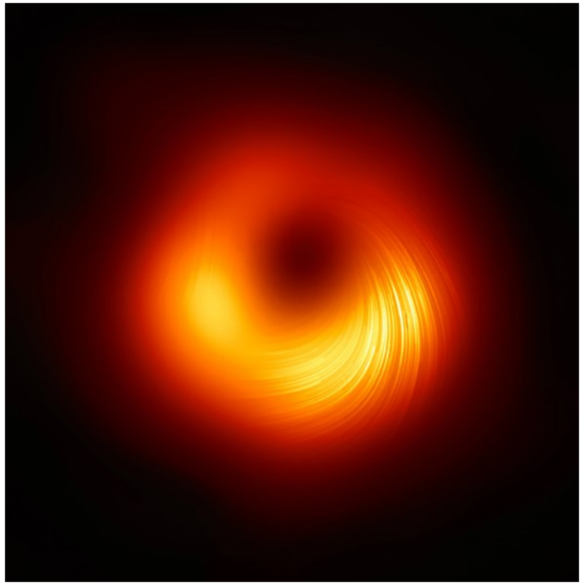
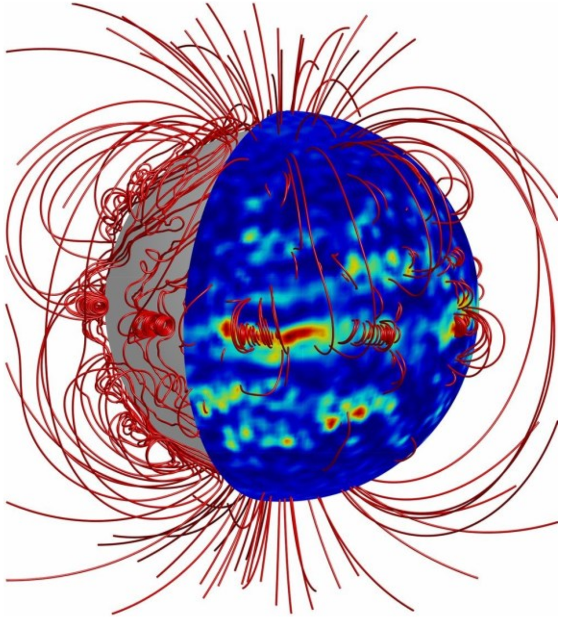


MHD in extreme Astrophysical Environments



Konstantinos Gourgouliatos

5th HelAS Summer School, Ioannina, 16-20 September 2024

Outline

- **Neutron Stars**:
  - **Magnetosphere**: Magnetic field dominates the dynamics. Framework: **Special Relativistic MHD**.
  - **Crust**: A high pressure and conductivity crystal. Magnetic field dynamics is subdominant (except for magnetars) evolution resembles that of a conductor. Framework: **Hall effect, Ohmic decay, crust yielding**.
  - **Core**: Consists of neutrons, protons and exotic particles. The protons may be **superconducting**, while the neutrons are **superfluid**. Framework: **superconducting** evolution or **ambipolar** diffusion.
  - In the limit of strong magnetic field, we may need to consider Quantum Electrodynamics.
- **Black Hole Environment:**
  - **No Hair theorem:** The properties of a BH can be uniquely determined only through its mass, rotation and electric charge. Framework: **Black Hole Electrodynamics**.
  - **Surroundings are crucial**: Magnetic field at the exterior is affected by the gravity and the rotation of BH, charge is unlikely. Framework: General Relativistic MHD in **disks and jets**.

Main tools:

Maxwell Equations (with charges and currents):

$$\nabla \cdot \mathbf{E} = 4\pi\rho \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$$

Force-free condition (current + charge):

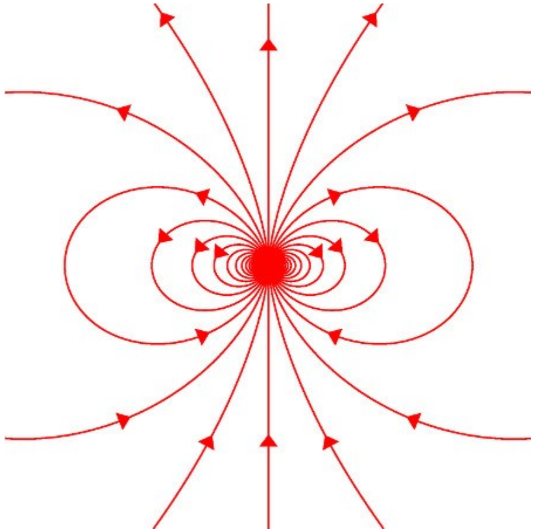
$$\frac{\boldsymbol{J} \times \boldsymbol{B}}{c} + \rho\boldsymbol{E} = \mathbf{0}$$

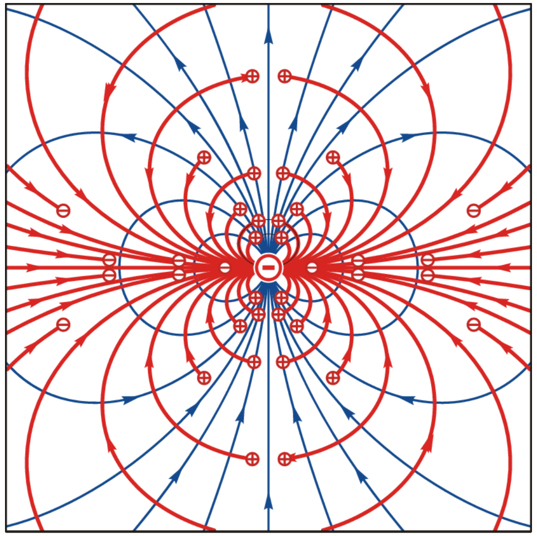
Ohm's law (arising from the momentum equation):

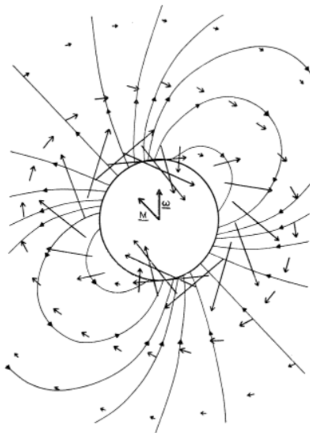
$$\boldsymbol{E} = -\frac{\boldsymbol{v} \times \boldsymbol{B}}{c} + \frac{\boldsymbol{J}}{\sigma}$$

Neutron Star Magnetosphere

- Vacuum: the magnetic field of an oblique rotating dipole (Deutsch 1955).
- Plasma-filled magnetosphere (Goldreich & Julian 1969). Force-free approximation: inertia, pressure and gravity are negligible compared to electromagnetic forces. **Force-free Electrodynamics**.
- Kinetic approach: the force generated by the particles acts on particles that carry the charge, accelerating them. **Particle in Cell**.
- MHD: the magnetic field is frozen into a low-density fluid, where the dynamics are solved through **special relativistic MHD**.
- **General relativistic** effects may be implemented through changes in the metric.







$$\begin{cases} H_r = \frac{\omega}{c} a^3 R_1(a) \frac{1}{r^2} \sin\chi \sin\theta \sin\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right] \\ H_\theta = \frac{1}{2}\frac{\omega^2}{c^2} a^3 R_1(a) \frac{1}{r} \sin\chi \cos\theta \cos\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right] \\ H_\varphi = -\frac{1}{2}\frac{\omega^2}{c^2} a^3 R_1(a) \frac{1}{r} \sin\chi \sin\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right], \end{cases}$$

$$\begin{cases} E_r = 0 \\ E_\theta = -\frac{1}{2}\frac{\omega^2 \mu_0}{c} a^3 R_1(a) \frac{1}{r} \sin\chi \sin\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right] \\ E_\varphi = -\frac{1}{2}\frac{\omega^2 \mu_0}{c} a^3 R_1(a) \frac{1}{r} \sin\chi \cos\theta \cos\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right]. \end{cases}$$

Vacuum

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

-Aligned rotator: nothing happens!

-Oblique rotator: generation of electric and magnetic field (Deutsch 1955).

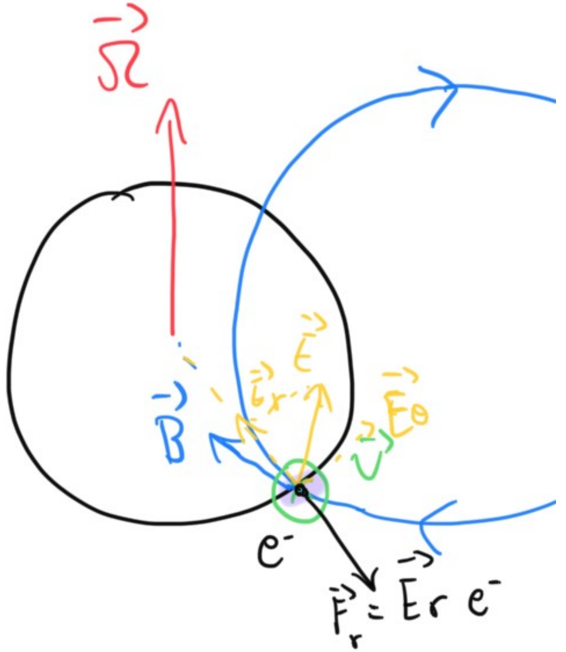
-Basic model still used to assign a magnetic field to a neutron star.

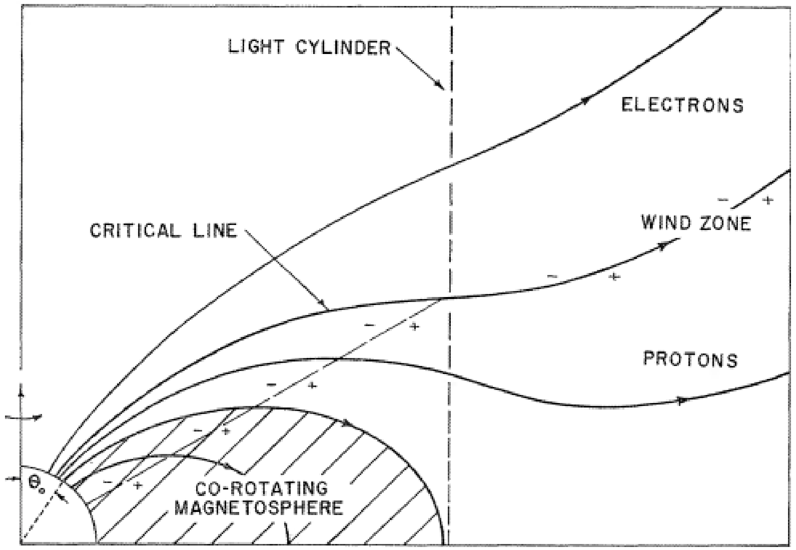
$$\mathbf{E} = -\frac{(\mathbf{\Omega}\times\mathbf{r})\times\mathbf{B}}{c} = -\frac{\Omega r \sin\theta}{c}\hat{\phi}\times\mathbf{B}$$

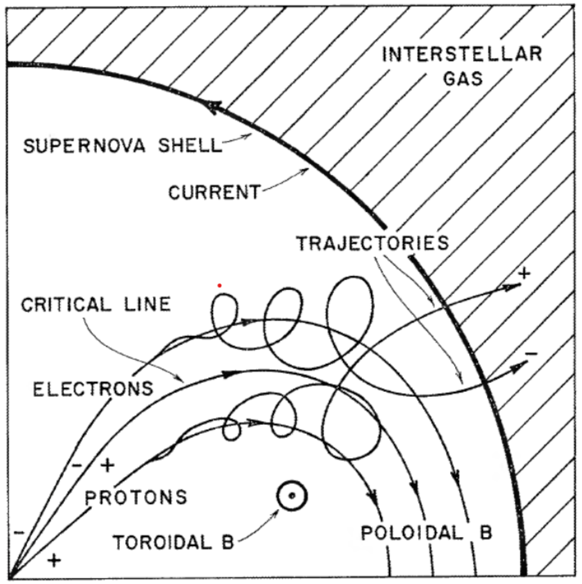
$$\mathbf{j} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

$$\rho = \frac{\nabla\cdot\mathbf{E}}{4\pi}$$

$$\frac{\mathbf{j}\times\mathbf{B}}{c} + \rho\mathbf{E} = \mathbf{0}$$



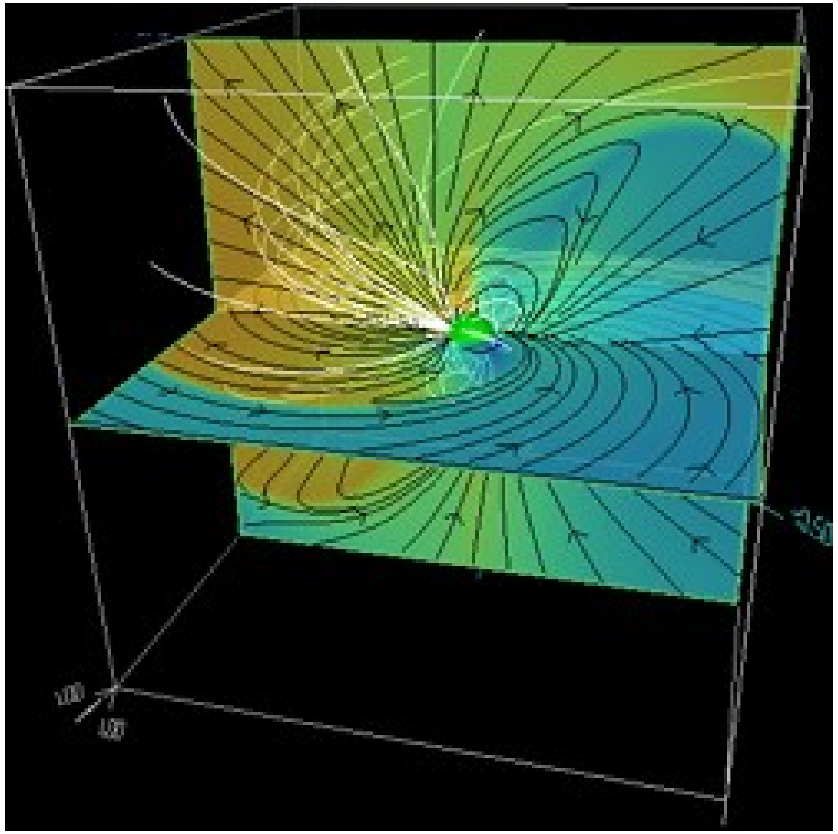




Plasma-filled magnetosphere:

- Aligned rotator: charges and currents fill the magnetosphere: energy emitted to infinity. A steady-state non-trivial solution is possible. The power radiated scales to an oblique magnetic dipole in vacuum (Goldreich & Julian 1969, Contopoulos et al. 1999)
- Oblique rotation: a steady-state is feasible in the corotating frame.

Force-free electrodynamics



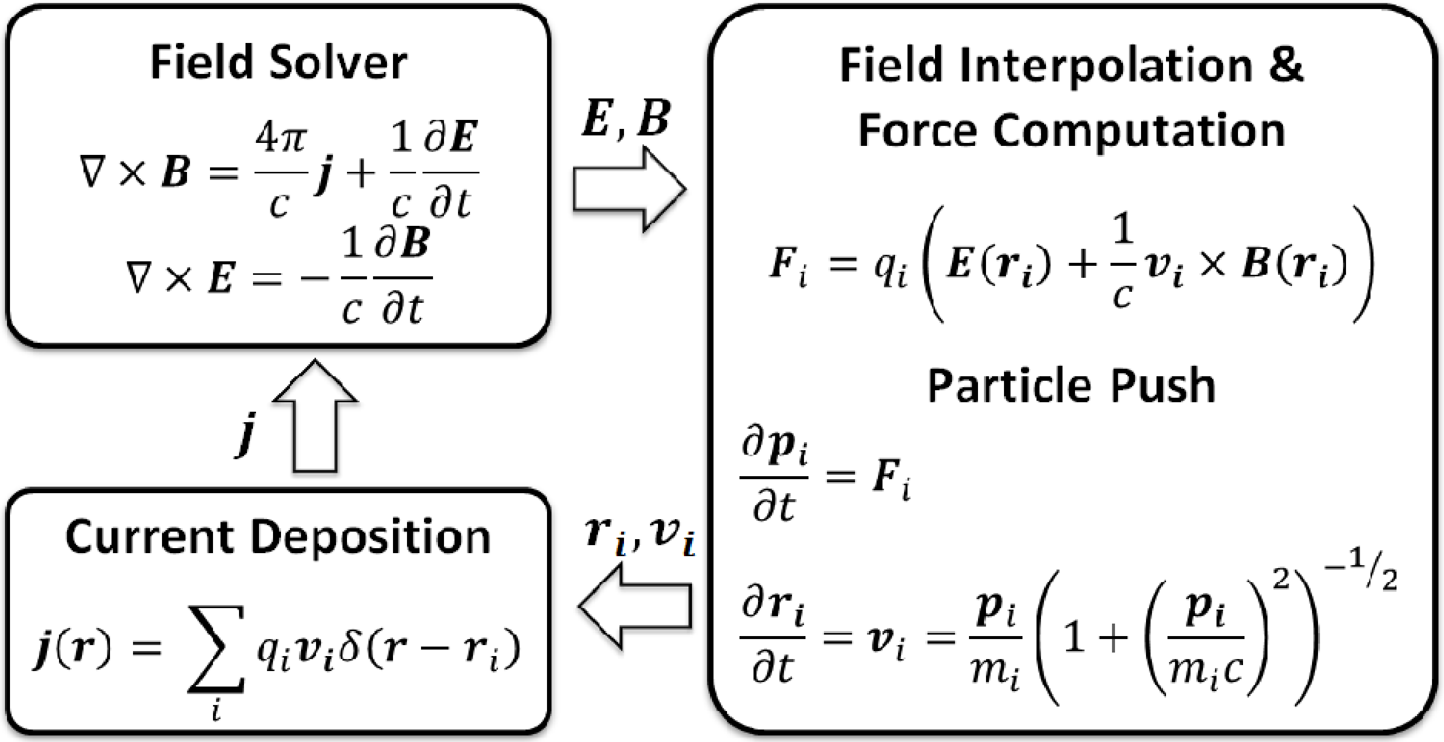
(Spitkovsky 2004)

$$\mathbf{J} = \frac{c}{4\pi}\nabla\cdot\mathbf{E}\,\frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{c}{4\pi}\frac{(\mathbf{B}\cdot\nabla\times\mathbf{B} - \mathbf{E}\cdot\nabla\times\mathbf{E})}{B^2}\,\mathbf{B}$$

$$\frac{\partial\mathbf{E}}{\partial t} = c\nabla\times\mathbf{B} - 4\pi\mathbf{J}\,, \quad \frac{\partial\mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}\,,$$

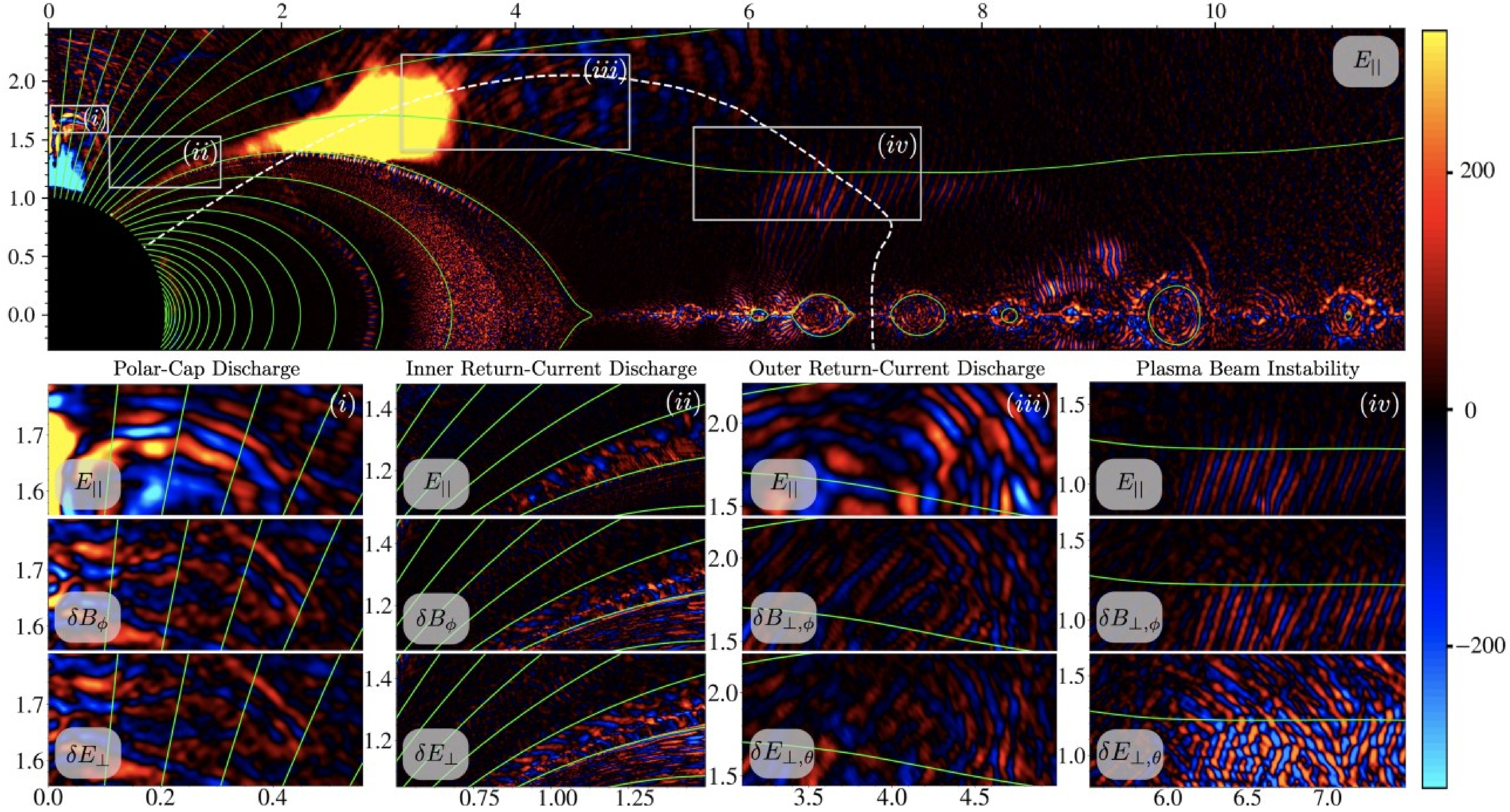
$$\nabla\cdot\mathbf{B} = 0\,, \quad \mathbf{E}\cdot\mathbf{B} = 0\,, \quad \rho_{\mathrm{e}}\mathbf{E} + \frac{1}{c}\mathbf{J}\times\mathbf{B} = 0$$

Kinetic Approach



- Evaluate the electric and magnetic field corresponding to charged particles.
- Evaluate the force-acceleration-motion of the particles due to the fields.
- Repeat!

- Sounds simple but technically it is very demanding!



Bransgrove et al. 2023

MHD solution

- A rotating dipole is embedded in a low-density, but clearly non-vacuum, plasma.
- After a few rotations relaxes to a magnetosphere resembling the force-free solutions.

$$\partial_t(\alpha\sqrt{\gamma}\,\rho u^t) + \partial_i(\alpha\sqrt{\gamma}\rho u^i) = 0,$$

$$\partial_t\left(\alpha\sqrt{\gamma}\,T^t{}_\nu\right) + \partial_i\left(\alpha\sqrt{\gamma}\,T^i{}_\nu\right) = \tfrac{1}{2}\partial_\nu(g_{\alpha\beta})T^{\alpha\beta}\alpha\sqrt{\gamma},$$

$$T^{\mu\nu}_{(\mathrm{e})} = \frac{1}{4\pi}\left[F^{\mu\gamma}F^\nu{}_\gamma - \tfrac{1}{4}\left(F^{\alpha\beta}F_{\alpha\beta}\right)g^{\mu\nu}\right]$$

$$T^{\mu\nu}_{(\mathrm{m})} = wu^\mu u^\nu - pg^{\mu\nu}$$

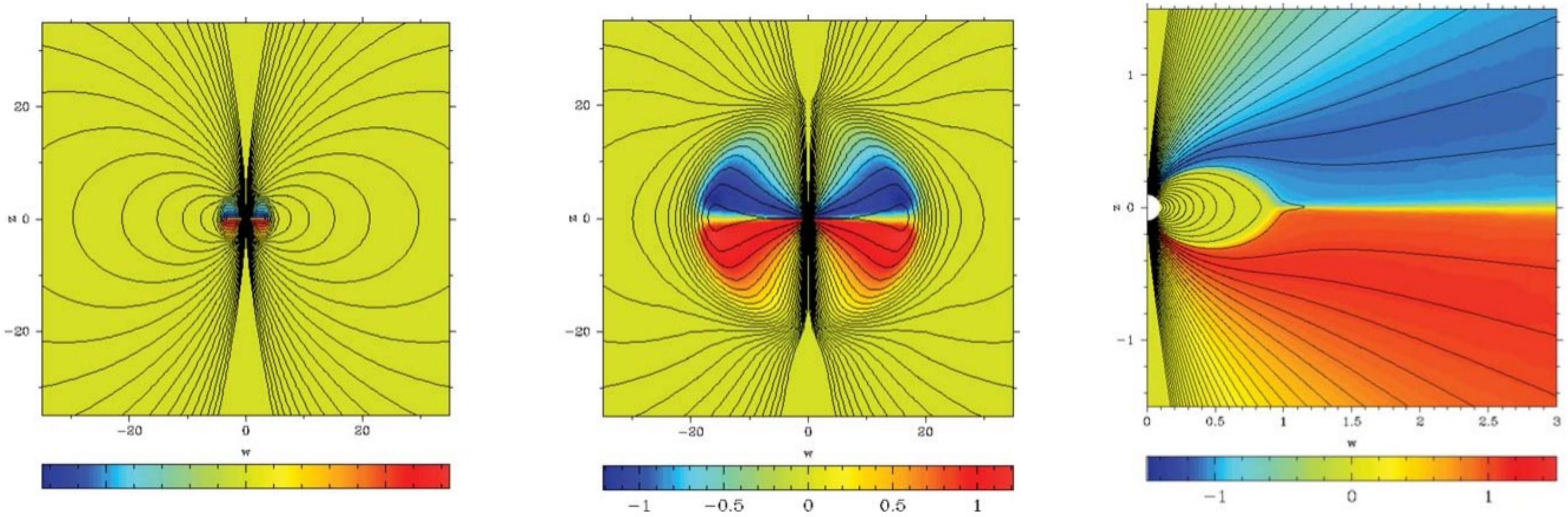
$$(1/c)\,\partial_t(B^i) + e^{ijk}\,\partial_j(E_k) = 0$$

$$\partial_i(\sqrt{\gamma}B^i) = 0.$$

$$E_i = e_{ijk}v^j B^k/c$$

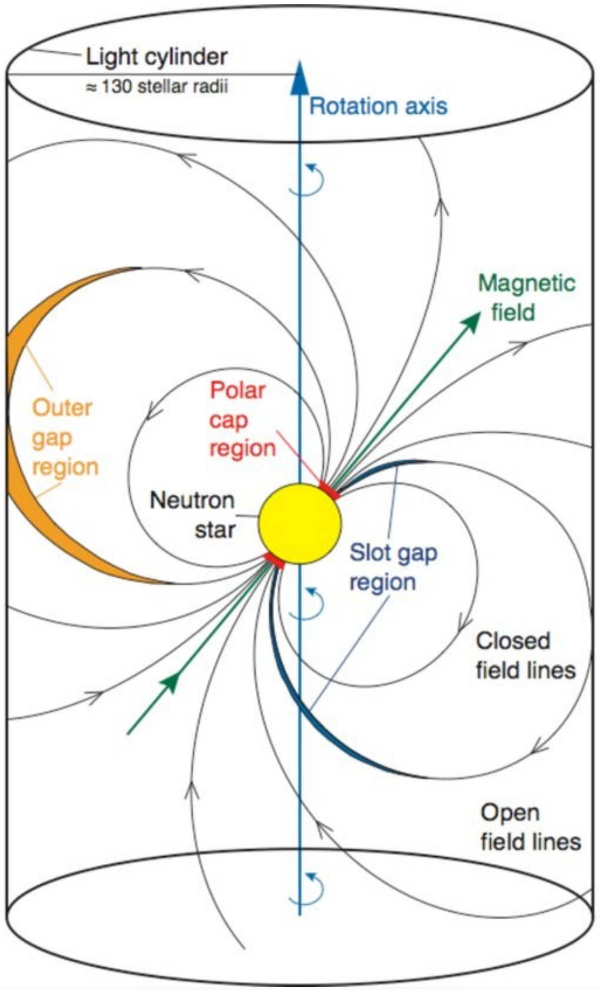
This approach is highly demanding as it tries deplete regions from matter.

(Komissarov 2005)



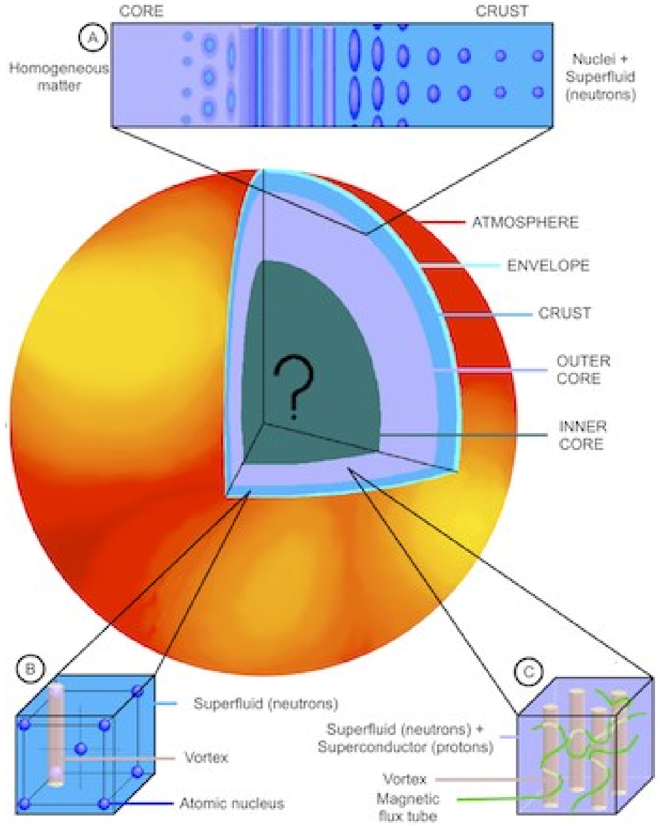
Consensus picture of the magnetosphere

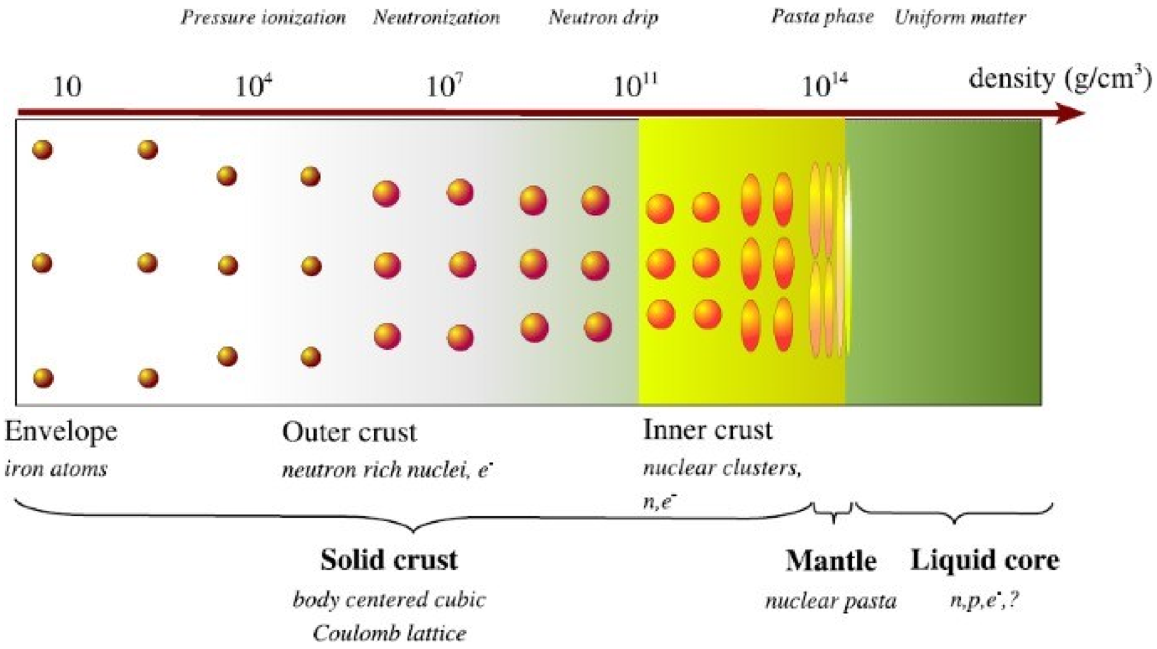
- The magnetosphere reaches an equilibrium.
- The light cylinder is a critical surface: field lines crossing the light cylinder are open (one end at infinity the other on the star).
- Still a lot to learn:
    - How is radiation produced?
    - Where is the energy dissipated?
    - Is the centered dipole picture sufficient?
    - Is the equilibrium the numerical models give unique?



Neutron star interior

- The outer layer of a neutron star is a BCC ion lattice (1km thick).
- It is highly conducting, and the current is mediated by the free electrons.
- The evolution is mediated through the Hall effect and Ohmic dissipation.
- The central part of the star ($0.9R_{NS}$) is the core.
- It contains neutron and protons, most likely in superfluid and superconducting state.





MHD inside the NS star

Momentum equation that leads to Ohm's law.

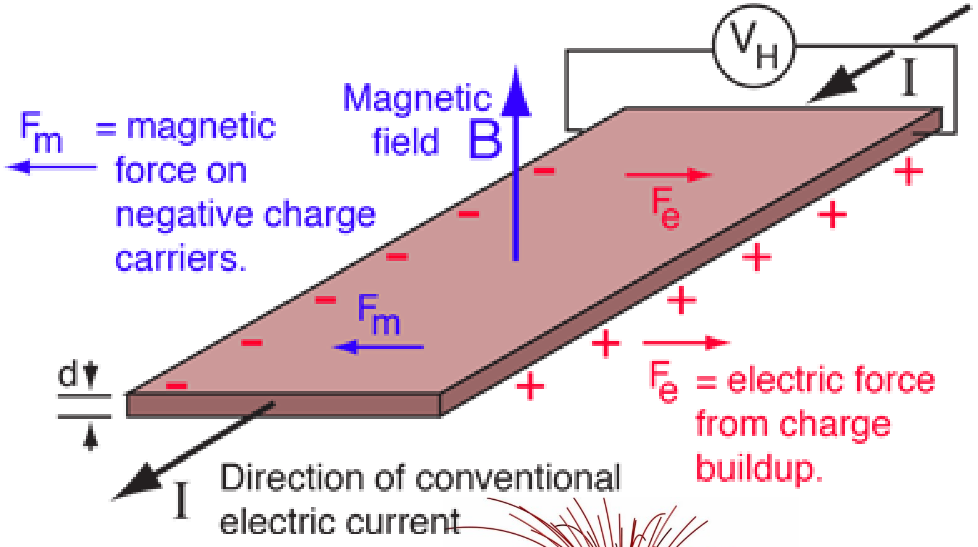
$$m_e^* \frac{\partial \boldsymbol{v_e}}{\partial t} + m_e^* (\boldsymbol{v_e} \cdot \nabla) \boldsymbol{v_e} = -\nabla \mu_e - m_e^* \nabla \Phi - e\left(\boldsymbol{E} + \frac{\boldsymbol{v_e}}{c} \times \boldsymbol{B}\right) - \frac{m_e^* (\boldsymbol{v_e} - \boldsymbol{v_n})}{\tau_{en}} - \frac{m_e^* (\boldsymbol{v_e} - \boldsymbol{v_p})}{\tau_{ep}} \qquad (1)$$

$$m_p^* \frac{\partial \boldsymbol{v_p}}{\partial t} + m_p^* (\boldsymbol{v_p} \cdot \nabla) \boldsymbol{v_p} = -\nabla \mu_p - m_p^* \nabla \Phi + e\left(\boldsymbol{E} + \frac{\boldsymbol{v_p}}{c} \times \boldsymbol{B}\right) - \frac{m_p^* (\boldsymbol{v_p} - \boldsymbol{v_n})}{\tau_{pn}} - \frac{m_p^* (\boldsymbol{v_p} - \boldsymbol{v_e})}{\tau_{pe}} \qquad (2)$$

$$m_n^* \frac{\partial \boldsymbol{v_n}}{\partial t} + m_n^* (\boldsymbol{v_n} \cdot \nabla) \boldsymbol{v_n} = -\nabla \mu_n - m_n^* \nabla \phi - \frac{m_n^* (\boldsymbol{v_n} - \boldsymbol{v_p})}{\tau_{np}} - \frac{m_n^* (\boldsymbol{v_n} - \boldsymbol{v_e})}{\tau_{ne}}, \qquad (3)$$

Goldreich & Reisenegger 1992

Hall effect and Ohmic decay (crust)

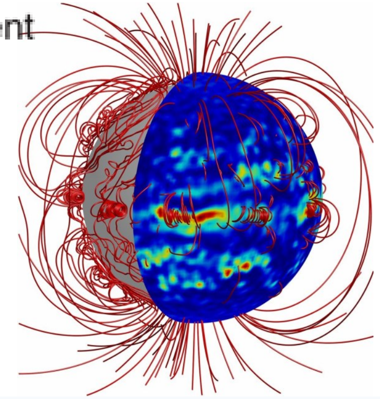


$$\boldsymbol{j} = -en_e\boldsymbol{v}$$

$$\boldsymbol{E} = -\frac{\boldsymbol{v}\times\boldsymbol{B}}{c} + \frac{\boldsymbol{j}}{\sigma}$$

$$\boldsymbol{j} = \frac{c}{4\pi}\nabla\times\boldsymbol{B}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = -c\nabla\times\boldsymbol{E}$$

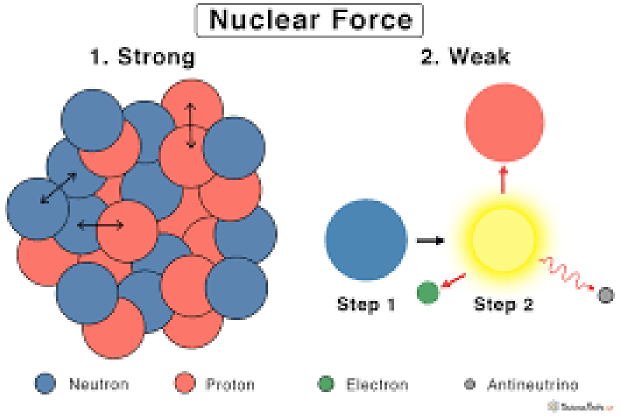


$$\frac{\partial\mathbf{B}}{\partial t} = -\frac{c}{4\pi \mathrm{e}}\nabla\times\left(\frac{\nabla\times\mathbf{B}}{n_\mathrm{e}}\times\mathbf{B}\right) - \frac{c^2}{4\pi}\nabla\times\left(\frac{\nabla\times\mathbf{B}}{\sigma}\right)$$

Gourgouliatos et al. 2016 (3D solution)

Core – Ambipolar Diffusion

- Evolution due to interaction between charged and neutral particles.
  - Electron-neutron interaction is mediated through the weak nuclear force.
  - Proton-neutron interaction is mediated through the strong nuclear force.



$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\frac{c}{4\pi e n_c}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \frac{c^2}{4\pi \sigma_0}\nabla \times \boldsymbol{B}\right] + \nabla \times \left[\frac{x_n^2 t_{pn}}{m_p^*}\left[\frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi n_c} - \nabla(\Delta\mu)\right] \times \boldsymbol{B}\right]$$

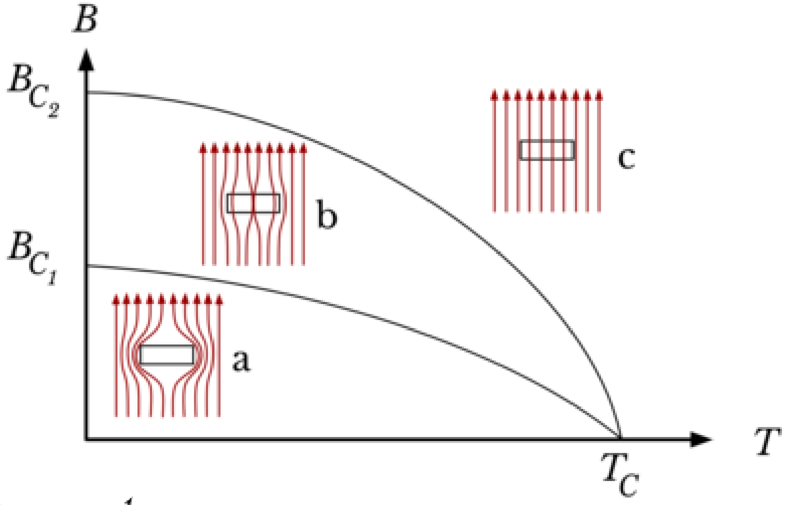
Skiathas & Gourgouliatos 2023

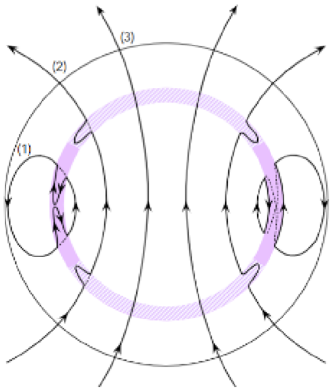
Core - Superconductivity

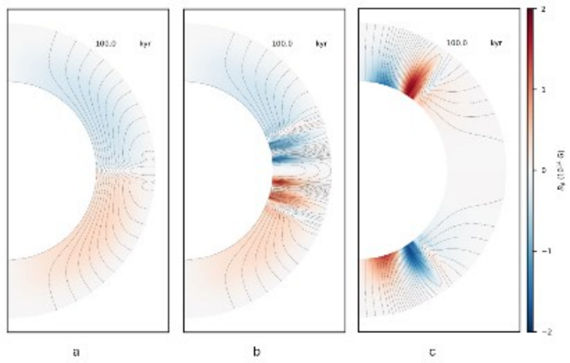
- Low temperature phenomenon. The temperature of neutron star core (~10^9 K) is low compared to the Fermi temperature (~10^{10} K), making favorable the superconducting state.
- Type-I vs Type-II superconductors:
  - Type-I fully expelled field.
  - Type-II superconducting flux-bundles.
- Strong magnetic field suppresses superconductivity.

$$\boldsymbol{F}_m = -\frac{1}{4\pi}\left[\boldsymbol{B} \times (\nabla \times \boldsymbol{B}_{c1}) + \rho_p \nabla\left(B\,\frac{\partial B_{c1}}{\partial \rho_p}\right)\right]$$

$$\boldsymbol{E} = \frac{\boldsymbol{F}_{\boldsymbol{m}}}{\boldsymbol{e}n_c} \qquad \partial_t \boldsymbol{B} \;=\; -c\,\nabla \times \boldsymbol{E},$$



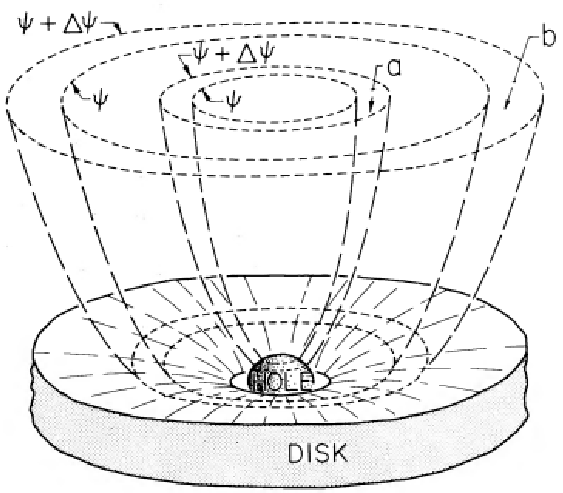




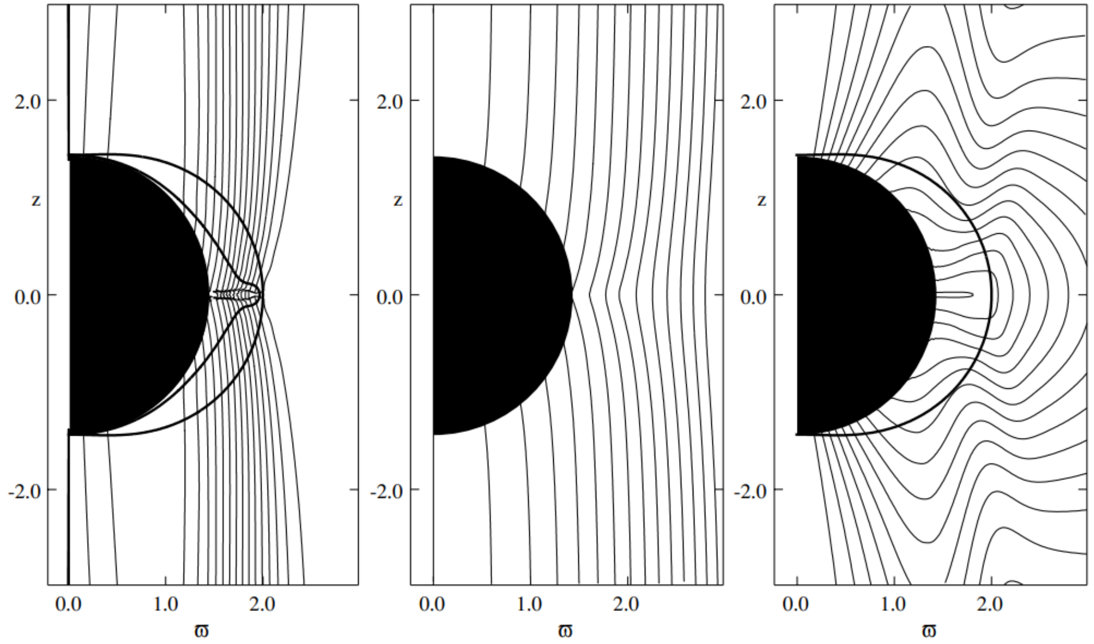
Lander, Gourgouliatos + 2024

Black Hole Electrodynamics

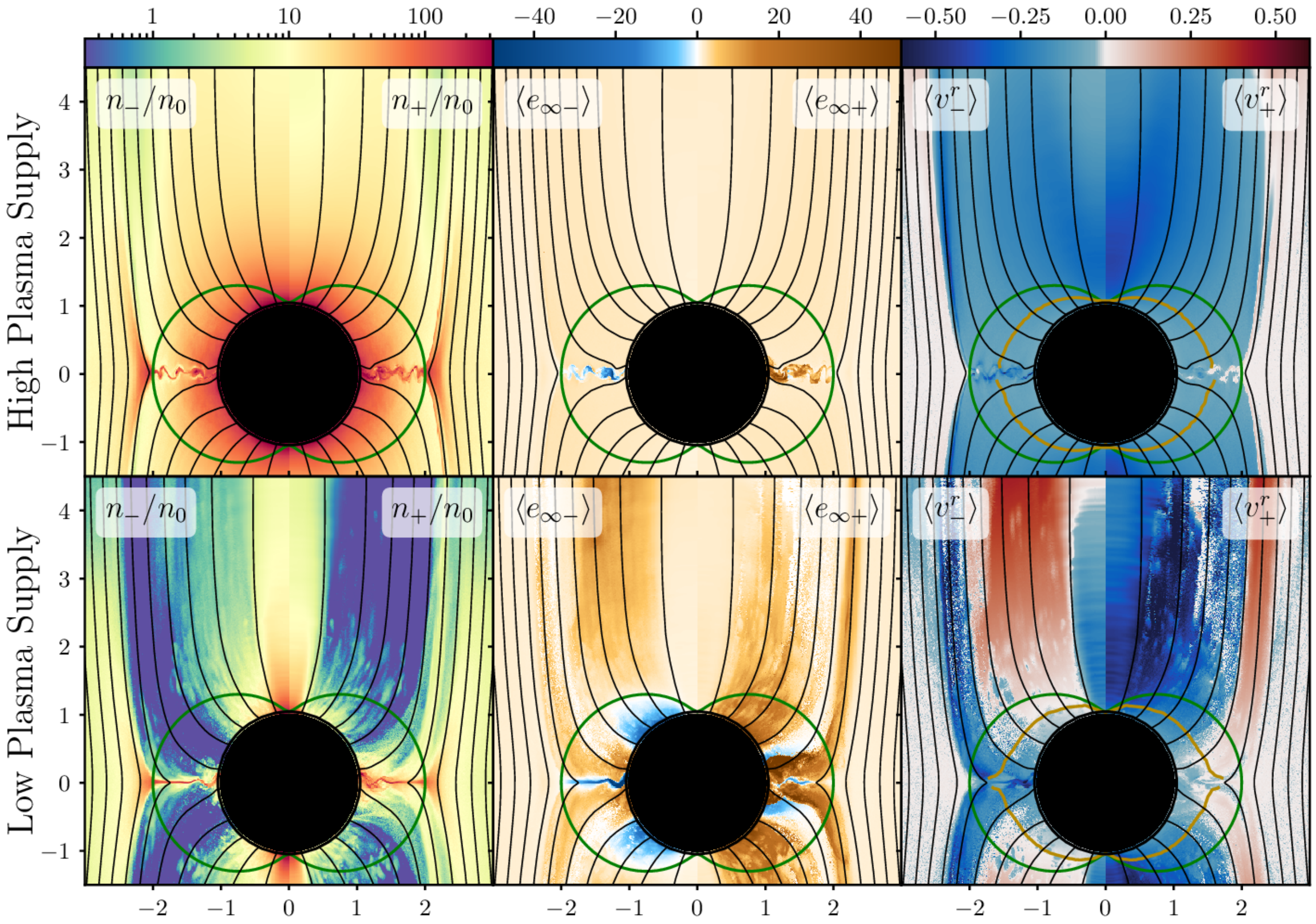
- Main motivation: how to extract energy from a (spinning) black hole, with the assistance of a magnetic field and an accretion disk.
- Force-free electrodynamics



Disk is required
(Blandford Znadjek 1977, MacDonald Thorne 1982)



Spinning black hole - ergosphere
(Komissarov 2004)



Kinetic simulations (Parfrey et al. 2018)

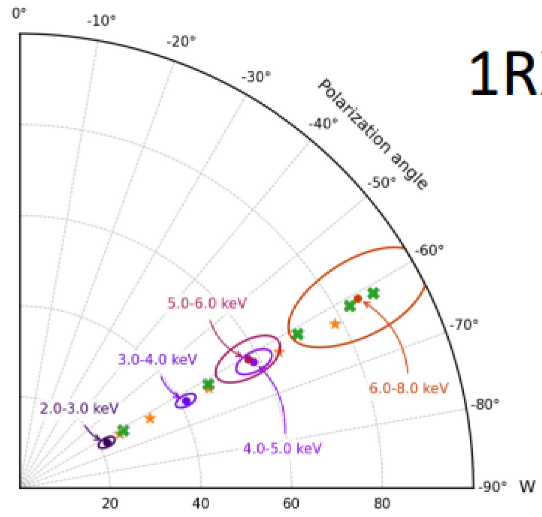
Quantum Electrodynamics

- The magnetic field in the vicinity of compact objects exceeds 4×10^{13}G, the QED limit:

$$\hbar \frac{eB_{\rm QED}}{m_e c} = m_e c^2$$

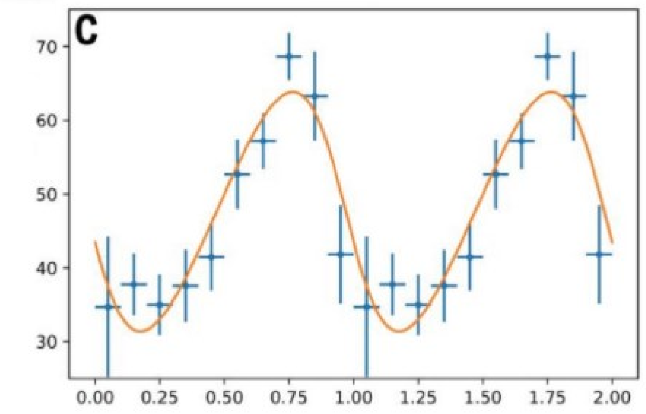
- QED effects are hinted in polarization observations of magnetars by IXPE (Taverna et al. 2022).

1RXS J1708



Polarization Degree 90%

Pol. Angle



Phase

Conclusions

- MHD processes are of core importance in extreme astrophysical environments.
- Rapid rotation, high density, compactness and strong gravity open new avenues for magnetic field evolution.
- MHD is of key importance to neutron stars and black hole environments: most of their observational manifestations are due to the presence of magnetic field.
- MHD in extreme enivornments is a key ingredient for our understanding of fundamental Physics!

- Still a lot to learn!

Thanks for your attention!