Neutron star magnetospheres with classical and modern methods

5^{th} Hel.A.S Summer School - MHD in Astrophysics

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1 Introduction

1 Introduction

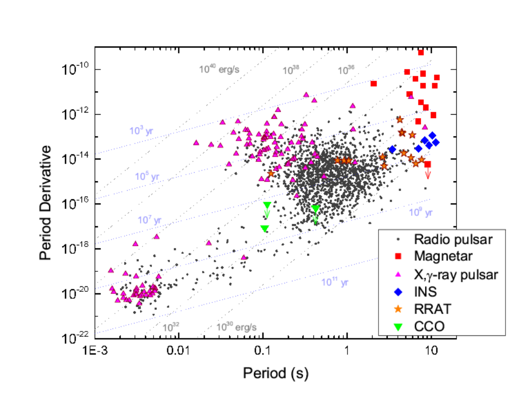
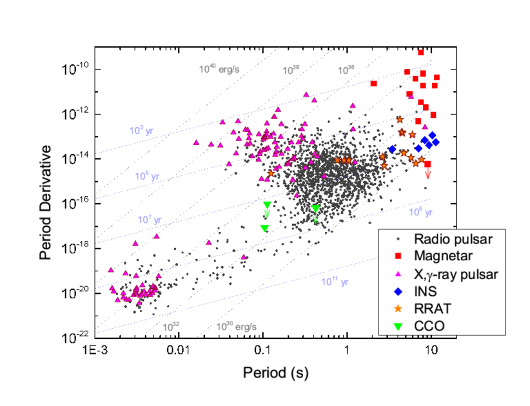


Figure 1: The $P - \dot{P}$ diagram (Harding, 2013)

Magnetars and Pulsars

Magnetars

- Slow rotation $P \gtrsim 1$ s
- Extreme magnetic field $B \gtrsim 10^{14}$ G
- Young age
- Powered by magnetic field decay



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Magnetars and Pulsars

Magnetars

- Slow rotation $P \gtrsim 1$ s
- Extreme magnetic field $B \gtrsim 10^{14}$ G
- Young age
- Powered by magnetic field decay

Pulsars

- Fast rotation $P < 1$ s
- Magnetic field $B \sim 10^{12}$ G
- Powered by rotational

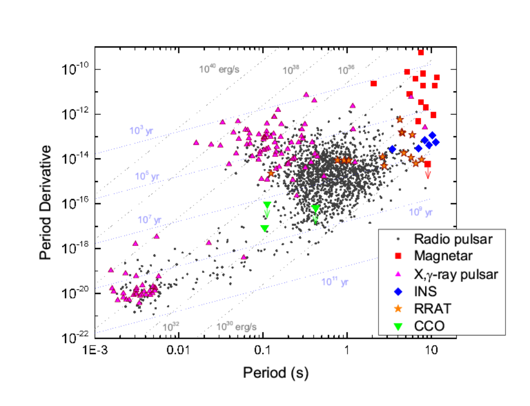

Figure 1: The $P - \dot{P}$ diagram (Harding, 2013)

Observations

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Magnetars

- Pulsed X-ray thermal emission
- Transient X-ray activity
  - Short duration bursts
  - Long duration outbursts
  - Highly energetic flares
- Reconfiguration of magnetic field
- Release of energy

Observations

Magnetars

- Pulsed X-ray thermal emission
- Transient X-ray activity
  - Short duration bursts
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- Reconfiguration of magnetic field
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Pulsars

- Pulsed non-thermal γ-ray emission
- Non-ideal regions
- Acceleration of particles

Magnetosphere

Magnetosphere

Definition

The region surrounding the neutron star where plasma flows in its magnetic field

Magnetosphere

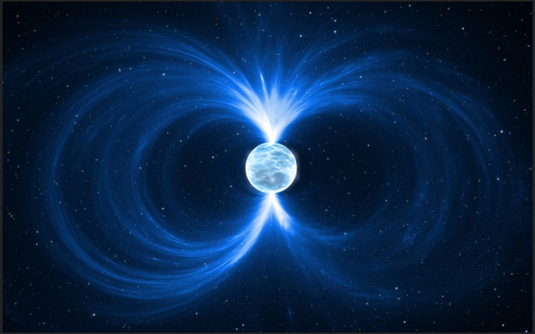

Figure 2: An artist's conception of a NS

Definition

The region surrounding the neutron star where plasma flows in its magnetic field

Magnetosphere

1 Introduction

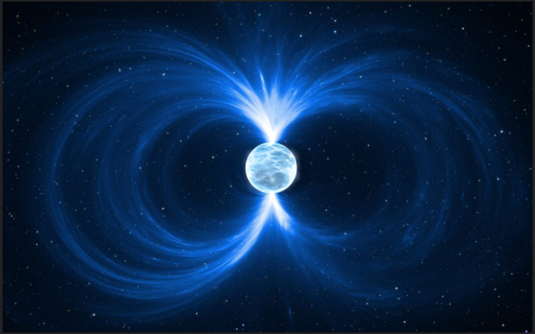

Figure 2: An artist's conception of a NS

Definition

The region surrounding the neutron star where plasma flows in its magnetic field

Characteristics

- Large scale field is roughly dipolar
- Charged particles move along field lines
- Corotation with the star
- Presence of toroidal fields
- Non-thermal emission

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2 Physical framework

Force-free approximation

Force-free approximation

Assumptions

- Dynamics is dominated by the electromagnetic field
- Gravity, plasma pressure and particle inertia are negligible
- Quasi-stationary regime

Definitions

$$\alpha = \frac{4\pi}{c}\left(\boldsymbol{J} - \rho_e c \boldsymbol{\beta}\right) \cdot \frac{\boldsymbol{B}}{B^2}$$

$$\boldsymbol{\beta} = \frac{\boldsymbol{\Omega} \times \boldsymbol{r}}{c}$$

3D Equations

$$\nabla \times \left(\boldsymbol{B} - \beta^2 \boldsymbol{B}_p\right) = \alpha \boldsymbol{B}$$

$$\boldsymbol{B} \cdot \nabla \alpha = 0$$

Properties

- α is constant along field lines
- Implicitly nonlinear equation
- Equilibrium solutions

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Motivation

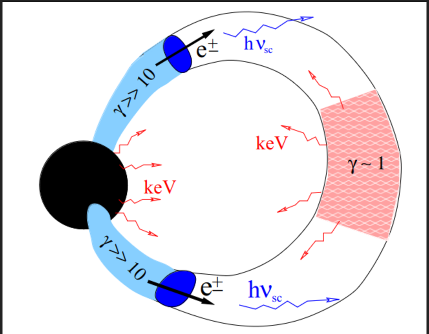

Figure 3: A sketch of a magnetar (Beloborodov, 2013)

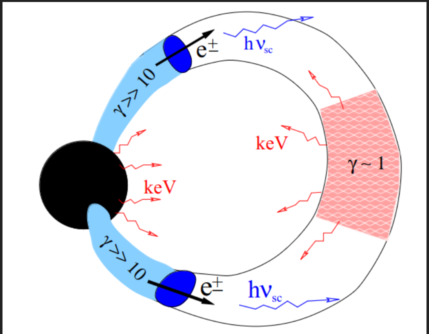

Figure 3: A sketch of a magnetar (Beloborodov, 2013)

Context

- Internal processes gradually displace field lines
- Energy and helicity flow into the magnetosphere
- Twisted magnetic loops appear in localised regions
- Critical point
- Release of magnetic energy
- Only axisymmetric solutions $\rightarrow$ novelty

Equations

Equations

Non-rotating limit

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \alpha \boldsymbol{B}$$
$$\boldsymbol{B} \cdot \boldsymbol{\nabla} \alpha = 0$$

Properties

- $\boldsymbol{J} \parallel \boldsymbol{B}$
- Nonlinear system if $\alpha \neq \text{const}$
- Only numerical solutions

The Grad-Rubin method

Hyperbolic part

$$\boldsymbol{B}^{(k)} \cdot \nabla \alpha^{(k+1)} = 0$$

- Solves for α with a given $\boldsymbol{B}$
- *Characteristics* Method

The Grad-Rubin method

Hyperbolic part

$$\boldsymbol{B}^{(k)} \cdot \nabla \alpha^{(k+1)} = 0$$

- Solves for α with a given $\boldsymbol{B}$
- *Characteristics* Method

Elliptic part

$$\nabla^2 \boldsymbol{A}^{(k+1)} = -\alpha^{(k+1)} \boldsymbol{B}^{(k)}$$

- Solves for $\boldsymbol{A}$ with a given α
- *Scheduled Relaxation Jacobi* method

Model

Model

Features

$$\alpha_{S+}(\theta,\phi) = \alpha_0 \exp\left[\frac{-(\theta-\theta_1)^2-(\phi-\phi_1)^2}{2\sigma^2}\right]$$

- Two hot spots connected by a twisted loop
- One spot has a Gaussian profile
- Dipole background
- Rest of the magnetosphere is current-free

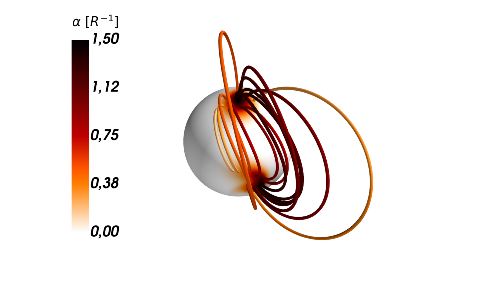

Figure 4: A model with two hotspots

Results

3 3D Magnetars

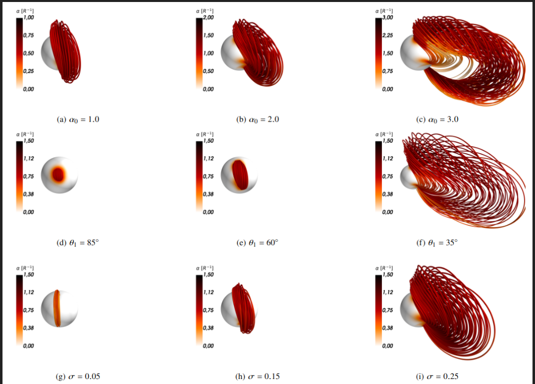

Figure 5: A selection of models for different values of $\alpha_0, \theta_1, \sigma$

Results

3 3D Magnetars

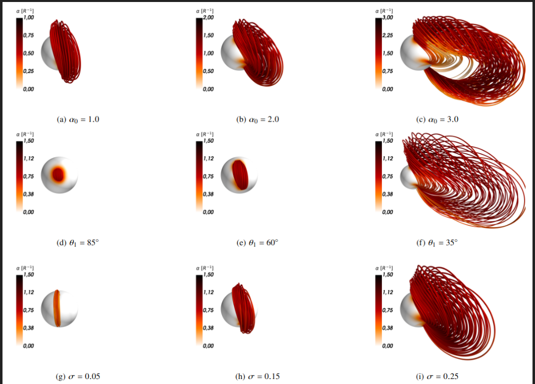

Figure 5: A selection of models for different values of $\alpha_0, \theta_1, \sigma$

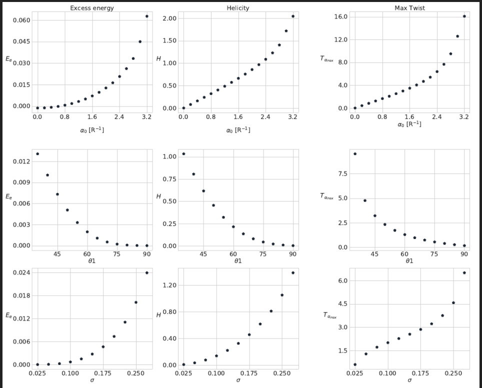

Figure 6: Dependence of energy, helicity and twist on $\alpha_0, \theta_1, \sigma$

Discussion

Discussion

Key points

- For certain values of $\alpha_0, \theta_1, \sigma$ the solver does not converge
- Non-existence of solution $\rightarrow$ release of energy
- Excess energy is enough to feed X-ray activity
- Currents heat the surface $\rightarrow$ X-ray pulses

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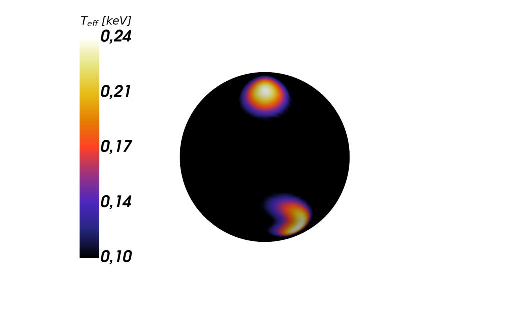

Figure 7: Effective temperature of the hotspots at the surface of the star

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PINNs

4 Magnetars with PINNs

PINNs

Basics

- Proposed in 1997
- Solve PDEs using neural networks
- Input: a set of points in some domain
- Output: an approximate solution
- Loss: the residuals of the PDE describing the system

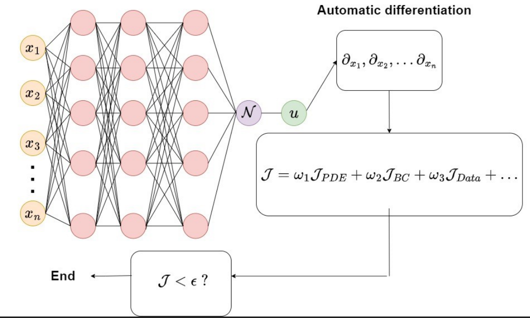

Figure 8: A physics-informed neural network

Motivation

4 Magnetars with PINNs

Motivation

Relevance

New, promising, exciting, actively developing method for solving PDEs

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Advantages

- Meshless
- Scalable
- Generalisable
- Very fast once trained

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Relevance

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Advantages

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- Scalable
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- Very fast once trained

Disadvantages (for now)

- Precision
- Efficiency
- Rigorous mathematical support
- Relies on trial and error

Motivation

4 Magnetars with PINNs

Motivation

Context

- PINNs for elliptic problems
- Axisymmetric magnetars
- Evaluate multiple solutions
- Identify possible caveats

Novelty

- Train **once** for a family of solutions
- Treat boundary conditions and source terms as **inputs** to the PINN
- Connect dissmilar domains

Equations

Equations

Axisymmetric magnetar magnetospheres

- Compactified polar coordinates $\rightarrow q = \frac{R}{r}, \mu = \cos\theta$
- Axisymmetry $\rightarrow \boldsymbol{B} = \frac{q}{\sqrt{1-\mu^2}}(\boldsymbol{\nabla}\mathcal{P} \times \hat{\phi} + \mathcal{T}\hat{\phi})$
- Grad-Shafranov equation

$$q^2 \frac{\partial}{\partial q}\left(q^2 \frac{\partial \mathcal{P}}{\partial q}\right) + \left(1-\mu^2\right) q^2 \frac{\partial^2 \mathcal{P}}{\partial \mu^2} + \mathcal{T}\frac{d\mathcal{T}}{d\mathcal{P}} = 0$$

- The solution is completely determined by the surface boundary condition

$$\mathcal{P}(q=1,\mu) = \left(1-\mu^2\right) \sum_{l=1}^{l_{\mathrm{max}}} \frac{b_l}{l} P_l'(\mu)$$

Results

4 Magnetars with PINNs

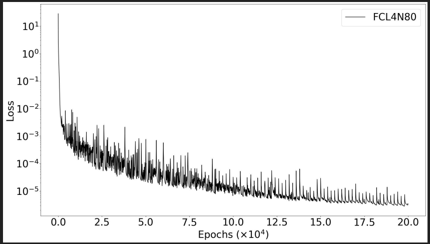

Figure 9: Loss function vs. iterations

	$E_{\mathcal{P}}$	E_{B_r}	E_{B_θ}	E_B
L_1 norm	0.017	0.017	0.025	0.015
L_2 norm	0.019	0.023	0.045	0.023

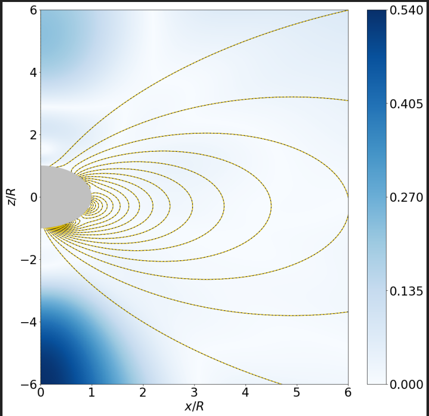

Figure 10: Contours of P

Application

- Couple magnetothermal evolution with a force-free magnetosphere
- Currents can thread the surface

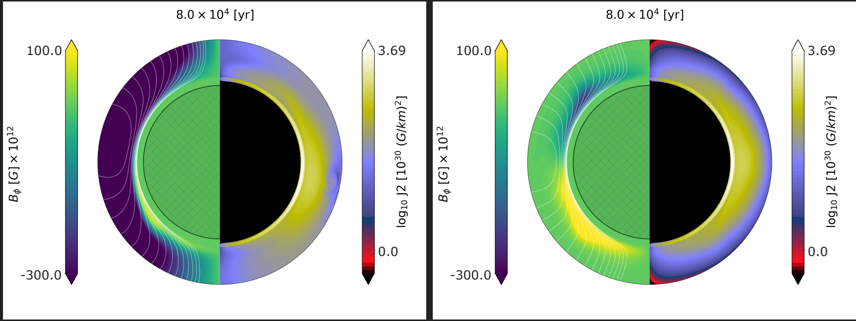

Figure 11: Snapshot of magnetic field and electric current

Discussion

4 Magnetars with PINNs

Discussion

Key points

- PINNs are suitable for elliptic problems
- Relatively accurate and reliable
- Can be trained for arbitrary solutions $\rightarrow$ speed advantage
- Errors can be estimated through discretisation
- Connect regions with vastly different physical conditions
- New solutions for magnetothermal evolution
- Pave the road for extension to 3D

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5 Pulsars with PINNs

Motivation

Motivation

Context

- Rotation is important
- Current-sheets
- $\mathcal{T}$ must be consistent, not prescribed
- Additional constraints
- Axisymmetry

Pulsar equation

$$\left(1-\beta^2\right)\Delta_{\mathrm{GS}}\mathcal{P}+2\beta^2 q^2\left(q\frac{\partial P}{\partial q}+\mu\frac{\partial \mathcal{P}}{\partial \mu}\right)+\mathcal{T}\frac{d\mathcal{T}}{d\mathcal{P}}=0$$

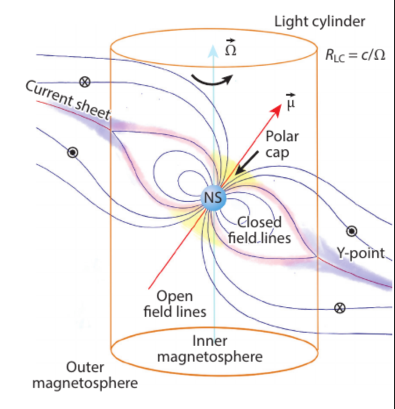

Figure 12: A sketch of a pulsar (Philippov and Kramer, 2022)

Results

5 Pulsars with PINNs

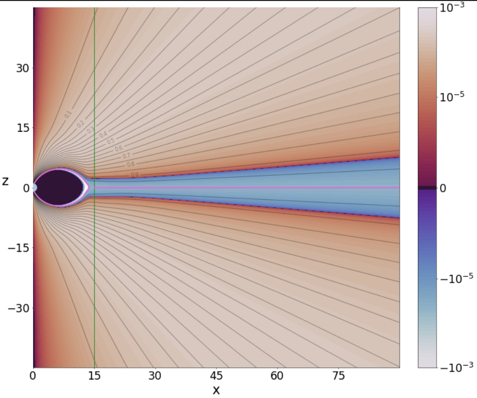

Figure 13: The classical pulsar magnetosphere

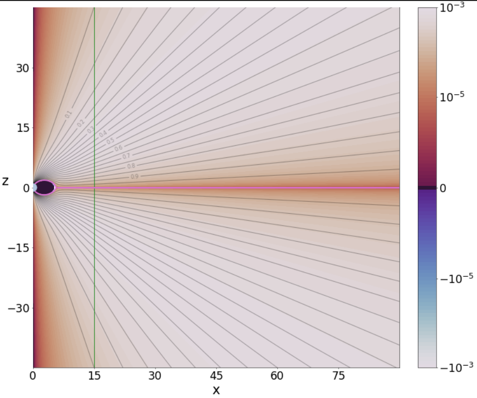

Figure 14: A pulsar magnetosphere with the Y-point close to the star

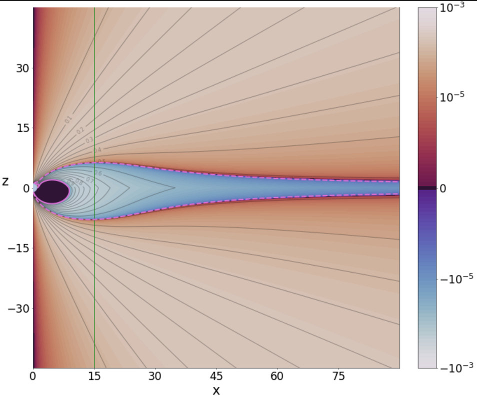

Figure 15: A multipolar pulsar magnetosphere

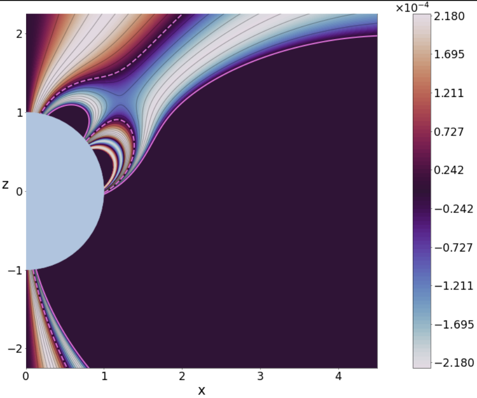

Figure 16: A close-up near the surface

Discussion

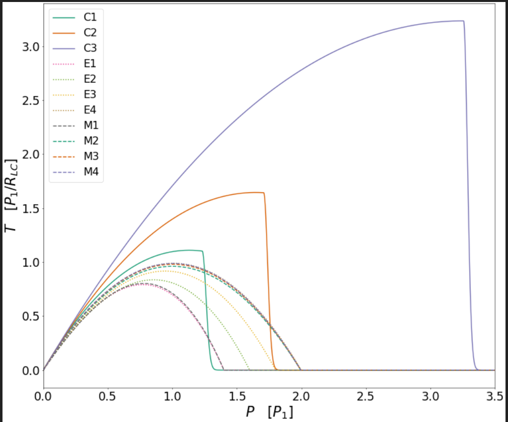

Figure 17: $T(P)$ for various models

Models

- Excess energy $< 3\%$
- Models with fixed P_c have less energy
- Area represents luminosity
- Luminosity $< 3L_{dip}$
- For models with fixed P_c, not all Poynting flux escapes

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6 Future plans

Ongoing and new projects

6 Future plans

PINNs

- Deeper understanding of how they work
- Improve PINN solvers

Magnetars

- Study stability of solutions
- Solve the 3D problem with PINNs
- Couple with 3D interior evolution

Pulsars

- Solve the full 3D problem

New projects

- Include magnetospheric radiation mechanisms
- Extend PINNs to other problems

Neutron star magnetospheres with classical and modern methods

Thank you

Stefanou, Petros, Jose A. Pons, and Pablo Cerdá-Durán (Feb. 2023). "Modelling 3D force-free neutron star magnetospheres". In: *MNRAS* 518.4, pp. 6390–6400. DOI: 10.1093/mnras/stac3570. arXiv: 2211.08957 [astro-ph.HE].

Urbán, Jorge F., Petros Stefanou, Clara Dehman, and José A. Pons (Sept. 2023). "Modelling force-free neutron star magnetospheres using physics-informed neural networks". In: *MNRAS* 524.1, pp. 32–42. DOI: 10.1093/mnras/stad1810. arXiv: 2303.11968 [astro-ph.HE].

Stefanou, Petros, Jorge F. Urbán, and José A. Pons (Nov. 2023). "Solving the pulsar equation using physics-informed neural networks". In: *MNRAS* 526.1, pp. 1504–1511. DOI: 10.1093/mnras/stad2840. arXiv: 2309.06410 [astro-ph.HE].

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Training

Optimisation process

- Trainable parameters $\boldsymbol{\Omega}$
- Loss function $\mathcal{J} = ||\boldsymbol{u}(\boldsymbol{\Omega}) - \tilde{\boldsymbol{u}}||$
- $\boldsymbol{\Omega}$ are adjusted so that $\mathcal{J}(\boldsymbol{\Omega}) \to 0$
- Corrections to $\boldsymbol{\Omega}$ based on $\nabla_{\boldsymbol{\Omega}} \mathcal{J}$
- Automatic differentiation

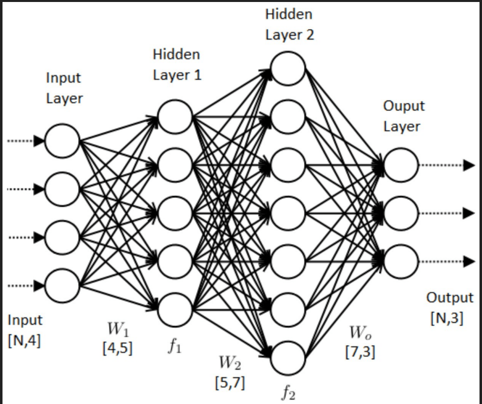

Figure 18: A neural network

Results

10 Backup slides

Force-free case

- $\mathcal{T}(\mathcal{P}) = s_1\mathcal{P} + s_2\mathcal{P}^2$
- Up to seven multipoles $1 \leq l \leq 7$
- s_1, s_2 so that solutions exist
- Estimate error through discretisation

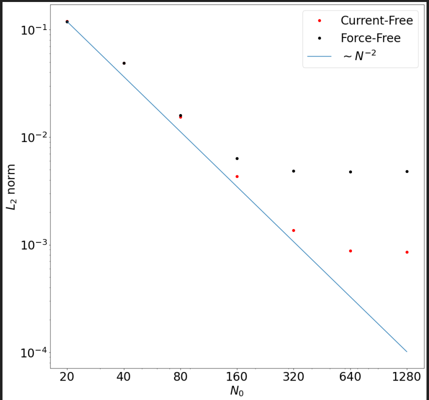

Figure 19: Discretisation error