Numerical Solution of Burger's Equation

The Minmod Flux Limiter

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Outline

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- Summary and Cocnlusions

Burger's Equation

The Burger's Equation

$$\frac{\partial \Psi}{\partial t} = -\Psi \frac{\partial \Psi}{\partial x} + \eta \frac{\partial^2 \Psi}{\partial x^2} \tag{1}$$

[Bonkile et al., 2023] In our case (for minmod flux limiter)

$$\frac{\partial \Psi}{\partial t} = -\Psi \frac{\partial \Psi}{\partial x} \tag{2}$$

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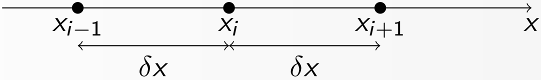
- Transportation equation
- Trasportation speed of Ψ depends on Ψ

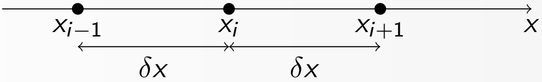
Use of Burger's Equation in Astrophysical Plasmas

- Force free fields with poloidal and toroidal components
- The simplest non linear wave equation
- Describes convection and diffusion
- Converts to transport equation for $\eta = 0$

Numerical Derivative



Numerical Derivative

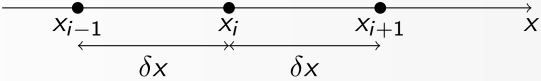


forward derivative

$$x' = \frac{x_{i+1} - x_i}{\delta x}$$

provides information in front of the point of interest

Numerical Derivative



forward derivative

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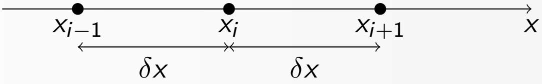
backward derivative

$$x' = \frac{x_i - x_{i-1}}{\delta x}$$

provides information behind the point of interest

Numerical Derivative



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backward derivative

$$x' = \frac{x_i - x_{i-1}}{\delta x}$$

provides information behind the point of interest

central derivative

$$x' = \frac{x_{i+1} - x_{i-1}}{2\delta x}$$

the better approach but sensitive to extreme variations

Forward Derivative

Initial condition: Gaussian

Periodic Boundary Conditions

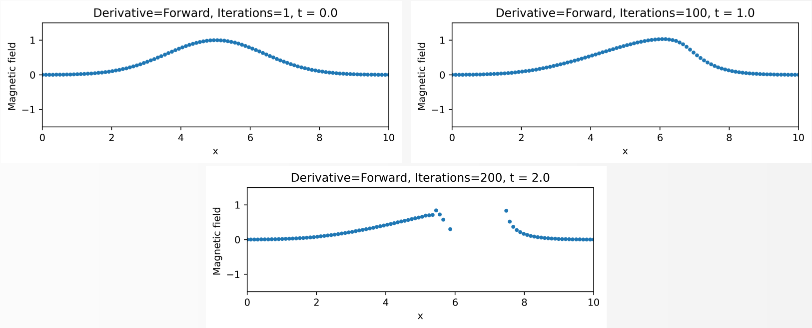

Figure: Time evolution of Burger's equation using numerical forward derivative. Top: steps=1. Middle: steps=100. Bottom: steps=200

Backward Derivative

Initial condition: Gaussian

Periodic Boundary Conditions

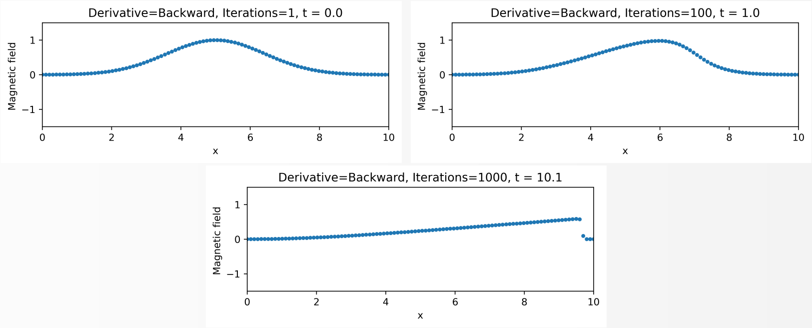

Figure: Time evolution of Burger's equation using numerical backward derivative. Top: steps=1. Middle: steps=100. Bottom: steps=1000

Central Derivative

Initial condition: Gaussian

Periodic Boundary Conditions

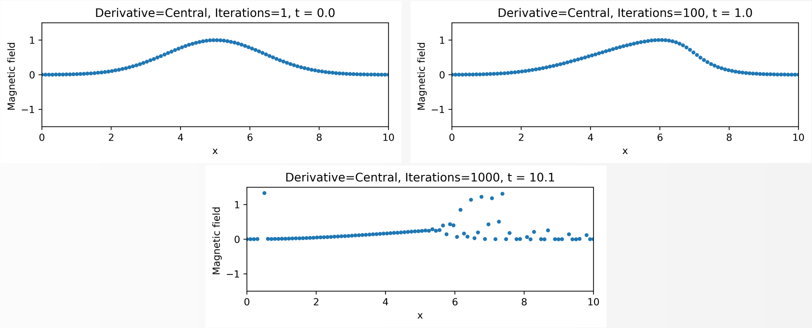

Figure: Time evolution of Burger's equation using numerical central derivative. Top: steps=1. Middle: steps=100. Bottom: steps=1000

Numerical Derivative

Backward:
We miss the points in front

Forward:
We miss the points at the back

Central:
Half of the info is from the points the wave has not reached yet

Flux Limiter (Minmod)

Flux limiter: Use ideal proportions of backward and forward derivative

Minmod:

$$\Phi_{mm}(x) = max[0, min(1, x)] \tag{3}$$

[Durran, 2010]

Flux Limiter (Minmod)

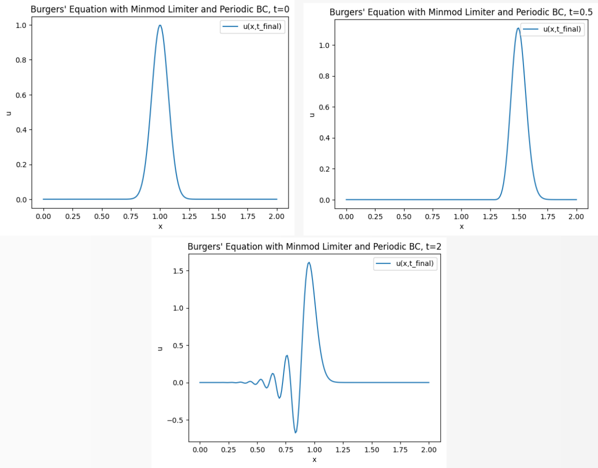

Figure: Time evolution of Burger's equation using minmod flux limiter

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- We can use numerical analysis to solve problems involving the Burger equation
- There are 3 numerical derivatives (forward, backward, central) but each has its own disadvantages due to the fact that they do not calculate the derivative to the actual point of interest
- The use of flux limiters can help deal with this problem, but there are other disadvantages introduced
- The use of the minmod flux limiter eliminates the discontinuities but it alters the wave front due to information diffusion

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Thank You!

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