MHD TURBULENCE AND PARTICLE ENERGISATION

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The dream team

The dream team

Introduction

MHD EQUATIONS

The core MHD equations consist of: (Krall & Trivelpiece 1973)

Theorem

Continuity Equation (Mass Conservation):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

where ρ is the mass density, and $\mathbf{v}$ is the fluid velocity.

Theorem

Momentum Equation (Navier-Stokes Equation with Lorentz Force):

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{v}$$

where p is the pressure, $\mathbf{B}$ is the magnetic field, $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ is the current density, ν is the kinematic viscosity, and μ_0 is the permeability of free space.

MHD EQUATIONS

Theorem

Induction Equation (Magnetic Field Evolution):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

where $\eta = \frac{1}{\sigma \mu_0}$ *is the magnetic diffusivity, and* σ *is the electrical conductivity.*

Theorem

Incompressibility Condition (if applicable):

$$\nabla \cdot \mathbf{v} = 0$$

Many MHD turbulence studies often assume this condition to simplify the analysis.

MHD Equations

Theorem

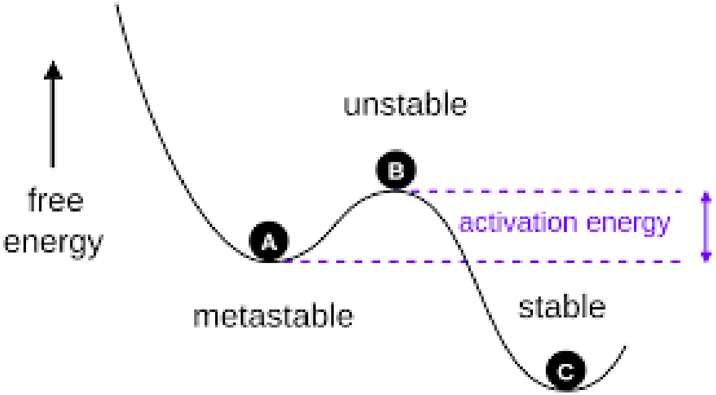
Ohm's Law for a Moving Conductor:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

where $\mathbf{E}$ *is the electric field,* $\mathbf{J}$ *is the current density, and* η *is the resistivity of the fluid.*

Stability of MHD Equations

Dropping the derivative and searching for the stable solution $(\vec{B}_0(\vec{r}), \rho_0(\vec{r}), \vec{v}_0(\vec{r}))$, if any, from the magnetic field topology, the density and the velocity of the fluid



Life close to equilibrium and wave turbulence

Perturbation theory

(see details in Chen (2016))
We perturb the stability

$$\rho_1(\vec{r}, t) << \rho_0$$

,

$$\vec{B}_1(\vec{r}, t) << \vec{B}_0$$

,

$$\vec{v}_1(\vec{r}, t) << \vec{v}_0$$

end searching for wave-like solutions, assuming that the perturbations have the form

$$\rho_1, B_1, v_1 \sim \exp(-i(\vec{k} \cdot \vec{r} + \omega t))$$

discover a particular dispersion relation,

$$\omega_A(k) = kV_A$$

where $V_A = B_0^2/4\pi\rho_0$ is the $Alfvén$ speed

Wave turbulence

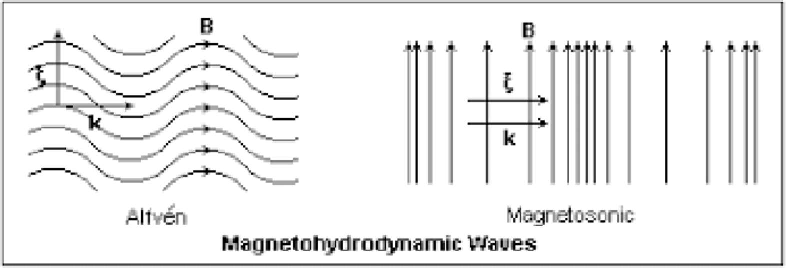


Figure: Propagation of the $Alfv\acute{e}n$ Waves

Wave turbulence

$$\vec{B}_1(\vec{r},t) = \sum_k B_{0k} \exp(-i(\vec{k}\cdot\vec{r} + \omega_A(k)t + \phi_k))$$

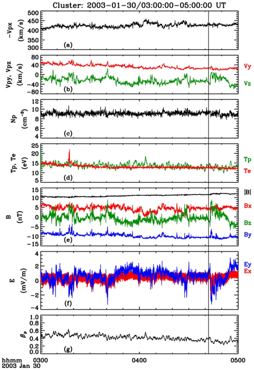
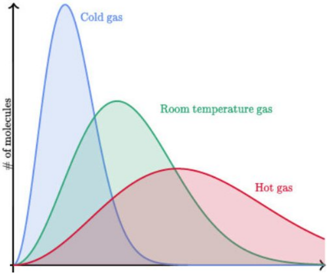


Figure: *Alfvén* Waves turbulence on the solar wind

"Particle distributions near equilibrium."

Based on the kinetic theory, when a gas is in equilibrium, its velocity distribution follows a Maxwellian distribution.

$$f(v) = 4\pi \left(\frac{m}{2\pi K_B T}\right)^{1/2} v^2 e^{-\frac{mv^2}{2K_B T}}$$



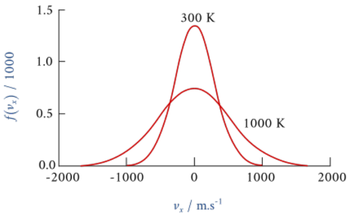


Figure: Particle distribution in equilibrium

Instabilities

We return to the MHD equations and look for wave solutions, but this time, we assume that the wave has an imaginary part.

$$\omega = \omega_A(k) + i\gamma_k$$

introducing this term on the wave formulae

$$B_1 = B_0 \exp(-i(\vec{k} \cdot \vec{r} + \omega_A(k)t) + \gamma t) = [B_0 exp(\gamma t)] exp(-i(\vec{k} \cdot \vec{r} + \omega_A(k)t))$$

$\gamma > 0$ growth and $\gamma < 0$ damping

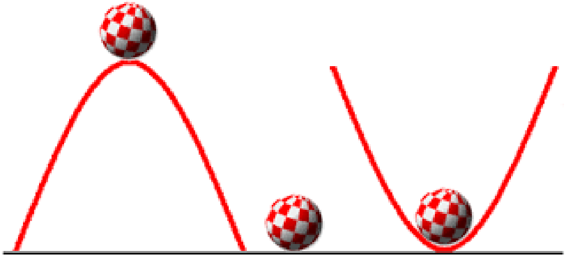


Figure: Unstable $Alfvén$ Waves

How to stabilize the unstable wave spectrum?

We deal with nonlinear wave-wave coupling within weak (wave) turbulence (Vedenov, 1963).
Let's represent the three interacting waves using their amplitudes, denoted as B_1, B_2, and B_3. Each wave is characterized by its frequency ω_i and wavenumber $\vec{k}_i$, for $i = 1, 2, 3$. The conditions for a resonant three-wave interaction are:
Frequency Matching Condition:

$$\omega_1 = \omega_2 + \omega_3$$

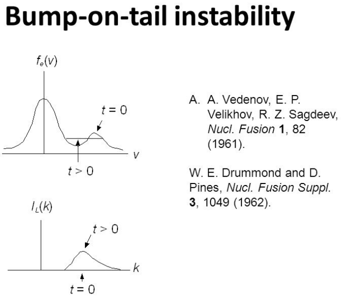
Wavenumber Matching Condition:

$$\vec{k}_1 = \vec{k}_2 + \vec{k}_3$$

These conditions ensure that energy and momentum are conserved in the interaction.

Stabilizing wave growth in weak turbulence

Governing Equations for the wave amplitudes: A set of coupled differential equations governs the evolution of the complex amplitudes $\vec{B}_1(t)$, $\vec{B}_2(t)$ and $\vec{B}_3(t)$. These are typically derived from the governing equations of the medium (e.g., fluid equations, Maxwell's equations) using perturbation theory or a multi-scale analysis. see wave interaction
Collisions and wave-particle interactions thermalize the particles in the end.
Example:



Life far from equilibrium and Strong turbulence

Unstable waves and the way to strong turbulence

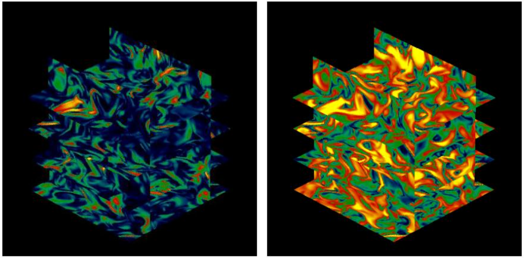
Questions

- What happens if the wave amplitude grows exponentially and reaches an amplitude $B_1 \geq B_0$
- Let us find out by using the MHD equations and drive them initially with a wave spectrum with large amplitude magnetic fluctuations

Vlahos & Isliker (2023)

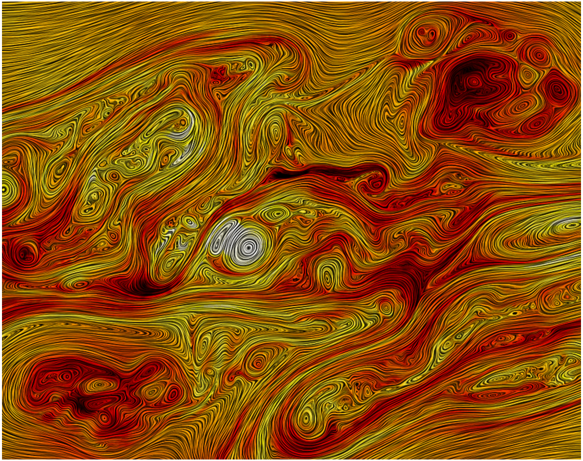
Simulation of large amplitude fluctuation

Electric field in 3D MHD turbulence

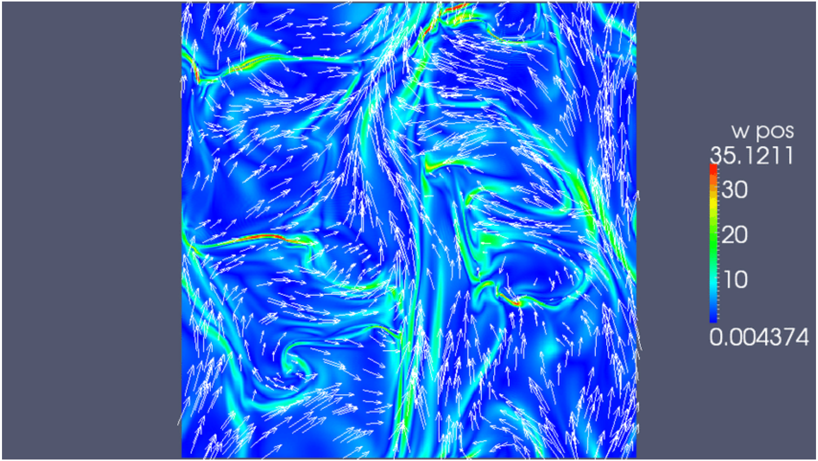


Electric field — Magnetic field magnitude

Simulation of large amplitude fluctuation

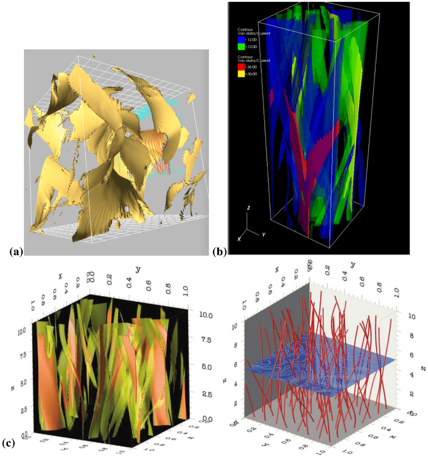


Simulation of large amplitude fluctuation

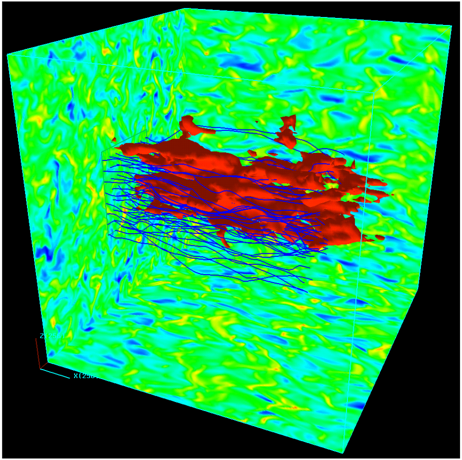


Simulation of externally driven large amplitude fluctuation

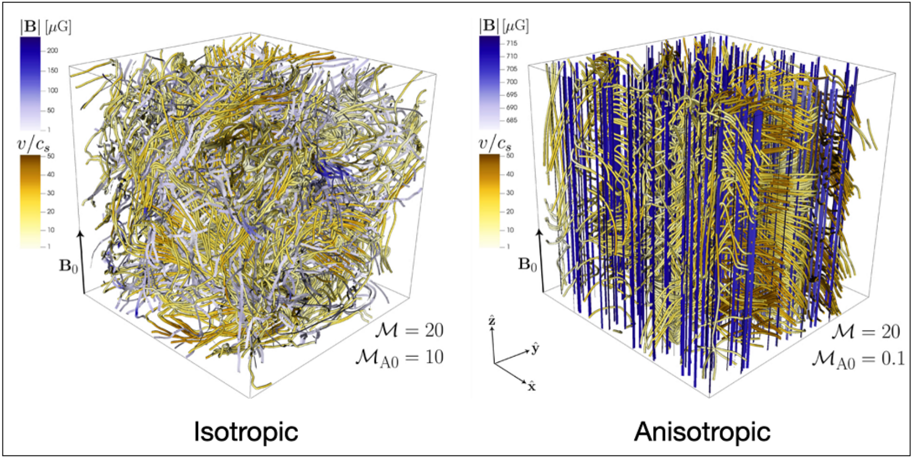
Galsgaard & Nordlund (1996); Rappazzo et al. (2013)



Simulation of large amplitude fluctuation in magnetized plasmas

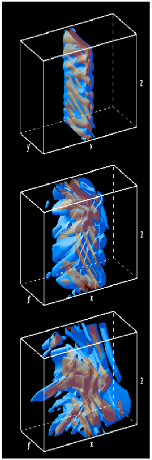


Simulation of large amplitude fluctuation



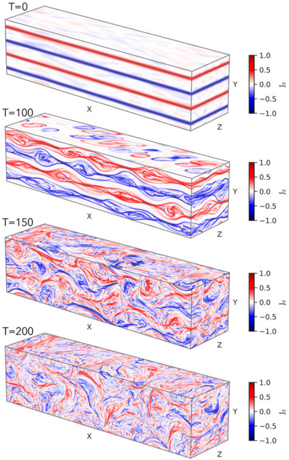
Current sheets and the way to strong turbulence

Onofri et al. (2006)



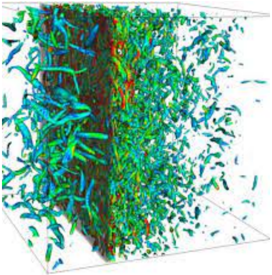
Current sheets and the way to strong turbulence

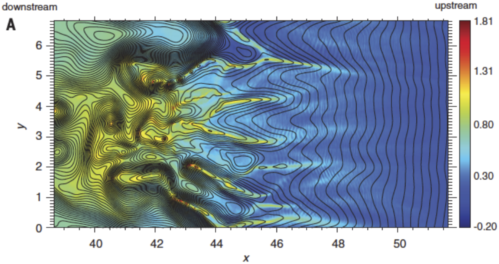
Nakanotani et al. (2022)



Simulation of large amplitude fluctuation in the vicinity of a shock

Matsumoto et al. (2015)



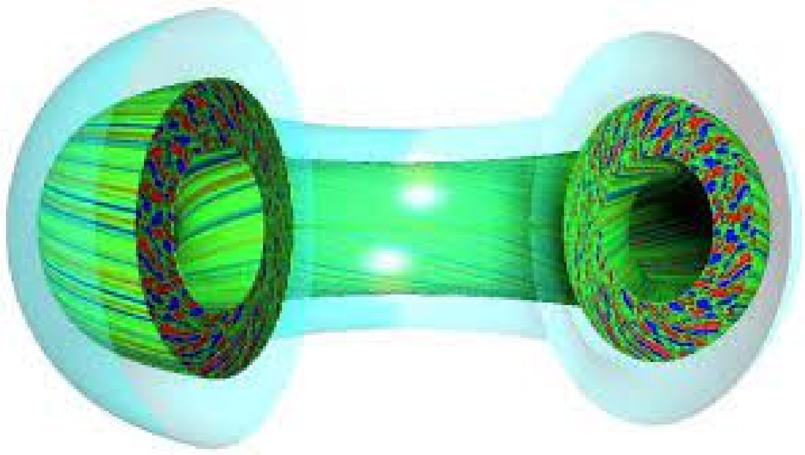


Space, Astrophysical and Laboratory systems in the state of strong turbulence

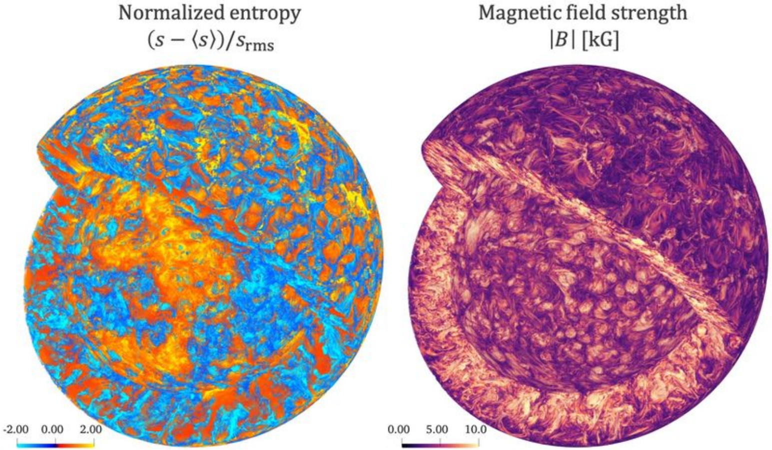
A short list of Space, Astrophysical, and Laboratory systems in the state of strong turbulence

1. Edge of Tokamak
2. Convection zone
3. Solar Atmosphere
4. Solar Wind
5. Solar wind-Earths Bow Shock-Magnetosheath-Magnetotail-Heliopause
6. Astrophysical Jets
7. Accretion Disks
8. Interstellar space in our galaxy
9. NS-NS collisions
10. SNR

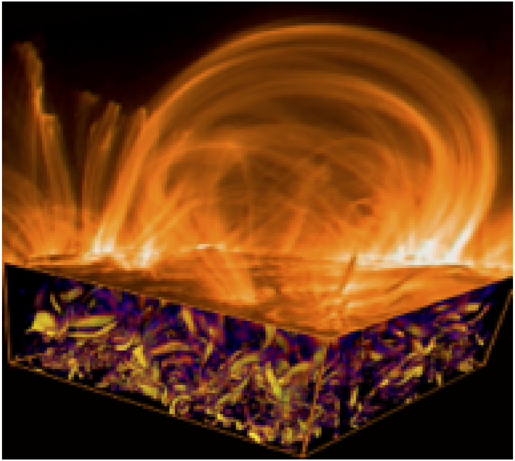
Edge of Tokamak



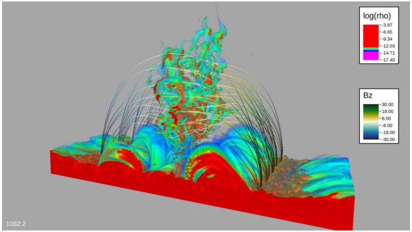
Solar Convection Zone

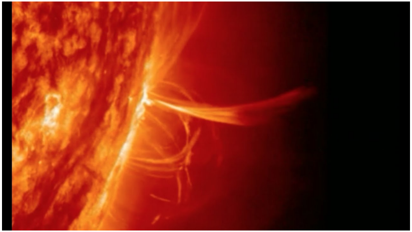


Solar Active Regions as convection zone driven systems



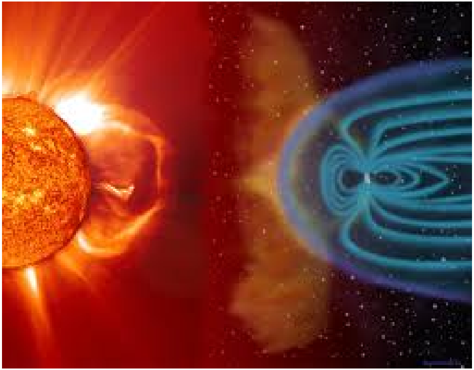
Solar Active Regions driven to strong Turbulence by convection zone



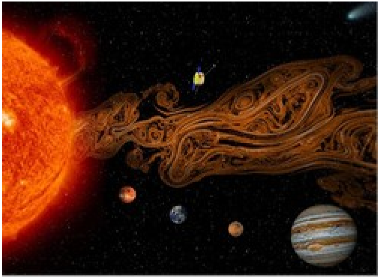


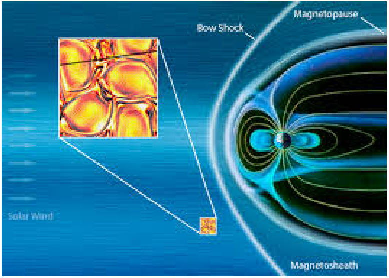
Solar Active Regions driven to strong Turbulence by convection zone

Solar Wind Turbulence

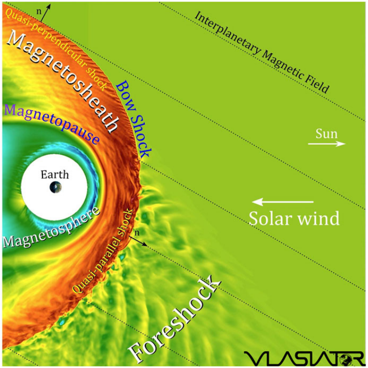


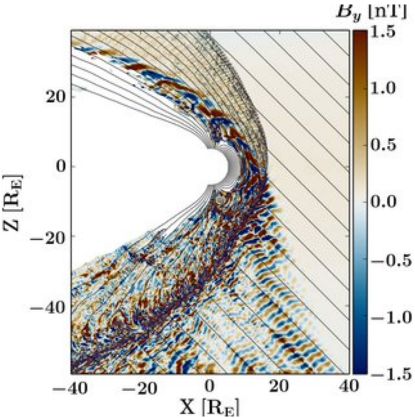
Solar Wind Turbulence



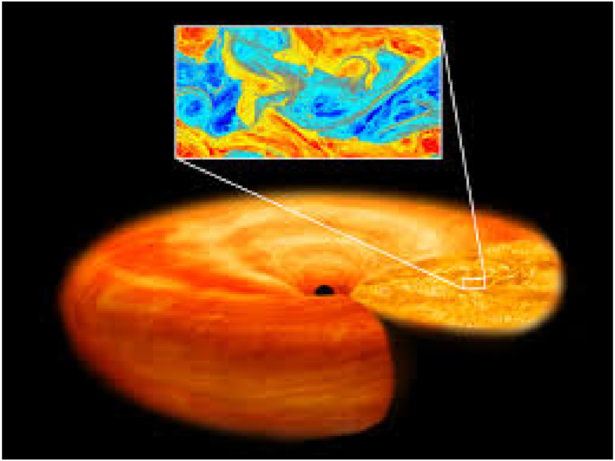


Soar Wind turbulence and Earths Bow Shock interaction

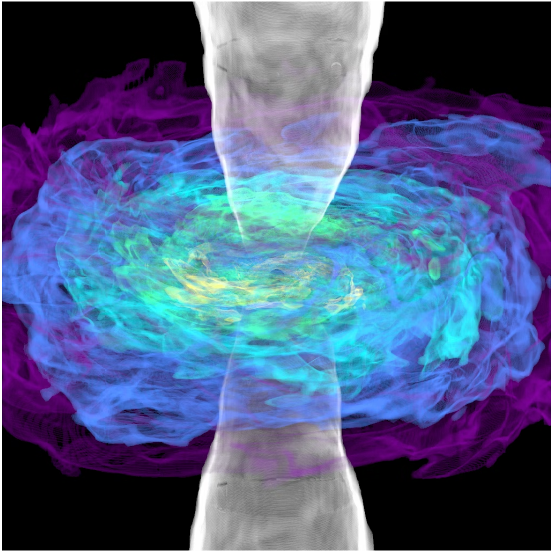




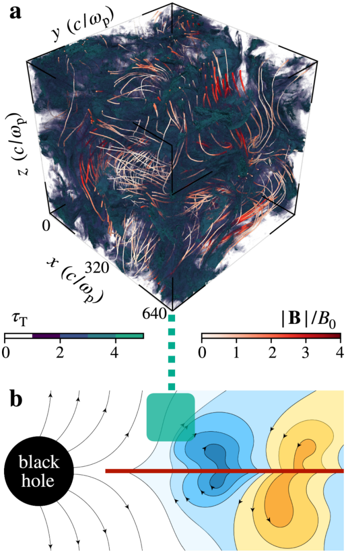
Simulation of large amplitude fluctuation in accretion disks



Simulation of accretion disk and turbulent jets

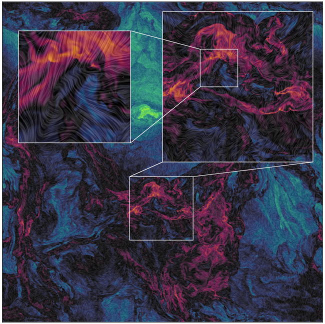


Simulation of large amplitude fluctuation in accretion disks

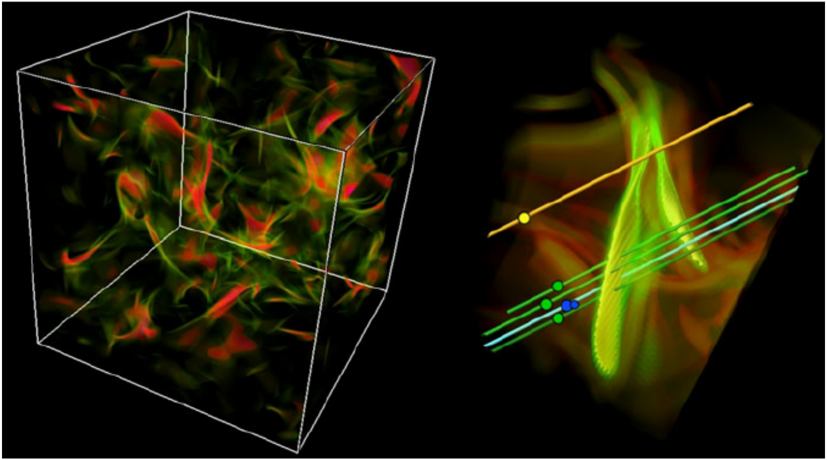


Main points on life far from equilibrium

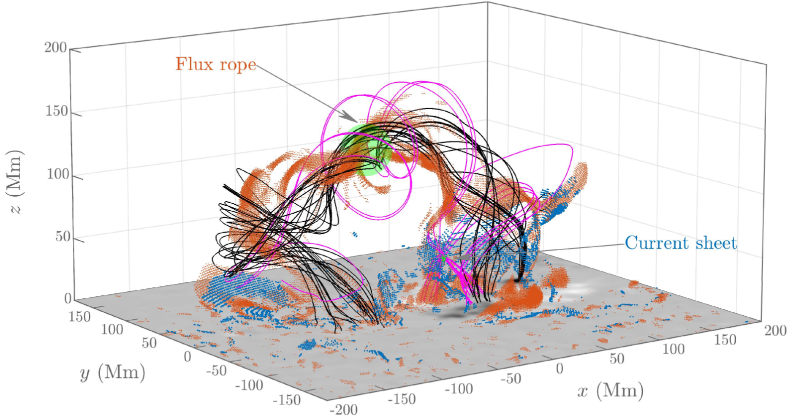
- When in an unstable system, the magnetic fluctuations are equal or larger than the ambient magnetic (strong turbulence), coherent structures replace the linear waves
- Coherent structures are Current Sheets (**CS**), magnetic filaments, large amplitude magnetic disturbances, vortices, and shocks (Vlahos & Isliker 2023)



Coherent Structures



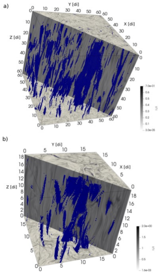
Coherent Structures



The Movies

PROPOSAL: Identification and Characterisation of Coherent Structures (CoS) in 3D strong turbulent plasma

We will try to identify and characterize Coherent Structures (CoSs) appearing in 3D magnetohydrodynamic (MHD) turbulent plasma and satellite data. We propose developing numerical tools based on trained and physics-informed neural networks to identify and characterize all CoSs inside the 3D turbulent simulations and observations.



How will couple the Coherent Structures in Strong MHD turbulence with particle energization

Strong Turbulence, dynamo and energization of particles

Let us go back to the MHD equations

Theorem

Induction Equation (Magnetic Field Evolution):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

where $\eta = \frac{1}{\sigma \mu_0}$ is the magnetic diffusivity, and σ is the electrical conductivity.

Theorem

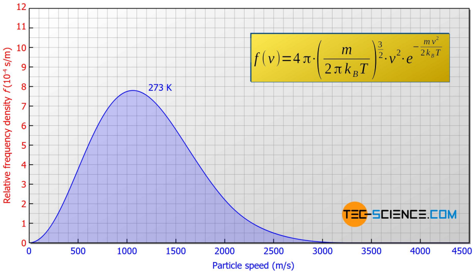
Ohm's Law for a Moving Conductor:

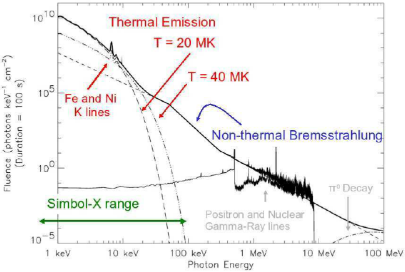
$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

where $\mathbf{E}$ is the electric field, $\mathbf{J}$ is the current density, and η is the resistivity of the fluid.

Strong Turbulence and particle heating and acceleration

In systems close to equilibrium, the energy distribution is close to Maxwell's equation (Thermal distribution). In systems far from equilibrium where strong turbulence is present, the nonthermal particles appear in the tail of the energy distribution.





The non-thermal Universe

- Solar Atmosphere (Coronal Heating, Flares, Coronal mass ejections)
- The solar wind, Bow shock magnetosheath, and magnetotail
- Cosmic Rays
- Nonthermal radiation (Gamma and X radiation sources)

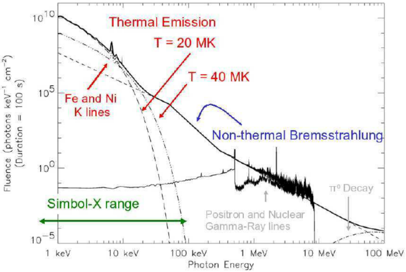


Figure: Coherent structures and strong turbulence

Cosmic Rays and Gamma rays from the Cosmos

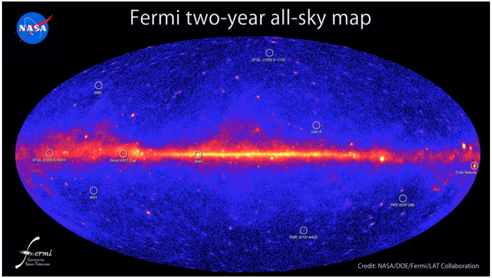


Figure: Coherent structures and strong turbulence

Cosmic Ray spectrum

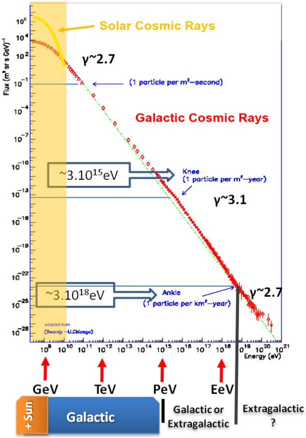


Figure: Coherent structures and strong turbulence

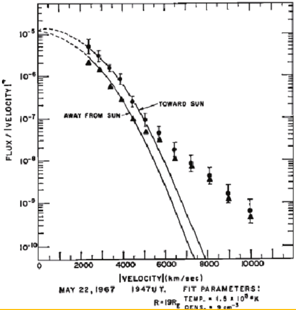
Heating of the bulk and Acceleration of the tail

Theorem

Ohm's Law for a Moving Conductor:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

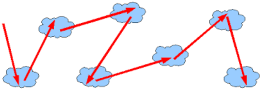
where $\mathbf{E}$ *is the electric field,* $\mathbf{J}$ *is the current density, and* η *is the resistivity of the fluid.*

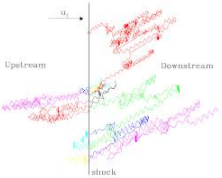


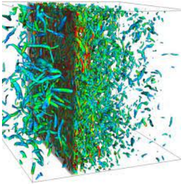
Fermi acceleration from the Coherent structures in strongly turbulent plasma's

The legacy of Enrico Fermi on particle energization is space plasmas

Stochastic and systematic acceleration (Fermi 1949, 1954)

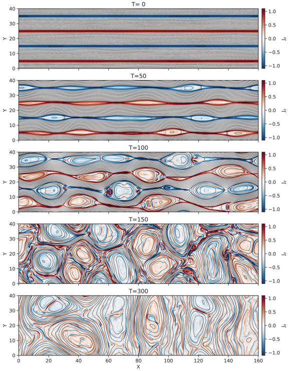


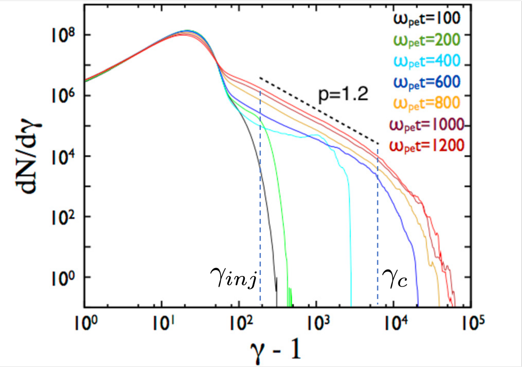




Systematic Acceleration from Magnetic reconnection

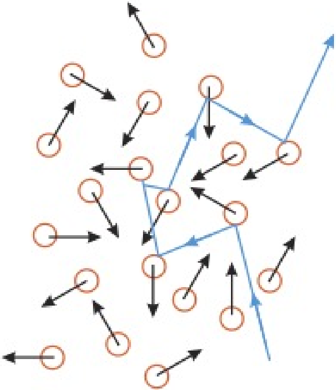
Guo et al. (2024)

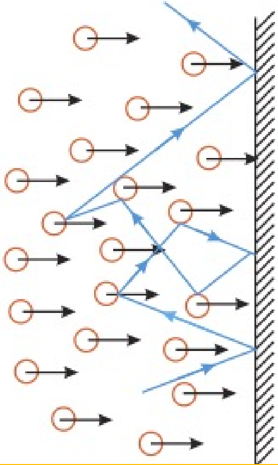




Stochastic and Systematic acceleration

Heating and acceleration of the tail





Numerical tools to explore the MHD simulation and the energization of particles

Test particle motion in MHD fields

Isliker et al. (2019)

Main Topics How photospheric turbulence drive the active region in the state of turbulent reconnection **Statistical Properties of Ur**

Level 3 (MHD+kinetic): Test particle evolution inside the MHD fields

The relativistic guiding center equations (without collisions) are for the evolution of the position **r** and the parallel component $u_{||}$ of the relativistic 4-velocity of the particles,

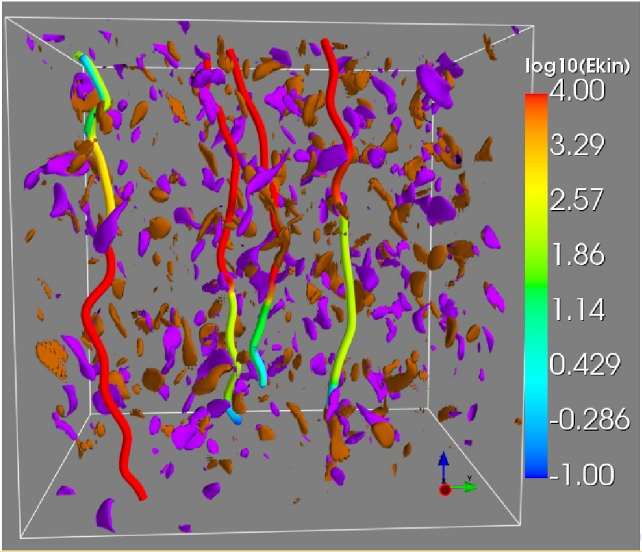
$$\frac{d\mathbf{r}}{dt} = \frac{1}{B^*_{||}}\left[\frac{u_{||}}{\gamma}\mathbf{B}^* + \hat{\mathbf{b}} \times \left(\frac{\mu}{q\gamma}\nabla B - \mathbf{E}^*\right)\right] \qquad (1)$$

$$\frac{du_{||}}{dt} = -\frac{q}{m_0 B^*_{||}}\mathbf{B}^* \cdot \left(\frac{\mu}{q\gamma}\nabla B - \mathbf{E}^*\right) \qquad (2)$$

where $\mathbf{B}^* = \mathbf{B} + \frac{m_0}{q}u_{||}\nabla \times \hat{\mathbf{b}}$, $\mathbf{E}^* = \mathbf{E} - \frac{m_0}{q}u_{||}\frac{\partial \hat{\mathbf{b}}}{\partial t}$, $\mu = \frac{m_0 v_\perp^2}{2B}$ is the magnetic moment, $\gamma = \sqrt{1 + \frac{u^2}{c^2}}$, $B = |\mathbf{B}|$, $\hat{\mathbf{b}} = \mathbf{B}/B$, $u_\perp$ is the perpendicular component of the relativistic 4-velocity, and q, m_0 are the particle charge and rest-mass, respectively.

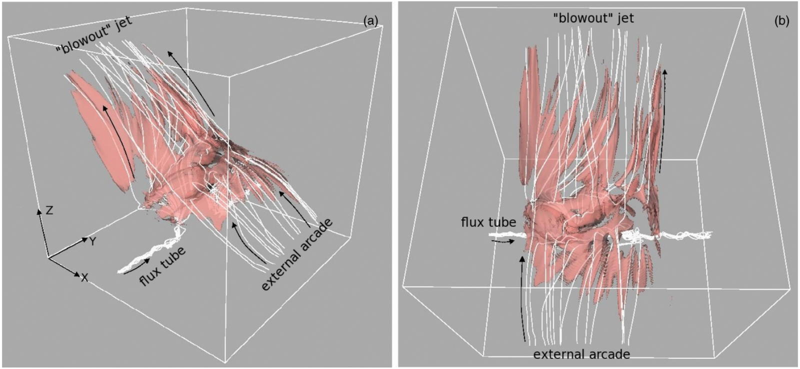
Test particle motion in MHD fields

Isliker et al. (2017)



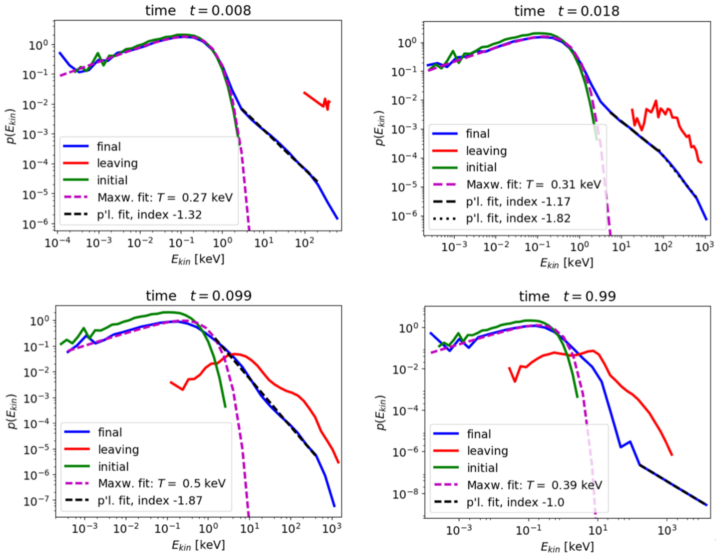
Emerging magnetic flux and current sheet fragmentation

Archontis & Hood (2013)



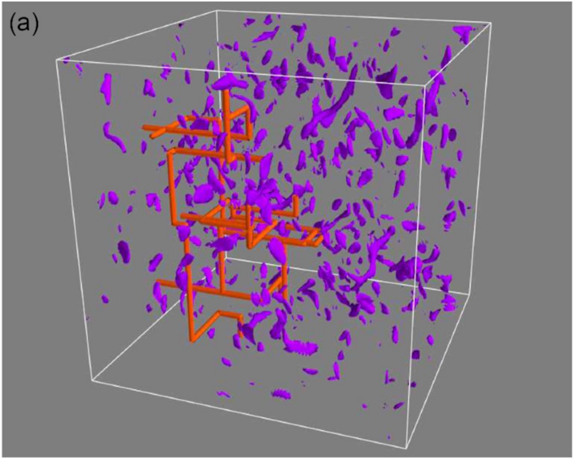
Test particle motion in MHD fields

Isliker et al. (2019)



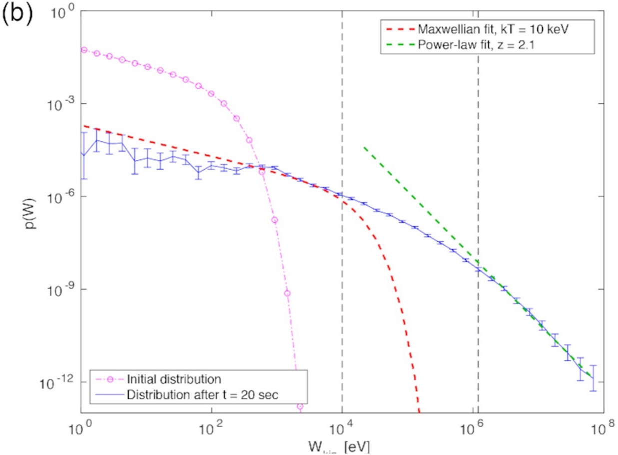
Test particle motion in MHD fields: Monde Carlo simulations

Sioulas et al. (2022)



Test particle motion in MHD fields: Monde Carlo simulations

Sioulas et al. (2022)



Main points to take home from this talk

Life close to equilibrium

Linearly unstable waves and weak (wave) turbulence $B_1/B_0 << 1$ are mathematically tractable but not realistic for very active space, astrophysical, and laboratory plasmas

Life far from equilibrium

When the unstable waves reach the level where the magnetic fluctuations $B_1/B_0 > 1$, coherent structures appear and are distributed in space

Coherent structures are heating the bulk and acceleration of the tail on most space, astrophysical, and laboratory sources which are far from equilibrium

Fermi and the two energization mechanisms: Stochastic and systematic

The stochastic interaction of particles with coherent structures is behind the bulk heating, and systematic acceleration is responsible for the formation of the high-energy tail.

Tracing particles inside strong turbulence, we discover the Kappa distribution (heating and the high energy tail).

Thank you!

References

Archontis, V. & Hood, A. W. 2013, Astrophysical Journal, 769, L21

Chen, F. F. 2016, Introduction to Plasma Physics and Controlled Fusion

Fermi, E. 1949, Physical Review, 75, 1169

Fermi, E. 1954, Astrophys. J., 119, 1

Galsgaard, K. & Nordlund, Å. 1996, Journal Geoph. Res, 101, 13445

Guo, F., Liu, Y.-H., Zenitani, S., & Hoshino, M. 2024, Space Scielce Reviw, 220, 43

Isliker, H., Archontis, V., & Vlahos, L. 2019, The Astrophysical Journal, 882, 57

Isliker, H., Vlahos, L., & Constantinescu, D. 2017, Physical Review Letters, 119, 045101

Krall, N. A. & Trivelpiece, A. W. 1973, Principles of plasma physics

Matsumoto, Y., Amano, T., Kato, T. N., & Hoshino, M. 2015, Science, 347, 974

Nakanotani, M., Zank, G. P., & Zhao, L. 2022, Frontiers in Astronomy and Space Sciences, 9, 954040

Onofri, M., Isliker, H., & Vlahos, L. 2006, Physical Review Letters, 96, 151102

Rappazzo, A. F., Velli, M., & Einaudi, G. 2013, Astroph. J., 771, 76

Sioulas, N., Isliker, H., & Vlahos, L. 2022, , 657, A8

Vlahos, L. & Isliker, H. 2023, Physics of Plasmas, 30, 040502