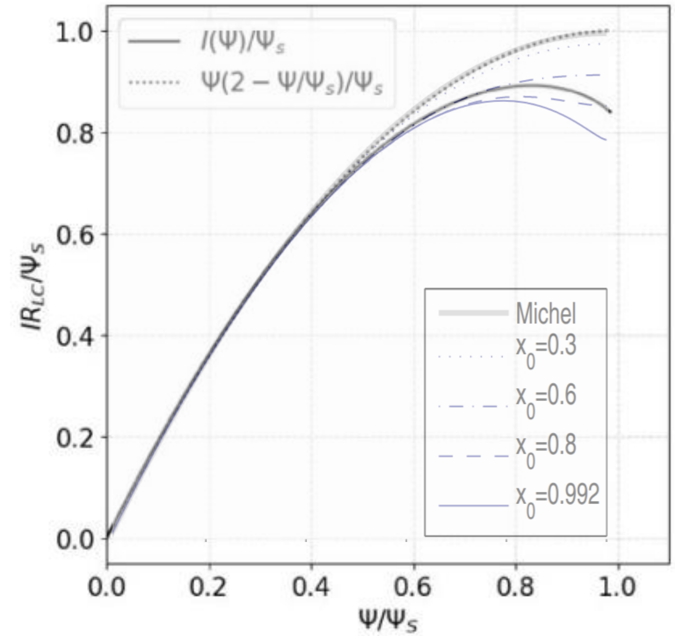
- Equatorial CS restored in the end!

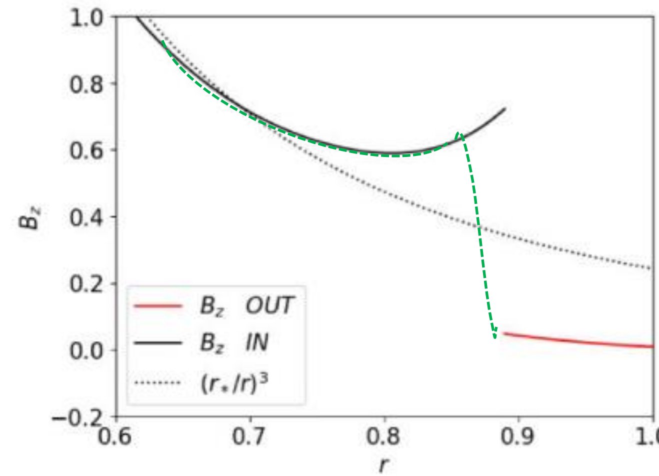


10^1
10^{-1}
10^{-3}
10^{-5}
10^{-7}
0
1
2
3
4
5
epochs
1e5
pde_in
pde_out
paral
sep_in
sep_out

Dimitropoulos et al. 2024

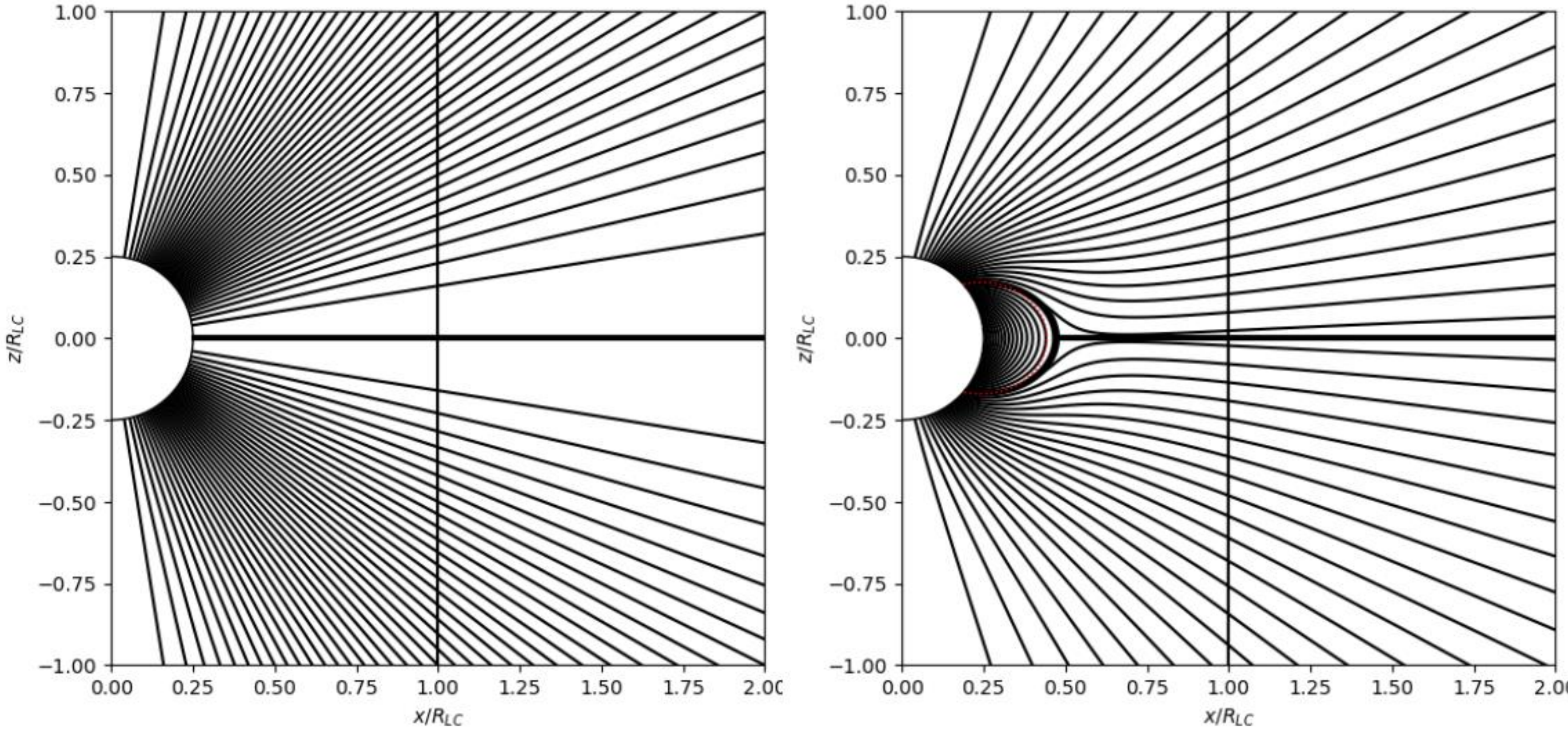






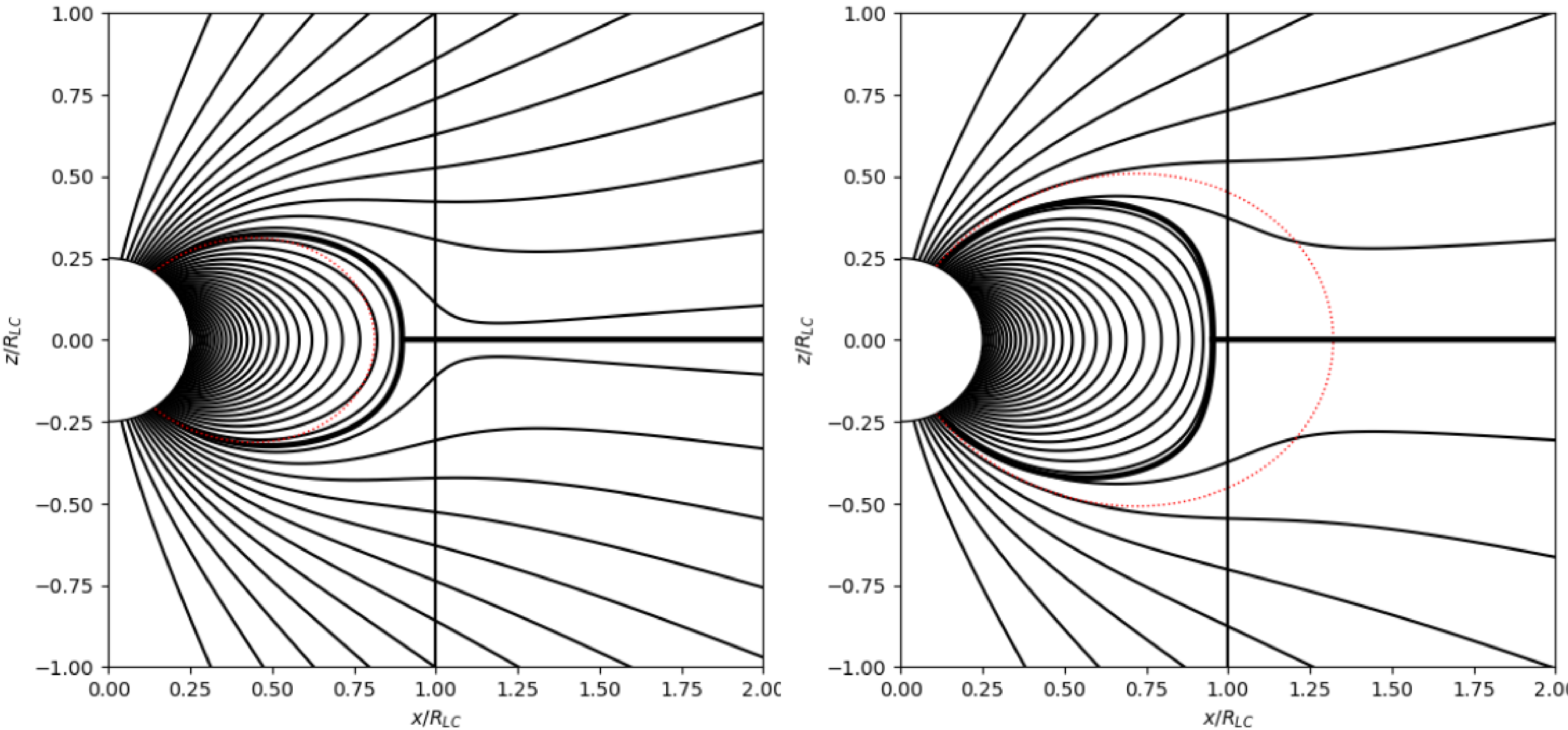
$$\left.(B^2 - E^2)\right|_{\mathrm{IN}} = \left.(B^2 - E^2)\right|_{\mathrm{OUT}}$$

Contopoulos et al. 2024



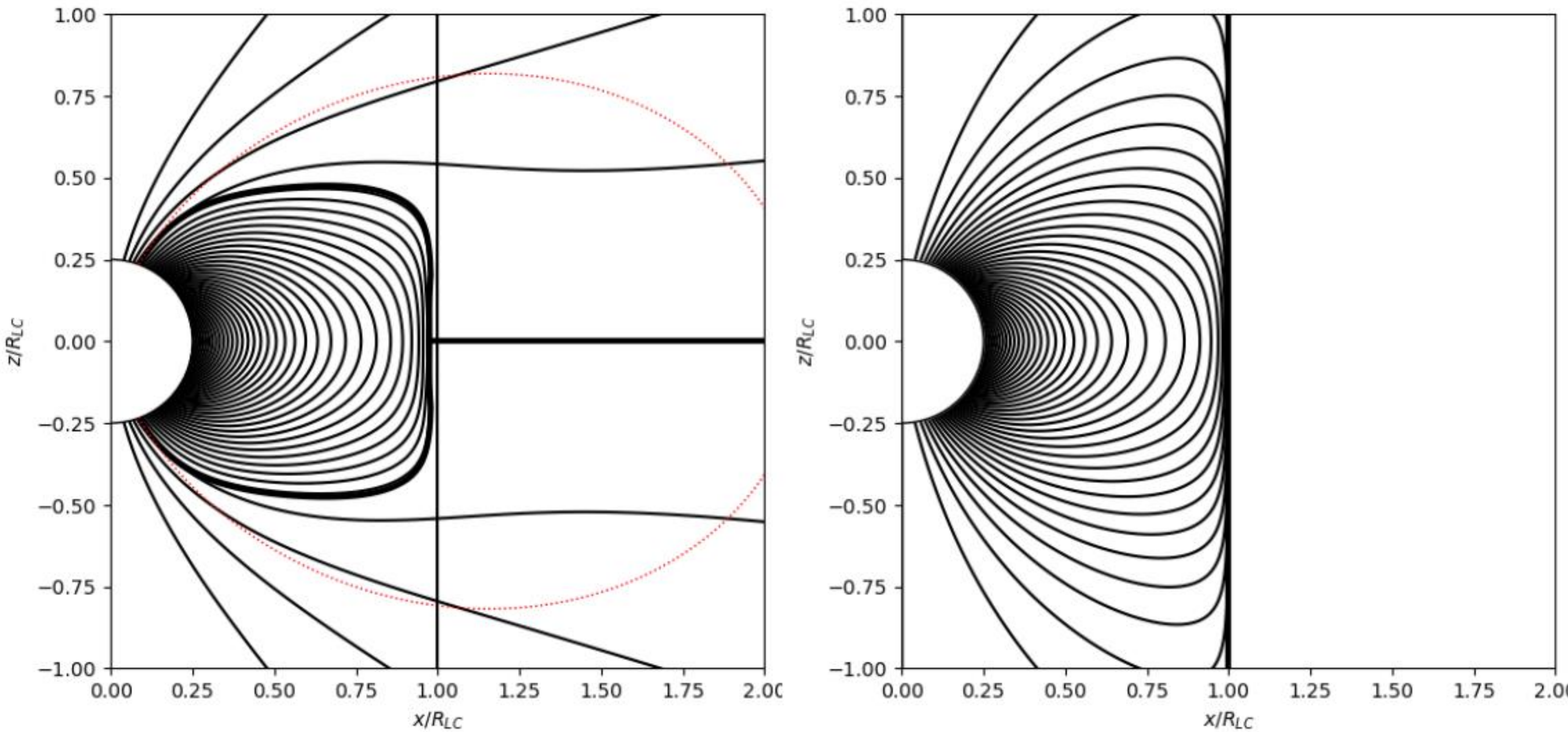
$$B_p^2(1-x^2)\Big|_{\mathrm{IN}} = B_p^2(1-x^2)\Big|_{\mathrm{OUT}} + \frac{I(\Psi_{\mathrm{S}})^2}{x^2}$$

Contopoulos et al. 2024



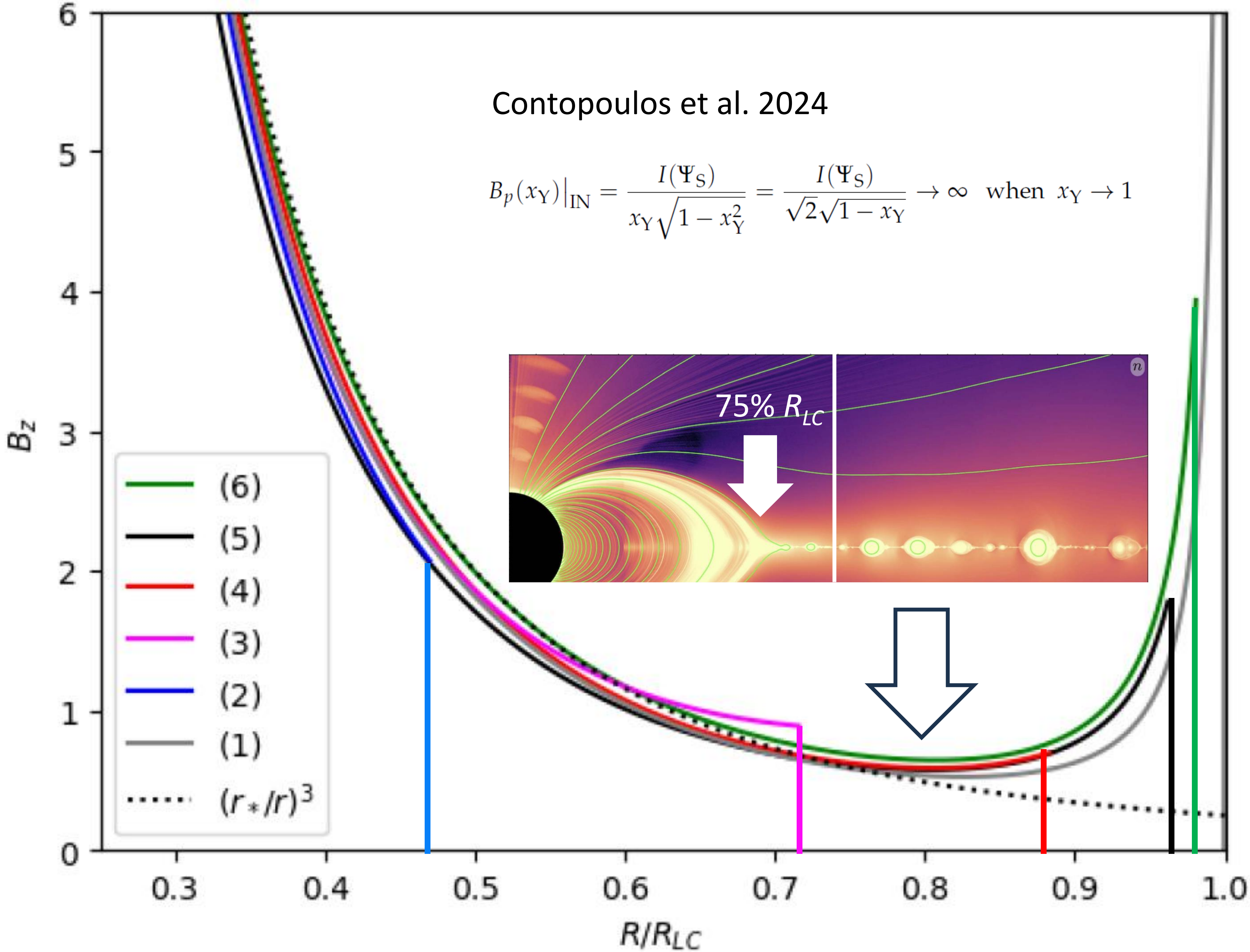
$$B_p(x_{\rm Y})\Big|_{\rm IN} = \frac{I(\Psi_{\rm S})}{x_{\rm Y}\sqrt{1-x_{\rm Y}^2}} = \frac{I(\Psi_{\rm S})}{\sqrt{2}\sqrt{1-x_{\rm Y}}} \to \infty \quad \text{when} \quad x_{\rm Y} \to 1$$

Contopoulos et al. 2024

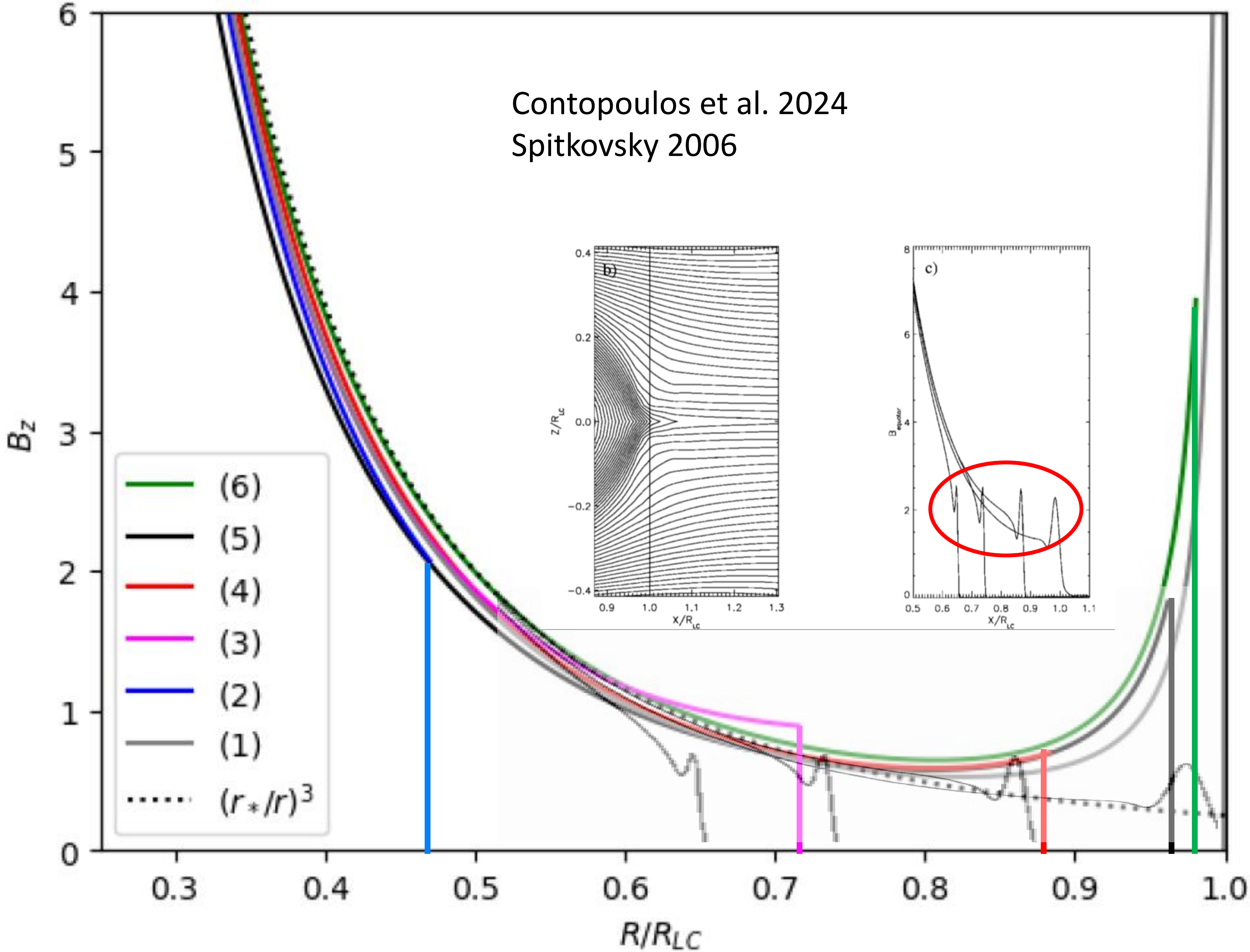


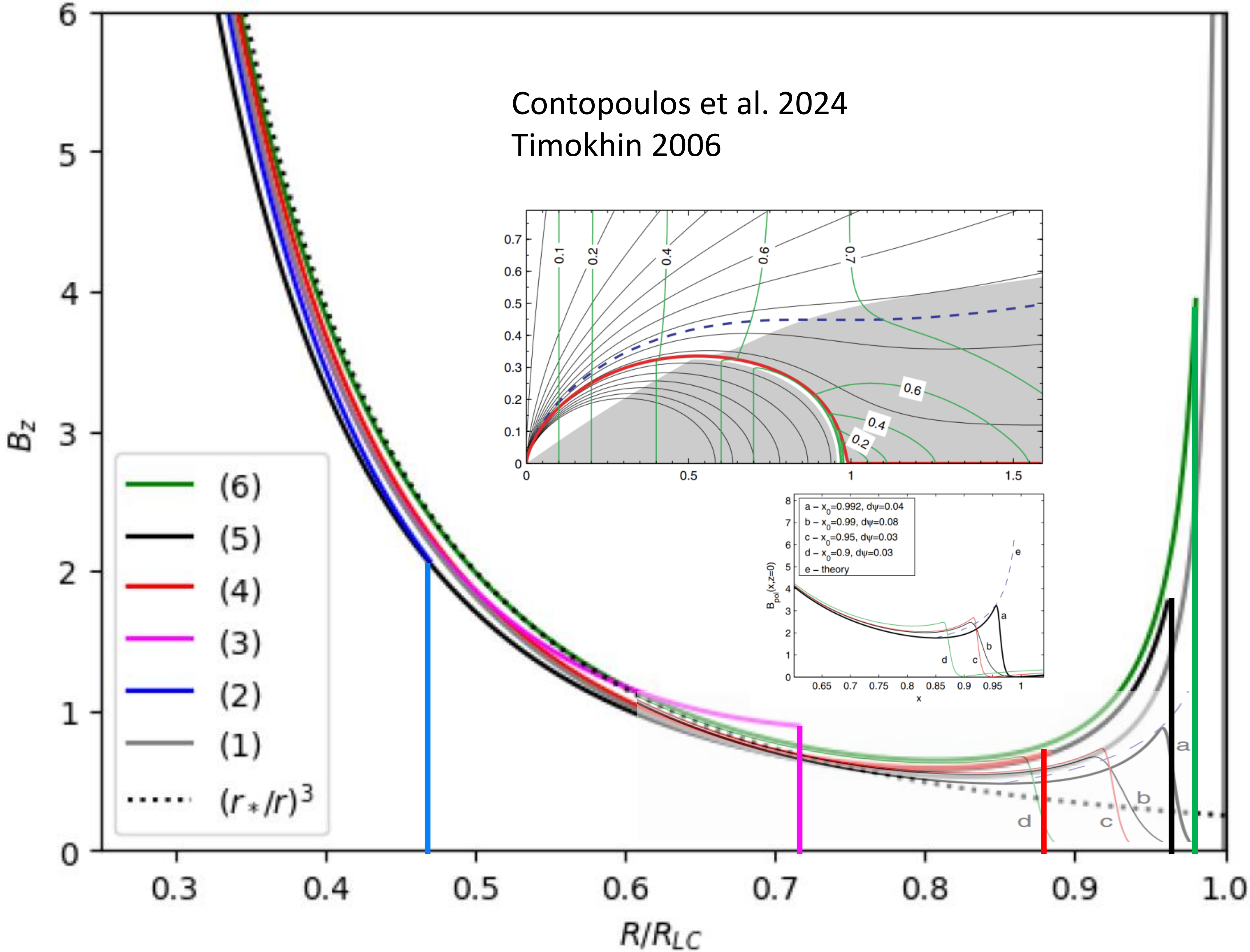
$$B_p(x_{\rm Y})\big|_{\rm IN} = \frac{I(\Psi_{\rm S})}{x_{\rm Y}\sqrt{1-x_{\rm Y}^2}} = \frac{I(\Psi_{\rm S})}{\sqrt{2}\sqrt{1-x_{\rm Y}}} \rightarrow \infty \quad \text{when} \quad x_{\rm Y} \rightarrow 1$$

We discovered a new solution!



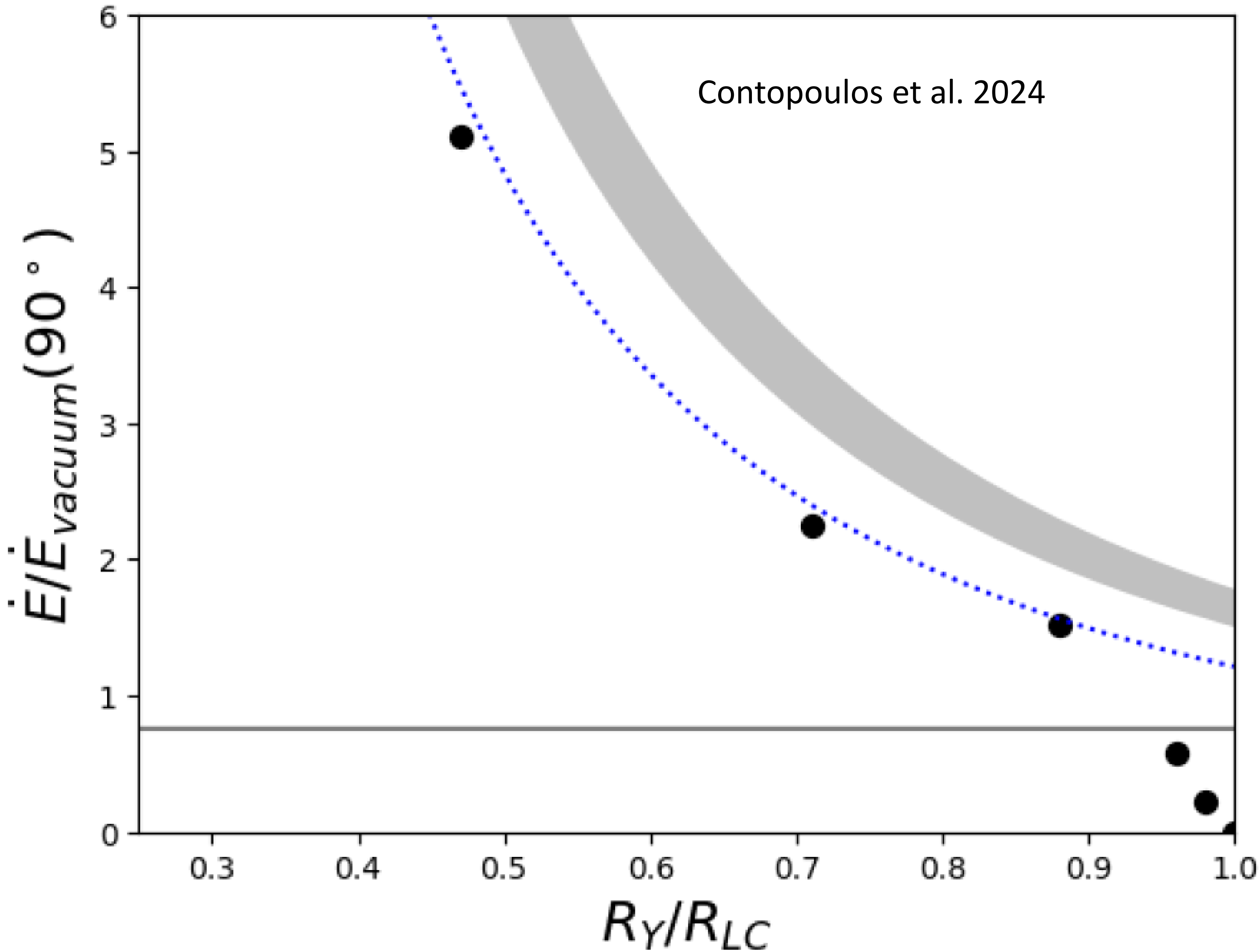
Contopoulos et al. 2024
$$B_p(x_Y)|_{IN} = \frac{I(\Psi_S)}{x_Y\sqrt{1-x_Y^2}} = \frac{I(\Psi_S)}{\sqrt{2}\sqrt{1-x_Y}} \to \infty \quad \text{when} \quad x_Y \to 1$$
75% R_{LC}
B_z
R/R_{LC}
(6)
(5)
(4)
(3)
(2)
(1)
$(r_*/r)^3$
0
1
2
3
4
5
6
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


Contopoulos et al. 2024
Spitkovsky 2006
B_z
R/R_{LC}
0
1
2
3
4
5
6
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
(6)
(5)
(4)
(3)
(2)
(1)
$(r_*/r)^3$
b)
c)
Z/R_{LC}
x/R_{LC}
$B_{equator}$


Contopoulos et al. 2024
Timokhin 2006
B_z
R/R_{LC}
(6)
(5)
(4)
(3)
(2)
(1)
$(r_*/r)^3$
a – x_0=0.992, dψ=0.04
b – x_0=0.99, dψ=0.08
c – x_0=0.95, dψ=0.03
d – x_0=0.9, dψ=0.03
e – theory
$B_{pol}(x,z=0)$
x

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Intermittent pulsars

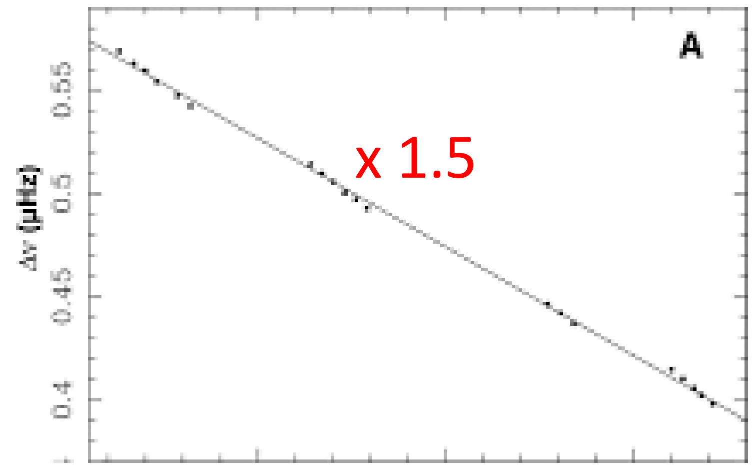


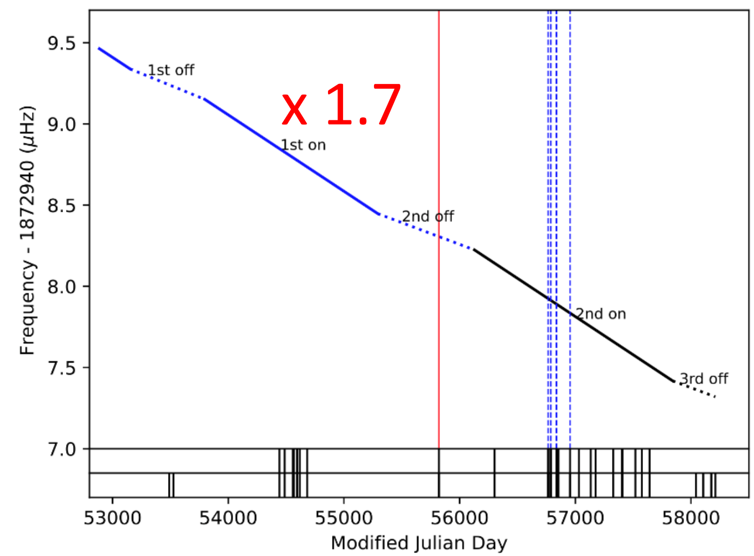
Contopoulos et al. 2024
$\dot{E}/\dot{E}_{vacuum}(90^\circ)$
R_Y/R_{LC}
0
1
2
3
4
5
6
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

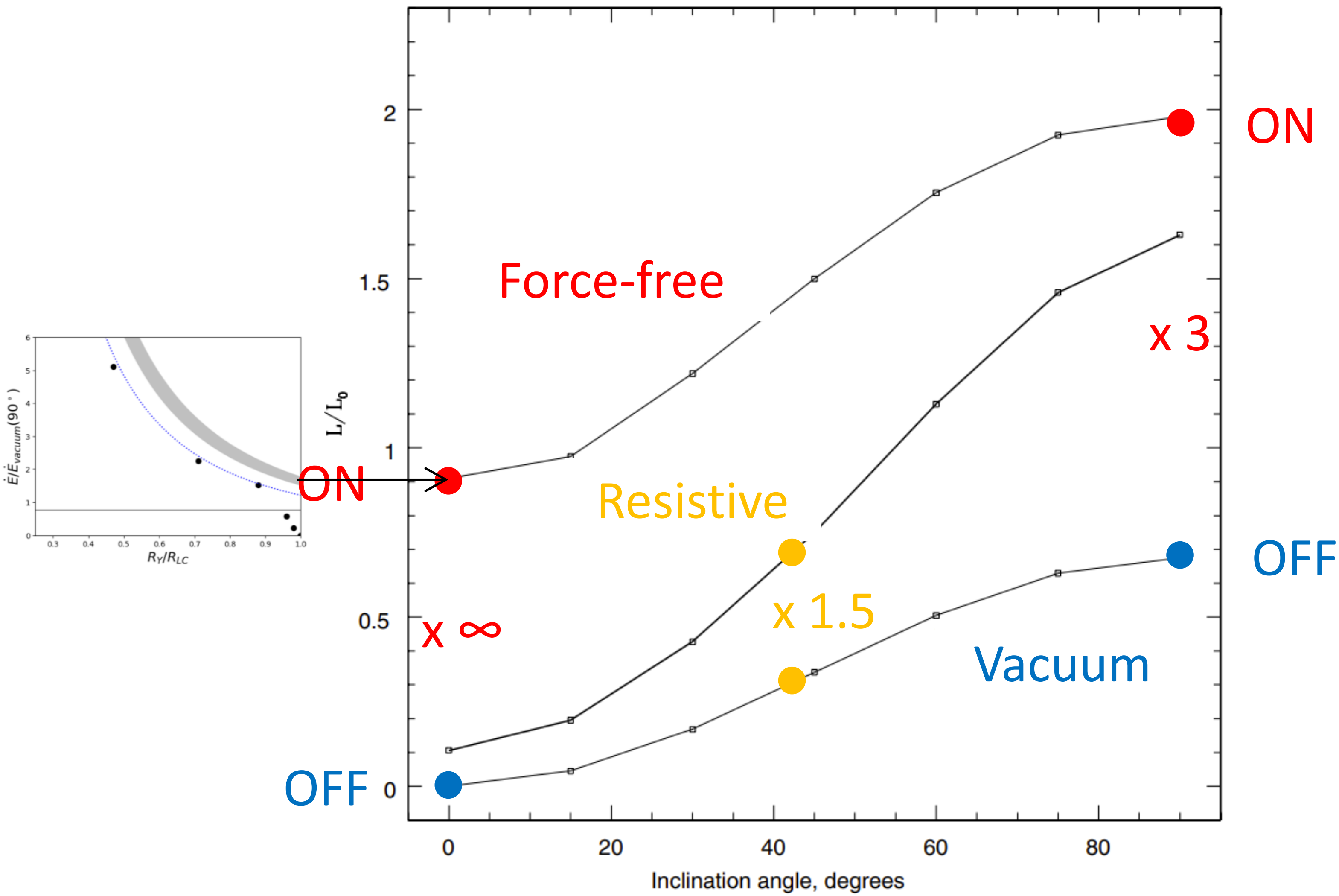
We also show the solution for $\theta_{\rm pc} = 0.94(r_*/R_{\rm LC})^{1/2}$ that yields $\Psi_{\rm open} = 0.87\Psi_{\rm dipole\ LC}$ and $\dot{E} = 0.75\dot{E}_{\rm vacuum}(90°)$. Here, $\dot{E}_{\rm vacuum}(\lambda)$ is the spindown rate of a vacuum dipole rotator with inclination angle λ. We may tentatively generalize this solution for non-zero pulsar inclination angles according to Spitkovsky (2006) as $\dot{E}(\lambda) \approx 0.75\dot{E}_{\rm vacuum}(90°)(1 + \sin^2\lambda)$, and since $\dot{E}_{\rm vacuum}(\lambda) = \dot{E}_{\rm vacuum}(90°)\sin^2\lambda$, we obtain that

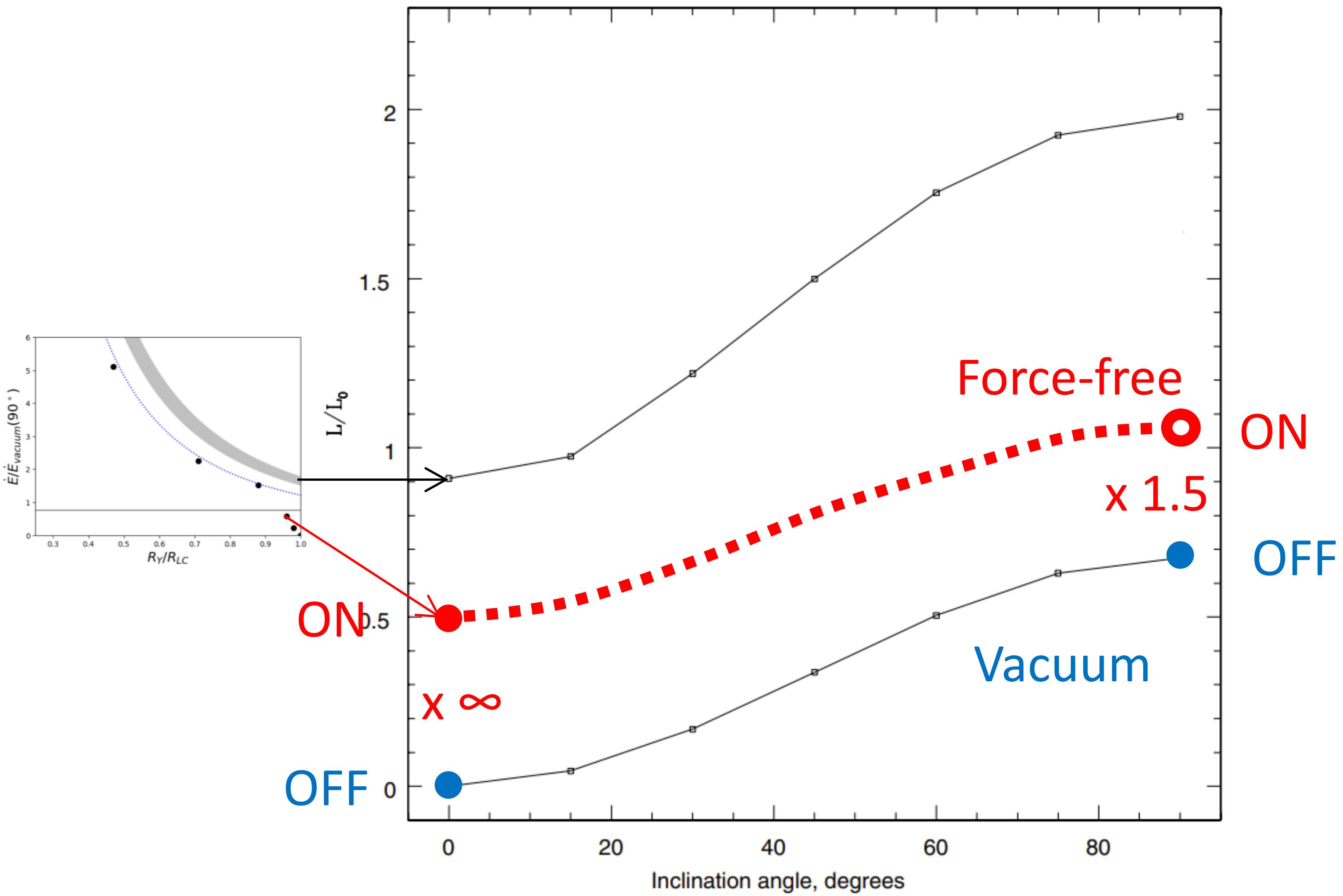
$$\frac{\dot{E}(\lambda)}{\dot{E}_{\rm vacuum}(\lambda)} \approx 0.75\,\frac{1+\sin^2\lambda}{\sin^2\lambda} \gtrsim 1.5\,. \qquad (38)$$

It is interesting that in all previous solutions of the FFE pulsar magnetosphere, the above ratio was found to be greater than 3 (e.g. Li, Spitkovsiky & Tchekhovskoy 2012). This value is significantly larger than the ratio of spindown rates $\dot{E}_{\rm ON}/\dot{E}_{\rm OFF}$ observed in the intermittent pulsars PSR B1931+24, PSR J1832+0029 and PSR J1841-0500 (1.5, 1.7 and 2.5 respectively; e.g. Rea et al. 2008, Wang et al. 2020). The inability to account for observed values lower than 3 is what led to the development of resistive magnetospheric solutions (e.g. Kalapotharakos et al. 2012b, Li, Spitkovsky & Tchekhovskoy 2012). With our new solutions it seems that there is no need for resistivity to explain intermittent pulsars. This result certainly merits further investigation.






ON
Force-free
x 3
x ∞
Resistive
x 1.5
OFF
Vacuum
OFF
ON
L/L0
Inclination angle, degrees
0
20
40
60
80
0
0.5
1
1.5
2
R_Y/R_LC



Ė/Ė_vacuum(90°)
R_Y/R_LC
L/L_0
2
1.5
1
0.5
0
0
20
40
60
80
Inclination angle, degrees
Force-free
ON
x 1.5
OFF
Vacuum
ON
x ∞
OFF

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Solution of the 3D Pulsar Equation with PINNs

Dimitropoulos, Contopoulos 2024 (work in progress)

Steady-state in 3D

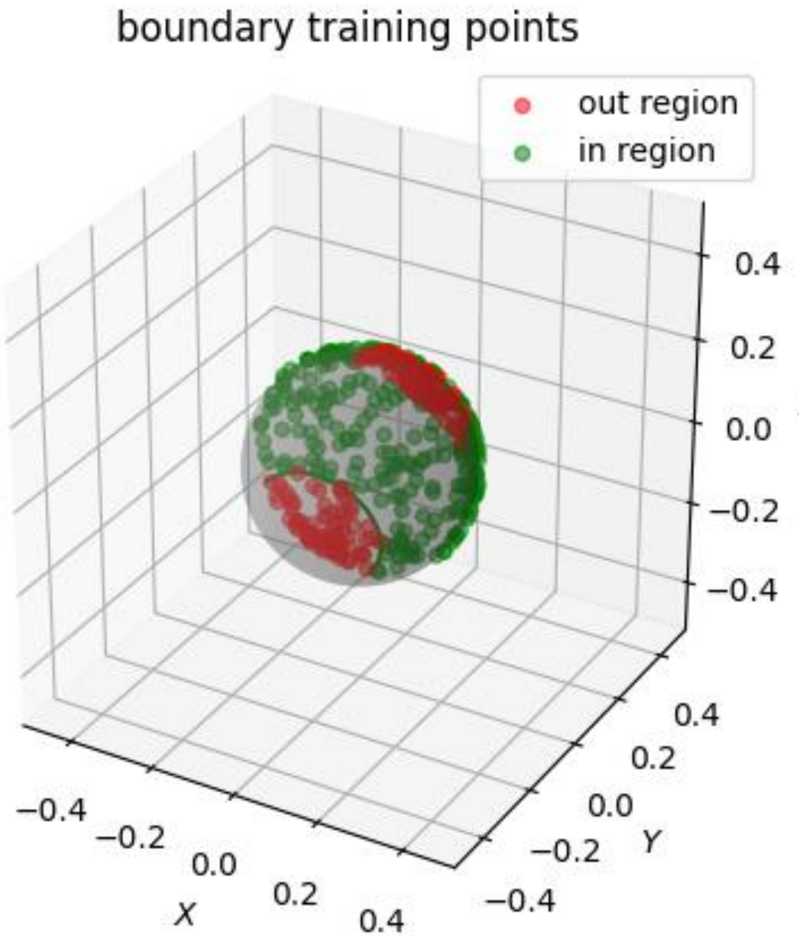
$$\rho_\mathrm{e}\boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}/c = 0$$

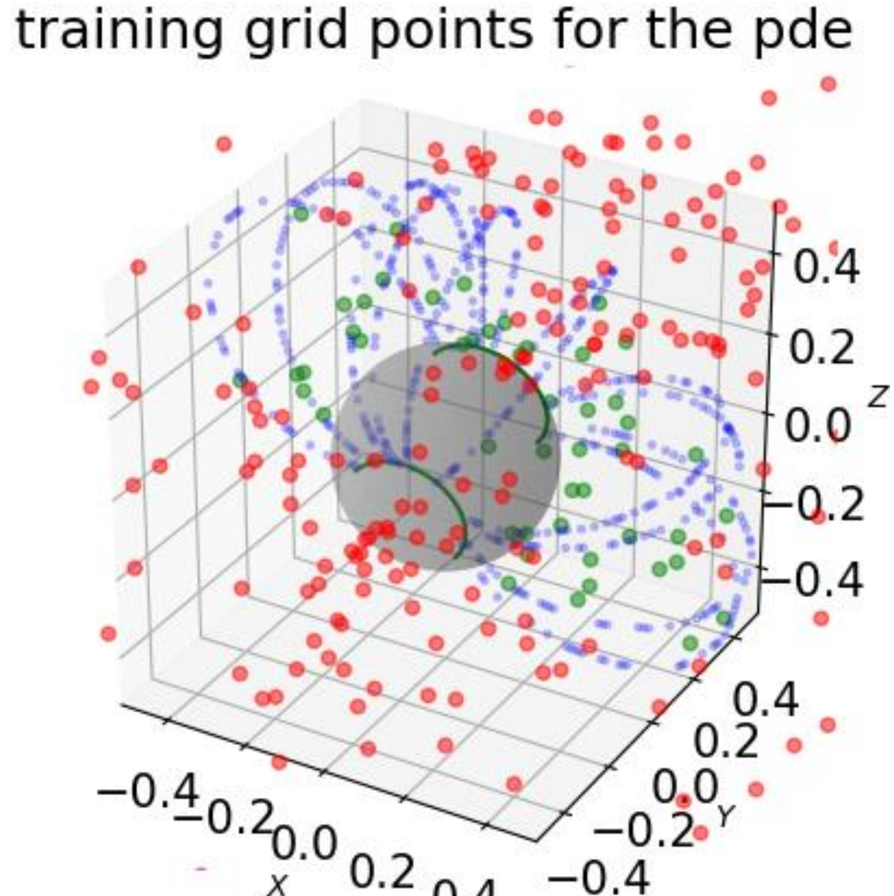
$$\nabla \times \boldsymbol{B} \quad \times \boldsymbol{B} \quad = 0$$

$$\nabla \times \boldsymbol{B} \quad = \alpha \boldsymbol{B} \qquad\qquad \boldsymbol{B} \cdot \nabla\alpha = 0$$

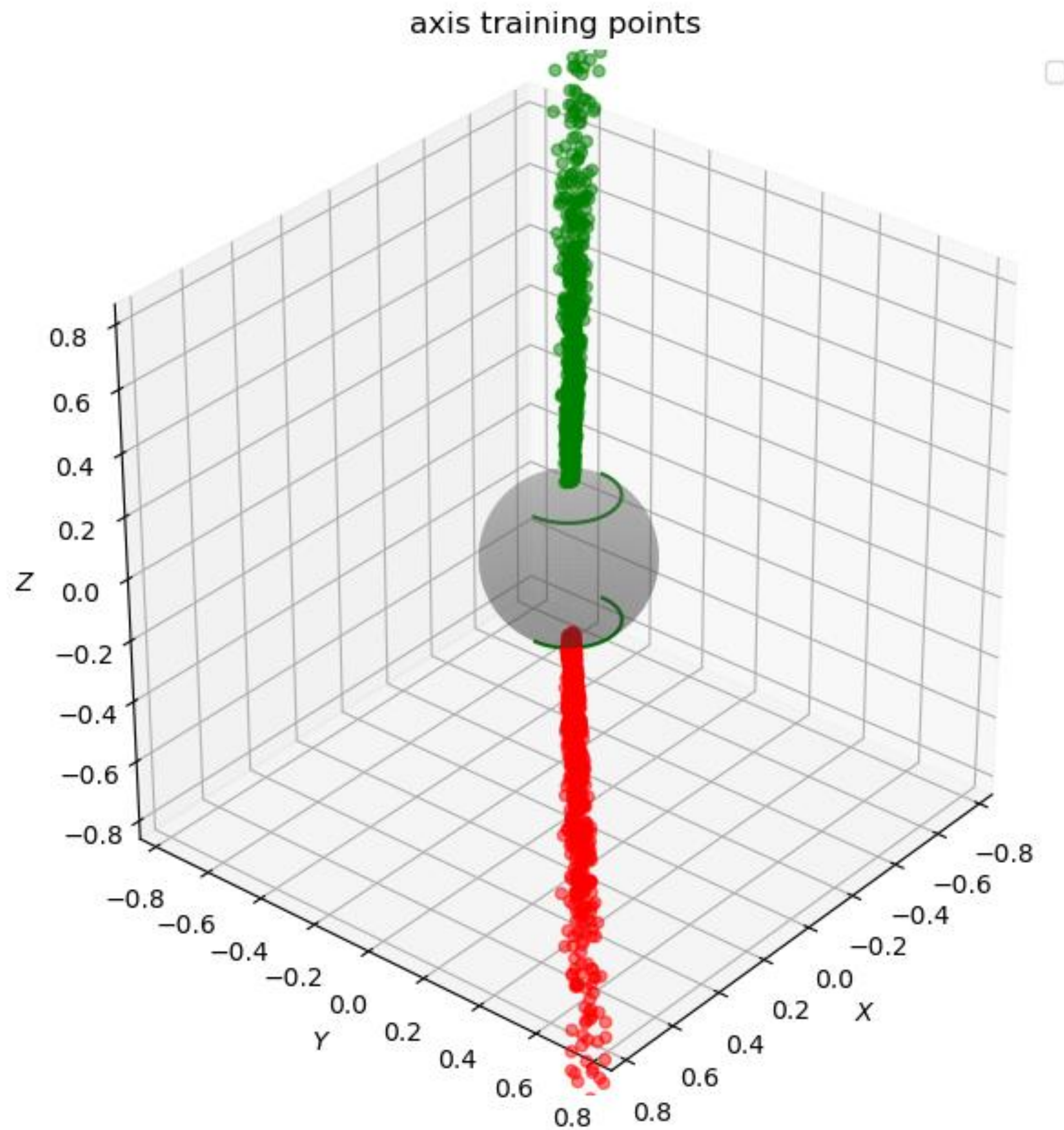
$$\nabla \times \left\{ \boldsymbol{B}_\mathrm{p} \left(1 - \left(\frac{r \sin\theta}{R_\mathrm{LC}} \right)^2 \right) + \boldsymbol{B}_\phi \right\} = \alpha \boldsymbol{B}$$

Dimitropoulos, Contopoulos 2024 (work in progress)

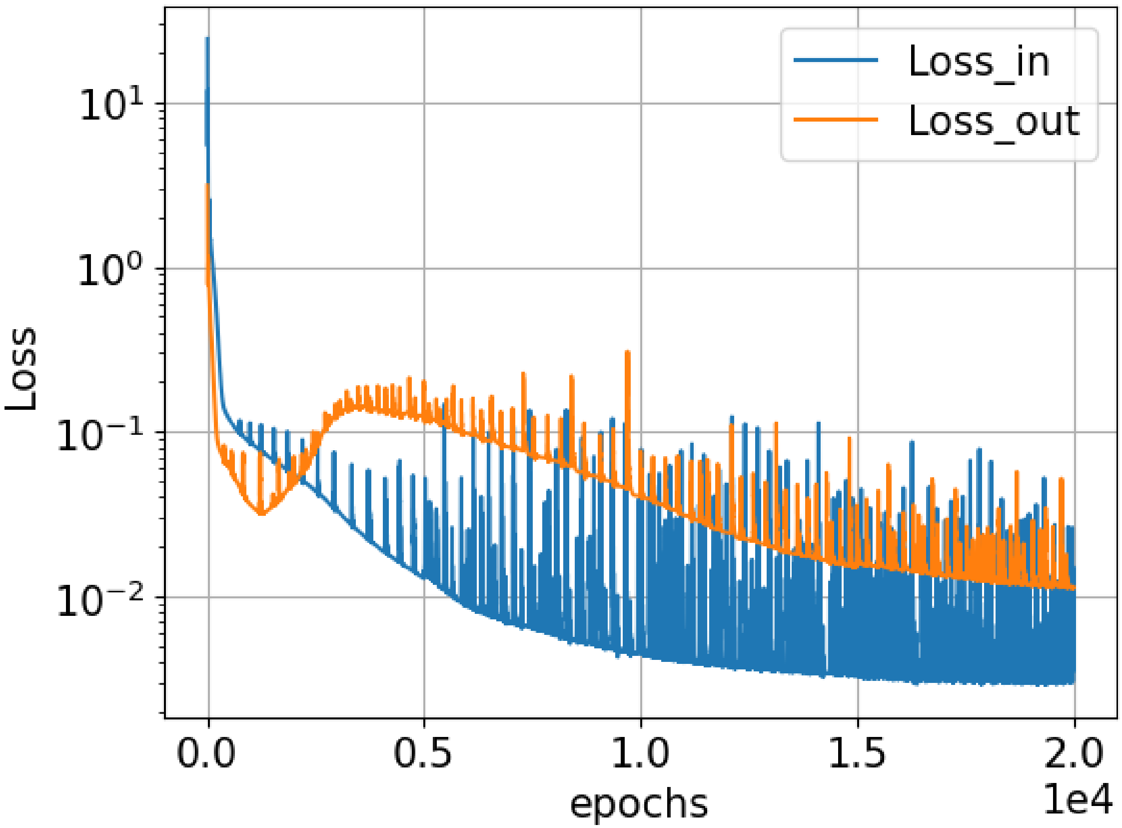


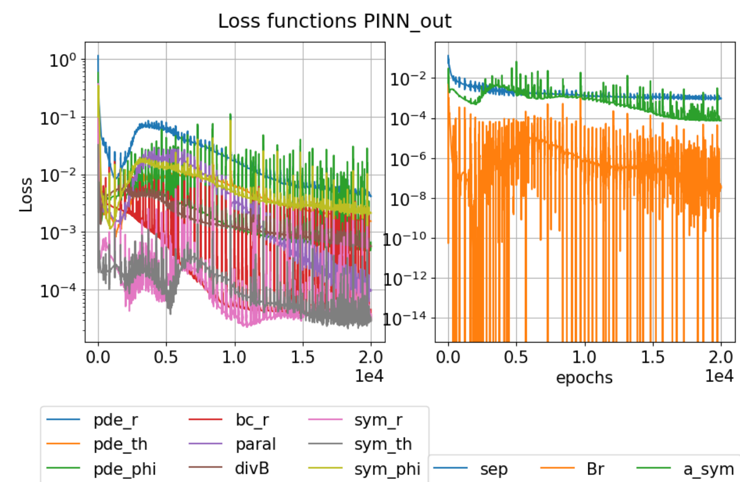


Dimitropoulos, Contopoulos 2024 (work in progress)

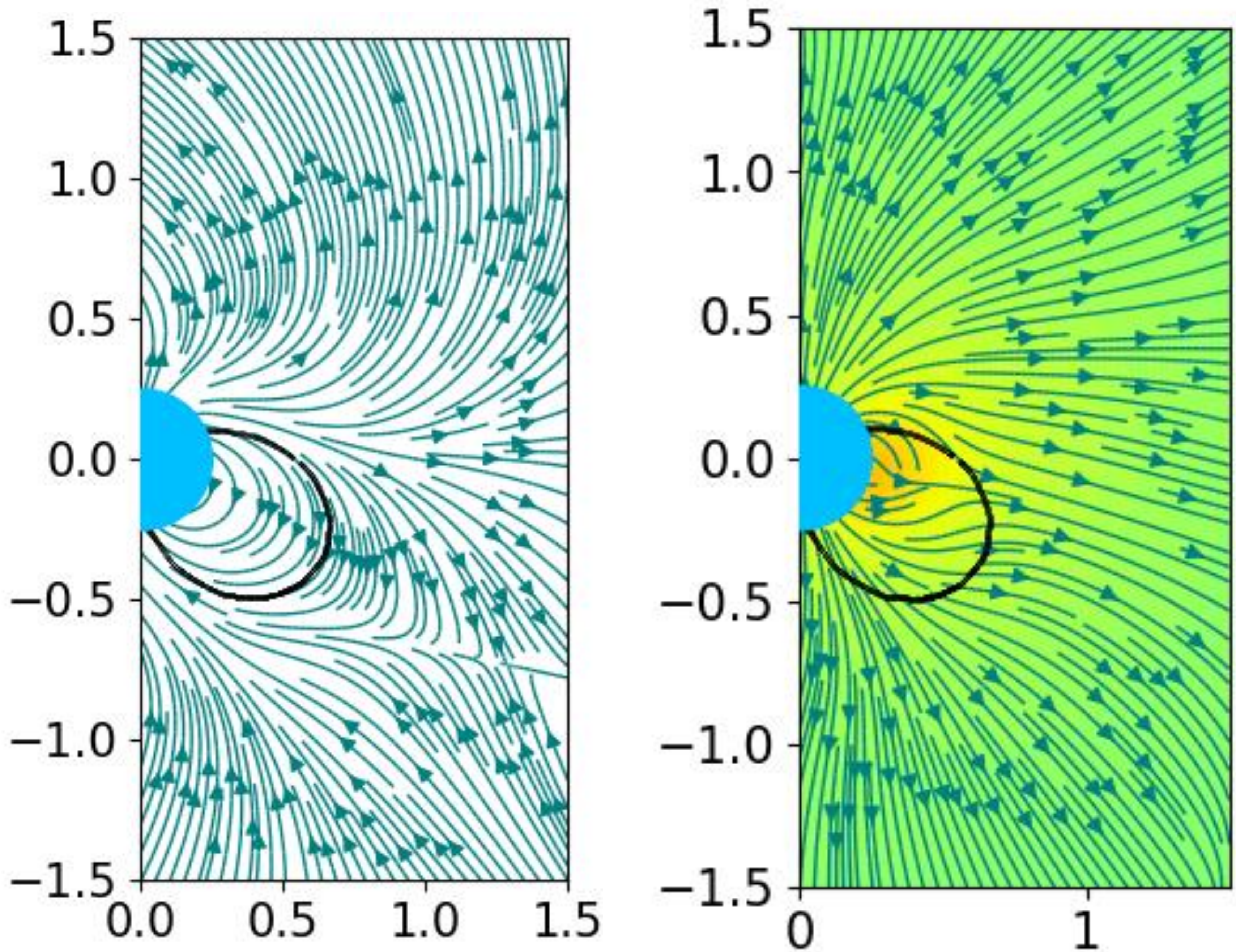


Dimitropoulos, Contopoulos 2024 (work in progress)

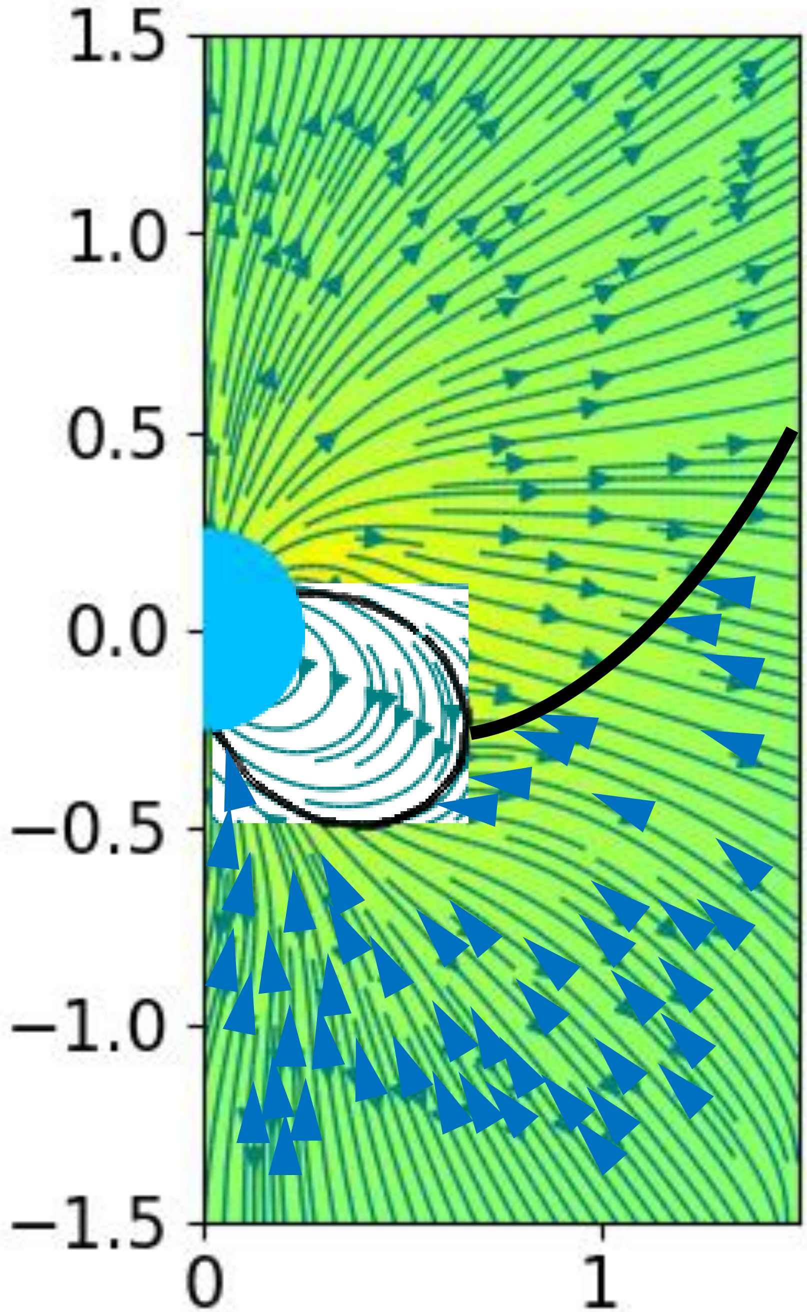




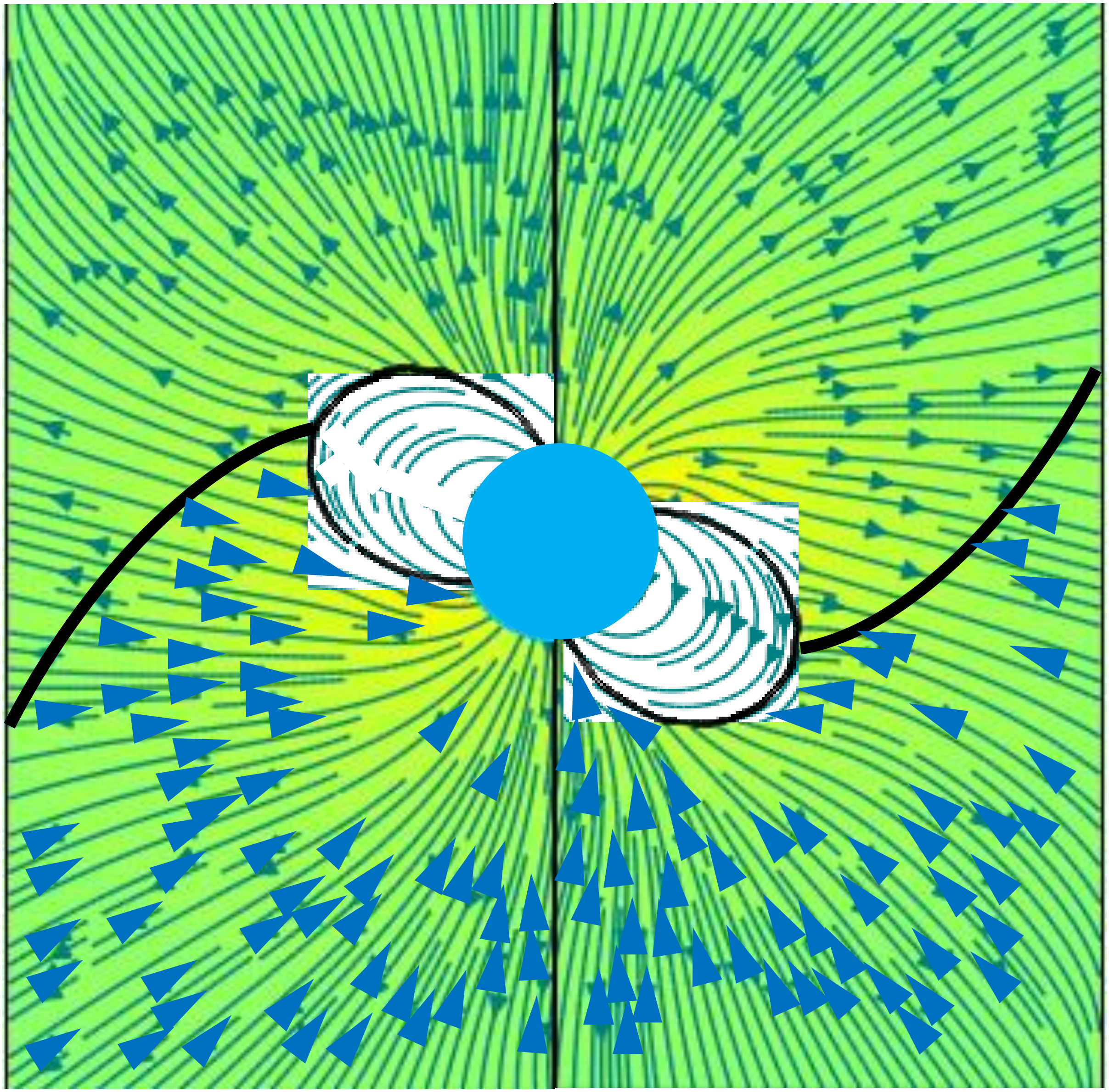
Dimitropoulos, Contopoulos 2024 (work in progress)



Dimitropoulos, Contopoulos 2024 (work in progress)



Dimitropoulos, Contopoulos 2024 (work in progress)



Dimitropoulos, Contopoulos 2024 (work in progress)

Summary

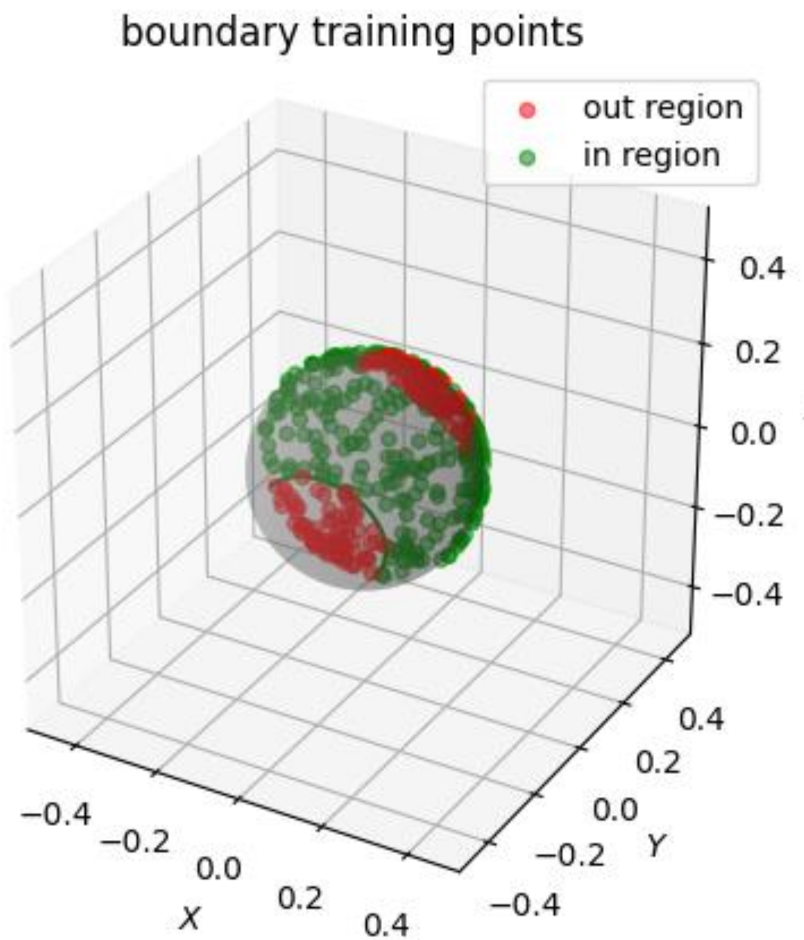
- PINNs possess considerable power
  - It is important to recognize their inherent challenges and limitations (must never be left unmonitored)
- Minimum energy solution when x_Y=0.85
  - Seen in all global PIC simulations since 2014
- New solutions: as $x_Y \rightarrow 1$, $\dot{E} \rightarrow 0$
  - New possibilities for pulsar spindown
- We need to investigate and resolve the above issues before embarking into more of the same "grand ab-initio" simulations

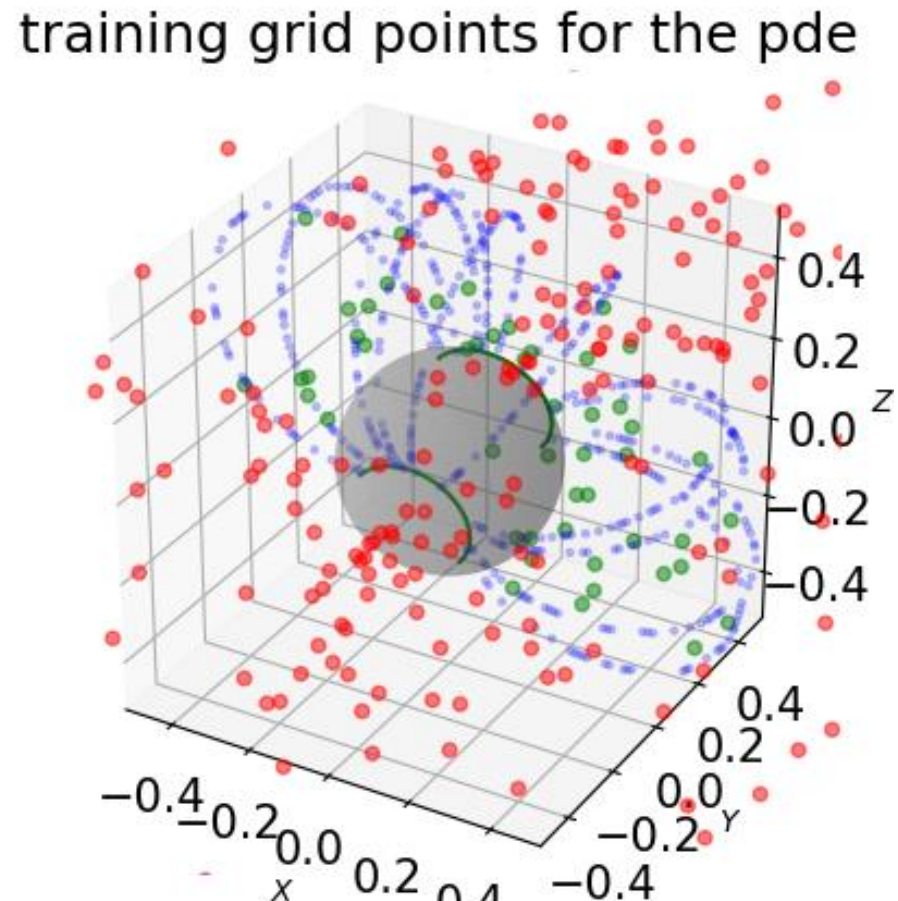
ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Future PINN applications

Future PINN applications

- Completion of the 3D pulsar magnetosphere
  + PIC investigation of Y-point and separatrix
- 3D reconstruction of the force-free magnetic field in solar active regions (direct spinoff)





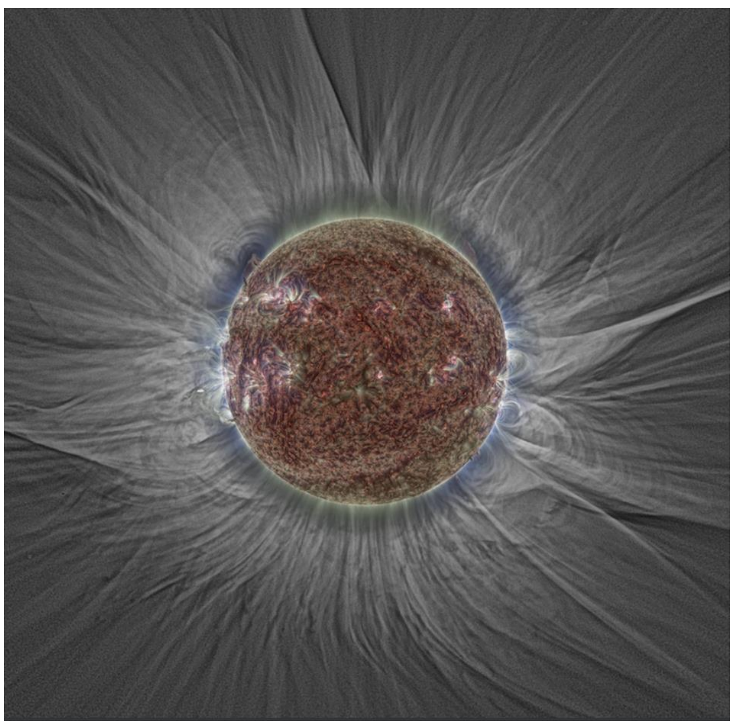
Dimitropoulos, Contopoulos 2024 (work in progress)

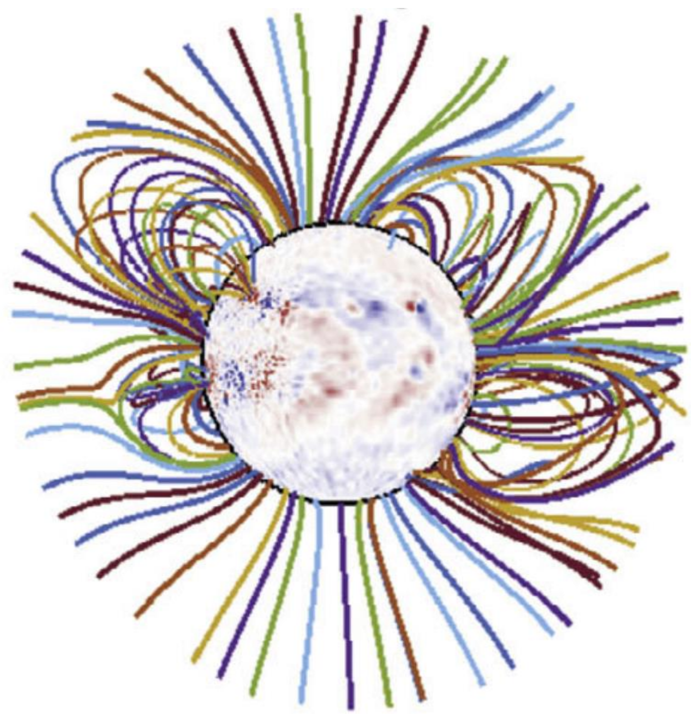
PINN applications in MHD

Space Sci Rev (2018) 214:99
https://doi.org/10.1007/s11214-018-0534-1

Global Non-Potential Magnetic Models of the Solar Corona During the March 2015 Eclipse

Anthony R. Yeates[1] · Tahar Amari[2] · Ioannis Contopoulos[3,4] · Xueshang Feng[5] ·





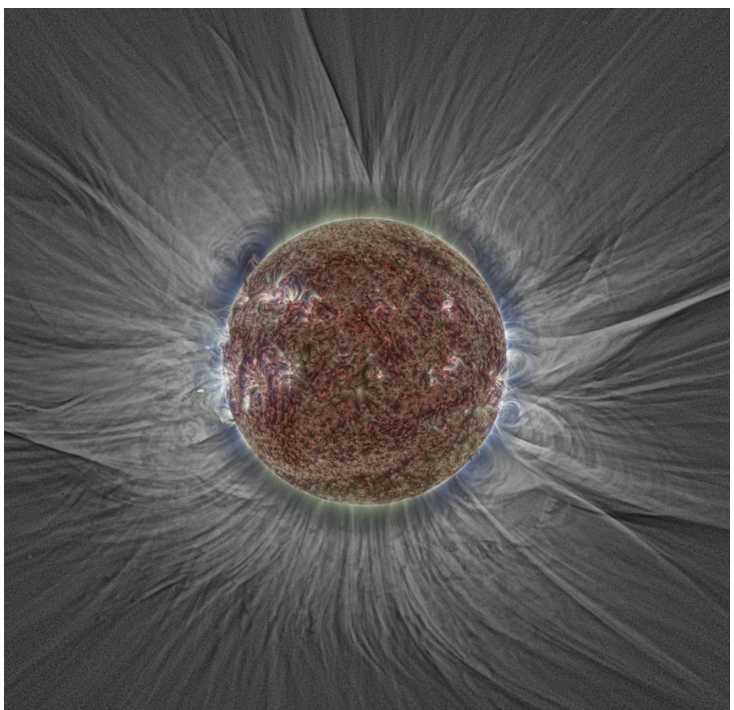
Contopoulos, Dimitropoulos, Gontikakis in preparation

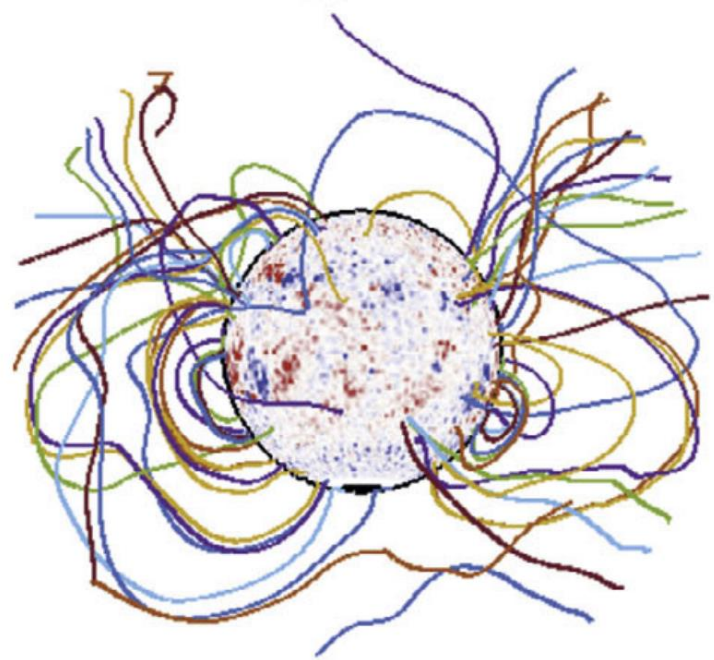
PINN applications in MHD

Space Sci Rev (2018) 214:99
https://doi.org/10.1007/s11214-018-0534-1

Global Non-Potential Magnetic Models of the Solar Corona During the March 2015 Eclipse

Anthony R. Yeates[1] · Tahar Amari[2] · Ioannis Contopoulos[3,4] · Xueshang Feng[5] ·





Contopoulos, Dimitropoulos, Gontikakis in preparation

ΑΚΑΔΗΜΙΑ ΑΘΗΝΩΝ

Thank you!