ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ
UNIVERSITY OF CRETE

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MHD of the Interstellar Medium

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The Interstellar Medium...

...is everything that exists between the stars.

1. Gas
2. Dust
3. Radiation
4. Cosmic Rays
5. Magnetic Fields

Structure of the ISM

Ionized Gas:

- Very Hot Ionized Medium (HIM)
  $n \sim 0.01\ cm^{-3}$ $T \sim 10^5$ K
- Warm Ionized Medium (WIM)
  $n \sim 0.1\ cm^{-3}$ $T \sim 10^4$ K
- HII Regions: $T \sim 10^4$ K $n \sim 0.1\text{-}10^4\ cm^{-3}$

Neutral Gas:

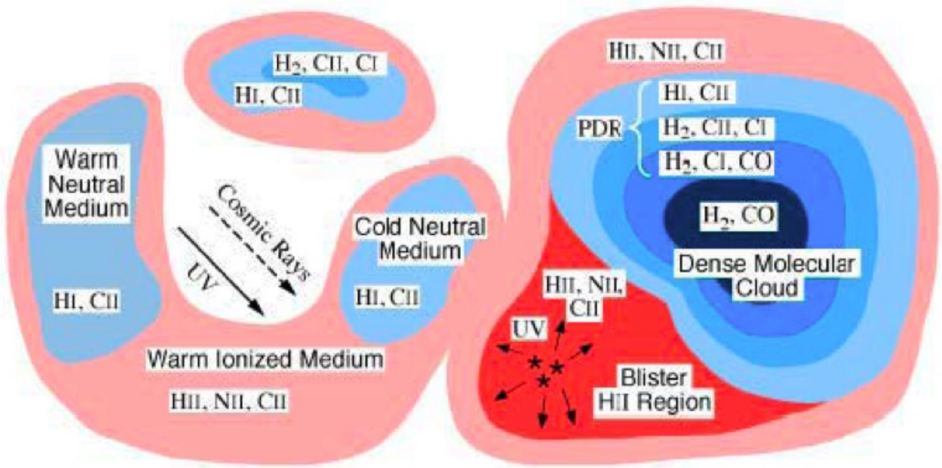
- Warm Neutral Medium (WNM)
  $n \sim 1\ cm^{-3}$ $T \sim 6000$ K
- Cold Neutral Medium (CNM)
  $n \sim 50\ cm^{-3}$ $T \sim 100$ K

Molecular Gas:

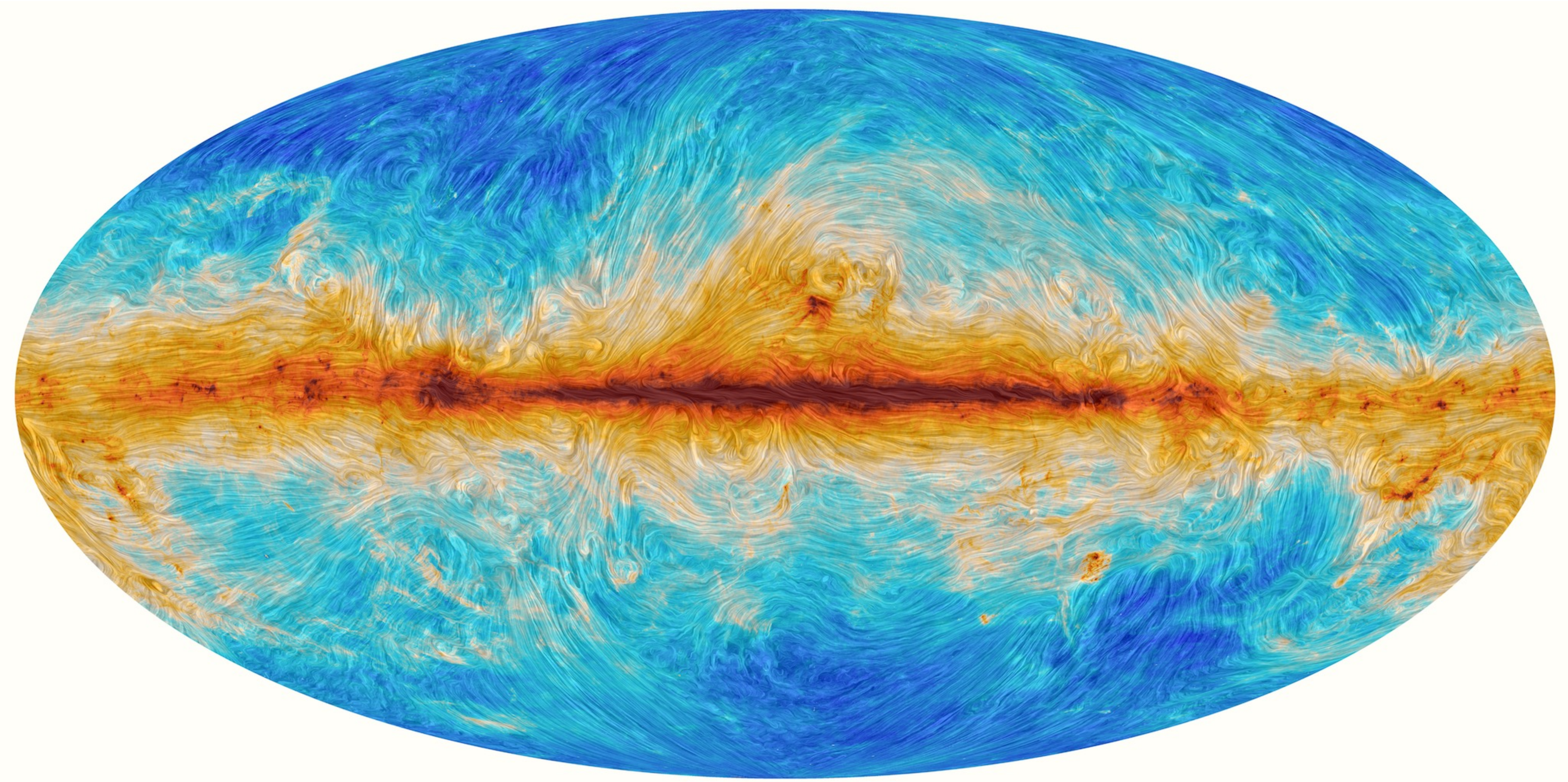
$n \sim 1000\ cm^{-3}$ $T \sim 10$ K

"cores": $n \sim 10^4 - 10^6\ cm^{-3}$ $T \sim 10$ K

(Ultra High Vacuum: $n \sim 10^6\ cm^{-3}$)



Dust Thermal Emission



Star Formation

Molecular Clouds (Birthplaces of Stars):

- Very Cold

  $M_{T,crit} = 5.8\ M_{\odot}$ ($n = 10^3\ cm^{-3}$, $T = 10\ K$)

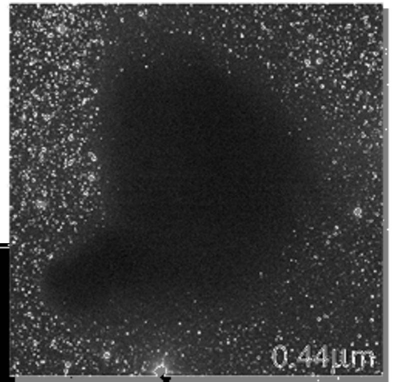
  but $M_{cloud} \approx 10^3 - 10^6 M_{\odot}$

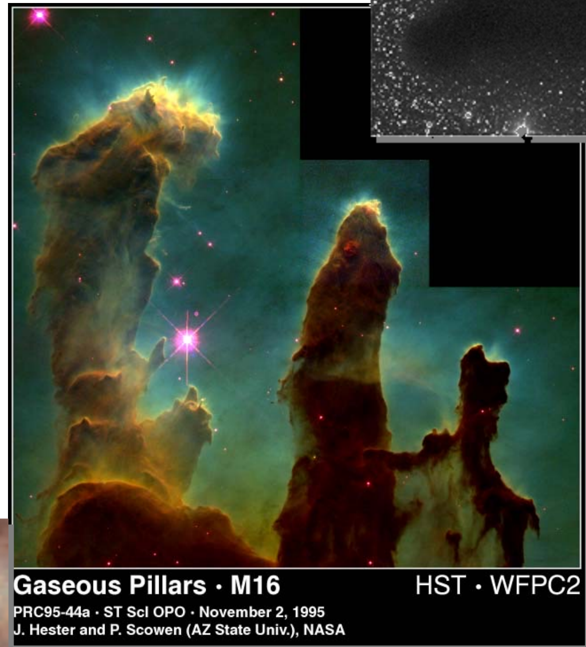
- Weakly Ionized

  $x_i \approx 10^{-4}$

- Magnetized

  $B \approx$ several μG





Read more in Review here

Magneto Hydro Dynamics

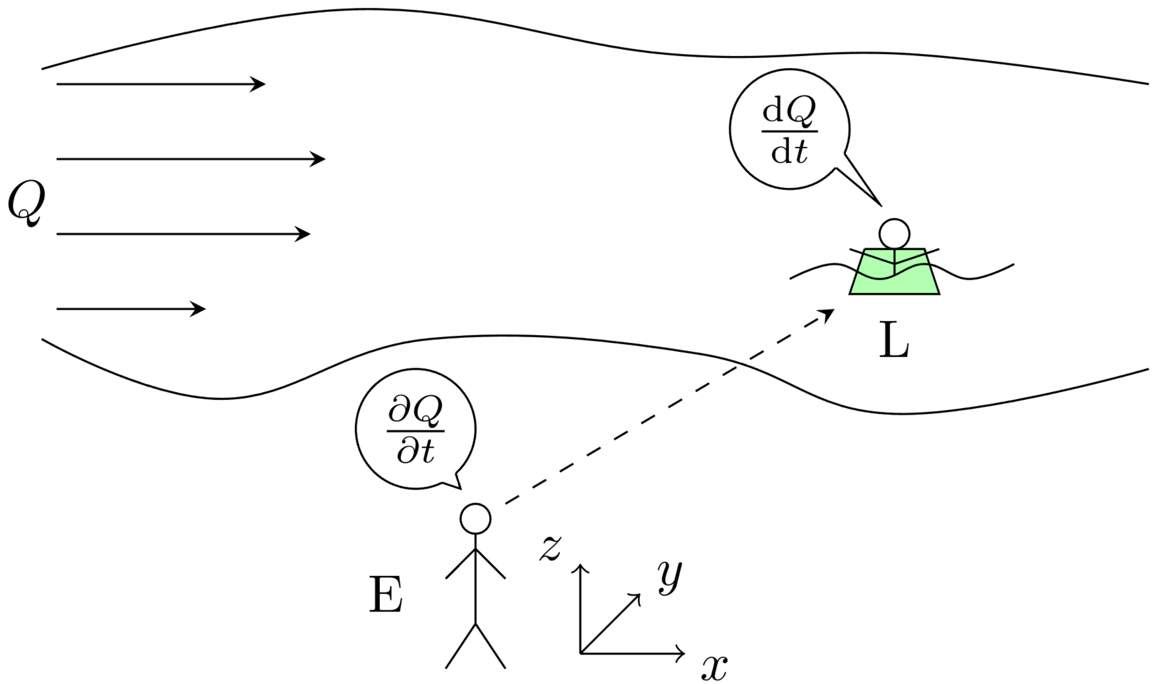
Gaseous ⇨ fluid description HD ✓

Magnetized ⇨ MHD ✓

but weakly ionized ⇨ Non-Ideal MHD

Hydro Dynamics

Lagrangian vs. Eulerian reference frames



$$\frac{\mathrm{D}Q}{\mathrm{D}t} = \frac{\partial Q}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) Q$$

Hydro Dynamics

Continuity Equation represents local mass conservation

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{u} \cdot \vec{\nabla} \rho = -\rho \vec{\nabla} \cdot \vec{u},$$

The change in density of a Lagrangian fluid element is produced by a change in the specific volume

Hydro Dynamics

Force Equation represents local momentum conservation

$$\frac{\mathrm{D}\vec{u}}{\mathrm{D}t} = \frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) \vec{u} = \vec{g} - \frac{1}{\rho}\vec{\nabla}P$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho$$

$$P = k\rho^{\gamma}$$

Magneto HD

Lorenz force for charged particles

$$\vec{F} = -e(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B})$$

+

Maxwell's Equations

$$\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial B}{\partial t} &= 0 \\
\vec{\nabla} \cdot \vec{E} &= 4\pi\rho \\
\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial E}{\partial t} &= \frac{4\pi}{c}\vec{J}.
\end{aligned}$$

3 (n, i, e) fluid MHD

$$\left(\frac{D}{Dt}\right)_s \rho_s = -\rho_s \nabla \cdot \mathbf{v}_s \qquad s = n, i, e$$

$$\rho_e \left(\frac{D}{Dt}\right)_e \mathbf{v}_e = -\nabla P_e - \rho_e g - n_e e(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}) + \mathbf{F}_{ei} + \mathbf{F}_{en}$$

$$\rho_i \left(\frac{D}{Dt}\right)_i \mathbf{v}_i = -\nabla P_i - \rho_i g - n_i Ze(\mathbf{E} + \frac{\mathbf{v}_i}{c} \times \mathbf{B}) + \mathbf{F}_{ie} + \mathbf{F}_{in}$$

$$\rho_n \left(\frac{D}{Dt}\right)_n \mathbf{v}_n = -\nabla P_n - \rho_n g + \mathbf{F}_{ne} + \mathbf{F}_{ni}$$

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

$$P = k\rho^\gamma \qquad \nabla \cdot \mathbf{g} = 4\pi G \rho_{tot} \qquad \mathbf{F}_{12} = n_1 \gamma_{12} \left[\frac{m_1 m_2}{m_1 + m_2}(\mathbf{v}_2 - \mathbf{v}_1)\right]$$

Force/volume on fluid 1 due to collisions with species 2 ($\mathbf{F}_{12}$)

Collision frequency of a particle of species 1 with particles of species 2 (γ_{12})

Momentum exchange per collision

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi(n_i Ze - n_e e)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$c\nabla \times \mathbf{B} = 4\pi \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t} \qquad \mathbf{j} = n_i Ze\mathbf{v}_i - n_e e\mathbf{v}_e$$

3 (n, i, e) fluid MHD

$$\left(\frac{D}{Dt}\right)_s \rho_s = -\rho_s \nabla \cdot \mathbf{v}_s \qquad s = n, i, e$$

$$\rho_e \left(\frac{D}{Dt}\right)_e \mathbf{v}_e = -\nabla P_e - \rho_e g - n_e e(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}) + \mathbf{F}_{ei} + \mathbf{F}_{en}$$

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$$\rho_n \left(\frac{D}{Dt}\right)_n \mathbf{v}_n = -\nabla P_n - \rho_n g + \mathbf{F}_{ne} + \mathbf{F}_{ni}$$

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$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi(n_i Ze - n_e e)$$ =0

$$Zen_i \approx en_e$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$c\nabla \times \mathbf{B} = 4\pi \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{j} = n_e e(\mathbf{v}_i - \mathbf{v}_e)$$

3 (n, i, e) fluid MHD

$$\left(\frac{D}{Dt}\right)_s \rho_s = -\rho_s \nabla \cdot \mathbf{v}_s$$

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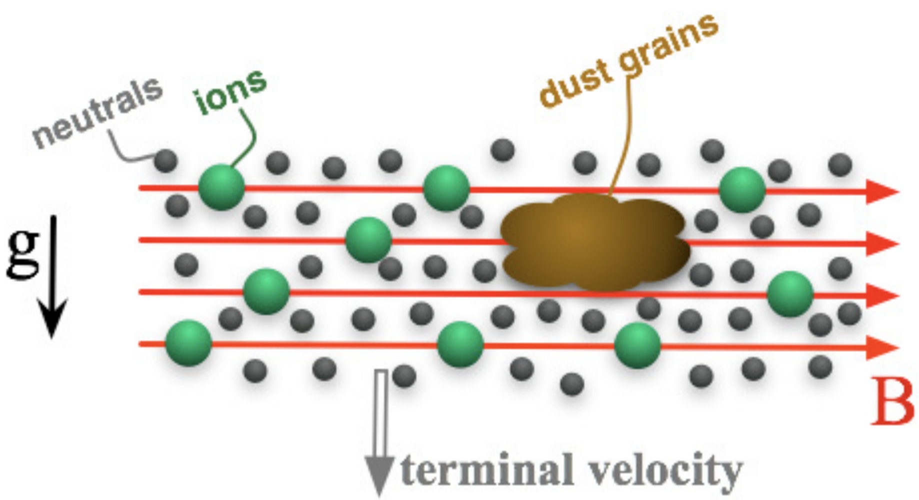
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$



3 (n, i, e) fluid MHD

$$\left(\frac{D}{Dt}\right)_s \rho_s = -\rho_s \nabla \cdot \mathbf{v}_s$$

$$0 = (n_i Ze - n_e e)\mathbf{E} + \frac{1}{c}\overbrace{(n_i Ze\mathbf{v}_i - n_e e\mathbf{v}_e)}^{\mathbf{j}} \times \mathbf{B} + \mathbf{F}_{ei} + \mathbf{F}_{ie} + \mathbf{F}_{en} + \mathbf{F}_{in}$$

$$\rho_n \left(\frac{D}{Dt}\right)_n \mathbf{v}_n = -\nabla P_n - \rho_n g + \mathbf{F}_{ne} + \mathbf{F}_{ni}$$

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$$Zen_i \approx en_e$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$

$$\mathbf{j} = n_e e(\mathbf{v}_i - \mathbf{v}_e)$$

2 (n, i+e) fluid MHD

$$\left(\frac{D}{Dt}\right)_s \rho_s = -\rho_s \nabla \cdot \mathbf{v}_s$$

$$0 = \frac{1}{c}\mathbf{j} \times \mathbf{B} + \mathbf{F}_{en} + \mathbf{F}_{in}$$

$$\rho_n \left(\frac{D}{Dt}\right)_n \mathbf{v}_n = -\nabla P_n - \rho_n g + \mathbf{F}_{ne} + \mathbf{F}_{ni}$$

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$$P = k\rho^\gamma \qquad \nabla \cdot \mathbf{g} = 4\pi G \rho_{tot} \qquad \mathbf{F}_{12} = n_1 \gamma_{12} \left[\frac{m_1 m_2}{m_1 + m_2}(\mathbf{v}_2 - \mathbf{v}_1)\right]$$

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2 (n, i+e) fluid MHD

$$\left(\frac{D}{Dt}\right)_s \rho_s = -\rho_s \nabla \cdot \mathbf{v}_s$$

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$$\rho_n \left(\frac{D}{Dt}\right)_n \mathbf{v}_n = -\nabla P_n - \rho_n g + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

Neutral particles experience a magnetic force due to collisions with charged particles

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

$$P = k\rho^\gamma \qquad \nabla \cdot \mathbf{g} = 4\pi G \rho_{tot} \qquad \mathbf{F}_{12} = n_1 \gamma_{12} \left[\frac{m_1 m_2}{m_1 + m_2}(\mathbf{v}_2 - \mathbf{v}_1)\right]$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

$$Zen_i \approx en_e$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$

$$\mathbf{j} = n_e e(\mathbf{v}_i - \mathbf{v}_e)$$

2 (n, i+e) fluid MHD

$$\left.\begin{aligned}\nabla\times\mathbf{B}&=\frac{4\pi}{c}\mathbf{j}\\ \mathbf{j}&=n_e e(\mathbf{v}_i-\mathbf{v}_e)\end{aligned}\right\}\qquad \mathbf{v}_i-\mathbf{v}_e=\frac{1}{n_e e}\frac{c}{4\pi}\nabla\times\mathbf{B}$$

For Molecular Clouds (diffuse parts)

$n_e \approx n_i \approx 10^{-1}$ cm^{-3}

$B \sim 10$ μG ➡️ $|v_i - v_e| \sim 10^{-4}$ cm/s $= 10^{-9}$ km/s

$L \sim 10^{19}$ cm

But in ISM 0.1 km/s < $|v_i| \backsim |v_e| \backsim |v_n|$ < 10 km/s

$$V_i \approx V_e$$

Since $m_e \ll m_i$ => Plasma Density $\backsim \rho_i$

Single fluid MHD

If coupling due to collisions very effective $V_n \approx V_i \approx V_e$

$$\left(\frac{D}{Dt}\right)\rho = -\rho\nabla\cdot\mathbf{v}$$

$$\rho\left(\frac{D}{Dt}\right)\mathbf{v} = -\nabla P - \rho\mathbf{g} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}$$

$$\nabla\cdot\mathbf{g} = 4\pi G\rho$$

$$P = k\rho^{\gamma}$$

$\mathbf{E} = ?$

Can not use $\nabla\cdot\mathbf{E} = 0$

Single fluid MHD

$$0 = -n_e e(\mathbf{E} + \frac{\mathbf{v}_e}{c} \times \mathbf{B}) + \mathbf{F}_{ei} + \mathbf{F}_{en}$$

$V_n \approx V_i \approx V_e$

$$\mathbf{E} = -\frac{\mathbf{v}_e}{c} \times \mathbf{B} = -\frac{\mathbf{v}}{c} \times \mathbf{B}$$

$$\mathbf{F}_{12} = n_1 \gamma_{12} \left[\frac{m_1 m_2}{m_1 + m_2} (\mathbf{v}_2 - \mathbf{v}_1) \right]$$

Ideal MHD

$$\left(\frac{D}{Dt}\right) \rho = -\rho \nabla \cdot \mathbf{v}$$

$$\rho \left(\frac{D}{Dt}\right) \mathbf{v} = -\nabla P - \rho \mathbf{g} + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\nabla \cdot \mathbf{g} = 4\pi G \rho$$

$$P = k \rho^\gamma$$

Alfvén's Theorem

$$\left(\frac{D}{Dt}\right)\rho = -\rho\nabla\cdot\mathbf{v}$$

$$\rho\left(\frac{D}{Dt}\right)\mathbf{v} = -\nabla P - \rho\mathbf{g} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

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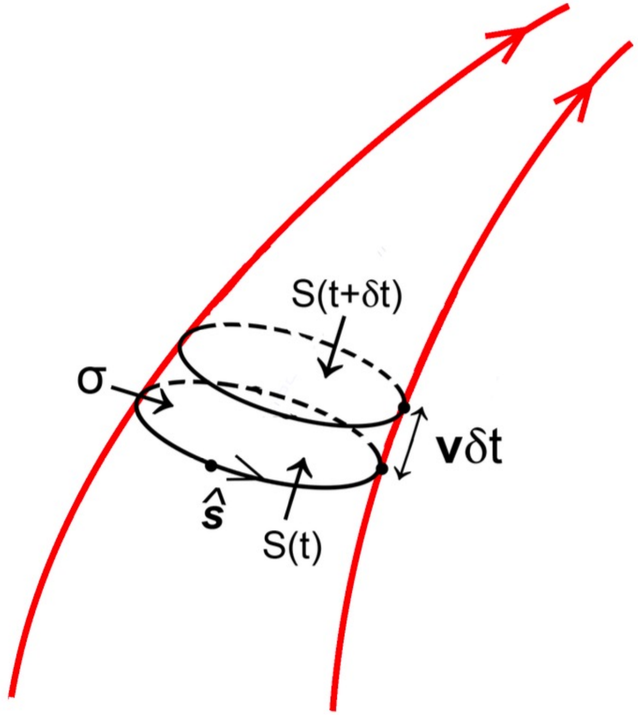
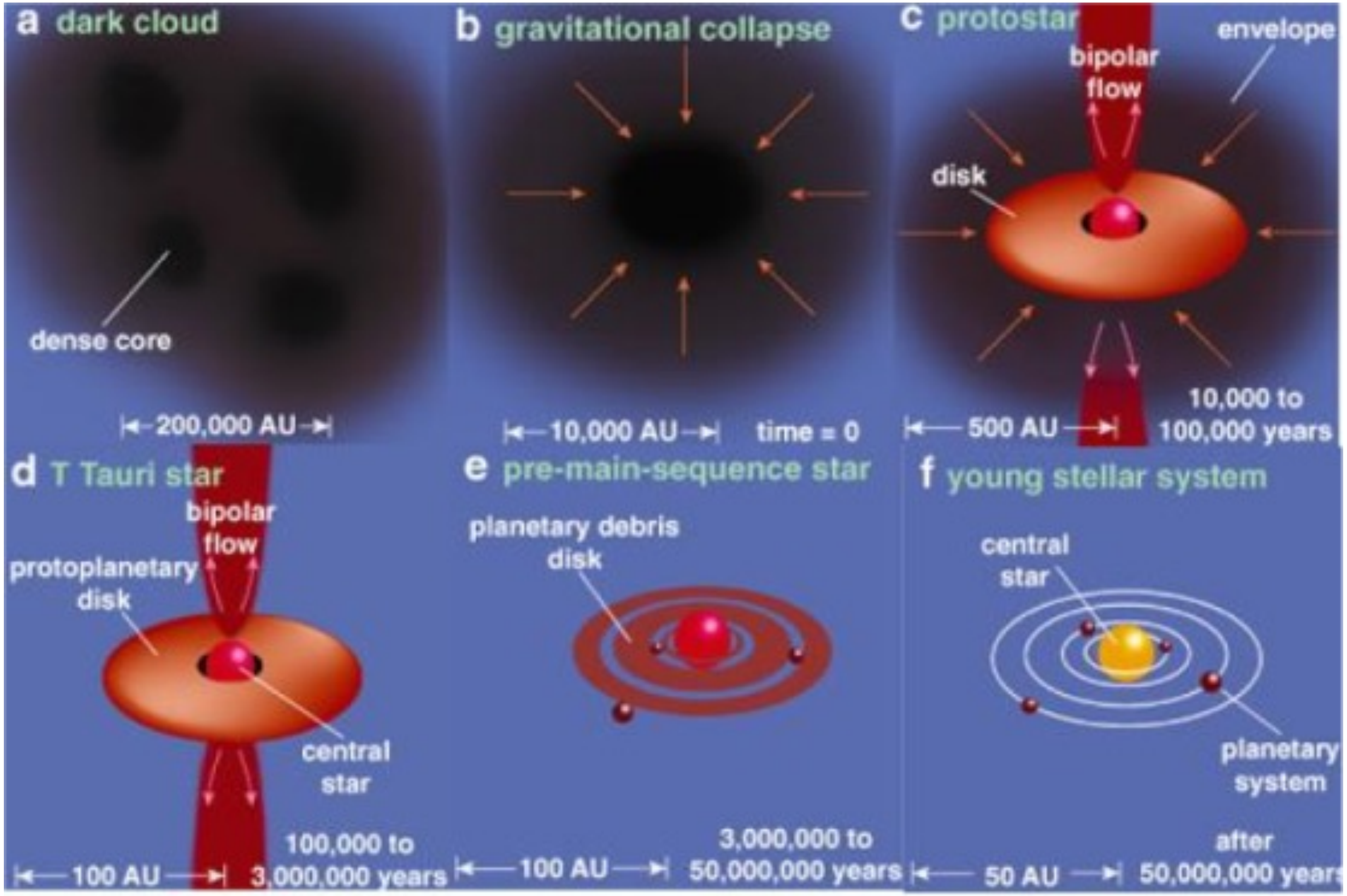


$$\frac{d\mathbf{B}}{dt} = 0$$

Ideal MHD => "Flux Freezing"

Naive picture of Star Formation

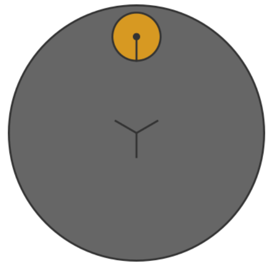


Greene 2001

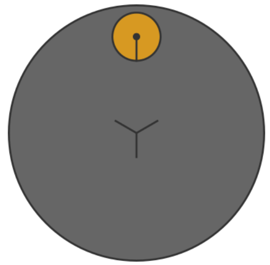
Angular Momentum Problem of Star Formation

Interstellar Cloud

J_{cloud} $(2M_{\odot}) \sim 10^{55}$ g cm^2 s^{-1} ($n \sim 1$ cm^{-3})

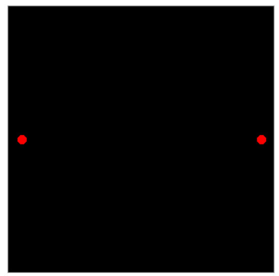


Angular Momentum Problem of Star Formation



Interstellar Cloud

$J_{cloud}\,(2M_{\odot}) \sim 10^{55}$ g cm^2 s^{-1} ($n \sim 1$ cm^{-3})



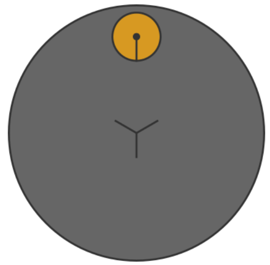
Wide Binary

$J_{binary}\,(2M_{\odot}$, 100yr$)$: $\sim 10^{53}$ g cm^2 s^{-1}

Angular Momentum Problem of Star Formation

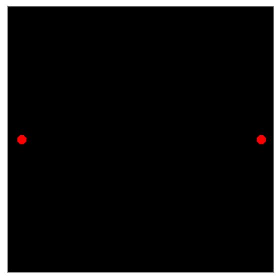
Interstellar Cloud

$J_{cloud}\,(2M_{\odot}) \sim 10^{55}\ \mathrm{g\ cm^2\ s^{-1}}$ $(n \sim 1\ \mathrm{cm^{-3}})$



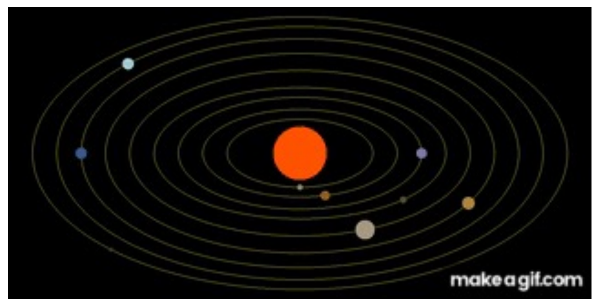
Wide Binary

$J_{binary}\,(2M_{\odot},\ 100\mathrm{yr}): \sim 10^{53}\ \mathrm{g\ cm^2\ s^{-1}}$



Solar System

$J_{solar\ system}: \sim 10^{51}\ \mathrm{g\ cm^2\ s^{-1}}$



Magnetic Braking

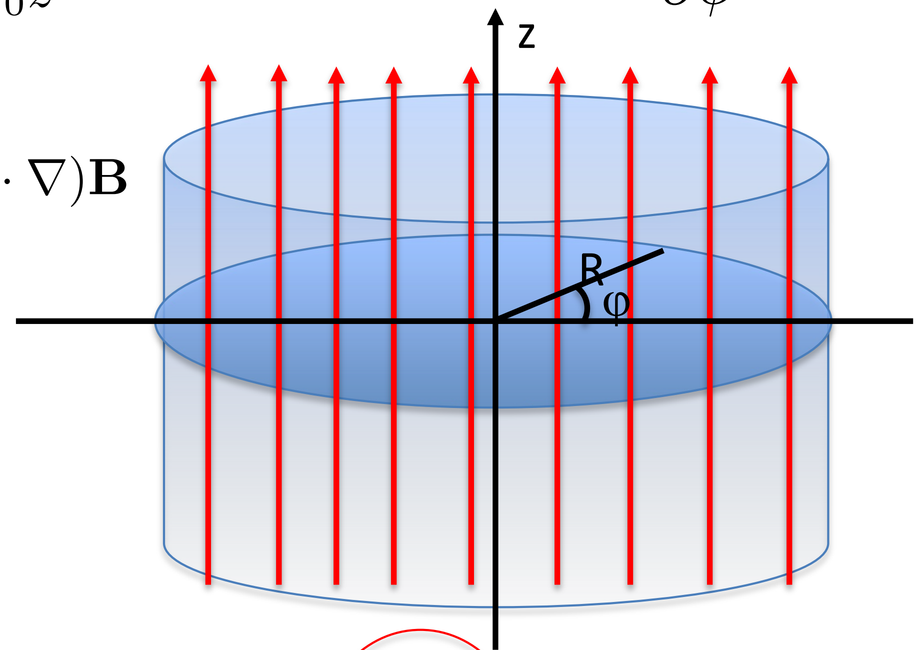
Early stages of SF => B Flux "Frozen" in the gas

$$\mathbf{B} = B_0\hat{z}, \rho = \rho_0, \frac{\partial}{\partial \phi} = 0$$

$$\mathbf{v}(t) = v_\phi(r,z)\hat{\phi} \quad \mathbf{B}(t) = B_\phi(r,z)\hat{\phi} + B_0\hat{z}$$

$$\rho\frac{D\mathbf{v}}{Dt} = \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} = -\nabla\left(\frac{B^2}{8\pi}\right) + \frac{1}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B})$$



$$\rho\frac{\partial^2 v_\phi}{\partial t^2} = \frac{1}{4\pi}B_0\frac{\partial^2 B_\phi}{\partial z\partial t}$$

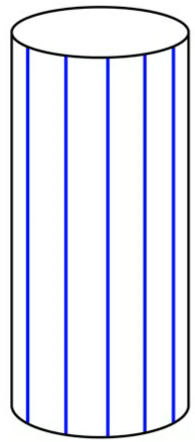
$$\frac{\partial^2 B_\phi}{\partial z\partial t} = B_0\frac{\partial^2 v_\phi}{\partial z^2}$$

$$\frac{\partial^2 v_\phi}{\partial t^2} = \left(\frac{B_0^2}{4\pi\rho}\right)\frac{\partial^2 v_\phi}{\partial z^2}$$

V_A

Magnetic Braking

Torsional Alfvén waves propagate above and below the fragment transfering angular momentum



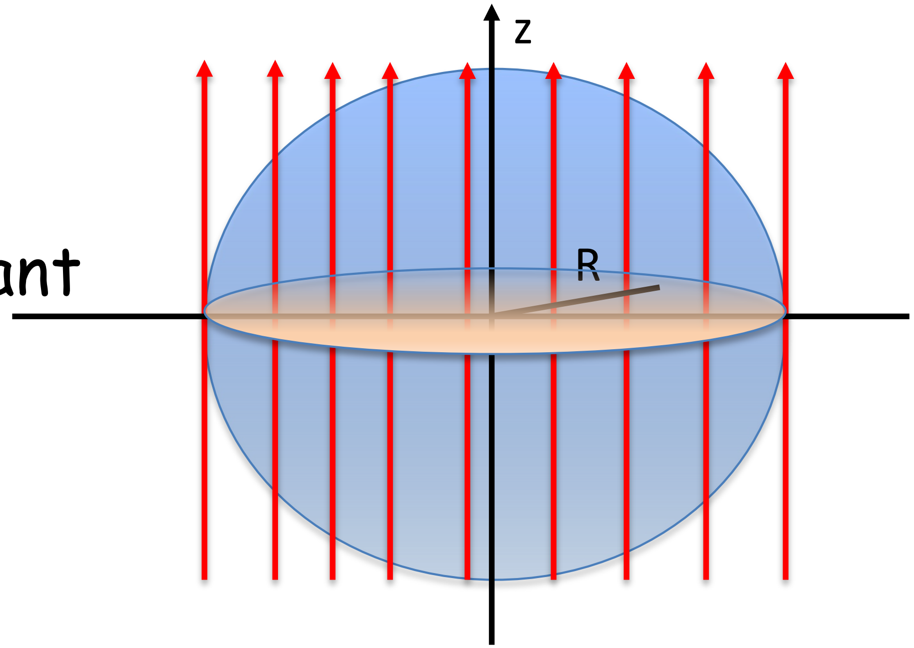
Mouschovias & Paleoplogou in the 80's prooved analytically that magnetic braking very efficient in solving the angular momentum problem of SF in the early stages

Magnetic Flux Problem of Star Formation

Mass = $\rho\,(4/3\ \pi\ R^3)$ = constant

Flux freezing

Flux = B Area = $B\ \pi\ R^2$ = constant



$$\left.\begin{array}{l}\rho \sim R^{-3} \\ B \sim R^{-2}\end{array}\right\} B \sim \rho^{2/3}$$

Interstellar medium

$n_{ISM} \sim 1\ cm^{-3} \Rightarrow \rho_{ISM} \sim 10^{-24}\ g\ cm^{-3}$

$B_{ISM} \sim 6\ \mu G$

Star $\rho_\star \sim 1\ g\ cm^{-3}$

$$B_\star = B_{ISM}\left(\frac{\rho_\star}{\rho_{ISM}}\right)^{2/3} \sim 10^{10} G$$

Magnetic Flux Problem of Star Formation

In the deep interiors of cloud cores

$V_i \approx V_e$ but $V_i \not\approx V_n$

2 fluid MHD

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B})$$

Flux frozen in the plasma!

$$\rho_n \frac{D\mathbf{v}_n}{Dt} = -\nabla P - \rho_n \mathbf{g} + \mathbf{F}_{ni}$$

$$0 = \frac{1}{c}\mathbf{j} \times \mathbf{B} + \mathbf{F}_{in}$$

Coupled through collisions

$$\mathbf{F}_{in} = \frac{\rho_i}{\tau_{in}}(\mathbf{v}_n - \mathbf{v}_i) = -\mathbf{F}_{ni}$$

$$\mathbf{v}_D = \mathbf{v}_i - \mathbf{v}_n$$

$$\mathbf{F}_{in} = -\frac{1}{c}\mathbf{j} \times \mathbf{B}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$

$$\mathbf{v}_D = \frac{\tau_{ni}}{c\rho_n}\mathbf{j} \times \mathbf{B} = \frac{\tau_{ni}}{4\pi\rho_n}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

Magnetic Flux Problem of Star Formation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B})$$

add and subtract v_n

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v}_n \times \mathbf{B}) = \nabla \times [(\mathbf{v}_i - \mathbf{v}_n) \times \mathbf{B}] = \nabla \times (\mathbf{v}_D \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}_n) = \nabla \times \{\frac{\tau_{ni}}{4\pi \rho_n}[(\nabla \times \mathbf{B}) \times \mathbf{B}] \times \mathbf{B}\}$$

Diffusion Equation albeit non-linear
(set $v_n = 0$ to see it)

<u>**Ambipolar Diffusion**</u>

Magnetic Flux Problem of Star Formation

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}_n) = \nabla \times \{\frac{\tau_{ni}}{4\pi\rho_n}[(\nabla \times \mathbf{B}) \times \mathbf{B}] \times \mathbf{B}\}$$

$$\frac{\partial \phi(\mathbf{r}, t)}{\partial t} = \nabla \cdot [D(\phi, \mathbf{r}) \nabla \phi(\mathbf{r}, t)]$$

Diffusion coefficient

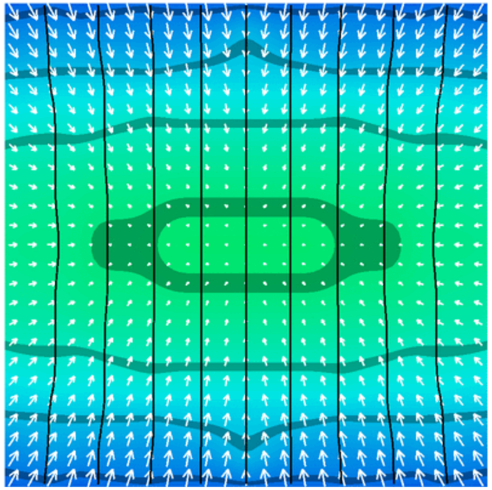
$$D \propto \frac{\tau_{ni} B^2}{\rho_n} \propto v_A^2 \tau_{ni}$$

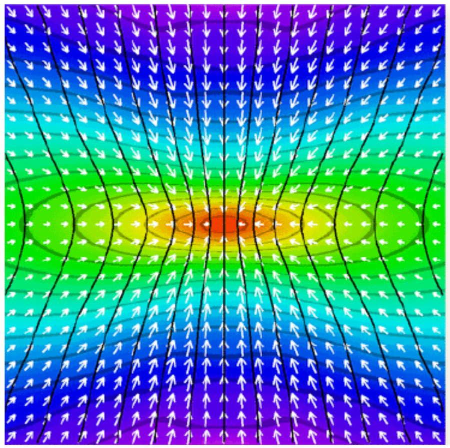
Diffusion timescale

$$\tau_{AD} \propto \frac{L^2}{D} \propto \frac{L^2}{\tau_{ni} v_A^2} \propto \frac{\tau_A^2}{\tau_{ni}}$$

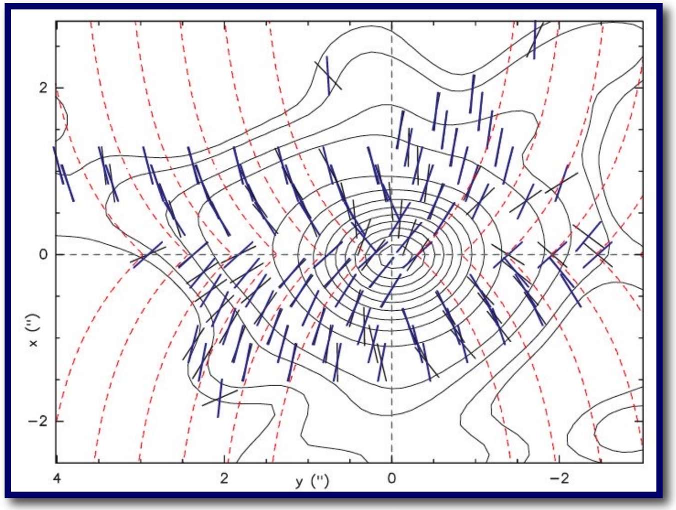
Ambipolar Diffusion timescale
small in high-density and low-ionization
molecular cloud cores

B-Field in Molecular Cloud Cores

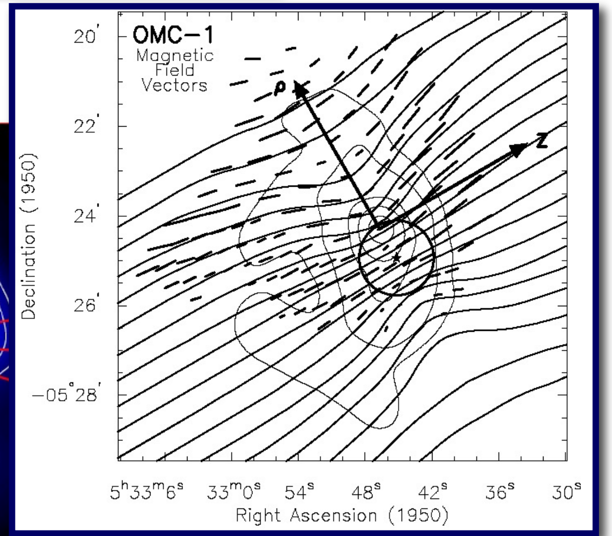




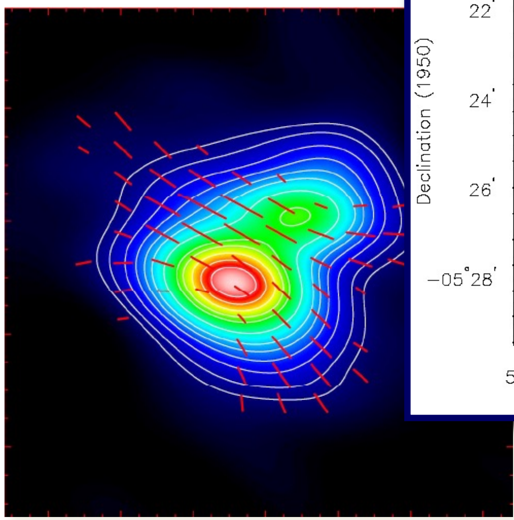
Kunz & Mouschovias 2009



Girart et al. 2009



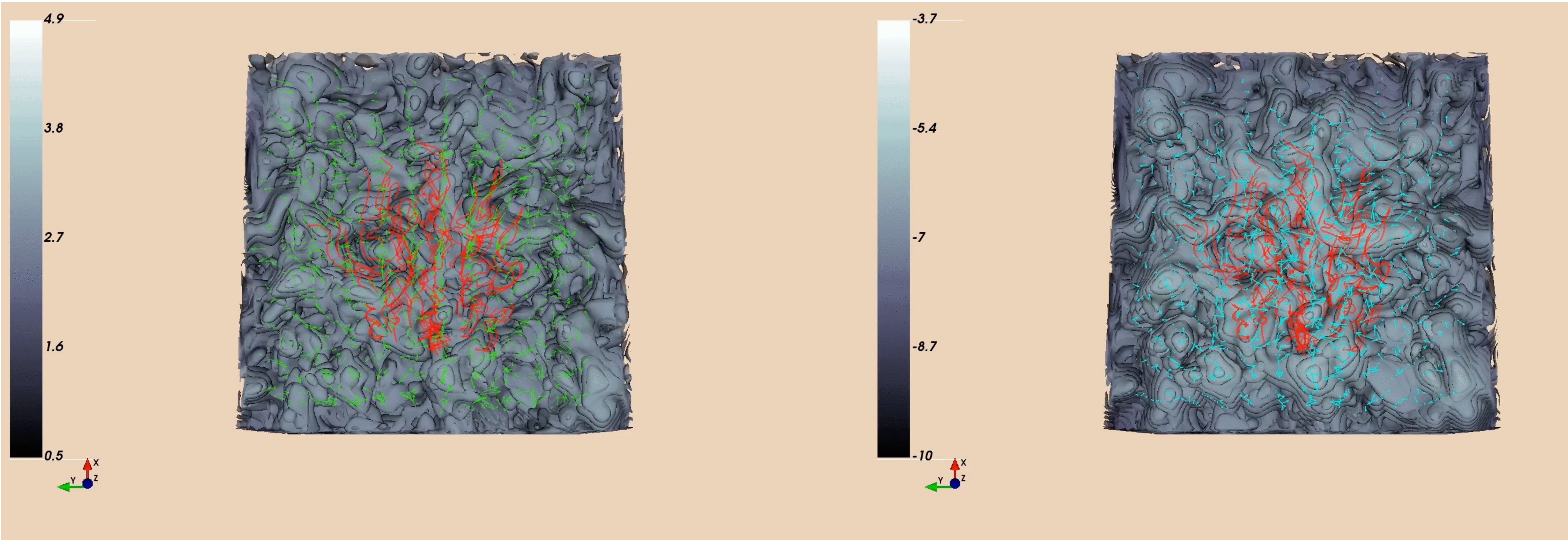
Schleuning 1998



Girart, Rao, & Marrone 2006

3D Non-Ideal MHD Simulation

H_2 HCO^+



Tritsis et al. in prep.

Conclusions

- MHD essential to understand the process of Star Formation
- Ideal MHD (good coupling of neutral and charged particles) => flux freezing => magnetic braking efficiently removes angular momentum allowing the collapse
- Non-Ideal MHD => AD (and other diffusive processes) => redistribution of magnetic flux

Thank you!